Université de Montpellier - Master Informatique

Théorie des bases de connaissances (HAI933I)

Contrôle n° 1 - Novembre 2024

Durée de l'épreuve : 1 heure

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Le barème est donné à titre indicatif et peut varier légèrement.

Exercice 1 (Homomorphismes et conséquence logique) - 3 pts

On considère ces 2 ensembles d'atomes, où a est la seule constante :
$$A_1 = \{p(x_1, y_1), p(y_1, u_1), p(x_1, z_1), p(z_1, u_1)\}$$
$$A_2 = \{p(x_2, a), p(x_2, y_2), p(y_2, a)\}$$

Question 1.1. Y a-t-il un homomorphisme de A_1 dans A_2 ? de A_2 dans A_1 ? Justifiez une réponse positive en donnant un homomorphisme et une réponse négative par un argument montrant qu'il n'y a pas d'homomorphisme.

Question 1.2. On rappelle qu'un ensemble d'atomes A est un *core* s'il n'existe pas d'homomorphisme de A dans $A' \subset A$. A_1 et A_2 sont-ils des *cores* ? En cas de réponse négative, donner un homomorphisme justifiant votre réponse.

Question 1.3. On voit A_1 et A_2 comme des conjonctions d'atomes fermées existentiellement (par exemple des requêtes conjonctives booléennes). A-t-on $A_1 \models A_2$? A-t-on $A_2 \models A_1$? Justifiez vos réponses en vous appuyant sur vos réponses à la question 1.

Exercise 2 (Natural Language to RA/FO Queries) - 5 pts

Consider the following database :

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
Internet's Own Boy	Knappenberger	Swartz
Internet's Own Boy	Knappenberger	Lessig
Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck

Venues

Cinema	Address	Phone
UFA	St. Petersburger Str. 24	4825825
Schauburg	Königsbrücker Str. 55	8032185
...	...	...

Program

Cinema	Title	Time
Schauburg	The Imitation Game	19 :30
Schauburg	Dogma	20 :45
UFA	The Imitation Game	22 :45

Question 2.1. Write these queries using the relational algebra. (3 pts)

1. Return every director that has worked with "Damon" or "Affleck".

2. Return every actor that (1) has worked with director "Tyldum" but (2) has never worked with director "Smith".

3. Return every actor that has worked with exactly one director.

Question 2.2. Write the following queries as first-order queries. (2 pts)

3. Return the addresses of the cinemas that are playing some film directed by "Tyldum".

4. Return the titles of films that do not feature "Damon" as an actor.

Exercise 3 (FO to RA Queries) - 2 pts

Question 3.1. Translate the following first-order queries into equivalent relation algebra expressions. Use the schema $R[U_1, U_2, U_3]$, $C[V]$, and $D[W]$ to define the latter.

$$\exists x, z.(D(y) \wedge R(x, y, z)) \qquad \neg C(x) \vee D(x)$$

Exercise 4 (Equivalent Queries) - 2 pts

Question 4.1. Is it the case that $\pi_X(R - S) \equiv \pi_X(R) - \pi_X(S)$? Assume that X and Y are the only attributes of R and S. If the equivalence holds, explain why; otherwise, provide a database for which these two queries yield different results.

Exercice 5 (Acyclic conjunctive queries) - 4 pts

Consider the conjunctive query $\exists z.R_1(x, y) \wedge R_2(y, z) \wedge R_3(z, u, v)$.

Question 5.1. What is the hypergraph describing this query?

Question 5.2. What is its primal graph?

Question 5.3. Is this query acyclic? Explain using one of the ways we learned in class (variable elimination, ear removal, or join trees).

Exercice 6 (Yannakakis Algorithm) - 1 pts

Question 6.1. Consider the conjunctive query $\exists x, y, z.R_1(x, y) \wedge R_2(y, z)$ and the following database. Which facts will be removed by the Yannakakis algorithm?
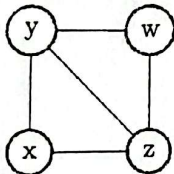
R_1 :

a	b
a	c
d	d

R_2 :

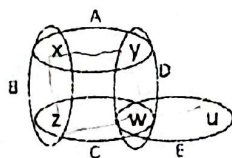| b	c
d	d

Exercice 7 (Treewidth) - 1 pts

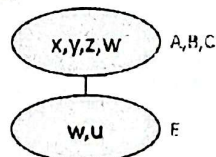**Question 7.1.** What is the treewidth of the following graph?

Exercice 8 (Generalised hypertree width) - 2 pts

Consider the following hypergraph H and its generalised hypertree decomposition D.

Question 8.1. What is the width of D?

Question 8.2. Is there a generalised hypertree decomposition for H with a lower width? (If so, give such a decomposition; otherwise, explain why not.)