# Contents

# 1 Introduction

## 1.1. Aim

Aim is to evaluate the aeroelastic stability of a typical section with a control surface.

This will be done by building up a state space model, taking unsteady aerodynamics on a 2D typical section with 3 degrees of freedom: heave ($h$), pitch ($\theta$) and control surface angle ($\beta$).

Once this model is setup, you have to calculate the open-loop flutter speed and verify this value with the value reported in the Technical Report of Professor Karpel, and report the flutter mode

## 1.2. State Space Model

State space model is a tool that is used to evaluate the stability of a system. It lowers the order of the working equation, making it easier to work with the system.

$$\dot{X} = AX + Bu \tag{1.1}$$

**X** here is the state vector, **A** matrix is the state matrix, which will be used to calculate the stability of the system, **'u'** is the input vector, **B** is the input matrix.

Since we are dealing with linear systems, the input does not contribute to the stability and hence can be ignored.

$$\dot{X} = AX \tag{1.2}$$

Now using unsteady aerodynamics, we can start to setup the **A** matrix.

## 1.3. State Vector

Degrees of freedom in the 2D typical section with a control surface :

$$[x] = \begin{bmatrix} h \\ \theta \\ \beta \end{bmatrix} \tag{1.3}$$

When working with unsteady aerodynamics, we use the Theodorsen or Wagner function to account for the circulatory components, hence aerodynamic lag states are introduced to complete the system of equations.

Hence the State vector is defined as :

$$[X] = \begin{bmatrix} \dot{x} \\ x \\ w \end{bmatrix} = \begin{bmatrix} \dot{h} \\ \dot{\theta} \\ \dot{\beta} \\ h \\ \theta \\ \beta \\ w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} \tag{1.4}$$

Hence the complete State Space equation Equation 1.2 is defined by:

$$\begin{bmatrix} \ddot{h} \\ \ddot{\theta} \\ \ddot{\beta} \\ \dot{h} \\ \dot{\theta} \\ \dot{\beta} \\ \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \\ \dot{w}_4 \\ \dot{w}_5 \\ \dot{w}_6 \end{bmatrix} = [A] \begin{bmatrix} \dot{h} \\ \dot{\theta} \\ \dot{\beta} \\ h \\ \theta \\ \beta \\ w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} \tag{1.5}$$

## 1.4. Aeroelastic Equation for State Space

For unsteady aerodynamics, the aeroelastic equation of motion for a 2D typical section can be written as:

$$\begin{aligned} M_s\ddot{x} + K_s x &= M_a\ddot{x} + K_a x + C_a\dot{x} + W w \\ (M_s - M_a)\ddot{x} &= (K_a - K_s)x + C_a\dot{x} + W w \\ \ddot{x} &= (M_s - M_a)^{-1}(K_a - K_s)x + (M_s - M_a)^{-1}C_a\dot{x} + (M_s - M_a)^{-1}W w \end{aligned}$$

Taking $(M_s - M_a) = M_{ae}$ and $(K_s - K_a) = K_{ae}$

$$\ddot{x} = M_{ae}^{-1}K_{ae}x + M_{ae}^{-1}C_a\dot{x} + M_{ae}^{-1}W w \tag{1.6}$$

Using the first derivative of the Aerodynamic Lag equation:

