

9-5-8

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Question:

Solve the following differential equation $x \frac{dy}{dx} = y - x \sin \frac{y}{x}$

Theoretical solution:

given equation

$$x \frac{dy}{dx} = y - x \sin \frac{y}{x} \quad (1)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \sin \frac{y}{x} \quad (2)$$

$$(3)$$

let $y=vx$ by doing partial differentiation with respect to x

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad (4)$$

substitute in given equation

$$v + x \frac{dv}{dx} = v - \sin v \quad (5)$$

$$x \frac{dv}{dx} = -\sin v \quad (6)$$

$$\frac{dv}{\sin v} = -\frac{1}{x} dx \quad (7)$$

$$(8)$$

integrating on both sides

$$\int \csc v dv = - \int \frac{1}{x} dx \quad (9)$$

$$\log \tan \frac{v}{2} = -\log x + \log k \quad (10)$$

$$\log \tan \frac{v}{2} = \log \frac{k}{x} \quad (11)$$

$$(12)$$

let k be any constant

$$\tan \frac{v}{2} = \frac{1}{x} \quad (13)$$

$$\tan \frac{y}{2x} = \frac{1}{x} \quad (14)$$

$$\frac{y}{2x} = \tan^{-1} \frac{1}{x} \quad (15)$$

$$y = 2x \tan^{-1} \frac{1}{x} \quad (16)$$

method of finite differences

The derivative of $f(x)$ can be written as

$$\frac{df}{dx} = \frac{f(x+h) - f(x)}{h} \quad (17)$$

$$\implies f(x+h) = f(x) + h \cdot \frac{dy}{dx} \quad (18)$$

from the above question

$$\frac{dy}{dx} = \frac{y}{x} - \sin \frac{y}{x} \quad (19)$$

$$\implies y(x+h) = y(x) + h \cdot \frac{y}{x} - \sin \frac{y}{x} \quad (20)$$

for $x \in [x_0, x_n]$ divide into equal parts by difference h

Let $x_1 = x_0 + h$ then

$$y_1 = y_0 + h \cdot \frac{y_0}{x_0} - \sin \frac{y_0}{x_0} \quad (21)$$

To obtain the graph repeat the process until sufficient points to plot the graph and the general equation will be

$$x_{n+1} = x_n + h \quad (22)$$

$$y_{n+1} = y_n + h \cdot \frac{y_n}{x_n} - \sin \frac{y_n}{x_n} \quad (23)$$

The curve generalised using the method of finite differences for the given question taking $x_0 = 1, y_0 = 1, h = 0.1$ and running iterations for 100 times

