

# 12-9-6-2

EE24BTECH11060

## Question:

Find the general equation of

$$\frac{dy}{dx} + 3y = e^{-2x} \quad (0.1)$$

## Theoretical solution by laplace transform:

Laplace transform of the derivative  $\frac{dy}{dx}$  is given by

$$\mathcal{L}\left\{\frac{dy}{dx}\right\} = sY(s) - y(0) \quad (0.2)$$

where  $Y(s)$  is the laplace transform of  $y(x)$ , and  $y(0)$  is the initial condition.  
taking laplace transform on both sides of the equation:

$$\mathcal{L}\left\{\frac{dy}{dx} + 3y\right\} = \mathcal{L}\{e^{-2x}\} \quad (0.3)$$

$$\Rightarrow \mathcal{L}\left\{\frac{dy}{dx}\right\} + \mathcal{L}\{3y\} = \mathcal{L}\{e^{-2x}\} \quad (0.4)$$

$$\mathcal{L}\{e^{-2x}\} = \frac{1}{s+2} \quad (0.5)$$

$$\Rightarrow sY(s) - y(0) + 3Y(s) = \frac{1}{s+2} \quad (0.6)$$

rearranging the equation

$$Y(s) = \frac{1}{(s+2)(s+3)} + \frac{y(0)}{s+3} \quad (0.7)$$

$$Y(s) = \frac{1}{(s+2)(s+3)} + \frac{y(0)}{s+3} \quad (0.8)$$

Taking the inverse laplace transform of each term:

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} \quad (0.9)$$

thus, the solution by laplace transform is

$$y(x) = e^{-2x} - e^{-3x} + y(0)e^{-3x} \quad (0.10)$$

$$\Rightarrow y(x) = e^{-2x} + (y(0) - 1)e^{-3x} \quad (0.11)$$

**Method of finite differences** The derivative of  $f(x)$  can be written as

$$\frac{df}{dx} = \frac{f(x+h) - f(x)}{h} \quad (0.12)$$

$$\Rightarrow f(x+h) = f(x) + h \cdot \frac{df}{dx} \quad (0.13)$$

from the above question

$$\frac{dy}{dx} + 3y = e^{-2x} \quad (0.14)$$

$$\Rightarrow \frac{dy}{dx} = e^{-2x} - 3x \quad (0.15)$$

$$\Rightarrow y(x+h) = y(x) + h(e^{-2x} - 3x) \quad (0.16)$$

for  $x \in [x_0, x_n]$  divide into equal parts by difference  $h$

Let us assume that  $x_0 = 0, y_0 = 1$

Let  $x_1 = x_0 + h$  then

$$y_1 = y_0 + h(e^{-2x_0} - 3x_0) \quad (0.17)$$

To obtain the graph repeat the process until sufficient points to plot the graph and the general equation will be

$$x_{n+1} = x_n + h \quad (0.18)$$

$$y_{n+1} = y_n + h(e^{-2x_n} - 3x_n) \quad (0.19)$$

The curve generalised using the method of finite differences for the given question taking  $x_0 = 0, y_0 = 1, h = 0.01$  and running iterations for 100 times