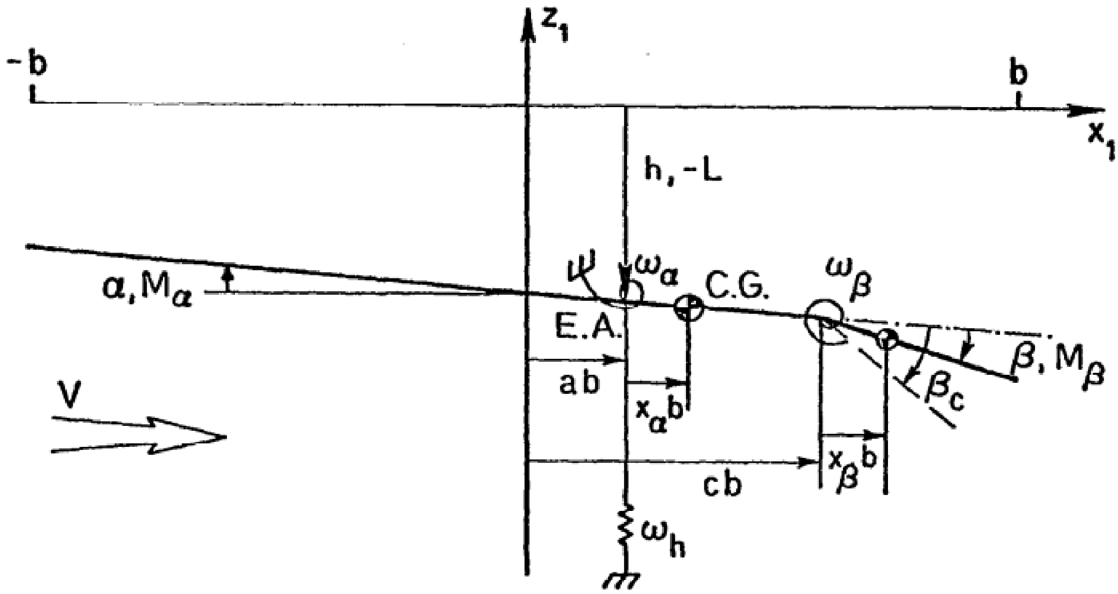
$$\dot{w} = x - W_0 w \tag{1.7}$$

We can write the complete State Space Equation and assemble the A matrix as:

$$\begin{bmatrix} \ddot{x} \\ \dot{x} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -M_{ae}^{-1}C_a & -M_{ae}^{-1}K_{ae} & -M_{ae}^{-1}W \\ I & 0 & 0 \\ 0 & I_1 & -W_0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ x \\ w \end{bmatrix} \tag{1.8}$$

# 2 Structural Matrices

The properties of the structure is taken from the Thesis paper by Professor Karpel. The values of the parameters is taken based on the 2D Theodorsen Airfoil profile:




From Prof. Karpel's Thesis paper[1], the following parameters will be used:

TABLE 1

Three DOF typical section structural parameters

| | | |
|---|---|---|
| $\omega_h$ = 50 rad/sec | a = -0.4 | $x_\alpha$ = 0.2 |
| $\omega_\alpha$ = 100 rad/sec | b = 1. | $x_\beta$ = -0.025 |
| $\omega_\beta$ = 300 rad/sec | c = 0.6 | $r_\alpha^2$ = 0.25 |
| $\mu$ = 40 | | $r_\beta^2$ = 0.00625 |

$M_s$ and $K_s$ are the **Structural Mass and Stiffness** Matrices respectively

$$[M_s] = m_s b^2 \begin{bmatrix} 1 & x_\alpha & x_\beta \\ x_\alpha & r_\alpha^2 & r_\beta^2 + (c-a)x_\beta \\ x_\beta & r_\beta^2 + (c-a)x_\beta & r_\beta^2 \end{bmatrix} \tag{2.1}$$

$$[K_s] = m_s b^2 \begin{bmatrix} \omega_h^2 & 0 & 0 \\ 0 & r_\alpha^2 \omega_\alpha^2 & 0 \\ 0 & 0 & r_\beta^2 \omega_\beta^2 \end{bmatrix} \tag{2.2}$$

# 3 Aerodynamic Matrices

- Unsteady aerodynamics is modelled by Theodorsen. The Lift, Moment due to pitch and Moment due to control surface angle equations are modelled by splitting it into 2 components, Circulatory and Non Circulatory.
- Theodorsen's expressions for circulatory components is dependent on a function of reduced frequency (k). Since we are using Aerodynamic Lag states, we need to convert this function into the time domain, since all the lag states are functions of time ($\tau$).
- We can use Wagner's indicial function to replace the Theodorsen Function, which is in the frequency domain, into the time domain, with an approximation.

Theodorsen Function can be approximated as:

$$C(k) = 1 - \frac{\psi_1 ik}{ik + \epsilon_1} - \frac{\psi_2 ik}{ik + \epsilon_2}$$

Applying the Inverse Fourier Transformation of the step function (here, Theodorsen Function):

$$\frac{C(k)}{ik} = \frac{1}{ik} - \frac{\psi_1}{ik + \epsilon_1} - \frac{\psi_2}{ik + \epsilon_2}$$

We then apply the convolution integral to convert the equation into the time domain with the Wagner's Function:

$$\frac{C(k)}{ik} = \int_{-\infty}^{t} \phi(t-\tau).d\tau$$

Hence, Wagner's Function can be approximated as:

$$\phi(t-\tau) = 1 - \psi_1 e^{-\epsilon_1 (t-\tau)\frac{V}{b}} - \psi_2 e^{-\epsilon_2 (t-\tau)\frac{V}{b}} \tag{3.1}$$

## 3.1. Lift Equation

Following the initial points, the Lift equation can be split into 2 components, Circulatory and Non Circulatory.

$$L = L^{nc} + L^{c} \tag{3.2}$$

Since the non circulatory component is independent of time, we will include it at the end. For now we will evaluate the circulatory component of Lift.

$$L^c = 2\pi\rho V b C(k).\omega_{\frac{3c}{4}}$$

$$L^c = 2\pi\rho V^2 b C(k).\alpha$$

$$L^c = 2\pi\rho V^2 b \frac{C(k)}{ik}.\hat{\alpha}$$

Applying the Inverse Fourier Transform:

$$L^c = 2\pi\rho V^2 b \int_{-\infty}^{t} \phi(t-\tau)\frac{\partial\alpha}{\partial\tau} d\tau \tag{3.3}$$

