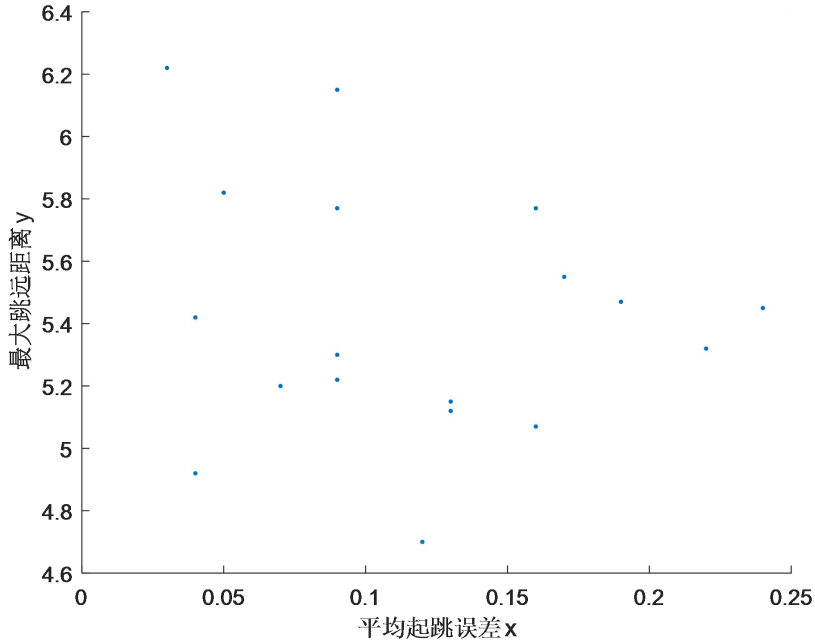
5.1

$Q(b)=\sum_{i=1}^{n}(y_i-bx_i)^2$, 令 $\frac{\partial Q}{\partial b}=-2\sum_{i=1}^{n}(y_i-bx_i)x_i=0$, 经整理得到: $\sum_{i=1}^{n}(x_iy_i-bx_i^2)=0$.

解方程可得到，b的最小二乘估计$\hat{b}=\frac{\sum_{i=1}^{n}x_iy_i}{\sum_{i=1}^{n}x_i^2}$.

5.3

(1)



(2) 本题中$n=18, \sum_{i=1}^{18}x_i=2.11, \sum_{i=1}^{18}y_i=97.62, \bar{x}=0.1172, \bar{y}=5.4233, \sum_{i=1}^{18}x_iy_i=11.3695,$

$$\sum_{i=1}^{18}x_i^2=0.3143, \hat{b}=\frac{\sum_{i=1}^{18}x_iy_i-n\bar{x}\bar{y}}{\sum_{i=1}^{18}x_i^2-n\bar{x}^2}=-1.0662, \hat{a}=\bar{y}-\hat{b}\bar{x}=5.5483.$$

经验线性回归方程： $\hat{y}=5.5483-1.0662x$.

(3) 提出假设$H_0: b=0$

$$(\hat{\sigma}^*)^2=\frac{\sum_{i=1}^{n}(y_i-\bar{y})^2-\hat{b}^2\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-2}=\frac{2.7646-1.0662*1.0662*0.067}{16}=0.1680$$

$l_{xx}=\sum_{i=1}^{n}(x_i-\bar{x})^2=0.0670, t=\frac{\hat{b}}{\hat{\sigma}^*}\sqrt{l_{xx}}=-0.6733$，而$t_{0.025}(18-2)=2.1199>|t|$，故接受原假设，认为线性回归关系不显 著。

(4) $r=\frac{l_{xy}}{\sqrt{l_{xx}}\sqrt{l_{yy}}}=\frac{-0.0737}{\sqrt{0.067}\sqrt{2.7646}}=-0.1712.$

5.4

(1)

本题中$n=10,\sum_{i=1}^{n}x_i=425,\sum_{i=1}^{n}y_i=186,\sum_{i=1}^{n}x_i^2=20125,\sum_{i=1}^{n}y_i^2=3564.06,\sum_{i=1}^{n}x_iy_i=8365,$

$\bar{x}=42.5,\bar{y}=18.6,$故$\hat{b}=\frac{\sum_{i=1}^{n}x_iy_i-n\bar{x}\bar{y}}{\sum_{i=1}^{n}x_i^2-n\bar{x}^2}=0.2230$，$\hat{a}=\bar{y}-\hat{b}\bar{x}=9.1212.$

(2) 提出假设$H_0:b=0$

$\hat{\sigma}^2=\frac{1}{n}\sum_{i=1}^{n}(y_i-\bar{y})^2-\frac{\hat{b}^2}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2=0.1894,\hat{\sigma}^{*2}=\frac{n}{n-2}\hat{\sigma}^2=0.2367$.

