Newton-Rapshon Method Example

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Exersice

A trough of length L has a cross section in the shape of a semicircle with radius r. When filled with water to within a distance h of the top, the volume V of water is

$$V = L\left[0{,}5\pi r^2 - r^2 \arcsin\left(\frac{h}{r}\right) - h\left(r^2 - h^2\right)^{1/2}\right], \tag{1}$$

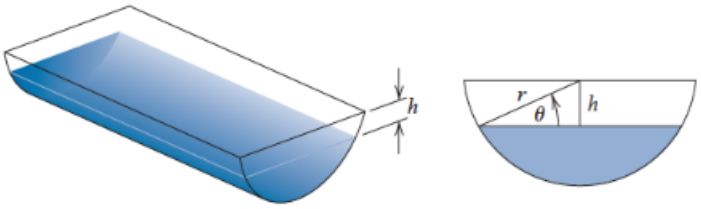

Figura 1: Graphical representation of the problem

Suppose $L = 10\ ft,\ r = 1\ ft,$ and $V = 12{,}4\ ft.$ Find the depth of water in the trough. This is a problem from chapter 2 of Richard Burden's numerical analysis text(Burden, Faires, and Burden 2015).

Solution

Note that the equation can be written as:

$$\arcsin\left(\frac{h}{r}\right) + \frac{h}{r}\left(1 - \left(\frac{h}{r}\right)^2\right)^{1/2} + \frac{V}{L} - 0{,}5\pi = 0, \tag{2}$$

let me do $x = \frac{h}{r},$ and we can write the equation as

$$\arcsin(x) + x(1 - x^2)^{1/2} + C = 0, \tag{3}$$

where

$$C = \frac{V}{L} - 0{,}5\pi. \tag{4}$$

We can solve equation 3 with Newton-Rapshon method. In this way we can obtain the value of $h,$ and thus we can determine the depth of the water which is given by

$$depth = r - h. \tag{5}$$

We found that the water depth is, $depth = 0{,}833\ ft$

Referencias

Burden, R., J. Faires, and A. Burden (2015). *Numerical Analysis*. Cengage Learning.