The equation for Angle of Attack for a 2D typical section with 3 Degrees of Freedom =

$$\alpha(t) = \theta + \frac{\dot{h}}{V} + \frac{\dot{\theta}}{V} b(\frac{1}{2} - a) + \frac{1}{\pi} T_{10}\beta + \frac{b}{2\pi} T_{11}\frac{\dot{\beta}}{V} \tag{3.4}$$

$$L^c = 2\pi\rho V^2 b\left(\phi(t).\alpha(0) + \int_0^t \phi(t-\tau)\frac{\partial\alpha}{\partial\tau}d\tau\right)$$

Using integration by parts:

$$\frac{L^c}{2\pi\rho V^2 b} = \left(\phi(t).\alpha(0) + \int_0^t \phi(t-\tau)\frac{\partial\alpha}{\partial\tau}d\tau\right)$$

$$\frac{L^c}{2\pi\rho V^2 b} = \left[\phi(t).\alpha(0) + \left[\left(\alpha(t).\phi(0)\right) - \left(\phi(t).\alpha(0)\right) - \int_0^t \alpha(\tau)\frac{\partial\phi(t-\tau)}{\partial\tau}d\tau)\right]\right]$$

$$\frac{L^c}{2\pi\rho V^2 b} = \alpha(t).\phi(0) - \int_0^t \alpha(\tau)\frac{\partial\phi(t-\tau)}{\partial\tau}d\tau)$$

We can substitute Equation 3.4, and replacing $\phi(0) = 1 - \psi_1 - \psi_2 = 0.5$, the only terms that do not vanish from the integral are:

$$\int_0^t \alpha(\tau)\frac{\partial\phi(t-\tau)}{\partial\tau}d\tau)$$

We can write $\frac{\partial\phi(t-\tau)}{\partial\tau} = -\psi_1\epsilon_1\frac{V}{b}e^{-\epsilon_1\frac{V}{b}(t-\tau)} - \psi_2\epsilon_2\frac{V}{b}e^{-\epsilon_2\frac{V}{b}(t-\tau)}$ , and multiply with Equation 3.4, we get 6 terms, defined as Lag States:

$$w_1 = \int_0^t h e^{-\epsilon_1\frac{V}{b}(t-\tau)}d\tau$$
$$w_2 = \int_0^t h e^{-\epsilon_2\frac{V}{b}(t-\tau)}d\tau$$
$$w_3 = \int_0^t \theta e^{-\epsilon_1\frac{V}{b}(t-\tau)}d\tau$$
$$w_4 = \int_0^t \theta e^{-\epsilon_2\frac{V}{b}(t-\tau)}d\tau$$
$$w_5 = \int_0^t \beta e^{-\epsilon_1\frac{V}{b}(t-\tau)}d\tau$$
$$w_6 = \int_0^t \beta e^{-\epsilon_2\frac{V}{b}(t-\tau)}d\tau$$

Hence we get Circulatory Lift as:

$$
\begin{aligned}
L^c = (2\pi\rho V^2 b)\Bigg[0.5\left(\theta + \frac{\dot{h}}{V} + \frac{\dot{\theta}}{V} b\left(\frac{1}{2} - a\right) + \frac{1}{\pi} T_{10}\beta + \frac{b}{2\pi} T_{11}\frac{\dot{\beta}}{V}\right) - \\
\Bigg[h\left(\frac{-\psi_1\epsilon_1 - \psi_2\epsilon_2}{b}\right) + \theta\left(\left(-\psi_1\epsilon_1 - \psi_2\epsilon_2\right)\left(\frac{1}{2} - a\right)\right) + \beta\left(\frac{-\psi_1\epsilon_1 - \psi_2\epsilon_2}{2\pi} T_{11}\right) - \frac{-\psi_1{\epsilon_1}^2 V}{b^2} w_1 - \frac{-\psi_2{\epsilon_2}^2 V}{b^2} w_2 + \\
w_3\left(\frac{-\psi_1\epsilon_1 V}{b} + \frac{\psi_1{\epsilon_1}^2 V}{b}\left(\frac{1}{2} - a\right)\right) + w_4\left(\frac{-\psi_2\epsilon_2 V}{b} + \frac{\psi_2{\epsilon_2}^2 V}{b}\left(\frac{1}{2} - a\right)\right) + w_5\left(\frac{-\psi_1\epsilon_1 V}{\pi b} T_{10} + \frac{\psi_1{\epsilon_1}^2 V}{2\pi b} T_{11}\right) + \\
w_6\left(\frac{-\psi_2\epsilon_2 V}{\pi b} T_{10} + \frac{\psi_2{\epsilon_2}^2 V}{2\pi b} T_{11}\right)\Bigg]\Bigg]
\end{aligned}
\tag{3.5}
$$

