DL HW1

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1 Gradient Descent

1.

$$\underset{\mathbf{w}}{\operatorname{argmin}} f(\mathbf{w}^t) + < \mathbf{w} - \mathbf{w}^t, \nabla f(\mathbf{w}^t) + \frac{\lambda}{2}(||\mathbf{w} - \mathbf{w}^t||)^2$$

Lets say

$$F(\mathbf{w*}) = f(\mathbf{w}^t) + < \mathbf{w} - \mathbf{w}^t, \nabla f(\mathbf{w}^t) + \frac{\lambda}{2}(||\mathbf{w} - \mathbf{w}^t||)^2$$

We want to find $\mathbf{w*}$ where the value of F is the minimum. To do this we can set F'($\mathbf{w*}$) $= 0$ and solve for $\mathbf{w*}$
First lets solve for $F'(\mathbf{w*})$

$$F'(\mathbf{w*}) = \frac{df(\mathbf{w}^t)}{d\mathbf{w*}} + \frac{d(\mathbf{w} - \mathbf{w}^t)}{d\mathbf{w*}} \cdot (\nabla f(\mathbf{w}^t)) + (\mathbf{w} - \mathbf{w}^t) \cdot \frac{d(\nabla f(\mathbf{w}^t))}{d\mathbf{w*}} + \frac{\lambda}{2} \frac{(d||\mathbf{w} - \mathbf{w}^t||)^2}{d\mathbf{w*}}$$

$$F'(\mathbf{w*}) = 0 + \nabla f(\mathbf{w}^t) + \frac{\lambda}{2}(2(\mathbf{w} - \mathbf{w}^t)$$

$$F'(\mathbf{w*}) = \nabla f(\mathbf{w}^t) + \lambda(\mathbf{w} - \mathbf{w}^t)$$

Now lets set $F'(\mathbf{w*}) = 0$ and solve for $\mathbf{w*}$

$$\nabla f(\mathbf{w}^t) + \lambda(\mathbf{w} - \mathbf{w}^t) = 0$$

$$\mathbf{w*} = \mathbf{w}^t - \frac{\nabla f(\mathbf{w}^t)}{\lambda}$$

We can see the gradient descent update rule ($\mathbf{w}^{(t+1)} = \mathbf{w}^t - \eta \nabla f(\mathbf{w}^t)$) in this solution. This tells us that the gradient descent update rule *is* in-fact optimally minimizing the function $f(\mathbf{w})$ at each step along the way.
In gradient descent, we define η to be the step size that we take in the direction of the negative gradient. In the optimization problem, we define λ as the trade-off between the approximation being close to $\mathbf{w}^t$ and obtaining a lower bound.

We can see from this equation that the term η from the gradient descent update rule is equal to $\frac{1}{\lambda}$ from our l2 proximity term. Intuitively, we can say that the amount we step in the direction of the negative gradient should be enough so that we are still faithfully close to our starting point $\mathbf{w}^t$ while still achieving a minimum along our convex function.

2.

We want to prove that

$$\sum_{t=1}^{T} < \mathbf{w}^t - \mathbf{w}^*, \mathbf{v}_t > \leq \frac{||\mathbf{w}^*||^2}{2\eta} + \frac{\eta}{2}\sum_{t=1}^{T}||\mathbf{v}_t||^2$$

First lets apply some algebraic manipulations on the LHS of the inequality. By defn of inner product,

$$< \mathbf{w}^t - \mathbf{w}^*, \mathbf{v}_t > = \frac{1}{\eta} < \mathbf{w}^t - \mathbf{w}^*, \eta\mathbf{v}_t > \quad (1)$$

We also know that

$$||x-y||^2 = < x-y, x-y > = ||x||^2 - 2 < x, y > + ||y||^2$$

Therefore,

$$||\mathbf{w}^t - \mathbf{w}^* - \eta\mathbf{v}_t||^2 = ||\mathbf{w}^t - \mathbf{w}^*||^2 - 2 < \mathbf{w}^t - \mathbf{w}^*, \eta\mathbf{v}_t > + ||\eta\mathbf{v}_t||^2 \quad (2)$$

So we can replace the LHS in (1) with (2).

$$< \mathbf{w}^t - \mathbf{w}^*, \mathbf{v}_t > = \frac{1}{2\eta}(||\mathbf{w}^t - \mathbf{w}^*||^2 + ||\eta\mathbf{v}_t||^2 - ||\mathbf{w}^t - \mathbf{w}^* - \eta\mathbf{v}_t||^2) \quad (3)$$

By our gradient update rule $\mathbf{w}^{(t+1)} = \mathbf{w}^t - \eta\mathbf{v}_t$), we can simplify this to

$$< \mathbf{w}^t - \mathbf{w}^*, \mathbf{v}_t > = \frac{1}{2\eta}(||\mathbf{w}^t - \mathbf{w}^*||^2 + \eta^2||\mathbf{v}_t||^2 - ||\mathbf{w}^{t+1} - \mathbf{w}^*||^2) \quad (4)$$

Now lets add our summation back in

$$\sum_{t=1}^{T} < \mathbf{w}^t - \mathbf{w}^*, \mathbf{v}_t > = \frac{1}{2\eta}\sum_{t=1}^{T}(||\mathbf{w}^t - \mathbf{w}^*||^2 + \eta^2||\mathbf{v}_t||^2 - ||\mathbf{w}^{t+1} - \mathbf{w}^*||^2)$$

$$\sum_{t=1}^{T} < \mathbf{w}^t - \mathbf{w}^*, \mathbf{v}_t > = \frac{1}{2\eta}\sum_{t=1}^{T}(||\mathbf{w}^t - \mathbf{w}^*||^2 - ||\mathbf{w}^{t+1} - \mathbf{w}^*||^2) + \frac{1}{2\eta}\sum_{t=1}^{T}(\eta^2||\mathbf{v}_t||^2) =$$

$$\sum_{t=1}^{T} < \mathbf{w}^t - \mathbf{w}^*, \mathbf{v}_t > = \frac{1}{2\eta}\sum_{t=1}^{T}(||\mathbf{w}^t - \mathbf{w}^*||^2 - ||\mathbf{w}^{t+1} - \mathbf{w}^*||^2) + \frac{\eta}{2}\sum_{t=1}^{T}(||\mathbf{v}_t||^2)$$

The summation of the first part of the RHS of the equation will reduce because terms will cancel out resulting in

$$\sum_{t=1}^{T} < \mathbf{w}^t - \mathbf{w}^*, \mathbf{v}_t > = \frac{1}{2\eta}(||\mathbf{w}^1 - \mathbf{w}^*||^2 - ||\mathbf{w}^{T+1} - \mathbf{w}^*||^2) + \frac{\eta}{2}\sum_{t=1}^{T}(||\mathbf{v}_t||^2)$$

Now to form our inequality, observe that

$$||\mathbf{w}^{T+1} - \mathbf{w}^*||^2 \geq 0$$

because any real number squared can never achieve a negative value. Therefore, removing this term from the left side leaves us with something greater (since it was subtracted out) so

$$\sum_{t=1}^{T} < \mathbf{w}^t - \mathbf{w}^*, \mathbf{v}_t > \leq \frac{1}{2\eta}(||\mathbf{w}^1 - \mathbf{w}^*||^2) + \frac{\eta}{2}\sum_{t=1}^{T}(||\mathbf{v}_t||^2)$$

Finally, since we are told that $\mathbf{w}^1 = 0$, we obtain

$$\sum_{t=1}^{T} < \mathbf{w}^t - \mathbf{w}^*, \mathbf{v}_t > \leq \frac{1}{2\eta}(||\mathbf{w}^*||^2) + \frac{\eta}{2}\sum_{t=1}^{T}(||\mathbf{v}_t||^2)$$

3.