$t=\frac{\hat{b}}{\hat{\sigma}^*}\sqrt{l_{xx}}=\frac{0.223}{\sqrt{0.2367}}\sqrt{2062.5}=20.8163>t_{0.025}(10-2)=2.3060$.故拒绝假设$H_0$，. 认为线性回归显著。

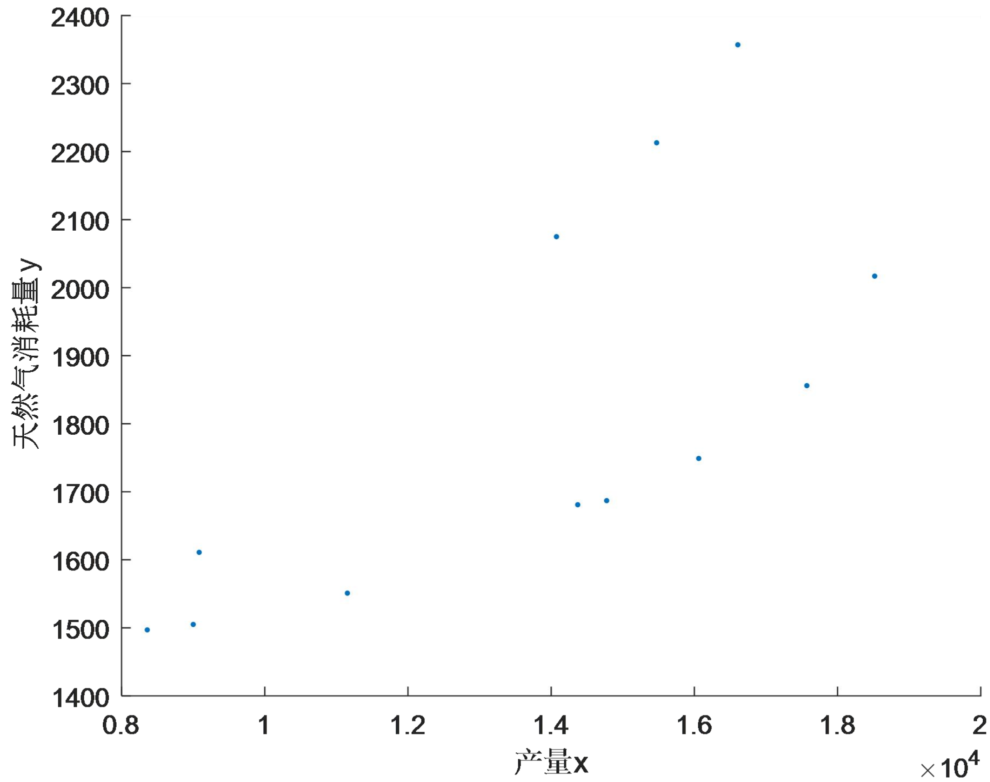
(3) $x=42^\circ\mathrm{C}$时，产量预测值$:y_0=18.6+0.223(42-42.5)=18.4885.$

$\delta(x_0)=t_{\alpha/2}(n-2)\hat{\sigma}^*\sqrt{1+\frac{1}{n}+\frac{(x_0-\bar{x})^2}{l_{xx}}}=1.1767$.

预测区间：(17.3118,19.6652).

5.6

(1)



(2)

这里$n=12,\sum_{i=1}^{n}x_i=165048,\sum_{i=1}^{n}y_i=21799,\sum_{i=1}^{n}x_i^2=2.4055\times10^9,\sum_{i=1}^{n}y_i^2=4050915,\sum_{i=1}^{n}x_iy_i=30771845,$

$$\bar{x}=13754,\bar{y}=1816,\hat{b}=\frac{\sum_{i=1}^{n}x_iy_i-n\bar{x}\bar{y}}{\sum_{i=1}^{n}x_i^2-n\bar{x}^2}=0.0583,\quad \hat{a}=\bar{y}-\hat{b}\bar{x}=1014.7.$$

经验回归方程$y=1014.7+0.0583x.$

(3)

提出假设H_0：$b=0$

$$\hat{\sigma}^2=\frac{1}{n}\sum_{i=1}^{n}(y_i-\bar{y})^2-\frac{\hat{b}^2}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2=37416,\quad \hat{\sigma}^{*2}=\frac{n}{n-2}\hat{\sigma}^2=44899,$$

$$l_{xx}=\sum_{i=1}^{n}(x_i-\bar{x})^2=135473468,t=\frac{\hat{b}}{\hat{\sigma}^*}\sqrt{l_{xx}}=\frac{0.0583}{\sqrt{44899}}\sqrt{135473468}=3.2024.$$

