Neural Networks -

Programming Club

IIT Kanpur

pclub.iitk@gmail.com

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Why Neural Networks?

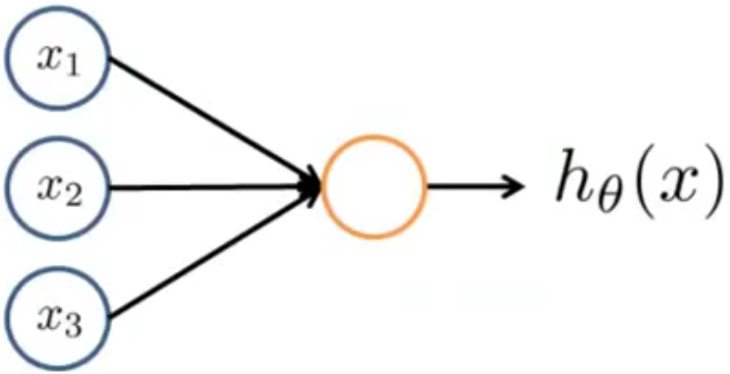
- Hard to incorporate higher order terms in our hypothesis in regressions
- Number of higher order terms increases rapidly with order
- Example -
  If number of features $= 100$
  Number of 2nd order features ≈ 5000
  Number of 3rd order features $\approx 1{,}70{,}000$
- Hard to decide which of the higher order terms are relevant.

Why Neural Networks

Won't it be awesome if our learning algorithm could decide by itself which festures to use in our hypothesis and which ones to neglect?

That's exactly what Neural Networks do

Neuron Model

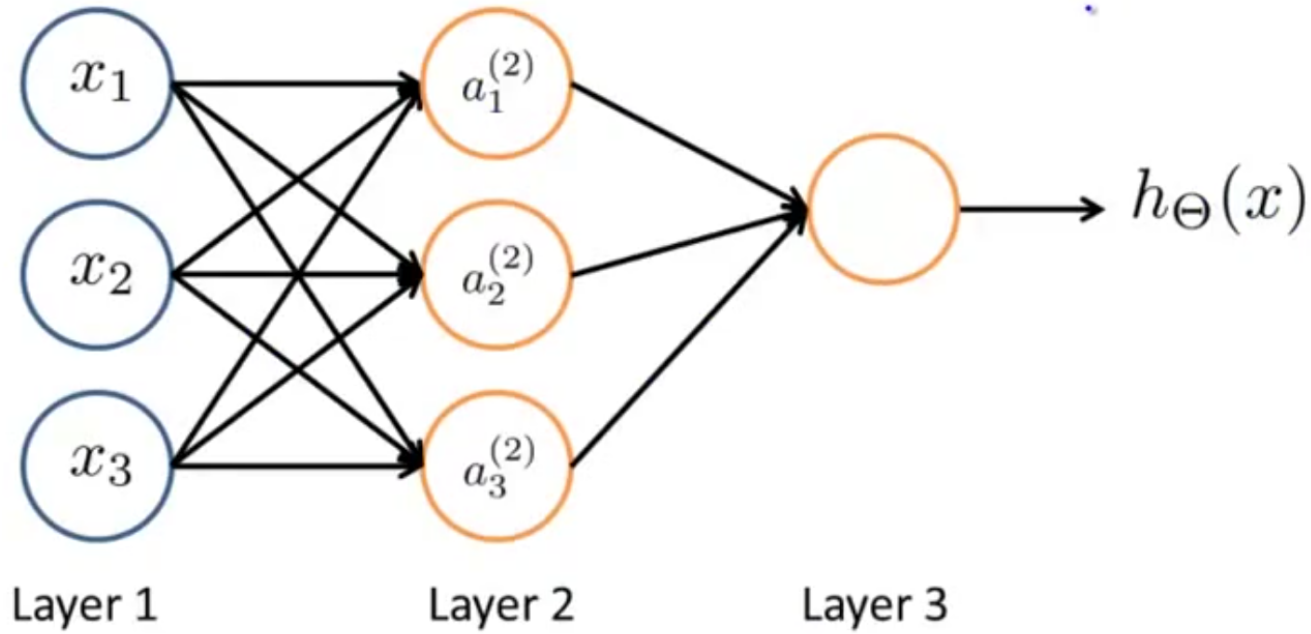


$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

- x is the data vector that is fed as input to our neuron model
- θ is the parameter or weight vector that we need to learn for the model
- The yellow unit of neuron model computes the sigmoid function on the input, and gives the value of $h_\theta(x)$ as the output
- $h_\theta(x)$ is the sigmoid or logistic activation function