**We repeat the steps for $M_\theta$ and $M_\beta$:**

$$
\begin{aligned}
M_\theta^c = \Bigg[(2\pi\rho V^2 b^2)(\frac{1}{2} + a)\Bigg\{0.5\left(\theta + \frac{\dot{h}}{V} + \frac{\dot{\theta}}{V} b\left(\frac{1}{2} - a\right) + \frac{1}{\pi} T_{10}\beta + \frac{b}{2\pi} T_{11}\frac{\dot{\beta}}{V}\right) \\
\Bigg[h\left(\frac{-\psi_1\epsilon_1}{b} + \frac{-\psi_2\epsilon_2}{b}\right) + \theta\left(\left(-\psi_1\epsilon_1 - \psi_2\epsilon_2\right)\left(\frac{1}{2} - a\right)\right) + \beta\left(\frac{-\psi_1\epsilon_1 - \psi_2\epsilon_2}{2\pi} T_{11}\right) - \frac{-\psi_1{\epsilon_1}^2 V}{b^2} w_1 - \frac{-\psi_2{\epsilon_2}^2 V}{b^2} w_2 \\
+ w_3\left(\frac{-\psi_1\epsilon_1 V}{b} + \frac{\psi_1{\epsilon_1}^2 V}{b}\left(\frac{1}{2} - a\right)\right) + w_4\left(\frac{-\psi_2\epsilon_2 V}{b} + \frac{\psi_2{\epsilon_2}^2 V}{b}\left(\frac{1}{2} - a\right)\right) \\
+ w_5\left(\frac{-\psi_1\epsilon_1 V}{\pi b} T_{10} + \frac{\psi_1{\epsilon_1}^2 V}{2\pi b} T_{11}\right) + w_6\left(\frac{-\psi_2\epsilon_2 V}{\pi b} T_{10} + \frac{\psi_2{\epsilon_2}^2 V}{2\pi b} T_{11}\right)\Bigg]\Bigg\}\Bigg]
\end{aligned}
\tag{3.6}
$$

$$
\begin{aligned}
M_\beta^c = \Bigg[(\rho V^2 b^2 T_{12})\Bigg\{0.5\left(\theta + \frac{\dot{h}}{V} + \frac{\dot{\theta}}{V} b\left(\frac{1}{2} - a\right) + \frac{1}{\pi} T_{10}\beta + \frac{b}{2\pi} T_{11}\frac{\dot{\beta}}{V}\right) - \\
\Bigg[h\left(\frac{-\psi_1\epsilon_1}{b} + \frac{-\psi_2\epsilon_2}{b}\right) + \theta\left(\left(-\psi_1\epsilon_1 - \psi_2\epsilon_2\right)\left(\frac{1}{2} - a\right)\right) + \beta\left(\frac{-\psi_1\epsilon_1 - \psi_2\epsilon_2}{2\pi} T_{11}\right) - \frac{-\psi_1{\epsilon_1}^2 V}{b^2} w_1 - \frac{-\psi_2{\epsilon_2}^2 V}{b^2} w_2 \\
+ w_3\left(\frac{-\psi_1\epsilon_1 V}{b} + \frac{\psi_1{\epsilon_1}^2 V}{b}\left(\frac{1}{2} - a\right)\right) + w_4\left(\frac{-\psi_2\epsilon_2 V}{b} + \frac{\psi_2{\epsilon_2}^2 V}{b}\left(\frac{1}{2} - a\right)\right) \\
w_5\left(\frac{-\psi_1\epsilon_1 V}{\pi b} T_{10} + \frac{\psi_1{\epsilon_1}^2 V}{2\pi b} T_{11}\right) + w_6\left(\frac{-\psi_2\epsilon_2 V}{\pi b} T_{10} + \frac{\psi_2{\epsilon_2}^2 V}{2\pi b} T_{11}\right)\Bigg]\Bigg\}\Bigg]
\end{aligned}
\tag{3.7}
$$

Values of $T_1$ to $T_{13}$ are taken from the Theodorsen Paper[2].

We have 9 degrees of freedom $(h, \theta, \beta, w_1, w_2, w_3, w_4, w_5, w_6)$ and 3 equations $(L, M_\theta, M_\beta)$. To complete the system, we can use the Libnitz Rule of Integration for the lag states:

$$\frac{d}{dx}\left(\int_{a(x)}^{b(x)} f(x,t)\,dt\right) = f(x,b(x)) \cdot \frac{d}{dx}b(x) - f(x,a(x)) \cdot \frac{d}{dx}a(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,t)\,dt \tag{3.8}$$