For a convex function, Jensen's inequality states that $E[f(x)] \geq f(E[x])$ Therefore, we have that

$$f(\bar{\mathbf{w}}) - f(\mathbf{w}^*) = f(\frac{1}{T}\sum_{t=1}^{T}\mathbf{w}^t) - f(\mathbf{w}^*) \leq \frac{1}{T}\sum_{t=1}^{T} f(\mathbf{w}^t) - f(\mathbf{w}^*)$$

Since $f(\mathbf{w}^*)$ is not affected by t, we can simplify this to

$$f(\bar{\mathbf{w}}) - f(\mathbf{w}^*) \leq \frac{1}{T}\sum_{t=1}^{T}(f(\mathbf{w}^t) - f(\mathbf{w}^*))$$

Furthermore, because of our convexity assumption, we know that

$$f(\bar{\mathbf{w}}) - f(\mathbf{w}^*) \leq \frac{1}{T}\sum_{t=1}^{T} < \mathbf{w}^t - \mathbf{w}^*, \nabla f(\mathbf{w}^t) >$$

Therefore, combining this inequality with what we proved in part 2 gives us

$$f(\bar{\mathbf{w}}) - f(\mathbf{w}^*) \leq \frac{1}{2T\eta}(||\mathbf{w}^*||^2) + \frac{\eta}{2T}\sum_{t=1}^{T}(||\nabla f(\mathbf{w}^t)||^2)$$

Now lets substitute the upperbounds in for $||\mathbf{w}^*||^2$ and $||\mathbf{v}_t||^2$ and $\eta = \sqrt{\frac{B^2}{\rho^2 T}}$.

$$f(\bar{\mathbf{w}}) - f(\mathbf{w}^*) \leq \frac{1}{2T\sqrt{\frac{B^2}{\rho^2 T}}}(B^2) + \frac{\sqrt{\frac{B^2}{\rho^2 T}}}{2T}\sum_{t=1}^{T}(\rho^2)$$

$$f(\bar{\mathbf{w}}) - f(\mathbf{w}^*) \leq \frac{1}{T}(\frac{B^2 p\sqrt{T}}{2B} + \frac{BT(\rho^2)}{2p\sqrt{T}})$$

$$f(\bar{\mathbf{w}}) - f(\mathbf{w}^*) \leq \frac{3Bp}{\sqrt{T}}$$

Since 3, B, and ρ are constants, we know that

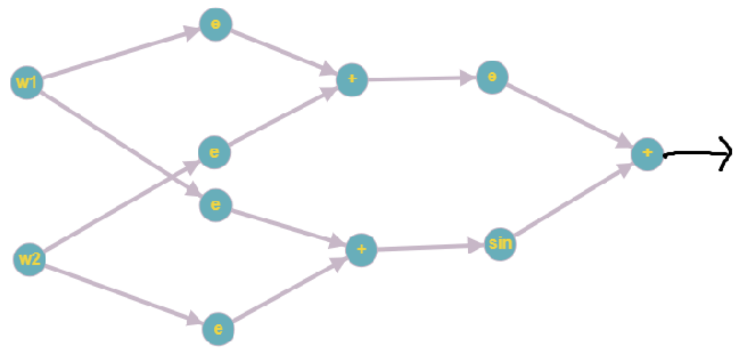
$$f(\bar{\mathbf{w}}) - f(\mathbf{w}^*) \propto \frac{1}{\sqrt{T}}$$

and therefore the convergence rate is $O(\frac{1}{\sqrt{T}})$

3 Automatic Differentiation

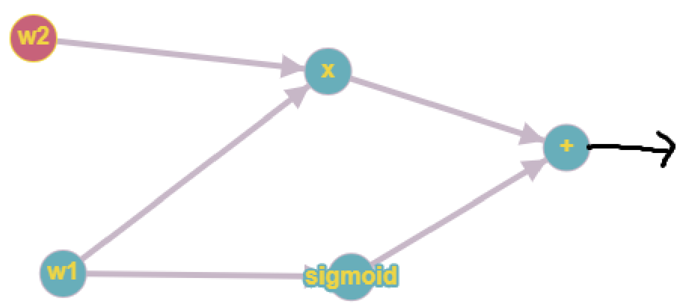
5.

a)



$$f_1(w_1, w_2) = e^{e^{w_1}+e^{2w_2}} + sin(e^{w_1} + e^{2w_2})$$

$$f_1(\mathbf{w}) = f_1(1, 2) = e^{e^1+e^{2*2}} + sin(e^1 + e^{2*2}) \approx 7.802e24$$



$$f_2(w_1, w_2) = w_1 * w_2 + \sigma(w_1)$$

$$f_2(\mathbf{w}) = f_2(1, 2) = 1 * 2 + \sigma(1) \approx 2.731$$

b)

i) First lets compute $\frac{\partial f_1}{\partial w_1}$

$$f_1(\mathbf{w} + (\Delta w, 0)) = f_1(1 + 0.01, 2) = 8.012e24$$

$$\frac{\partial f_1}{\partial w_1} = \frac{f_1(\mathbf{w} + (\Delta w, 0)) - f_1(\mathbf{w})}{\Delta w} = \frac{8.012e24 - 7.802e24}{0.01} = 2.162e25$$

ii) Now lets compute $\frac{\partial f_1}{\partial w_2}$

$$f_1(\mathbf{w} + (0, \Delta w)) = f_1(1, 2 + 0.01) = 2.351e25$$

$$\frac{\partial f_1}{\partial w_2} = \frac{f_1(\mathbf{w} + (0, \Delta w)) - f_1(\mathbf{w})}{\Delta w} = \frac{2.351e25 - 7.802e24}{0.01} = 1.571e27$$

iii) Now lets compute $\frac{\partial f_2}{\partial w_1}$

$$f_2(\mathbf{w} + (\Delta w, 0)) = f_2(1 + 0.01, 2) = 2.753$$

$$\frac{\partial f_2}{\partial w_1} = \frac{f_2(\mathbf{w} + (\Delta w, 0)) - f_2(\mathbf{w})}{\Delta w} = \frac{2.753 - 2.731}{0.01} = 2.2$$

iv) Now lets compute $\frac{\partial f_2}{\partial w_2}$

$$f_2(\mathbf{w} + (0, \Delta w)) = f_1(1, 2 + 0.01) = 2.741$$

$$\frac{\partial f_2}{\partial w_2} = \frac{f_2(\mathbf{w} + (0, \Delta w)) - f_2(\mathbf{w})}{\Delta w} = \frac{2.741 - 2.731}{0.01} = 1$$

c)

i) First lets compute $\frac{\partial f_1}{\partial w_1}$

Lets define several intermediate variables to simplify computations.
$w_1 = w_1 = 1$; $w_2 = w_2 = 2$; $w_3 = e^{w_1} = 2.718$; $w_4 = e^{2w_2} = 54.599$; $w_5 = w_3 + w_4 = 57.316$; $w_6 = e^{w_5} = 7.802e24$; $w_7 = w_3 = 2.718$; $w_8 = w_4 = 54.599$; $w_9 = w_5 = 57.316$; $w_{10} = sin(w_9) = 0.6945$; $f = w_{11} = w_{10} + w_6 = 7.802e24$;

Now lets go in order to find the derivatives of each w_i for $i = 1...w_11$ which will continuously build on top of each other.
$\partial w_1/w_1 = 1$, $\partial w_1/w_2 = 0$;
$\partial w_2/w_1 = 0$, $\partial w_2/w_2 = 1$;
$\partial w_3/w_1 = e^{w_1}$, $\partial w_3/w_2 = 0$;
$\partial w_4/w_1 = 0$, $\partial w_4/w_2 = 2e^{2w_2}$;
$\partial w_5/w_1 = e^{w_1}$, $\partial w_5/w_2 = 2e^{2w_2}$;
$\partial w_6/w_1 = e^{w_1}e^{w_5}$, $\partial w_6/w_2 = 2e^{2w_2}e^{w_5}$;
$\partial w_7/w_1 = e^{w_1}$, $\partial w_7/w_2 = 0$;
$\partial w_8/w_1 = 0$, $\partial w_8/w_2 = 2e^{2w_2}$;
$\partial w_9/w_1 = e^{w_1}$, $\partial w_9/w_2 = 2e^{2w_2}$;
$\partial w_{10}/w_1 = cos(w_9)e^{w_1}$, $\partial w_{10}/w_2 = 2e^{2w_2}cos(w_9)$;
$\partial f/w_1 = \partial w_{11}/w_1 = cos(w_9)e^{w_1} + e^{w_1}e^{w_5}$,
$\partial f/w_2 = \partial w_{11}/w_2 = 2e^{2w_2}cos(w_9) + 2e^{2w_2}e^{w_5}$;

So finally by substituting in w_1 and w_2 we get

$$\partial f(1,2)/w_1 = cos(w_9)e^{w_1} + e^{w_1}e^{w_5} = cos(57.316)e^1 + e^1e^{57.316} = 2.12e25$$

ii) Now lets compute $\frac{\partial f_1}{\partial w_2}$

Using our calculations above, we just substitute and get

$$\partial f(1,2)/w_2 = 2e^{2w_2}cos(w_9)+2e^{2w_2}e^{w_5} = 2e^{2*2}cos(57.316)+2e^{2*2}e^{57.316} = 8.516e26$$

iii) Now lets compute $\frac{\partial f_2}{\partial w_1}$

$\partial w_1/w_1 = 1$, $\partial w_2/w_1 = 0$, $\partial x_1/w_1 = w_2$, $\partial x_2/w_1 = \sigma(w_1)(1-\sigma(w_1))$ $\partial x_3/w_1 = \partial f_2/w_1 = \partial x_1/w_1 + \partial x_2/w_1 = w_2 + \sigma(w_1)(1-\sigma(w_1))$

$$\partial f_2(1,2)/w_1 = 2 + \sigma(1)(1-\sigma(1)) = 2.2$$

iii) Now lets compute $\frac{\partial f_2}{\partial w_2}$

$\partial w_1/w_2 = 0$, $\partial w_2/w_2 = 1$, $\partial x_1/w_2 = w_1$, $\partial x_2/w_2 = 0$ $\partial x_3/w_2 = \partial f_2/w_2 = \partial x_1/w_2 + \partial x_2/w_2 = w_1 + 0$