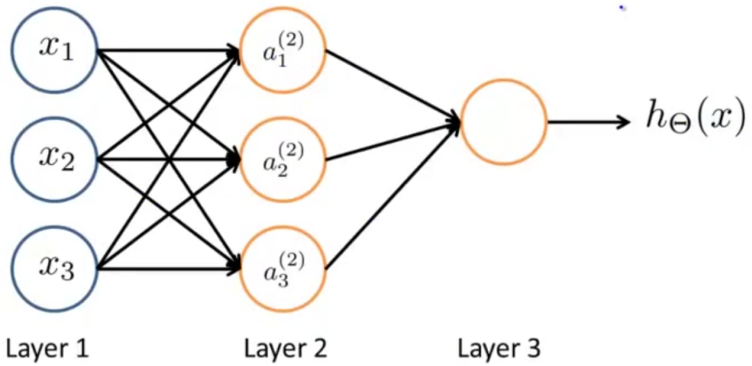
$$h_\theta(x) = \frac{1}{1+e^{(-\theta^T x)}}$$

Neural Network



- Layer 1 is the input layer
- Layer 3 is the output layer
- Layer 2 is called the "Hidden Layer"
- We can have more than one hidden layers in our neural network
- We can also add a bias units to our input and hidden layers, which can act as a constant input for the next layer.

Neural Network - Forward Propagation



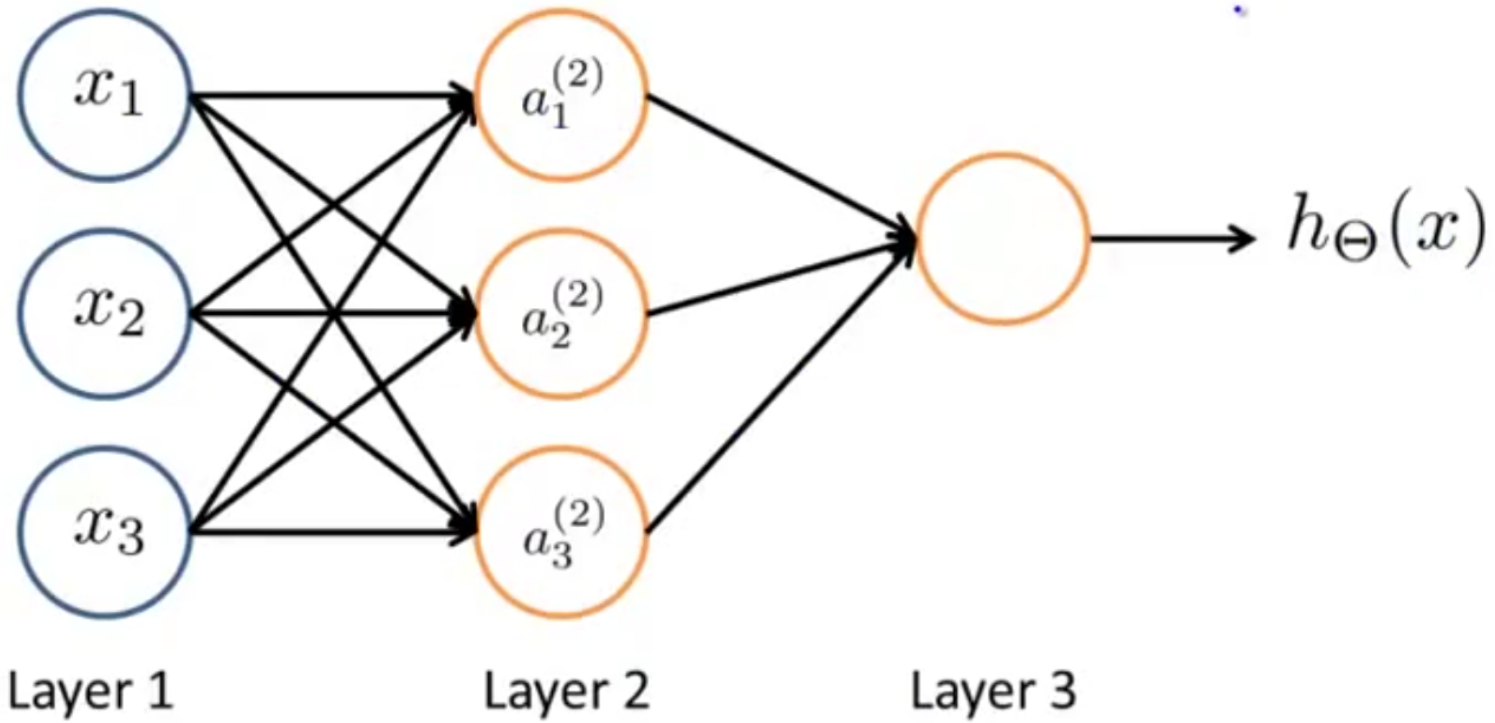
$a_i^{(j)}$ = "activation" of unit i in layer j

