12-9-6-2

EE24BTECH11060 - sruthi bijili

Question:
Find the general equation of

$$\frac{dy}{dx} + 3y = e^{-2x} \tag{1}$$

Theoretical solution: The given equation is in the form of

$$\frac{dy}{dx} + Py = Q \tag{2}$$

The integration factor is

$$I.F = e^{\int P dx} \tag{3}$$
$$\implies I.F = e^{\int 3 dx} \tag{4}$$
$$\implies I.F = e^{3x} \tag{5}$$

For the equation (0.2) the solution is

$$y(I.F) = \int (I.F) Q dx + C \tag{6}$$
$$\implies ye^{3x} = \int e^{3x} e^{-2x} dx + C \tag{7}$$
$$\implies ye^{3x} = \int e^{x} dx + C \tag{8}$$
$$\implies ye^{3x} = e^{x} + C \tag{9}$$

Let C be any constant so C=0 then the final equation is

$$y = e^{-2x} \tag{10}$$

Method of finite differences The derivative of f(x) can be written as

$$\frac{df}{dx} = \frac{f(x+h) - f(x)}{h} \tag{11}$$
$$\implies f(x+h) = f(x) + h \cdot \frac{df}{dx} \tag{12}$$

from the above question

$$\frac{dy}{dx} + 3y = e^{-2x} \tag{13}$$

$$\implies \frac{dy}{dx} = e^{-2x} - 3x \tag{14}$$

$$\implies y(x+h) = y(x) + h\left(e^{-2x} - 3x\right) \tag{15}$$

for $x \in [x_0, x_n]$ divide into equal parts by difference h
Let us assume that $x_0 = 0$,$y_0 = 1$
Let x_1=$x_0 + h$ then

$$y_1 = y_0 + h\left(e^{-2x_0} - 3x_0\right) \tag{16}$$

To obtain the graph repeat the process until sufficient points to pllot the graph and the general equation will be

$$x_{n+1} = x_n + h \tag{17}$$

$$y_{n+1} = y_n + h\left(e^{-2x_n} - 3x_n\right) \tag{18}$$

The curve generalised using the method of finite differences for the given question taking $x_0 = 0$,$y_0 = 1$, $h = 0.01$ and running iterations for 100 times