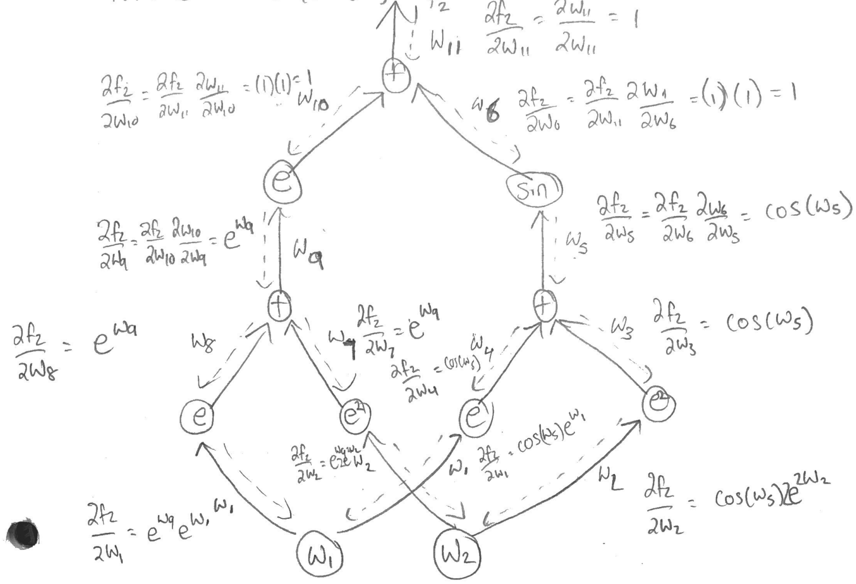
$$\partial f_2(1,2)/w_2 = 1$$

d)

i and ii

Reverse Mode AD $\frac{\partial f_1}{\partial w_1}$ AND $\frac{\partial f_1}{\partial w_2}$

$$f(1,2) = e^{e^{w_1}+e^{2w_2}} + \sin(e^{w_1}+e^{2w_2})$$



$$\frac{\partial f_2}{\partial w_1} = e^{w_1} e^{w_9} + e^{w_1}\cos(w_5)$$

$$w_9 = w_8 + w_7 = e^{w_1} + e^{2w_2} = e^1 + e^4 = 57.3164$$

$$w_5 = e^{w_4} + e^{w_3} = \sin(e^{w_1} + e^{2w_2}) = \sin(e^1 + e^4) = 0.6945$$

$$\boxed{\frac{\partial f_2}{\partial w_1} = e^1 e^{57.3164} + e^1 \cos(0.6945) = 2.12 \times 10^{25}}$$

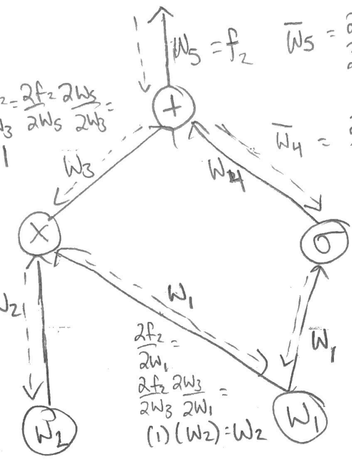
$$\frac{\partial f_2}{\partial w_2} = e^{w_9}\, 2e^{2w_2} + \cos(w_5)\, 2e^{2w_2}$$

$$\boxed{\frac{\partial f_2}{\partial w_2} = e^{57.3164}\, 2e^{2\cdot 2} + \cos(0.6945)\, 2e^{2\cdot 2} = 8.516 \times 10^{26}}$$

iii and iv

Reverse Mode AD $\frac{\partial f_2}{\partial w_1}$ AND $\frac{\partial f_2}{\partial w_2}$

$$f_2(w_1, w_2) = w_1 w_2 + \sigma(w_1)$$



$w_5 = f_2$ $\quad \bar{w}_5 = \frac{\partial f_2}{\partial w_5} = \frac{\partial w_5}{\partial w_5} = 1$

$\bar{w}_3 = \frac{\partial f_2}{\partial w_3} = \frac{\partial f_2}{\partial w_5}\frac{\partial w_5}{\partial w_3} = (1)(1) = 1$

$\bar{w}_4 = \frac{\partial f_2}{\partial w_4} = \frac{\partial f_2}{\partial w_5}\frac{\partial w_5}{\partial w_4} = (1)(1) = 1$

$\frac{\partial f_2}{\partial w_2} = \frac{\partial f_2}{\partial w_3}\frac{\partial w_3}{\partial w_2} = (1)(w_1) = w_1$

$\frac{\partial f_2}{\partial w_1} = \frac{\partial f_2}{\partial w_3}\frac{\partial w_3}{\partial w_1} = (1)(w_2) = w_2$

$\frac{\partial f_2}{\partial w_1} = \frac{\partial f_2}{\partial w_4}\frac{\partial w_4}{\partial w_1} = (1)\sigma(w_1)(1-\sigma(w_1))$

$$\frac{\partial f_2}{\partial w_1} = w_2 + \sigma(w_1)(1-\sigma(w_1)) =$$

$$\boxed{\frac{\partial f_2(1,2)}{\partial w_1} = 1 + \sigma(1)(1-\sigma(1)) = 2.2}$$

$$\boxed{\frac{\partial f_2(1,2)}{\partial w_2} = w_1 = 1}$$

e)

4 Convolutions

6.

a)

To prove that S and C_a commute, lets show that $SC_a = C_aS$ which means that $(SC_a)_{ij} = (C_aS)_{ij}$ for any row i and col j.

We know that since S is the shift matrix, the value at each row i and col j is only equal to 1 when i-j-1=0. Therefore, we can say that$S_{ij} = \delta_{i-j-1}$.
Furthermore, since we have the additional property that $a_i = a_{i+n}$, we know that for any element a_{ij} in the matrix C_a, $a_{ij} = a_{i-j}$

$$(SC_a)_{ij} = \sum_{b=0}^{n-1} S_{ib}(C_a)_{bj} = \sum_{b=0}^{n-1} \delta_{i-b-1}a_{b-j}$$

The expression δ_{i-b-1} is only equal to 1 when i-b-1=0, so b = i-1. This gives us

$$(SC_a)_{ij} = i - 1 - j$$

$$(C_aS)_{ij} = \sum_{b=0}^{n-1}(C_a)_{ib}S_{bj} = \sum_{b=0}^{n-1} a_{i-b}\delta_{b-j-1}$$

The expression δ_{b-j-1} is only equal to 1 when b-j-1=0, so b = j+1. This gives us

$$(SC_a)_{ij} = i - j - 1$$

Since we have proved that $(SC_a)_{ij} = (SC_a)_{ij}$, we have shown that the shift matrix and the circulant matrix are commutative.
*NOTE: This proof works even if our circulant matrix took the form of $(C_a)^T$ where C_a is the circulant matrix form given in the hw. This is because our assumption of $a_i = a_{i+n}$ is generalized to all types of circulant matrices so the computations do not change.

b)

Lets prove the bidirectional implication that A matrix is circular convolution IF and only IF it is a linear operation with shift invariance

IF there is a circular convolution => it is a linear operation with shift invariance

In 6a, we proved that f(Sx)=Sf(x) where f(x)=C_ax. This proves one direction that states that if C_a is a circulant convolution matrix, then it is shift invariant.

IF there is a linear operation with shift invariance => it is the circular convolution

Here, we get to assume that f(Sx)=Sf(x). The question is how do we show that C_a in f(x)=C_ax is the circulant convolution? Lets define $C_a = A$ to be a random matrix with column vectors = $[a_0, ..., a_{n-1}]$. We know that S is the shift matrix as shown above in the homework problem. Lets define the x vector with values $x = [x_0, ..., x_{n-1}]$.
Lets first look at ASx. First compute Sx. Since S is the shift matrix, we know that Sx = $[x_{n-1}, x_0, x_1, ..., x_{n-3}, x_{n-2}]$. Now lets apply A to this vector Sx.

$$[a_0, ..., a_{n-1}][x_{n-1}, x_0, x_1, ..., x_{n-3}, x_{n-2}] = [a_0x_{n-1}+a_1x_0+a_2x_1+...+a_{n-2}x_{n-3}+a_{n-1}x_{n-2}]$$

Now lets look at SAx. First compute Ax.

$$[a_0, ..., a_{n-1}][x_0, ..., x_{n-1}] = a_0x_0 + a_1x_1 + a_2x_2 + ... + a_{n-2}x_{n-2} + a_{n-1}, x_{n-1}$$

Now lets apply S to the vector Ax.

$$S[a_0x_0+a_1x_1+a_2x_2+...+a_{n-2}x_{n-2}+a_{n-1}, x_{n-1}] = [Sa_0x_0+Sa_1x_1+Sa_2x_2+...+Sa_{n-2}x_{n-2}+Sa_{n-1}, x_{n-1}]$$

Now since ASx = SAx, these two calculations are equivalent.

$$[a_0x_{n-1}+a_1x_0+a_2x_1+...+a_{n-2}x_{n-3}+a_{n-1}x_{n-2}] = [Sa_0x_0+Sa_1x_1+Sa_2x_2+...+Sa_{n-2}x_{n-2}+Sa_{n-1}, x_{n-1}]$$

This means that Sa_ix_i must be equivalent to $a_{(i+1)modn}x_i$. This means that $Sa_i = a_{(i+1)modn}$. If this is true, then we know that $a_{(i+1)modn}$ is simply the vector whose column is made up of the values in the column a_i with a circular shift. Since each subsequent column of A is obtained by a circular shift of the previous column, we know that this is a circulant convolution matrix.

c)

This tells us that deep learning architecture for spatio-temporal data like images and videos are only possible through convolutions IF you want the model to be shift invariant. This is because as we proved, convolutions are are the only shift invariant linear operators and therefore, there is no other linear operators that would allow a model to generalize to small shifts in input images/videos. Convolutions are thus ideal for finding features that are encoded in spatial processing and being invariant to location of the features relative to a standard xyz reference frame.