$\Theta^{(j)}$ = matrix of weights controlling function mapping from layer j to layer $j+1$

$$a_1^{(2)} = g(\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3)$$
$$a_2^{(2)} = g(\Theta_{20}^{(1)}x_0 + \Theta_{21}^{(1)}x_1 + \Theta_{22}^{(1)}x_2 + \Theta_{23}^{(1)}x_3)$$
$$a_3^{(2)} = g(\Theta_{30}^{(1)}x_0 + \Theta_{31}^{(1)}x_1 + \Theta_{32}^{(1)}x_2 + \Theta_{33}^{(1)}x_3)$$
$$h_\Theta(x) = a_1^{(3)} = g(\Theta_{10}^{(2)}a_0^{(2)} + \Theta_{11}^{(2)}a_1^{(2)} + \Theta_{12}^{(2)}a_2^{(2)} + \Theta_{13}^{(2)}a_3^{(2)})$$

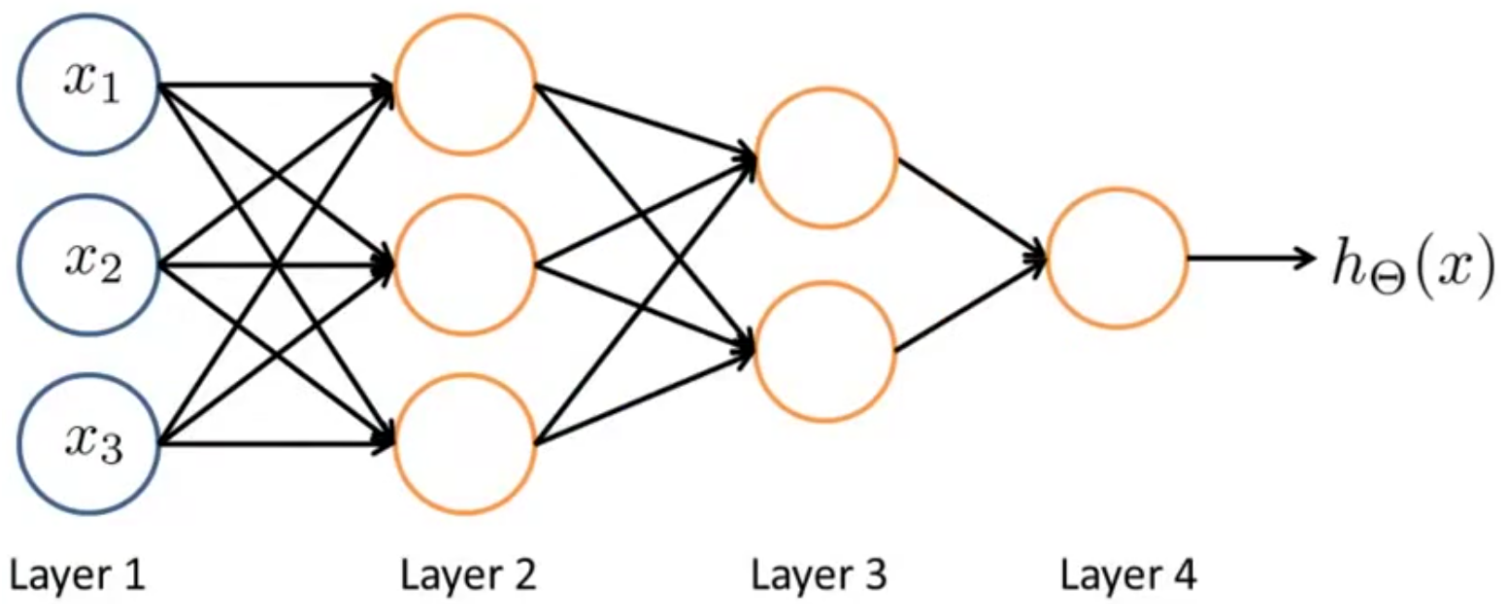
- If network has s_j units in layer j, s_{j+1} units in layer $j+1$, then θ^j will be $(s_{j+1})x(s_j+1)$ dimensional matrix