Using this for each lag state, we can rewrite them as:

$$\begin{aligned}
\dot{w}_1 &= h - \epsilon_1 \left(\frac{V}{b}\right) w_1 \\
\dot{w}_2 &= h - \epsilon_2 \left(\frac{V}{b}\right) w_2 \\
\dot{w}_3 &= \theta - \epsilon_1 \left(\frac{V}{b}\right) w_3 \\
\dot{w}_4 &= \theta - \epsilon_2 \left(\frac{V}{b}\right) w_4 \\
\dot{w}_5 &= \beta - \epsilon_1 \left(\frac{V}{b}\right) w_5 \\
\dot{w}_6 &= \beta - \epsilon_2 \left(\frac{V}{b}\right) w_6
\end{aligned}$$

We can write the **Coefficient Matrix for the Derivative of the Lag states** $(W_0)$ as:

$$\left[W_0\right] = \begin{bmatrix}
\epsilon_1 \frac{V}{b} & 0 & 0 & 0 & 0 & 0 \\
0 & \epsilon_2 \frac{V}{b} & 0 & 0 & 0 & 0 \\
0 & 0 & \epsilon_1 \frac{V}{b} & 0 & 0 & 0 \\
0 & 0 & 0 & \epsilon_2 \frac{V}{b} & 0 & 0 \\
0 & 0 & 0 & 0 & \epsilon_1 \frac{V}{b} & 0 \\
0 & 0 & 0 & 0 & 0 & \epsilon_2 \frac{V}{b}
\end{bmatrix}$$

## 3.2. Setting up A matrix

We can now define the Aerodynamic Mass, Stiffness, Damping and Lag State Matrices.
**Aerodynamic Mass Matrix ($M_a$)**

$$[M_a] = b^2 \begin{bmatrix} \pi & -\pi ab & -T_1 b \\ -\pi ab & \pi b^2(\frac{1}{8}+a^2) & -(T_7+(c-a)T_1)b^2 \\ -T_1 b & 2T_{13}b^2 & -\frac{1}{\pi}T_3 b^2 \end{bmatrix} \tag{3.9}$$

**Aerodynamic Stiffness Matrix ($K_a$)**

$$[K_{NonCirc}] = -b^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2(T_4+T_{10}) \\ 0 & 0 & \frac{2(T_5-T_4T_{10})}{\pi} \end{bmatrix}$$

$$[K_{Circ}] = \begin{bmatrix} 0 & -4\pi b & -4bT_{10} \\ 0 & 4\pi b^2(0.5+a) & 4b^2(0.5+a)T_{10} \\ 0 & -2b^2T_{12} & \frac{-2b^2T_{12}T_{10}}{\pi} \end{bmatrix}$$

$$[K_{Lag}] = \begin{bmatrix} 4\pi(-\psi_1\epsilon_1-\psi_2\epsilon_2) & 4\pi b((-\psi_1\epsilon_1-\psi_2\epsilon_2)(0.5-a)) & 2b((-\psi_1\epsilon_1-\psi_2\epsilon_2)T_{11}) \\ -4\pi b(0.5+a)(-\psi_1\epsilon_1-\psi_2\epsilon_2) & -4\pi b^2(0.5+a)(-\psi_1\epsilon_1-\psi_2\epsilon_2)(0.5-a) & -2b^2(0.5+a)((-\psi_1\epsilon_1-\psi_2\epsilon_2)T_{11}) \\ 2bT_{12}(-\psi_1\epsilon_1-\psi_2\epsilon_2) & 2b^2T_{12}(0.5-a)(-\psi_1\epsilon_1-\psi_2\epsilon_2) & \frac{b^2T_{12}(-\psi_1\epsilon_1-\psi_2\epsilon_2)T_{11}}{\pi} \end{bmatrix}$$

$$K_a = K_{NonCirc} + \phi(0).K_{Circ} + K_{Lag} \tag{3.10}$$

**Aerodynamic Damping Matrix ($C_a$)**

$$[C_{NonCirc}] = b^2 \begin{bmatrix} 0 & 2\pi & -2T_4 \\ 0 & 2\pi(0.5-a)b & (T_1-T_8-(c-a)T_4+2(\frac{T_{11}}{2}))b \\ 0 & 2((-2T_9)-T_1+T_4(a-0.5))b & \frac{-T_4T_{11}}{\pi} \end{bmatrix}$$

$$[C_{Circ}] = \begin{bmatrix} 4\pi b & 4\pi b^2(0.5-a) & 2b^2T_{11} \\ -4\pi b^2(0.5+a) & -4\pi b^3(0.5-a)(0.5+a) & -4b^3(0.5+a)\frac{T_{11}}{2} \\ 2b^2T_{12} & 2b^3T_{12}(0.5-a) & \frac{2b^3T_{12}T_{11}}{\pi} \end{bmatrix}$$

$$C_a = C_{NonCirc} + \phi(0).C_{Circ} \tag{3.11}$$

**Lag State Matrix ($W$)**

$$[W] = \begin{bmatrix} [A_1] & [A_2] & [A_3] & [A_4] & [A_5] & [A_6] \\ [B_1] & [B_2] & [B_3] & [B_4] & [B_5] & [B_6] \\ [C_1] & [C_2] & [C_3] & [C_4] & [C_5] & [C_6] \end{bmatrix}$$