5 Paper Review

7.

The central theme of this paper was trying to make deep learning scientist think about how regularization really affects generalization and what truly causes a deep neural network to generalize well to the held out test data. The authors ran an interesting experiment where they randomized the training labels such that they no longer corresponded to the correct data points and noticed that a model could still achieve a training loss of zero though there is no relationship between the data and the labels! They also tried an experiment where the input images were just noise and noticed the network again achieved zero training error. With such experiments, they concluded a that neural networks have the capability to memorize an entire dataset, and that architecture does indeed affect regularization. To figure out what part of the architecture affects the generalization of the model, the authors ran several studies with different regularization methods but the ultimatum was that there is no clear answer to this question and this subject requires further analysis.

8.

First, I realized that regularization is not the be all end all for ensuring a network is generalizeable. This paper showed that many times, a regularized network preforms well in unseen test data but the regularized network with the same hyper parameters has very little difference. This shows that there is a high chance the regularization may not be contributing anything to the outputs. Secondly, optimization's connection to generalization is not necessarily what we think it is. We add a regularization term so that the optimizer can converge but if this regularization term is not affecting the model's performance, then why is it able to optimize so quickly? This topic needs to be explored in further detail. Furthermore, a future discussion that could be brought up is how there is a possibility that large deep learning network architectures are actually just memorizing the training data in a 'smart' way but are still generalizing well to unseen data. There is no way we have a confirmation that this does not happen so I wonder if this is a possibility?

Modular neural nets

In the previous homework, we implemented modular neural networks for a two-layer neural network with fully connected layers. Now, we will do the same for convolutional layers. Once again, the benefit of modular design is that we can snap together different types of layers and loss functions in order to quickly experiment with different architectures.

In [45]:
```
# As usual, a bit of setup
import numpy as np
import matplotlib.pyplot as plt
from cs231n.gradient_check import eval_numerical_gradient_array, eval_numerical_gradient
from cs231n.conv_layers import *

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading external modules
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2

def rel_error(x, y):
  """ returns relative error """
  return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

```
The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
```

Convolution layer: forward naive

Implement the function `conv_forward_naive` in the file `cs231n/conv_layers.py`.

You don't have to worry too much about efficiency at this point; just write the code in whatever way you find most clear.

You can test your implementation by running the following:

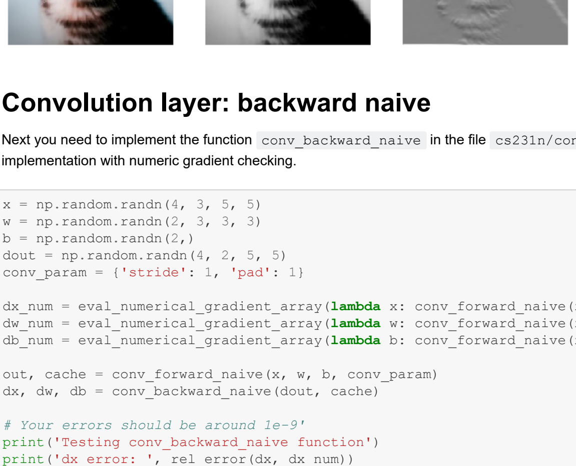
In [46]:
```
x_shape = (2, 3, 4, 4)
w_shape = (3, 3, 4, 4)
x = np.linspace(-0.1, 0.5, num=np.prod(x_shape)).reshape(x_shape)
w = np.linspace(-0.2, 0.3, num=np.prod(w_shape)).reshape(w_shape)
b = np.linspace(-0.1, 0.2, num=3)

conv_param = {'stride': 2, 'pad': 1}
out, _ = conv_forward_naive(x, w, b, conv_param)
correct_out = np.array([[[[-0.08759809, -0.10987781],
                           [-0.18387192, -0.2109216 ]],
                          [[ 0.21027089,  0.21661097],
                           [ 0.22847626,  0.23004637]],
                          [[ 0.50813986,  0.54309974],
                           [ 0.64082444,  0.67101435]]],
                         [[[-0.98053589, -1.03143541],
                           [-1.19128892, -1.24695841]],
                          [[ 0.69108355,  0.66880383],
                           [ 0.59480972,  0.56776003]],
                          [[ 2.36270298,  2.36904306],
                           [ 2.38090835,  2.38247847]]]])

# Compare your output to ours; difference should be around 1e-8
print('Testing conv_forward_naive')
print('difference: ', rel_error(out, correct_out))
```

```
Testing conv_forward_naive
difference:  2.2121476417505994e-08
```

Aside: Image processing via convolutions

As fun way to both check your implementation and gain a better understanding of the type of operation that convolutional layers can perform, we will set up an input containing two images and manually set up filters that perform common image processing operations (grayscale conversion and edge detection). The convolution forward pass will apply these operations to each of the input images. We can then visualize the results as a sanity check.

In [47]:
```
from PIL import Image
kitten, puppy = np.array(Image.open('kitten.jpg')), np.array(Image.open('puppy.jpg'))
# kitten is wide, and puppy is already square
d = kitten.shape[1] - kitten.shape[0]
kitten_cropped = kitten[:, d//2:-d//2, :]

img_size = 200   # Make this smaller if it runs too slow
x = np.zeros((2, 3, img_size, img_size))
x[0, :, :, :] = np.array(Image.fromarray(puppy).resize((img_size, img_size))).transpose((2, 0, 1))
x[1, :, :, :] = np.array(Image.fromarray(kitten_cropped).resize((img_size, img_size))).transpose((2, 0, 1))

# Set up a convolutional weights holding 2 filters, each 3x3
w = np.zeros((2, 3, 3, 3))

# The first filter converts the image to grayscale.
# Set up the red, green, and blue channels of the filter.
w[0, 0, :, :] = [[0, 0, 0], [0, 0.3, 0], [0, 0, 0]]
w[0, 1, :, :] = [[0, 0, 0], [0, 0.6, 0], [0, 0, 0]]
w[0, 2, :, :] = [[0, 0, 0], [0, 0.1, 0], [0, 0, 0]]

# Second filter detects horizontal edges in the blue channel.
w[1, 2, :, :] = [[1, 2, 1], [0, 0, 0], [-1, -2, -1]]

# Vector of biases. We don't need any bias for the grayscale
# filter, but for the edge detection filter we want to add 128
# to each output so that nothing is negative.
b = np.array([0, 128])

# Compute the result of convolving each input in x with each filter in w,
# offsetting by b, and storing the results in out.
out, _ = conv_forward_naive(x, w, b, {'stride': 1, 'pad': 1})

def imshow_noax(img, normalize=True):
    """ Tiny helper to show images as uint8 and remove axis labels """
    if normalize:
        img_max, img_min = np.max(img), np.min(img)
        img = 255.0 * (img - img_min) / (img_max - img_min)
    plt.imshow(img.astype('uint8'))
    plt.gca().axis('off')

# Show the original images and the results of the conv operation
plt.subplot(2, 3, 1)
imshow_noax(puppy, normalize=False)
plt.title('Original image')
plt.subplot(2, 3, 2)
imshow_noax(out[0, 0])
plt.title('Grayscale')
plt.subplot(2, 3, 3)
imshow_noax(out[0, 1])
plt.title('Edges')
plt.subplot(2, 3, 4)
imshow_noax(kitten_cropped, normalize=False)
plt.subplot(2, 3, 5)
imshow_noax(out[1, 0])
plt.subplot(2, 3, 6)
imshow_noax(out[1, 1])
plt.show()
```



Convolution layer: backward naive

Next you need to implement the function `conv_backward_naive` in the file `cs231n/conv_layers.py`. As usual, we will check your implementation with numeric gradient checking.

In [48]:
```
x = np.random.randn(4, 3, 5, 5)
w = np.random.randn(2, 3, 3, 3)
b = np.random.randn(2,)
dout = np.random.randn(4, 2, 5, 5)
conv_param = {'stride': 1, 'pad': 1}

dx_num = eval_numerical_gradient_array(lambda x: conv_forward_naive(x, w, b, conv_param)[0], x, dout)
dw_num = eval_numerical_gradient_array(lambda w: conv_forward_naive(x, w, b, conv_param)[0], w, dout)
db_num = eval_numerical_gradient_array(lambda b: conv_forward_naive(x, w, b, conv_param)[0], b, dout)

out, cache = conv_forward_naive(x, w, b, conv_param)
dx, dw, db = conv_backward_naive(dout, cache)