Neural Network - Forward Propagation



- Neural Network can be viewed as nested Logistic Regression in each layer.
- In each layer, the previous layer acts as input, and current layer performs Logistic Regression to generate a new set of inputs for further layers.

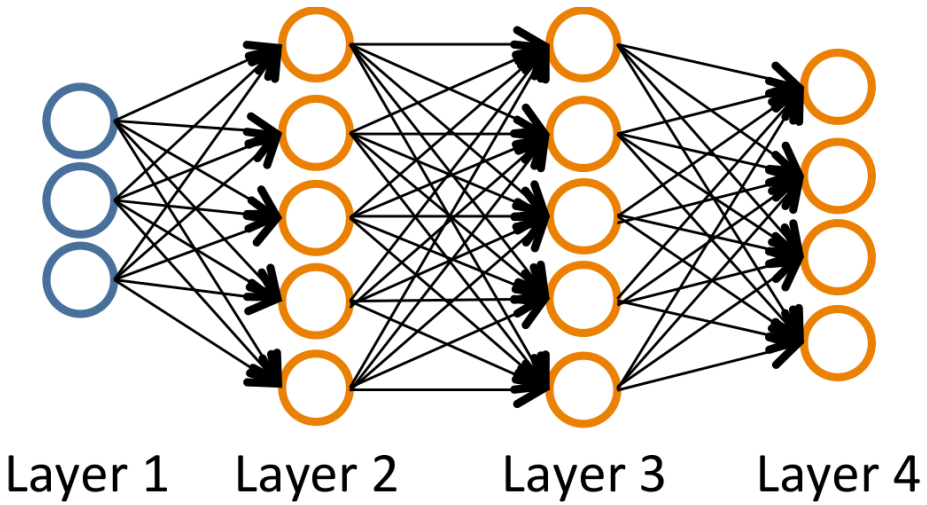
Neural Network - Other Architectures



An architecture with multiple hidden layers

Neural Network - Classification

- $\{(X^{(1)}, y^{(1)}), \ldots, (X^{(m)}, y^{(m)})\}$: Data Set
- L = Total no. of layers in the network
- s_l = no. of units (not counting bias unit) in layer l



- This network can be used for multi-class classification, as it has 4 units in the last layer, representing the 4 classes that can be used for classification

Neural Network - Cost Function

- Cost function for Logistic Regression -

$$J(\theta) = -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log(1 - h_\theta(x^{(i)}))\right] + \frac{\lambda}{2m}\sum_{j=1}^{n} \theta_j^2$$

