

Newton Rapshon Method Example

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Exersice

A trough of length L has a cross section in the shape of a semicircle with radius r . When filled with water to within a distance h of the top, the volume V of water is

$$V = L \left[0,5\pi r^2 - r^2 \arcsin\left(\frac{h}{r}\right) - h(r^2 - h^2)^{1/2} \right], \quad (1)$$

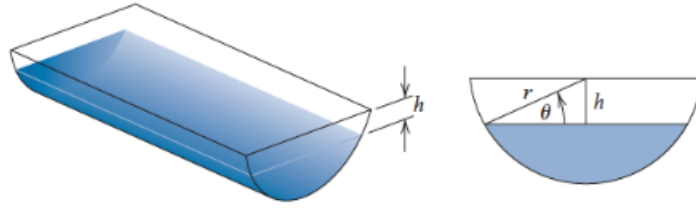


Figura 1: Graphical representation of the problem

Suppose $L = 10 \text{ ft}$, $r = 1 \text{ ft}$, and $V = 12,4 \text{ ft}$. Find the depth of water in the trough. This is a problem from chapter 2 of Richard Burden's numerical analysis text (Burden, Faires, and Burden 2015).

Solution

Note that the equation can be written as:

$$\arcsin\left(\frac{h}{r}\right) + \frac{h}{r} \left(1 - \left(\frac{h}{r}\right)^2\right)^{1/2} + \frac{V}{L} - 0,5\pi = 0, \quad (2)$$

let me to $x = \frac{h}{r}$, and we can write the equation as

$$\arcsin(x) + x(1 - x^2)^{1/2} + C = 0, \quad (3)$$

where

$$C = \frac{V}{L} - 0,5\pi. \quad (4)$$

We can solve equation 3 with Newton's method. In this way we can obtain the value of h , and thus we can determine the depth of the water which is given by

$$\text{depth} = r - h. \quad (5)$$

We found that the water depth is, $depth = 0,833 \text{ ft}$

Referencias

Burden, R., J. Faires, and A. Burden (2015). *Numerical Analysis*. Cengage Learning.