# Your errors should be around 1e-9'
print('Testing conv_backward_naive function')
print('dx error: ', rel_error(dx, dx_num))
print('dw error: ', rel_error(dw, dw_num))
print('db error: ', rel_error(db, db_num))
```

```
Testing conv_backward_naive function
dx error:  9.682029025656176e-10
dw error:  5.431767655780389e-10
db error:  2.929860525174844e-11
```

Max pooling layer: forward naive

The last layer we need for a basic convolutional neural network is the max pooling layer. First implement the forward pass in the function `max_pool_forward_naive` in the file `cs231n/conv_layers.py`.

In [49]:
```
x_shape = (2, 3, 4, 4)
x = np.linspace(-0.3, 0.4, num=np.prod(x_shape)).reshape(x_shape)
pool_param = {'pool_width': 2, 'pool_height': 2, 'stride': 2}

out, _ = max_pool_forward_naive(x, pool_param)

correct_out = np.array([[[[-0.26315789, -0.24842105],
                          [-0.20421053, -0.18947368]],
                         [[-0.14526316, -0.13052632],
                          [-0.08631579, -0.07157895]],
                         [[-0.02736842, -0.01263158],
                          [ 0.03157895,  0.04631579]]],
                        [[[ 0.09052632,  0.10526316],
                          [ 0.14947368,  0.16421053]],
                         [[ 0.20842105,  0.22315789],
                          [ 0.26736842,  0.28210526]],
                         [[ 0.32631579,  0.34105263],
                          [ 0.38526316,  0.4       ]]]])

# Compare your output with ours. Difference should be around 1e-8.
print('Testing max_pool_forward_naive function:')
print('difference: ', rel_error(out, correct_out))
```

```
Testing max_pool_forward_naive function:
difference:  4.1666665157267834e-08
```

Max pooling layer: backward naive

Implement the backward pass for a max pooling layer in the function `max_pool_backward_naive` in the file `cs231n/conv_layers.py`. As always we check the correctness of the backward pass using numerical gradient checking.

In [50]:
```
x = np.random.randn(3, 2, 8, 8)
dout = np.random.randn(3, 2, 4, 4)
pool_param = {'pool_height': 2, 'pool_width': 2, 'stride': 2}

dx_num = eval_numerical_gradient_array(lambda x: max_pool_forward_naive(x, pool_param)[0], x, dout)

out, cache = max_pool_forward_naive(x, pool_param)
dx = max_pool_backward_naive(dout, cache)

# Your error should be around 1e-12
print('Testing max_pool_backward_naive function:')
print('dx error: ', rel_error(dx, dx_num))
```

```
Testing max_pool_backward_naive function:
dx error:  3.2756192835058297e-12
```

Fast layers

Making convolution and pooling layers fast can be challenging. To spare you the pain, we've provided fast implementations of the forward and backward passes for convolution and pooling layers in the file `cs231n/fast_layers.py`.

The fast convolution implementation depends on a Cython extension; to compile it you need to run the following from the `cs231n` directory:

```
python setup.py build_ext --inplace
```

The API for the fast versions of the convolution and pooling layers is exactly the same as the naive versions that you implemented above: the forward pass receives data, weights, and parameters and produces outputs and a cache object; the backward pass recieves upstream derivatives and the cache object and produces gradients with respect to the data and weights.

NOTE: The fast implementation for pooling will only perform optimally if the pooling regions are non-overlapping and tile the input. If these conditions are not met then the fast pooling implementation will not be much faster than the naive implementation.

You can compare the performance of the naive and fast versions of these layers by running the following:

In [51]:
```
from cs231n.fast_layers import conv_forward_fast, conv_backward_fast
from time import time

x = np.random.randn(100, 3, 31, 31)
w = np.random.randn(25, 3, 3, 3)
b = np.random.randn(25,)
dout = np.random.randn(100, 25, 16, 16)
conv_param = {'stride': 2, 'pad': 1}

t0 = time()
out_naive, cache_naive = conv_forward_naive(x, w, b, conv_param)
t1 = time()
out_fast, cache_fast = conv_forward_fast(x, w, b, conv_param)
t2 = time()

print('Testing conv_forward_fast:')
print('Naive: %fs' % (t1 - t0))
print('Fast: %fs' % (t2 - t1))
print('Speedup: %fx' % ((t1 - t0) / (t2 - t1)))
print('Difference: ', rel_error(out_naive, out_fast))

t0 = time()
dx_naive, dw_naive, db_naive = conv_backward_naive(dout, cache_naive)
t1 = time()
dx_fast, dw_fast, db_fast = conv_backward_fast(dout, cache_fast)
t2 = time()

print('\nTesting conv_backward_fast:')
print('Naive: %fs' % (t1 - t0))
print('Fast: %fs' % (t2 - t1))
print('Speedup: %fx' % ((t1 - t0) / (t2 - t1)))
print('dx difference: ', rel_error(dx_naive, dx_fast))
print('dw difference: ', rel_error(dw_naive, dw_fast))
print('db difference: ', rel_error(db_naive, db_fast))
```

```
Testing conv_forward_fast:
Naive: 0.098736s
Fast: 0.015957s
Speedup: 6.187587x
Difference:  3.104202640264915e-11

Testing conv_backward_fast:
Naive: 0.221409s
Fast: 0.025931s
Speedup: 8.538363x
dx difference:  3.008014534569333e-12
dw difference:  1.0814979782973717e-13
db difference:  0.0
```

In [52]:
```
from cs231n.fast_layers import max_pool_forward_fast, max_pool_backward_fast

x = np.random.randn(100, 3, 32, 32)
dout = np.random.randn(100, 3, 16, 16)
pool_param = {'pool_height': 2, 'pool_width': 2, 'stride': 2}

t0 = time()
out_naive, cache_naive = max_pool_forward_naive(x, pool_param)
t1 = time()
out_fast, cache_fast = max_pool_forward_fast(x, pool_param)
t2 = time()

print('Testing pool_forward_fast:')
print('Naive: %fs' % (t1 - t0))
print('fast: %fs' % (t2 - t1))
print('speedup: %fx' % ((t1 - t0) / (t2 - t1)))
print('difference: ', rel_error(out_naive, out_fast))

t0 = time()
dx_naive = max_pool_backward_naive(dout, cache_naive)
t1 = time()
dx_fast = max_pool_backward_fast(dout, cache_fast)
t2 = time()

print('\nTesting pool_backward_fast:')
print('Naive: %fs' % (t1 - t0))
print('fast: %fs' % (t2 - t1))
print('speedup: %fx' % ((t1 - t0) / (t2 - t1)))
print('dx difference: ', rel_error(dx_naive, dx_fast))
```

```
Testing pool_forward_fast:
Naive: 0.014960s
fast: 0.004987s
speedup: 3.000143x
difference:  0.0

Testing pool_backward_fast:
Naive: 3.017929s
fast: 0.012967s
speedup: 232.732942x
dx difference:  0.0
```

Sandwich layers

There are a couple common layer "sandwiches" that frequently appear in ConvNets. For example convolutional layers are frequently followed by ReLU and pooling, and affine layers are frequently followed by ReLU. To make it more convenient to use these common patterns, we have defined several convenience layers in the file `cs231n/layer_utils.py`. Lets grad-check them to make sure that they work correctly:

In [53]:
```
from cs231n.layer_utils import conv_relu_pool_forward, conv_relu_pool_backward

x = np.random.randn(2, 3, 16, 16)
w = np.random.randn(3, 3, 3, 3)
b = np.random.randn(3,)
dout = np.random.randn(2, 3, 8, 8)
conv_param = {'stride': 1, 'pad': 1}
pool_param = {'pool_height': 2, 'pool_width': 2, 'stride': 2}

out, cache = conv_relu_pool_forward(x, w, b, conv_param, pool_param)
dx, dw, db = conv_relu_pool_backward(dout, cache)

dx_num = eval_numerical_gradient_array(lambda x: conv_relu_pool_forward(x, w, b, conv_param, pool_param
)[0], x, dout)
dw_num = eval_numerical_gradient_array(lambda w: conv_relu_pool_forward(x, w, b, conv_param, pool_param
)[0], w, dout)
db_num = eval_numerical_gradient_array(lambda b: conv_relu_pool_forward(x, w, b, conv_param, pool_param
)[0], b, dout)

print('Testing conv_relu_pool_forward:')
print('dx error: ', rel_error(dx_num, dx))
print('dw error: ', rel_error(dw_num, dw))
print('db error: ', rel_error(db_num, db))
```