- Cost function for Neural Network is a generalization of the above -

$$h_\Theta(x) \in \mathbb{R}^K \quad (h_\Theta(x))_i = i^{th} \text{ output}$$

$$J(\Theta) = -\frac{1}{m}\left[\sum_{i=1}^{m}\sum_{k=1}^{K} y_k^{(i)} \log(h_\Theta(x^{(i)}))_k + (1 - y_k^{(i)}) \log(1 - (h_\Theta(x^{(i)}))_k)\right]$$
$$+\frac{\lambda}{2m}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}(\Theta_{ji}^{(l)})^2$$

- The extra summation over k is the summation over the k output units

Neural Network - Cost Function

$$J(\Theta) = -\frac{1}{m}\left[\sum_{i=1}^{m}\sum_{k=1}^{K} y_k^{(i)} \log h_\theta(x^{(i)})_k + (1 - y_k^{(i)}) \log(1 - h_\theta(x^{(i)})_k)\right]$$
$$+\frac{\lambda}{2m}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}(\Theta_j^{(l)})^2$$

$$\min_{\Theta} J(\Theta)$$

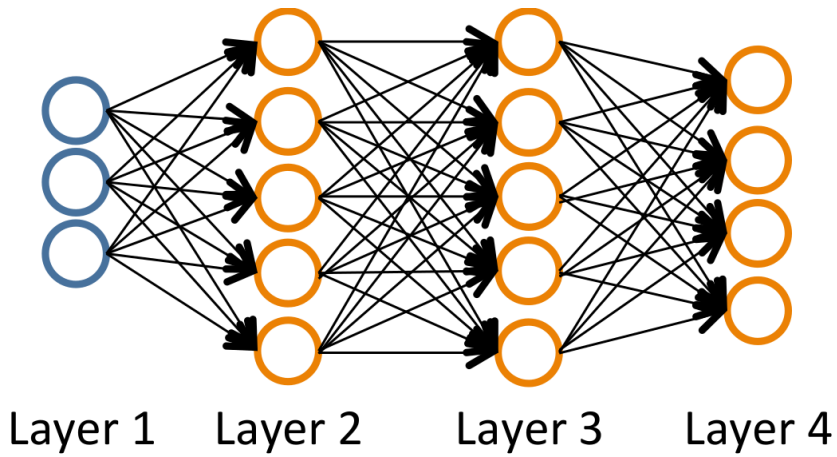
Need code to compute:

- $J(\Theta)$
- $\frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta)$

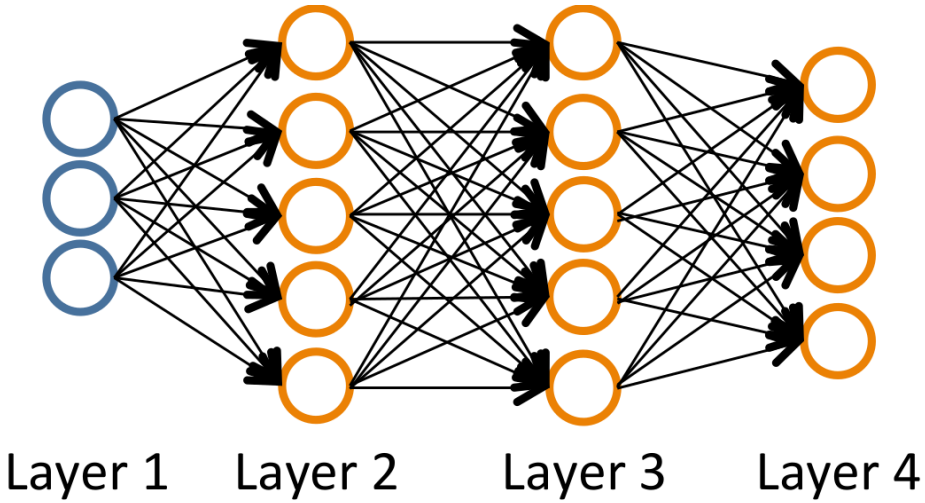
Neural Network - Forward Propagation

The Forward Propagation algorithm we saw previously -

$$
\begin{aligned}
a^{(1)} &= x \\
z^{(2)} &= \Theta^{(1)} a^{(1)} \\
a^{(2)} &= g(z^{(2)}) \quad (\text{add } a_0^{(2)}) \\
z^{(3)} &= \Theta^{(2)} a^{(2)} \\
a^{(3)} &= g(z^{(3)}) \quad (\text{add } a_0^{(3)}) \\
z^{(4)} &= \Theta^{(3)} a^{(3)} \\
a^{(4)} &= h_\Theta(x) = g(z^{(4)})
\end{aligned}
$$



