QPC Project Status to 07.2025: Perturbative Approach

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1 First Order Corrections

We consider the interaction between the QPC and the qubit to be the perturbative term. Therefore, the full Hamiltonian $H = H_0 + \Omega H_1$ is given by

$$H_0 = -\sum_{n}^{N} J(a_n^\dagger a_n + a_{n+1}^\dagger a_n) - t(d_1^\dagger d_0 + d_0^\dagger d_1) \tag{1}$$

and

$$H_1 = d_1^\dagger d_1(a_b^\dagger a_{b+1} + a_{b+1}^\dagger a_b) \tag{2}$$

where a, d are the annihilation operators of the QPC and qubit respectively, and the sub-index b refers to the QPC site where the bond is located.

The eigenstates of H_0 are the tensor product $\left|\phi_\nu^{(0)}(k)\right\rangle = |k\rangle \otimes |\nu\rangle$, where k is the label for the momentum states of the tight-binding model with open boundaries for the QPC, and $\nu = \pm$ refers to the symetric/anti-symmetric qubit states.

QPC eigenstates

$$\langle n|k\rangle = \sqrt{\frac{1}{N+1}}\sin(nk), \quad E(k) = -2J\cos(k), \quad k = \frac{n\pi}{N+1}, \quad n = 1, 2, ...N \tag{3}$$

Here $|n\rangle$ is the position basis.

Qubit eigenstates

$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\phi}|1\rangle\right), \quad \epsilon_+ = t \tag{4}$$

$$|-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - e^{i\phi}|1\rangle\right), \quad \epsilon_- = -t \tag{5}$$

From this setup, the zeroth order energies are

$$E_\nu(k)^{(0)} = E(k) + \epsilon_\nu = -2J\cos(k) \pm t, \tag{6}$$

meaning that, we now have two bands (symmetric anti-symmetric) of the tight-binding energy spectrum.

The following matrix element often appears in the later calculations

$$\left\langle \phi_\mu^{(0)}(p)\right| H_1 \left|\phi_\nu^{(0)}(k)\right\rangle = \langle\mu| d_0^\dagger d_0 |\nu\rangle \langle p| \left(a_b^\dagger a_{b+1} + a_{b+1}^\dagger a_b\right)|k\rangle . \tag{7}$$

The momentum part can be calculated by changing to the lattice basis by inserting $1 = \sum_n |n\rangle\langle n|$ two times and resolving the orthogonality conditions

$$\left\langle \phi_\mu^{(0)}(p)\right| H_1 \left|\phi_\nu^{(0)}(k)\right\rangle = \frac{1}{N+1}\left(\sin(bk+k)\sin(bp) + \sin(bp+p)\sin(bk)\right) = \frac{1}{N+1}\xi(k,p). \tag{8}$$

Notice that, since the interaction is local, only the sites near the bond contribute. In addition, this form makes it explicit that the perturbation couples the occupation at the state $|0\rangle$ of the qubit with the momentum of the QPC particle.

The first order corrections to the energy are

$$E_\nu^{(1)}(k) = \left\langle \phi_\nu^{(0)}(k) \right| H_1 \left| \phi_\nu^{(0)}(k) \right\rangle = \frac{1}{N+1}\xi(k,k). \tag{9}$$

Hence, to first order both bands are affected equally and due to the periodicity of $\xi(k,k)$ some energy states will not change, as is seen in figure 1.

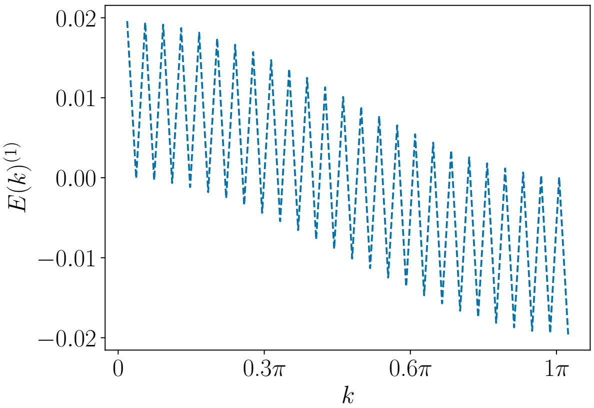

Figure 1: First order corrections to the energy for a system with 50 sites in the QPC.

