

11.16.2.3

EE24BTECH11060-Sruthi Bijili

Question:

An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:

A: the sum is greater than 8

B: 2 occurs on either die

C: the sum is at least 7 and a multiple of 3

Which pairs of these events are mutually exclusive?

Solution:

Let X_1, X_2 be the random variables taking values from 1 to 6 with probability $\frac{1}{6}$

The PMF of RV X_1 is defined as,

$$p_X(X_1) = \begin{cases} 0 & x < 0 \\ \frac{1}{6} & 1 \leq x \leq 6 \\ 0 & x > 6 \end{cases} \quad (1)$$

$$A : X_1 + X_2 > 8 \quad (2)$$

$$B : X_1 = 2 \text{ or } X_2 = 2 \quad (3)$$

$$C : X_1 + X_2 \geq 7, (X_1 + X_2) \% 3 = 0 \quad (4)$$

For A and B

$$X_1 + X_2 > 8 \quad (5)$$

$$\text{at } X_2 = 2 \implies X_1 + 2 > 8 \quad (6)$$

$$\implies X_1 > 6 \quad (7)$$

$$\implies P_{X_1}(x > 6) = 0 \quad (8)$$

$$P(AB) = 0 \quad (9)$$

Therefore, they are mutually exclusive.

For B and C

$$X_1 + X_2 \geq 7 \quad (10)$$

$$\text{at } X_2 = 2 \quad (11)$$

$$X_1 + 2 \geq 7 \quad (12)$$

$$\implies X_1 \geq 5 \quad (13)$$

$$Y = (X_1 + X_2) | ((X_1 \geq 5) \cap (X_1 + X_2) \% 3 = 0) \quad (14)$$

$$p_Y(X_1) = \begin{cases} 0 & X_1 = 5 \\ 0 & X_1 = 6 \end{cases} \quad (15)$$

therefore, $P(BC)=0$

$\Rightarrow B$ and C are mutually exclusive

For A and C

$$X = X_1 + X_2 \quad (16)$$

$$p_X(x) = \begin{cases} \frac{x-1}{36}, & 2 \leq x \leq 7 \\ \frac{13-x}{36}, & 8 \leq x \leq 12 \\ 0, & \text{otherwise} \end{cases} \quad (17)$$

Let $Y = (X_1 + X_2) | (X_1 + X_2) > 8 \cap (X_1 + X_2) \% 3 = 0$

$$p_Y(X) = \begin{cases} \frac{1}{9}, & \text{if } X_1 + X_2 = 9 \\ \frac{1}{36}, & \text{if } X_1 + X_2 = 12 \\ 0, & \text{otherwise} \end{cases} \quad (18)$$

Clearly $P(AC) \neq 0$

Therefore A and C are not mutually exclusive.

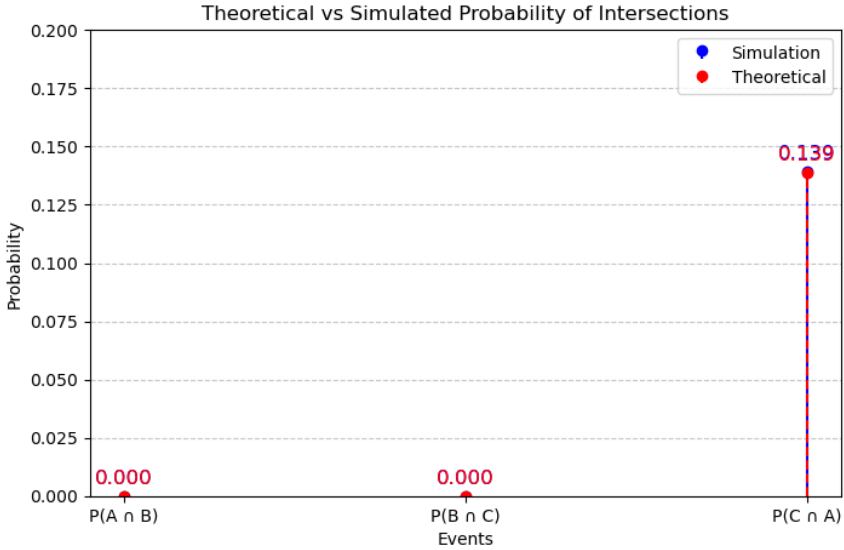


Fig. 1: Theoretical vs Simulation