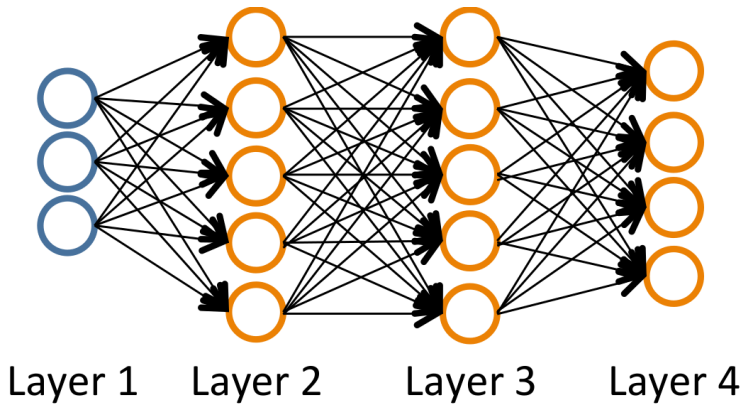
Neural Network - Backpropagation Algorithm



To compute the gradient, we use the backpropagation algorithm

- *Step 1* - Compute $\delta_j^{(l)}$ = "error" in node j of layer l
- $\delta_j^{(l)}$ is computed in a fashion similar to Forward Propagation, but in the reverse direction

Neural Network - Backpropagation Algorithm



- For each output unit (layer L=4)

$$\delta_j^{(4)} = a_j^{(4)} - y_j$$

- For units in hidden layers

$$\delta_j^{(i)} = (\Theta^{(i)})^T \delta^{(i+1)} .* g'(z^{(i)}) \quad i = 2, 3$$

- We don't compute $\delta_j^{(1)}$ as it is the input, which won't have any error
- Also, $g'(z^{(i)})$ is the derivative of the activation function g evaluated at input values given by $z^{(i)}$, given by

$$g'(z^{(i)}) = a^{(i)} .* (1 - a^{(i)})$$

Neural Network - Backpropagation Algorithm

For each example in $\{(X^{(1)}, y^{(1)}), \ldots, (X^{(m)}, y^{(m)})\}$ -

- Step 1 - Compute $\delta_j^{(l)}$ = "error" in node j of layer l
- Step 2 - Compute $\Delta_{ij}^{(l)}$

$$\Delta_{ij}^{(l)} := \Delta_{ij}^{(l)} + a_j^{(l)} \delta_i^{(l+1)} \quad \forall \ \ l, i, j$$

- Step 3 - Compute $D_{ij}^{(l)}$

$$D_{ij}^{(l)} := \frac{1}{m}\Delta_{ij}^{(l)} + \frac{\lambda}{m}\Theta_{ij}^{(l)} \quad \text{if } j \neq 0$$
$$D_{ij}^{(l)} := \frac{1}{m}\Delta_{ij}^{(l)} \quad \text{if } j = 0$$

- And in the last step

$$\frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta) = D_{ij}^{(l)}$$

Neural Network - Implementation Notes

- Use some in built library functions to minimize the cost function
- Initialize the parameters Θ (in range $[-\epsilon, \epsilon]$ for some value of ϵ) randomly, else all hidden layers will have same values after each iteration
- Implement some numerical gradient checking initially, to check if your gradient computation is correct

$$\frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta) = \frac{J(\Theta+\epsilon) - J(\Theta-\epsilon)}{2\epsilon}$$

- The numerical gradient computation is slow, hence, we use it only as a check for our implementation

Homework - Try to get the matrix based implementation of Forward Propagation and Backpropagation Algorithms