1-1.8-21

EE24BTECH11060 - Sruthi Bijili

Question:
Find a point which is equidistant from the points $(-5,4)$ and $(-1,6)$. How many such points are there ?

solution:

Variable	Description
$\mathbf{A}(-5,4)$	coordinates of first point
$\mathbf{B}(-1,6)$	coordinates of second point
$\mathbf{C}$	midpoint of $\mathbf{A}$ and $\mathbf{B}$

TABLE 0: Input parameters

$$\|\mathbf{C}-\mathbf{A}\| = \|\mathbf{C}-\mathbf{B}\| \tag{0.1}$$
$$\implies \|\mathbf{C}-\mathbf{A}\|^2 = \|\mathbf{C}-\mathbf{B}\|^2 \tag{0.2}$$
$$\implies \|\mathbf{C}\|^2 - 2\mathbf{C}^T\mathbf{A} + \|\mathbf{A}\|^2 = \|\mathbf{C}\|^2 - 2\mathbf{C}^T\mathbf{B} + \|\mathbf{B}\|^2 \tag{0.3}$$
$$\implies (A-B)^T\,\mathbf{C} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \tag{0.4}$$
$$\implies \left(\begin{pmatrix}-5\\4\end{pmatrix} - \begin{pmatrix}-1\\6\end{pmatrix}\right)^T \begin{pmatrix}x\\y\end{pmatrix} = \frac{\sqrt{(-5)^2+(4)^2} - \sqrt{(-1)^2+(6)^2}}{2} \tag{0.5}$$
$$\implies \begin{pmatrix}-4\\-2\end{pmatrix}^T \begin{pmatrix}x\\y\end{pmatrix} = \frac{41-37}{2} \tag{0.6}$$
$$\implies \begin{pmatrix}-4 & -2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = 2 \tag{0.7}$$
$$\implies 2x + y = -1 \tag{0.8}$$
$$\text{if} x = -3 \tag{0.9}$$
$$\implies y = 5 \tag{0.10}$$
$$\mathbf{C} = \begin{pmatrix}-3\\5\end{pmatrix} \tag{0.11}$$

There are infinitely many points that are equidistant from the points$(-5,4)$ and $(-1,6)$

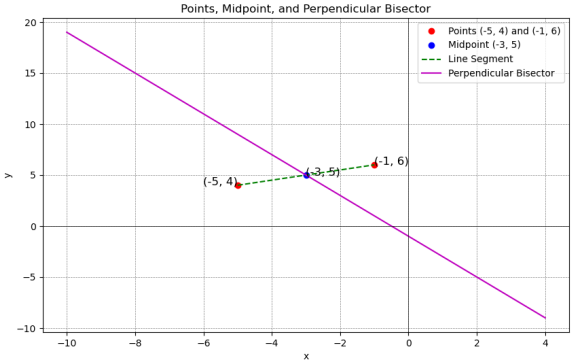

Fig. 0.1: line passing through the midpoint of AB