The second order corrections are given by

$$E_\nu^{(2)}(k) = \sum_{(\mu,p)\neq(\nu,k)} \frac{\left| \left\langle \phi_\mu^{(0)}(p) \right| H_1 \left| \phi_\nu^{(0)}(k) \right\rangle \right|^2}{E_\nu^{(0)}(k) - E_\mu^{(0)}(p)} \tag{10}$$

$$= \frac{1}{(N+1)^2} \sum_{(\mu,p)\neq(\nu,k)} \frac{\xi(k,p)^2}{E_\nu^{(0)}(k) - E_\mu^{(0)}(p)} \tag{11}$$

where the summation over the μ and p states is subject to the condition that both cannot be equal to ν and k at the same time. To simplify the calculation, it is convenient to separate the sum into three terms

$$E_\nu^{(2)}(k) = \frac{1}{(N+1)^2} \sum_{p\neq k} \left\{ \frac{\xi(k,p)^2}{-2J(\cos k - \cos p)} + \frac{\xi(k,p)^2}{2\epsilon_\nu - 2J(\cos k - \cos p)} \right\} + \frac{1}{(N+1)^2}\frac{\xi(k,k)^2}{2\epsilon_\nu}, \tag{12}$$

where the first term corresponds to $\mu = \nu, p \neq k$, the second to $\mu \neq \nu, p \neq k$ and the third to $\mu \neq \nu, p = k$. Notice that there are some degeneracies in this expression, when k gets close to the $0, \pi$ boundaries of the band and when $\epsilon_\nu = -2J(\cos k - \cos p)$. We shall discuss these in the following after calculating the corrections to the eigenstates as the meaning will be clearer there.

The first order corrections to the eigenstates are

$$\left| \phi_\nu^{(1)}(k) \right\rangle = \sum_{(\mu,p)\neq(\nu,k)} \frac{\left\langle \phi_\mu^{(0)}(p) \right| H_1 \left| \phi_\nu^{(0)}(k) \right\rangle}{E_\nu^{(0)}(k) - E_\mu^{(0)}(p)} |p\rangle \otimes |\mu\rangle \,. \tag{13}$$

Much like with the energy, it is convenient to separate the sums into three following three terms

$$\begin{aligned} \left| \phi_\nu^{(1)}(k) \right\rangle =& \frac{1}{(N+1)} \sum_{p\neq k} |p\rangle \otimes \left\{ \frac{\xi(k,p)}{-2J(\cos k - \cos p)} |\nu\rangle + \frac{\xi(k,p)^2}{2\epsilon_\nu - 2J(\cos k - \cos p)} |\mu\rangle \right\} \\ &+ \frac{1}{(N+1)} \frac{\xi(k,k)}{2\epsilon_\nu} |k\rangle \otimes |\mu\rangle \,, \end{aligned} \tag{14}$$

corresponding to the same ones from Eq.(12). Here, it is important to point out the convention that $\mu \neq \nu$ when it appears explicitly in such a form. This convention is held for the rest of this text.

Now, working with Eq.(14) it is easier to understand what the aforementioned degeneracies mean. The first term, mixes momentum states within the same band and becomes degenerate towards the edges ($k \approx 0, \pi$) due to the cosine shape of the energy levels.

On the other hand, the second term is more interesting. It mixes momentum states from different bands (ν and μ), and becomes degenerate when the difference of between the QPC energies is equal to the band-gap, which can happen at several momenta. If $\nu = +$ the denominator blows up at $p = \pm \arccos(-t/J + \cos k)$ and for $\nu = -$ it happens at $p = \pm \arccos(t/J + \cos k)$. The dependence on the arccos imposes additional restrictions to the momenta that can cause a degeneracy. Since it is only defined for an argument between -1 and 1, the only states that contribute are those that fulfill

$$-1 + \frac{t}{J} < \cos k < 1 - \frac{t}{J}. \tag{15}$$

This condition implies that, at larger values of t, there will be less available states that can become degenerate and hybridize the two bands.

1.1 Corrections to the Reduced Density Matrix

References