```
Testing conv_relu_pool_forward:
dx error:  1.5359714066853973e-08
dw error:  6.730359554066423e-10
db error:  3.0926810572024394e-11
```

In [54]:
```
from cs231n.layer_utils import conv_relu_forward, conv_relu_backward

x = np.random.randn(2, 3, 8, 8)
w = np.random.randn(3, 3, 3, 3)
b = np.random.randn(3,)
dout = np.random.randn(2, 3, 8, 8)
conv_param = {'stride': 1, 'pad': 1}

out, cache = conv_relu_forward(x, w, b, conv_param)
dx, dw, db = conv_relu_backward(dout, cache)

dx_num = eval_numerical_gradient_array(lambda x: conv_relu_forward(x, w, b, conv_param)[0], x, dout)
dw_num = eval_numerical_gradient_array(lambda w: conv_relu_forward(x, w, b, conv_param)[0], w, dout)
db_num = eval_numerical_gradient_array(lambda b: conv_relu_forward(x, w, b, conv_param)[0], b, dout)

print('Testing conv_relu_forward:')
print('dx error: ', rel_error(dx_num, dx))
print('dw error: ', rel_error(dw_num, dw))
print('db error: ', rel_error(db_num, db))
```

```
Testing conv_relu_forward:
dx error:  1.6412906626008716e-09
dw error:  1.8805714936176815e-09
db error:  2.183516243068678e-11
```

In [55]:
```
from cs231n.layer_utils import affine_relu_forward, affine_relu_backward

x = np.random.randn(2, 3, 4)
w = np.random.randn(12, 10)
b = np.random.randn(10)
dout = np.random.randn(2, 10)

out, cache = affine_relu_forward(x, w, b)
dx, dw, db = affine_relu_backward(dout, cache)

dx_num = eval_numerical_gradient_array(lambda x: affine_relu_forward(x, w, b)[0], x, dout)
dw_num = eval_numerical_gradient_array(lambda w: affine_relu_forward(x, w, b)[0], w, dout)
db_num = eval_numerical_gradient_array(lambda b: affine_relu_forward(x, w, b)[0], b, dout)

print('Testing affine_relu_forward:')
print('dx error: ', rel_error(dx_num, dx))
print('dw error: ', rel_error(dw_num, dw))
print('db error: ', rel_error(db_num, db))
```

```
Testing affine_relu_forward:
dx error:  1.6226945537770296e-09
dw error:  3.5321483340718618e-09
db error:  1.8928900002169046e-11
```

In [ ]:

Train a ConvNet!

We now have a generic solver and a bunch of modularized layers. It's time to put it all together, and train a ConvNet to recognize the classes in CIFAR-10. In this notebook we will walk you through training a simple two-layer ConvNet and then set you free to build the best net that you can to perform well on CIFAR-10.

Open up the file `cs231n/classifiers/convnet.py`; you will see that the `two_layer_convnet` function computes the loss and gradients for a two-layer ConvNet. Note that this function uses the "sandwich" layers defined in `cs231n/layer_utils.py`.

In [3]:
```
# As usual, a bit of setup

import numpy as np
import matplotlib.pyplot as plt
from cs231n.classifier_trainer import ClassifierTrainer
from cs231n.gradient_check import eval_numerical_gradient
from cs231n.classifiers.convnet import *

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading external modules
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2

def rel_error(x, y):
  """ returns relative error """
  return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

In [4]:
```
from cs231n.data_utils import load_CIFAR10

def get_CIFAR10_data(num_training=49000, num_validation=1000, num_test=1000):
    """
    Load the CIFAR-10 dataset from disk and perform preprocessing to prepare
    it for the two-layer neural net classifier. These are the same steps as
    we used for the SVM, but condensed to a single function.
    """
    # Load the raw CIFAR-10 data
    cifar10_dir = 'cs231n/datasets/cifar-10-batches-py'
    X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

    # Subsample the data
    mask = range(num_training, num_training + num_validation)
    X_val = X_train[mask]
    y_val = y_train[mask]
    mask = range(num_training)
    X_train = X_train[mask]
    y_train = y_train[mask]
    mask = range(num_test)
    X_test = X_test[mask]
    y_test = y_test[mask]

    # Normalize the data: subtract the mean image
    mean_image = np.mean(X_train, axis=0)
    X_train -= mean_image
    X_val -= mean_image
    X_test -= mean_image

    # Transpose so that channels come first
    X_train = X_train.transpose(0, 3, 1, 2).copy()
    X_val = X_val.transpose(0, 3, 1, 2).copy()
    x_test = X_test.transpose(0, 3, 1, 2).copy()

    return X_train, y_train, X_val, y_val, X_test, y_test


# Invoke the above function to get our data.
X_train, y_train, X_val, y_val, X_test, y_test = get_CIFAR10_data()
print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Train data shape:  (49000, 3, 32, 32)
Train labels shape:  (49000,)
Validation data shape:  (1000, 3, 32, 32)
Validation labels shape:  (1000,)
Test data shape:  (1000, 32, 32, 3)
Test labels shape:  (1000,)
```

Sanity check loss

After you build a new network, one of the first things you should do is sanity check the loss. When we use the softmax loss, we expect the loss for random weights (and no regularization) to be about `log(C)` for `C` classes. When we add regularization this should go up.

In [5]:
```
model = init_two_layer_convnet()

X = np.random.randn(100, 3, 32, 32)
y = np.random.randint(10, size=100)

loss, _ = two_layer_convnet(X, model, y, reg=0)

# Sanity check: Loss should be about log(10) = 2.3026
print('Sanity check loss (no regularization): ', loss)

# Sanity check: Loss should go up when you add regularization
loss, _ = two_layer_convnet(X, model, y, reg=1)
print('Sanity check loss (with regularization): ', loss)
```

```
Sanity check loss (no regularization):  2.3024942537348783
Sanity check loss (with regularization):  2.3443364674260807
```

Gradient check

After the loss looks reasonable, you should always use numeric gradient checking to make sure that your backward pass is correct. When you use numeric gradient checking you should use a small amount of artifical data and a small number of neurons at each layer.

In [6]:
```
num_inputs = 2
input_shape = (3, 16, 16)
reg = 0.0
num_classes = 10
X = np.random.randn(num_inputs, *input_shape)
y = np.random.randint(num_classes, size=num_inputs)

model = init_two_layer_convnet(num_filters=3, filter_size=3, input_shape=input_shape)
loss, grads = two_layer_convnet(X, model, y)
for param_name in sorted(grads):
    f = lambda _: two_layer_convnet(X, model, y)[0]
    param_grad_num = eval_numerical_gradient(f, model[param_name], verbose=False, h=1e-6)
    e = rel_error(param_grad_num, grads[param_name])
    print('%s max relative error: %e' % (param_name, rel_error(param_grad_num, grads[param_name])))
```

```
W1 max relative error: 1.416301e-06
W2 max relative error: 4.636411e-06
b1 max relative error: 1.130980e-08
b2 max relative error: 8.226398e-10
```