由于$t>t_{0.025}(12-2)=2.2281$,故拒绝假设H_0，认为线性回归显著。

(4) $y_0=\bar{y}+\hat{b}(x_0-\bar{x})=1816.6+0.0583(17000-13754)=2005.8$(百立方/周)。

5.8

(1) $X=\begin{pmatrix}1&57&64\\1&59&71\\1&49&53\\1&62&67\\1&51&55\\1&50&58\\1&52&56\\1&42&51\\1&61&76\\1&55&77\\1&48&57\\1&57&68\end{pmatrix}$, $Y=\begin{pmatrix}8\\10\\6\\11\\8\\7\\10\\6\\12\\10\\9\\9\end{pmatrix}$. $\hat{B}=(X^TX)^{-1}X^TY=\begin{pmatrix}3.651\\0.855\\1.506\end{pmatrix}$.

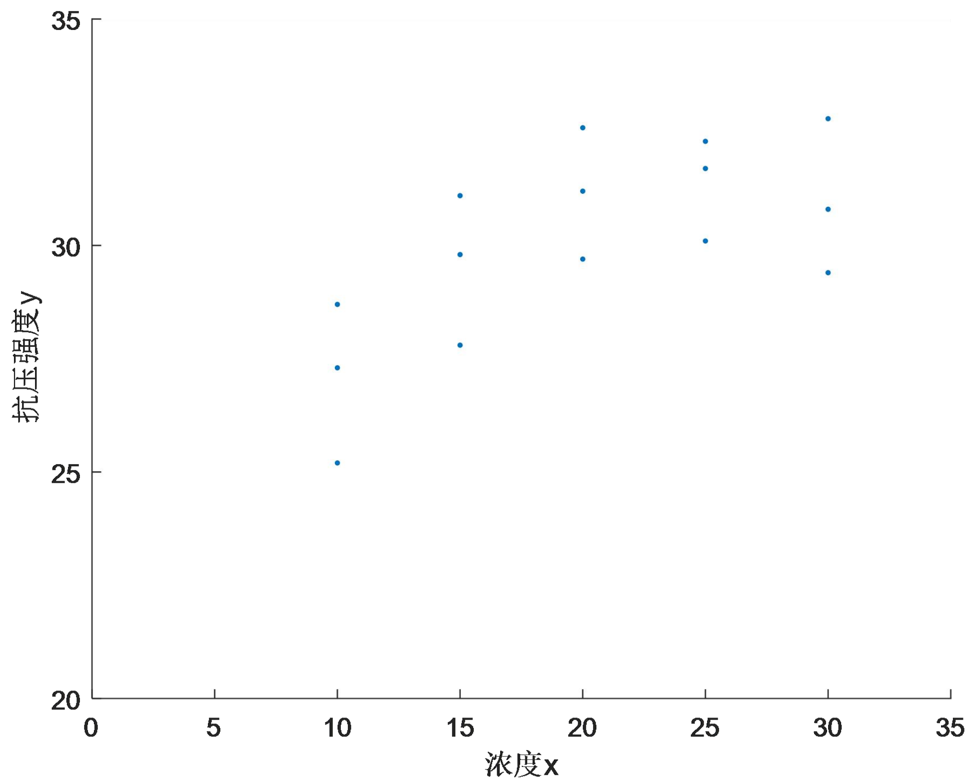
线性回归方程: $\hat{z}=\hat{b}_0+\hat{b}_1x+\hat{b}_2y=3.651+0.855x+1.506y$

(2) x=(137.16/2.54)cm=54cm, $\hat{z}=3.651+0.855\times54+1.506\times9=63.375$.

实际体重：63.375*0.45=28.52kg.

5.10

(1)



(2) $X=\begin{pmatrix}1 & 10 & 100\\ 1 & 15 & 225\\ 1 & 20 & 400\\ 1 & 25 & 625\\ 1 & 30 & 900\end{pmatrix}$， $Y=\begin{pmatrix}27.07\\ 29.57\\ 31.17\\ 31.37\\ 31\end{pmatrix}$. $\hat{B}=(X^TX)^{-1}X^TY=\begin{pmatrix}19.0333\\ 1.0086\\ -0.0204\end{pmatrix}$.

经验回归曲线方程：$\hat{y}=19.0333+1.0086x-0.0204x^2$.

(3)

方差来源	平方和	自由度	均方	F 值
回归	38.937143	2	19.468571	9.5449027
误差	24.47619	12	2.0396825	
总和	63.41333	14		

$F > F_{0.05}(2,12) = 3.89$,认为二项式回归显著。