**Where the values are:**

$$[A_1] = -2\pi b\psi_1(\frac{\epsilon_1}{b})^2$$
$$[A_2] = -2\pi b\psi_2(\frac{\epsilon_2}{b})^2$$
$$[A_3] = 2\pi b\psi_1\epsilon_1\frac{(1-\epsilon_1(0.5-a))}{b}$$
$$[A_4] = 2\pi b\psi_2\epsilon_2\frac{(1-\epsilon_2(0.5-a))}{b}$$
$$[A_5] = 2\pi b\psi_1\epsilon_1\frac{(T_{10}-\frac{\epsilon_1 T_{11}}{2})}{\pi b}$$
$$[A_6] = 2\pi b\psi_2\epsilon_2\frac{(T_{10}-\frac{\epsilon_2 T_{11}}{2})}{\pi b}$$

$$[B_1] = 2\pi b^2(0.5+a)\psi_1(\frac{\epsilon_1}{b})^2$$
$$[B_2] = 2\pi b^2(0.5+a)\psi_2(\frac{\epsilon_2}{b})^2$$
$$[B_3] = -2\pi b^2(0.5+a)\psi_1\epsilon_1\frac{(1-\epsilon_1(0.5-a))}{b}$$
$$[B_4] = -2\pi b^2(0.5+a)\psi_2\epsilon_2\frac{(1-\epsilon_2(0.5-a))}{b}$$
$$[B_5] = -2\pi b^2(0.5+a)\psi_1\epsilon_1\frac{(T_{10}-\epsilon_1\frac{T_{11}}{2})}{\pi b}$$
$$[B_6] = -2\pi b^2(0.5+a)\psi_2\epsilon_2\frac{(T_{10}-\epsilon_2\frac{T_{11}}{2})}{\pi b}$$

$$[C_1] = -b^2 T_{12}\psi_1(\frac{\epsilon_1}{b})^2$$
$$[C_2] = -b^2 T_{12}\psi_2(\frac{\epsilon_2}{b})^2$$
$$[C_3] = b^2 T_{12}\psi_1\epsilon_1\frac{(1-\epsilon_1(0.5-a))}{b}$$
$$[C_4] = b^2 T_{12}\psi_2\epsilon_2\frac{(1-\epsilon_2(0.5-a))}{b}$$
$$[C_5] = b^2 T_{12}\psi_1\epsilon_1\frac{(T_{10}-\epsilon_1\frac{T_{11}}{2})}{\pi b}$$
$$[C_6] = b^2 T_{12}\psi_2\epsilon_2\frac{(T_{10}-\epsilon_2\frac{T_{11}}{2})}{\pi b}$$

- We have all the matrices defined, and now it will be a matter of assembling them into the **A** matrix and perform an Eigen Value Analysis, which will be performed in the next chapter.

- The A matrix is assembled according to Equation 1.6 and Equation 1.8

** Here it is important to note, these matrices are built by taking certain variables common from all the values in each matrix. Refer to the code in the Appendix to see which variables were taken as common for each of the Mass, Damping, Stiffness and Lag matrices.

# 4 Evaluation of State Space Model

To evaluate the State Space Model, we perform an Eigen Value Analysis of the Matrix to assess stability of the system. This state space model was built using Linear Aeroelastic equations, hence the input term does not contribute to the Stability of the System.

**Method of Eigen Value Stability Analysis:**

- When the real part of the Eigen Value is negative, the system is damped and the oscillations die out when excited, When positive, the system is unstable and behaves as an unstable oscillator, with a growing amplitude.
- If the real part of the Eigen Value changes from Negative to Positive, that will be the point of transition from stable to unstable solution.
- Flutter is a harmonic phenomenon, that occurs at the boundary between stable and unstable, hence the point of sign change from Negative to Positive will be the point of flutter.