Overfit small data

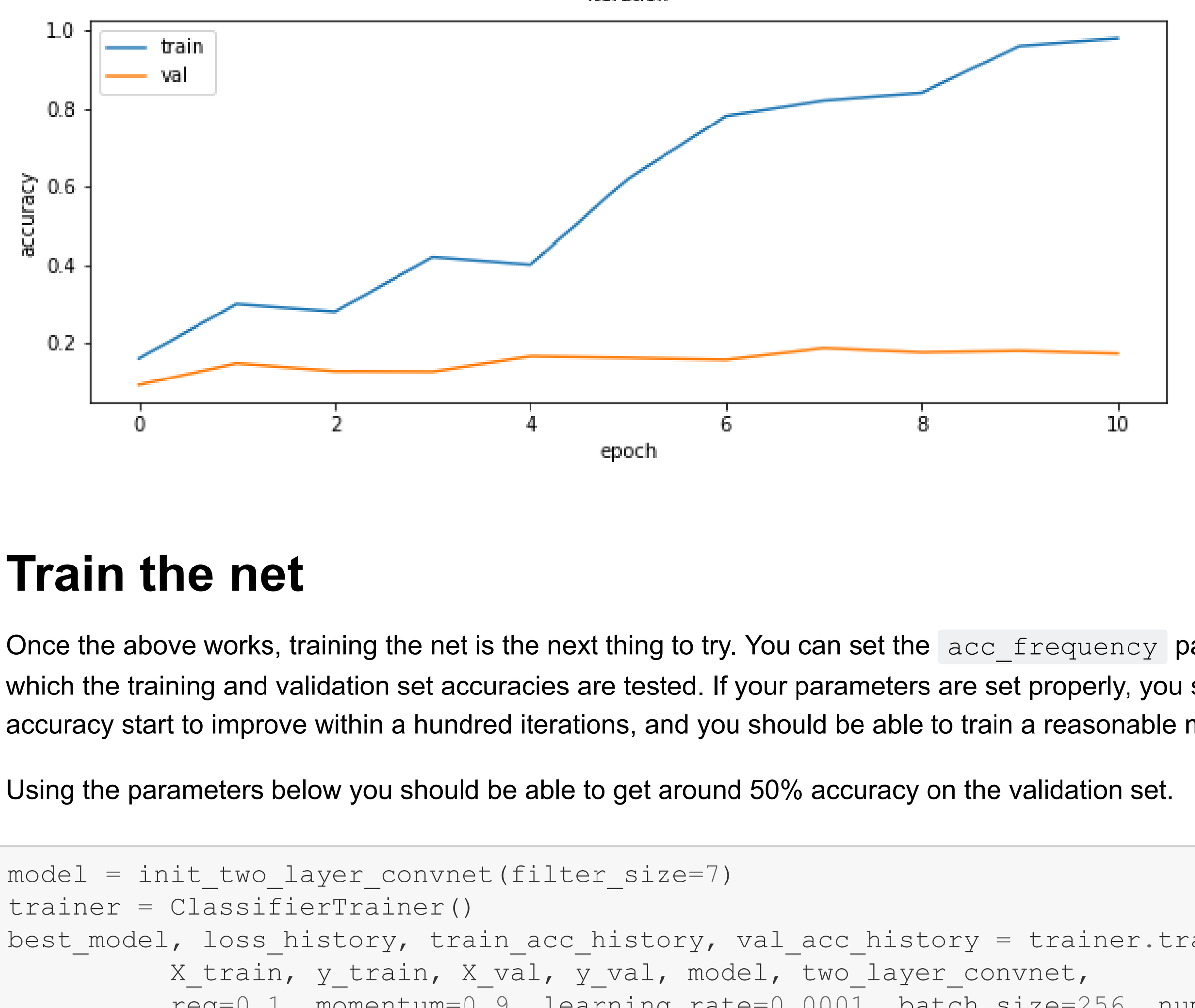
A nice trick is to train your model with just a few training samples. You should be able to overfit small datasets, which will result in very high training accuracy and comparatively low validation accuracy.

In [7]:
```
# Use a two-layer ConvNet to overfit 50 training examples.

model = init_two_layer_convnet()
trainer = ClassifierTrainer()
best_model, loss_history, train_acc_history, val_acc_history = trainer.train(
          X_train[:50], y_train[:50], X_val, y_val, model, two_layer_convnet,
          reg=0.001, momentum=0.9, learning_rate=0.0001, batch_size=10, num_epochs=10,
          verbose=True)
```

```
starting iteration  0
Finished epoch 0 / 10: cost 2.293451, train: 0.160000, val 0.093000, lr 1.000000e-04
Finished epoch 1 / 10: cost 2.194749, train: 0.300000, val 0.148000, lr 9.500000e-05
Finished epoch 2 / 10: cost 1.998509, train: 0.280000, val 0.128000, lr 9.025000e-05
starting iteration  10
Finished epoch 3 / 10: cost 1.878304, train: 0.420000, val 0.127000, lr 8.573750e-05
Finished epoch 4 / 10: cost 1.295153, train: 0.400000, val 0.166000, lr 8.145062e-05
starting iteration  20
Finished epoch 5 / 10: cost 1.550190, train: 0.620000, val 0.162000, lr 7.737809e-05
Finished epoch 6 / 10: cost 1.144751, train: 0.780000, val 0.157000, lr 7.350919e-05
starting iteration  30
Finished epoch 7 / 10: cost 0.466211, train: 0.820000, val 0.187000, lr 6.983373e-05
Finished epoch 8 / 10: cost 0.467357, train: 0.840000, val 0.176000, lr 6.634204e-05
starting iteration  40
Finished epoch 9 / 10: cost 0.191292, train: 0.960000, val 0.180000, lr 6.302494e-05
Finished epoch 10 / 10: cost 0.542685, train: 0.980000, val 0.173000, lr 5.987369e-05
finished optimization. best validation accuracy: 0.187000
```

Plotting the loss, training accuracy, and validation accuracy should show clear overfitting:

In [8]:
```
plt.subplot(2, 1, 1)
plt.plot(loss_history)
plt.xlabel('iteration')
plt.ylabel('loss')

plt.subplot(2, 1, 2)
plt.plot(train_acc_history)
plt.plot(val_acc_history)
plt.legend(['train', 'val'], loc='upper left')
plt.xlabel('epoch')
plt.ylabel('accuracy')
plt.show()
```



Train the net

Once the above works, training the net is the next thing to try. You can set the `acc_frequency` parameter to change the frequency at which the training and validation set accuracies are tested. If your parameters are set properly, you should see the training and validation accuracy start to improve within a hundred iterations, and you should be able to train a reasonable model with just one epoch.

Using the parameters below you should be able to get around 50% accuracy on the validation set.

In [11]:
```
model = init_two_layer_convnet(filter_size=7)
trainer = ClassifierTrainer()
best_model, loss_history, train_acc_history, val_acc_history = trainer.train(
          X_train, y_train, X_val, y_val, model, two_layer_convnet,
          reg=0.1, momentum=0.9, learning_rate=0.0001, batch_size=256, num_epochs=1,
          acc_frequency=50, verbose=True)
```

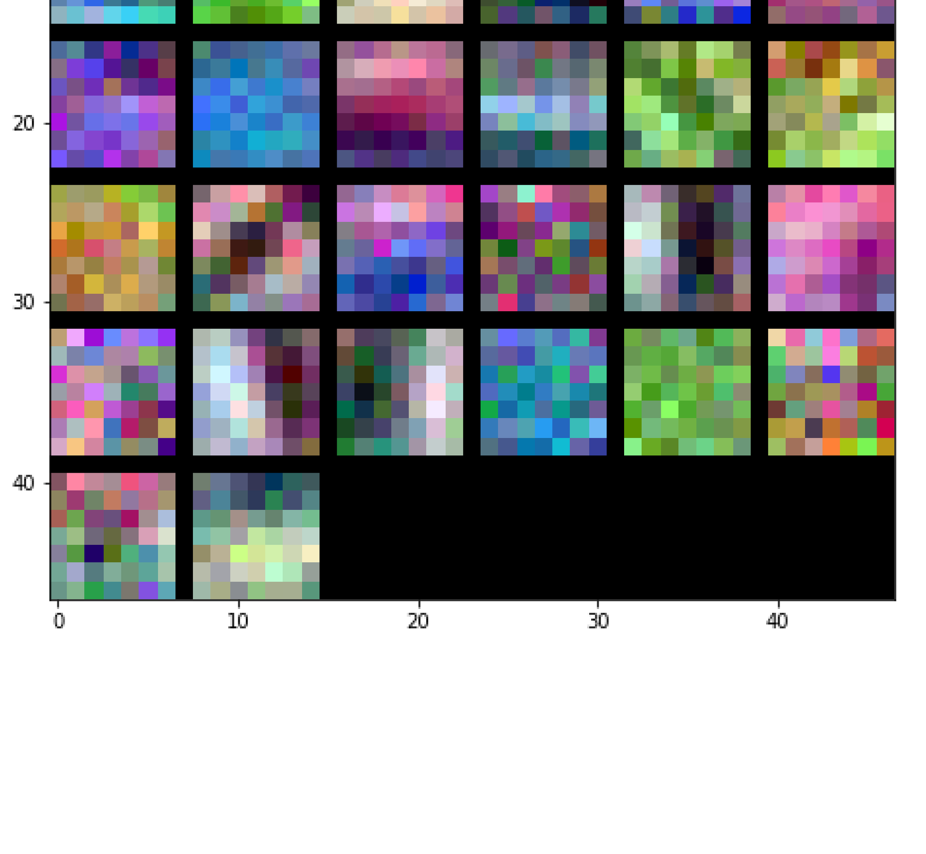
```
starting iteration  0
Finished epoch 0 / 1: cost 2.317788, train: 0.084000, val 0.098000, lr 1.000000e-04
starting iteration  10
starting iteration  20
starting iteration  30
starting iteration  40
starting iteration  50
Finished epoch 0 / 1: cost 1.789126, train: 0.375000, val 0.389000, lr 1.000000e-04
starting iteration  60
starting iteration  70
starting iteration  80
starting iteration  90
starting iteration  100
Finished epoch 0 / 1: cost 1.636210, train: 0.463000, val 0.474000, lr 1.000000e-04
starting iteration  110
starting iteration  120
starting iteration  130
starting iteration  140
starting iteration  150
Finished epoch 0 / 1: cost 1.547649, train: 0.478000, val 0.489000, lr 1.000000e-04
starting iteration  160
starting iteration  170
starting iteration  180
starting iteration  190
Finished epoch 1 / 1: cost 1.379095, train: 0.514000, val 0.520000, lr 9.500000e-05
finished optimization. best validation accuracy: 0.520000
```

Visualize weights

We can visualize the convolutional weights from the first layer. If everything worked properly, these will usually be edges and blobs of various colors and orientations.

