STA141C Homework 1

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Question 1

(a) This is a TRUE statement. The essence of a matrix is to do linear transformation. For instance, given a $n \times m$ rectangular matrix, we can use a $1 \times n$ vector to multiply with the matrix, which will transform the $1 \times n$ to a $1 \times m$ vector.

(b) This is a TRUE statement. The eigenvalue decomposition is only plausible for linearly independent square matrix. By the definition, the eigenvalue decomposition of a square matrix A is given by

$$A = V \Lambda V^{-1}$$

where V is the matrix attains all eigenvectors of A and Λ is the diagonal matrix with corresponding eigenvalues on its diagonal. This particular transformation cannot be applied to non-square matrix because the matrix of eigenvectors is not necessarily a square matrix, which implies that we are not able to find its inverse matrix. Therefore, we would not be able to do the eigenvalue decomposition for non-square matrices.

(c) This is a FALSE statement. While it is true that power method can be used to find

eigenvalues, it can also be used to find singular values as the essence of the process of calculating both values are the same.

For non-square matrices, we are not able to find its eigenvalue decomposition. Instead, we first take $A^* = A^T A$. Now we have a square matrix A^* to work with. Then we would proceed to find the eigenvalues of A^*. Last, the singular values are defined to be $\sigma_i = \sqrt{\lambda_i}$ where $\lambda_i' s$ are the non-zero eigenvalues of A^*.

Therefore, we can conclude that the process of finding singular values is basically analogue to the process of calculating eigenvalues with several more step to take in the beginning. Thus, we can certainly use the Power Method to calculate singular values.

(d) This is a FALSE statement. By the process of Singular Value Decomposition, we see that we can represent some non-square matrix A as

$$A = U \Sigma V^T$$

The columns of matrix U is regarded as the ***Left Singular Vector*** and the columns of matrix V is regarded as the ***Right Singular Vector***. We know that

$$V^T = ||eigenvector(A^T A)^T|| = \begin{bmatrix} v_1 \\ v_2 \\ \dots \\ v_i \end{bmatrix}$$

$$U = \begin{bmatrix} \frac{1}{\sigma_1} A v_1 & \frac{1}{\sigma_2} A v_2 & \cdots & \frac{1}{\sigma_i} A v_i & ||Nullspace(A^T)|| \end{bmatrix}, \text{ where } \sigma_i = \sqrt{\lambda_i}$$

Therefore, we can see that matrix V^T and U are orthonormal matrices and we can conclude that the left singular vectors are orthogonal to each other and the right singular vectors

are orthogonal to each other. There is, however, no guarantee in this process that the left singular vectors are orthogonal to the right singular vectors. Therefore, the generalized statement saying that singualr vectors are all orthogonal to each other is FALSE.

(e)