

1-1.8-21

EE24BTECH11060 - Sruthi Bijili

Question:

Find a point which is equidistant from the points $(-5, 4)$ and $(-1, 6)$. How many such points are there ?

solution:

Variable	Description	formula
$\mathbf{A}(-5, 4)$	coordinates of first point	—
$\mathbf{B}(-1, 6)$	coordinates of second point	—
\mathbf{C}	midpoint of \mathbf{A} and \mathbf{B}	—
k	ratio in which \mathbf{c} divides the line joining AB	$\frac{\mathbf{B}+k\mathbf{A}}{k+1}$

TABLE 0: Variables Used

$$\|\mathbf{C} - \mathbf{A}\| = \|\mathbf{C} - \mathbf{B}\| \quad (0.1)$$

$$\Rightarrow \|\mathbf{C} - \mathbf{A}\|^2 = \|\mathbf{C} - \mathbf{B}\|^2 \quad (0.2)$$

$$\Rightarrow \|\mathbf{C}\|^2 - 2\mathbf{C}^T \mathbf{A} + \|\mathbf{A}\|^2 = \|\mathbf{C}\|^2 - 2\mathbf{C}^T \mathbf{B} + \|\mathbf{B}\|^2 \quad (0.3)$$

$$\Rightarrow (\mathbf{A} - \mathbf{B})^T \mathbf{C} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \quad (0.4)$$

$$\Rightarrow \left(\begin{pmatrix} -5 \\ 4 \end{pmatrix} - \begin{pmatrix} -1 \\ 6 \end{pmatrix} \right)^T \begin{pmatrix} x \\ y \end{pmatrix} = \frac{\sqrt{(-5)^2 - (4)^2}}{2} \quad (0.5)$$

$$\Rightarrow \begin{pmatrix} -4 \\ 10 \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} = \frac{\sqrt{9}}{2} \quad (0.6)$$

$$\Rightarrow \begin{pmatrix} -4 & 10 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{3}{2} \quad (0.7)$$

$$\Rightarrow 2x + y = -1 \quad (0.8)$$

$$\text{if } x = -3 \quad (0.9)$$

$$\Rightarrow y = 5 \quad (0.10)$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ 5 \end{pmatrix} \quad (0.11)$$

There are infinitely many points that are equidistant from the points $(-5, 4)$ and $(-1, 6)$

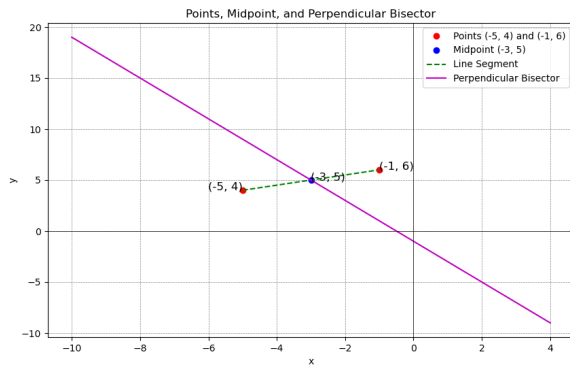


Fig. 0.1: line passing through the midpoint of AB