In [12]:
```
from cs231n.vis_utils import visualize_grid

grid = visualize_grid(best_model['W1'].transpose(0, 2, 3, 1))
plt.imshow(grid.astype('uint8'))
```

Out[12]: `<matplotlib.image.AxesImage at 0x7f1e3a71ef60>`



PyTorch data

PyTorch comes with a nice paradigm for dealing with data which we'll use here. A PyTorch `Dataset` knows where to find data in its raw form (files on disk) and how to load individual examples into Python datastructures. A PyTorch `DataLoader` takes a dataset and offers a variety of ways to sample batches from that dataset.

Take a moment to browse through the `CIFAR10` `Dataset` in `2_pytorch/cifar10.py`, read the `DataLoader` documentation linked above, and see how these are used in the section of `train.py` that loads data. Note that in the first part of the homework we subtracted a mean CIFAR10 image from every image before feeding it in to our models. Here we subtract a constant color instead. Both methods are seen in practice and work equally well.

PyTorch provides lots of vision datasets which can be imported directly from `torchvision.datasets`. Also see `torchtext` for natural language datasets.

ConvNet Classifier in PyTorch

In PyTorch Deep Learning building blocks are implemented in the neural network module `torch.nn` (usually imported as `nn`). A PyTorch model is typically a subclass of `nn.Module` and thereby gains a multitude of features. Because your logistic regressor is an `nn.Module` all of its parameters and sub-modules are accessible through the `.parameters()` and `.modules()` methods.

Now implement a ConvNet classifier by filling in the marked sections of `models/convnet.py`.

The main driver for this question is `train.py`. It reads arguments and model hyperparameter from the command line, loads CIFAR10 data and the specified model (in this case, softmax). Using the optimizer initialized with appropriate hyperparameters, it trains the model and reports performance on test data.

Complete the following couple of sections in `train.py`:

1. Initialize an optimizer from the torch.optim package
2. Update the parameters in model using the optimizer initialized above

At this point all of the components required to train the softmax classifer are complete for the softmax classifier. Now run

```
$ run_convnet.sh
```

to train a model and save it to `convnet.pt`. This will also produce a `convnet.log` file which contains training details which we will visualize below.

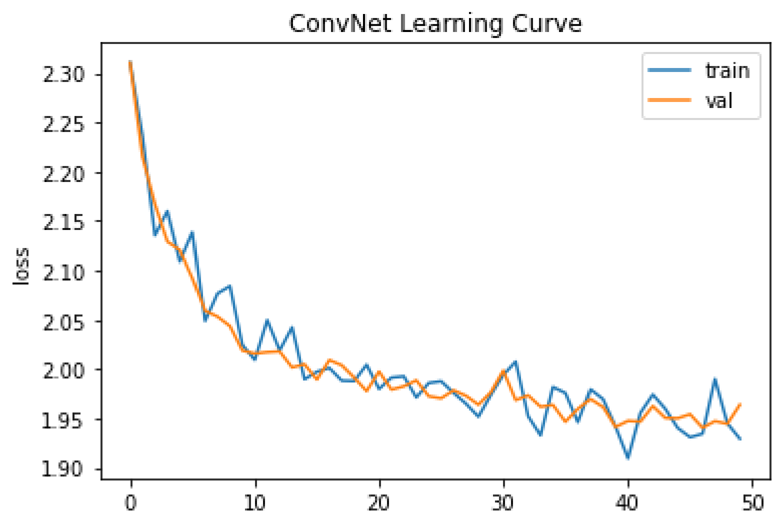
Note: You may want to adjust the hyperparameters specified in `run_convnet.sh` to get reasonable performance.

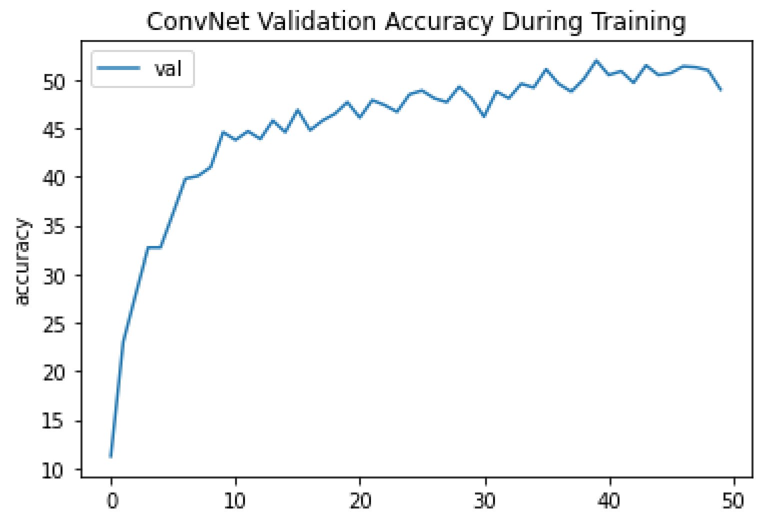
Visualizing the PyTorch model

In [1]:
```
# Assuming that you have completed training the classifer, let us plot the training loss vs. iteration. This is an
# example to show a simple way to log and plot data from PyTorch.

# we neeed matplotlib to plot the graphs for us!
import matplotlib
# This is needed to save images
matplotlib.use('Agg')
import matplotlib.pyplot as plt
%matplotlib inline
```

In [4]:
```
# Parse the train and val losses one line at a time.
import re
# regexes to find train and val losses on a line
float_regex = r'[-+]?(\d+(\.\d*)?|\.\d+)([eE][-+]?\d+)?'
train_loss_re = re.compile('.*Train Loss: ({})'.format(float_regex))
val_loss_re = re.compile('.*Val Loss: ({})'.format(float_regex))
val_acc_re = re.compile('.*Val Acc: ({})'.format(float_regex))
# extract one loss for each logged iteration
train_losses = []
val_losses = []
val_accs = []
# NOTE: You may need to change this file name.
with open('convnet.log', 'r') as f:
    for line in f:
        train_match = train_loss_re.match(line)
        val_match = val_loss_re.match(line)
        val_acc_match = val_acc_re.match(line)
        if train_match:
            train_losses.append(float(train_match.group(1)))
        if val_match:
            val_losses.append(float(val_match.group(1)))
        if val_acc_match:
            val_accs.append(float(val_acc_match.group(1)))
```

In [5]:
```
fig = plt.figure()
plt.plot(train_losses, label='train')
plt.plot(val_losses, label='val')
plt.title('ConvNet Learning Curve')
plt.ylabel('loss')
plt.legend()
fig.savefig('convnet_lossvstrain.png')

fig = plt.figure()
plt.plot(val_accs, label='val')
plt.title('ConvNet Validation Accuracy During Training')
plt.ylabel('accuracy')
plt.legend()
fig.savefig('convnet_valaccuracy.png')
```





In [ ]:

Visualizing the trained filters

In [59]:
```
# some startup!
import numpy as np
import matplotlib
# This is needed to save images
matplotlib.use('Agg')
import matplotlib.pyplot as plt
import torch
```

In [60]:
```
# load the model saved by train.py
# This will be an instance of models.convnet.CNN.
# NOTE: You may need to change this file name.
convnet_model = torch.load('convnet.pt')
```

In [61]:
```
# collect all the weights
w = None
###############################################################################
# TODO: Extract the weight matrix (without bias) from convnet_model, convert
# it to a numpy array with shape (10, 32, 32, 3), and assign this array to w.
# The first dimension should be for channels, then height, width, and color.
# This step depends on how you implemented models.convnet.CNN.
###############################################################################
w = convnet_model.conv1.weight.detach().reshape(10,3,7,7).permute(0,2,3,1).numpy()
###############################################################################
#                             END OF YOUR CODE                                #
###############################################################################
# obtain min,max to normalize
w_min, w_max = np.min(w), np.max(w)
# classes
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
# init figure
fig = plt.figure(figsize=(6,6))
for i in range(10):
    wimg = 255.0*(w[i].squeeze() - w_min) / (w_max - w_min)
    # subplot is (2,5) as ten filters are to be visualized
    fig.add_subplot(2,5,i+1).imshow(wimg.astype('uint8'))
# save fig!
fig.savefig('convnet_filt.png')
print('figure saved')
```

```
figure saved
```

In [62]:
```
# vis_utils.py has helper code to view multiple filters in single image. Use this to visuzlize
# neural network.
# import vis_utils
from vis_utils import visualize_grid
# saving the weights is now as simple as:
plt.imsave('convnet_gridfilt.png',visualize_grid(w, padding=3).astype('uint8'))
# padding is the space between images. Make sure that w is of shape: (N,H,W,C)
print('figure saved as a grid!')
```

```
figure saved as a grid!
```

In [ ]:
```
```