**Key for Degrees of Freedom**

- Control Surface Mode
- Pitch Mode
- Heave/Plunge Mode

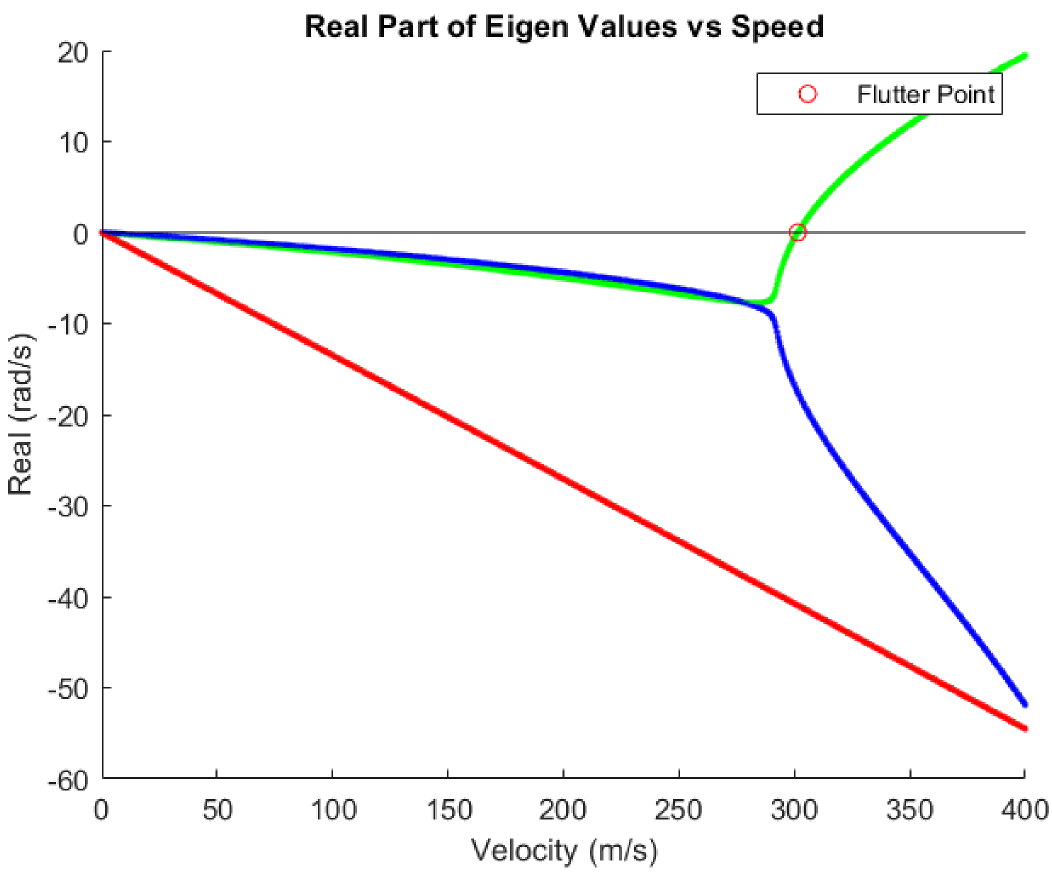



**Figure 4.1**

- Plotting the Real part of the Eigen value versus Velocity, we see that the Heave degree of freedom crosses the Real = 0 line from negative to positive.

