Applications of Derivatives

EE24BTECH11060-Sruthi Bijili

1) Prove the minimum value of $\frac{(a+x)(b+x)}{(c+x)}$, a,b>c, x>$-c$ is $\left(\sqrt{a-c}+\sqrt{b-c}\right)^2$.

(1979)

2) Let x and y be two real variables such that x>0and $xy = 1$.Find the minimum value of $x + y$.

(1981-2 Marks)

3) For all x in $[0, 1]$,let the second derivative $f''(x)$ of a function $f(x)$ exist and satisfy $|f''(x)| < 1$.If $f(0)$=$f(1)$,then show that$|f'(x)|$ <1for all x in $[0, 1]$.

(1981-4 Marks)

4) Use the function $f(x)$=$x^{\frac{1}{x}}$,$x > 0$.to determine the bigger of the two numbers e^{π} and π^{e}.

(1981-4 Marks)

5) If $f(x)$ and $g(x)$ are differentiable function for $0 \leq x \leq 1$ such that $f(0)$=2, $g(0)$=0, $f(0)$=6; $g(0)$=2, then show that there exist c satisfying 0<c<1and $f'(c)$=$2g'(c)$.

(1982-2 Marks)

6) Find the shortest distance of the point $(0, c)$from the parabola $y = x^2$ where $0 \leq c \leq 5$.

(1992-2 Marks)

7) If $ax^2+\frac{b}{x}$ for all positive x where a>0 and b>0 show that $27ab^2 \geq 4c^3$.

(1982-2 Marks)

8) Show that $1+x\ln\left(x+\sqrt{x^2+1}\right) \geq \sqrt{1+x^2}$ for all $x \geq 0$.

(1983-2 Marks)

9) Find the coordinates of the point on the curve y=$\frac{x}{1+x^2}$ where the tangent to the curve has the greatest slope.

(1984-4 Marks)

10) Find all the tangents to the curve y=$\cos(x + y)$, $-2\pi \leq x \leq 2\pi$, that are parallel to the line $x + 2y = 0$.

(1985-5 Marks)

11) Let $f(x)$=$\sin^3 x$+$\lambda \sin^2 x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Find the intervals in which λ should lie in order that $f(x)$ has exactly one minimum and exactly one maximum.

(1985-5 Marks)

12) Find the point on the curve $4x^2 + a^2y^2 = 4a^2$ that is farthest from the point$(0, -2)$.

(1987-4 Marks)

13) Investigate for the maxima and minima the function $f(x)$=$\int_1^x \left[2(t-1)(t-2)^3 + 3(t-1)^2(t-1)^2\right]$dt.

(1988-5 Marks)

14) Find all maxima and minima of the function y=$x(x-1)^2$,the y-axis and the line $y = 2$.

(1989-5 Marks)

15) Show that $2\sin x$+$\tan x \geq 3x$ where $0 \leq x \leq \frac{\pi}{2}$.

(1990-4 Marks)