10.3.6.1.8

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1. Problem
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PROBLEM STATEMENT

Solve the following pairs of equations by reducing them to a pair of linear equations:

$$\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$$
$$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$$

Numerical Method

Let's solve this using LU decomposition. First, let's substitute:

$$\frac{1}{3x+y} = u \tag{4.1}$$

$$\frac{1}{3y-y} = v \tag{4.2}$$

Then our equations become:

$$u + v = \frac{3}{4} \tag{4.3}$$

$$\frac{1}{2}u - \frac{1}{2}v = \frac{-1}{8} \tag{4.4}$$

$$4u + 4v = 3 \tag{4.5}$$

$$4u - 4v = -1 \tag{4.6}$$

This can be written in matrix form as:

$$\begin{pmatrix} 4 & 4 \\ 4 & -4 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \tag{4.7}$$

Numerical Method

Let A$=\begin{pmatrix} 4 & 4 \\ 4 & -4 \end{pmatrix}$

By multiplying A on both sides

$$\begin{pmatrix} 4 & 4 \\ 4 & -4 \end{pmatrix} \begin{pmatrix} 4 & 4 \\ 4 & -4 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 4 & -4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \end{pmatrix} \tag{4.8}$$

after solving

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \frac{1}{4} \\ \frac{1}{2} \end{pmatrix} \tag{4.9}$$

Therefore:

$$\frac{1}{3x+y} = \frac{1}{4} \implies 3x+y=4 \tag{4.10}$$

$$\frac{1}{3x-y} = \frac{1}{2} \implies 3x-y=2 \tag{4.11}$$

$$\tag{4.12}$$

Numerical Method

Any non-singular matrix can be represented as a product of a lower triangular matrix L and an upper triangular matrix U

$$\mathbf{Ax} = \mathbf{LUx} = \mathbf{b} \tag{4.13}$$

Factorization of LU:

Given a matrix $\mathbf{A}$ of size $n \times n$, LU decomposition is performed row by row and column by column. The update equations are as follows:

Start by initializing $\mathbf{L}$ as the identity matrix $\mathbf{L} = \mathbf{I}$ and $\mathbf{U}$ as a copy of $\mathbf{A}$.

For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j} \quad \forall \quad j \geq k \tag{4.14}$$

Numerical Method

For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right) \quad \forall \quad i > k \tag{4.15}$$

By doing the following steps and solving we get :

$$\begin{pmatrix} 3 & 1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \tag{4.16}$$

By doing the following factorization we get:

$$\mathbf{U} = \begin{pmatrix} 3 & 1 \\ 0 & -2 \end{pmatrix} and \mathbf{L} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \tag{4.17}$$

We can solve this using two steps:

$$L\mathbf{y} = \mathbf{b} \tag{4.18}$$

$$U\mathbf{x} = \mathbf{y} \tag{4.19}$$

Using forward substitution:

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \tag{4.20}$$

$$\implies y_1 = 4 \tag{4.21}$$

$$\implies y_2 = -2 \tag{4.22}$$

Now using back substitution:

$$\begin{pmatrix} 3 & 1 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \tag{4.23}$$

This gives:

$$\implies y = 1 \tag{4.24}$$

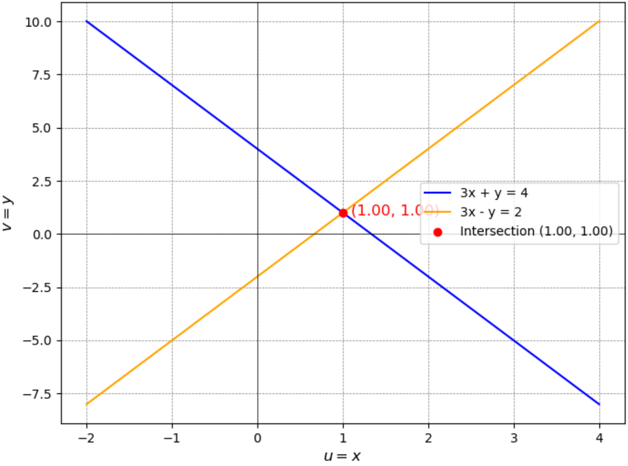
$$3x + 1 = 4 \tag{4.25}$$

$$x = 1 \tag{4.26}$$

The solution is:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{4.27}$$

Figure



10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−2
−1
0
1
2
3
4
$v = y$
$u = x$
(1.00, 1.00)
3x + y = 4
3x - y = 2
Intersection (1.00, 1.00)