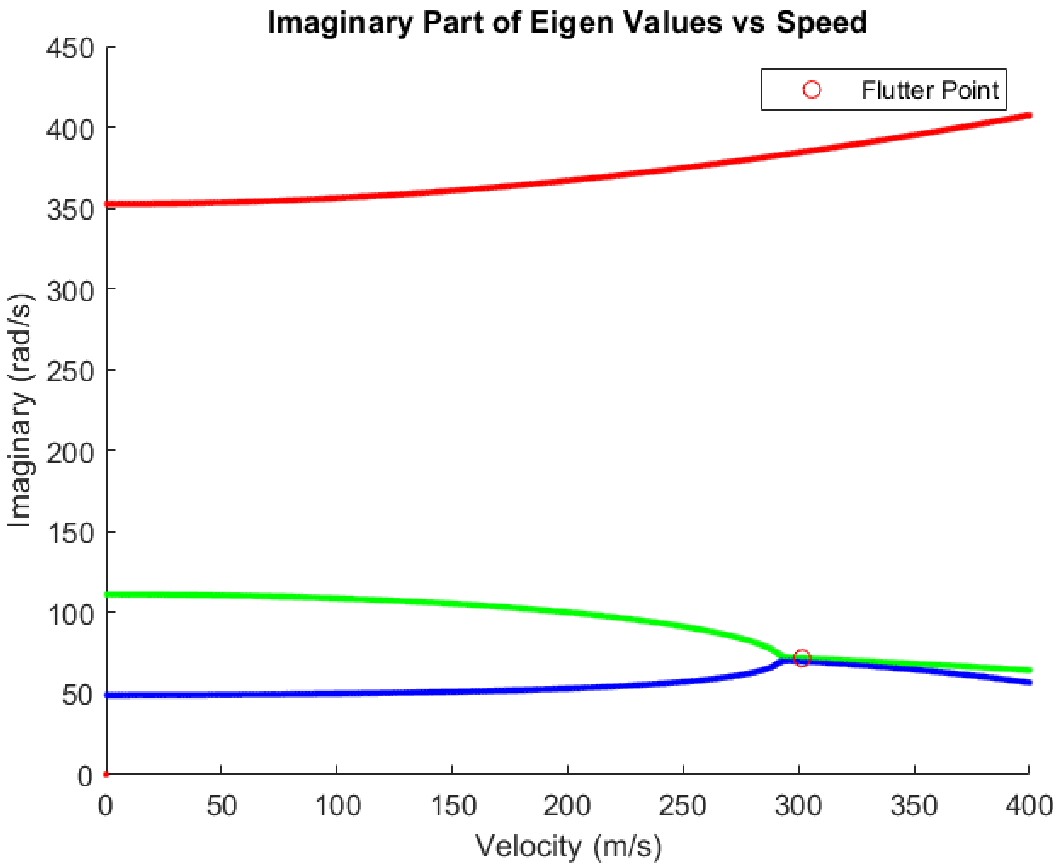



**Figure 4.2**

- This plot shows the **Modal Coalesce** of the **Pitch and Heave** modes. At the point of coalesce, the Aeroelastic frequencies of the Pitch and Heave modes come close enough to oscillate harmonically, which is what flutter is characterized.

- The control surface mode does not interact with any other modes.

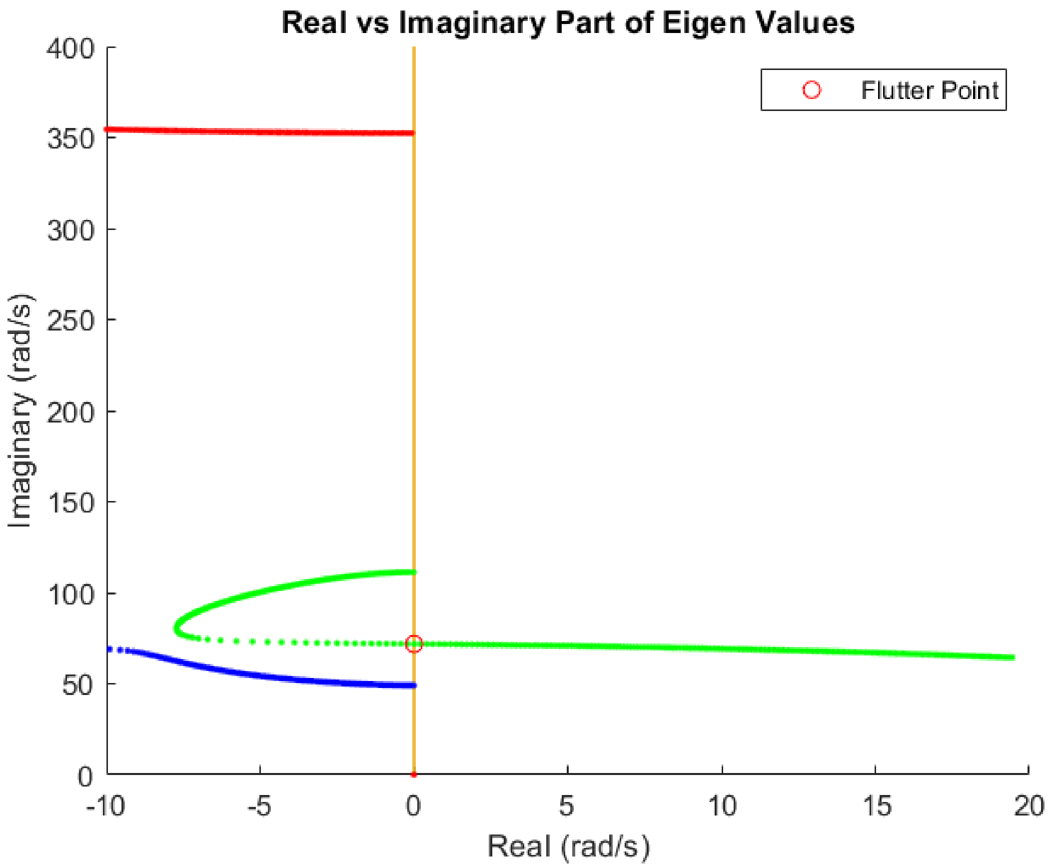



**Figure 4.3**

**Combining the 2 previous plots accurately shows the stability plot of the degrees of freedom.**

- Plotting the Real vs Imaginary values of each Eigen value, Figure 4.3 shows that beyond a certain speed, the Heave mode crosses into the Positive Real Half, hence becoming unstable.

## 4.1. Conclusion

Flutter Point at Velocity of **301.5m/s**
**Heave and Pitch** Degrees of Freedom interacting as the Flutter Mode

These results are in line with the obtained results from Prof. Karpel's paper[1].

# Bibliography

[1] Moti Karpel. *Design for Active and Passive Flutter Suppression and Gust Alleviation.* 1981.

[2] Theodore Theodorsen. *General Theory of Aerodynamic Instability and the Mechanism of Flutter.* 1940.