Quadrotor Attitude Dynamics and Constants

State: $x = [\phi,\ \theta,\ \psi,\ {}_B\omega_x,\ {}_B\omega_y,\ {}_B\omega_z]^T$

Inputs: $u = [\tau_x \quad \tau_y \quad \tau_z]^T$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & s(\phi)\cdot t(\theta) & c(\phi)\cdot t(\theta) \\ 0 & c(\phi) & -s(\phi) \\ 0 & \dfrac{s(\phi)}{c(\theta)} & \dfrac{c(\phi)}{c(\theta)} \end{bmatrix} \cdot \begin{bmatrix} {}_B\omega_x \\ {}_B\omega_y \\ {}_B\omega_z \end{bmatrix}$$

$$\begin{bmatrix} {}_B\dot{\omega}_x \\ {}_B\dot{\omega}_y \\ {}_B\dot{\omega}_z \end{bmatrix} = -J^{-1} \cdot \left(\begin{bmatrix} {}_B\omega_x \\ {}_B\omega_y \\ {}_B\omega_z \end{bmatrix} \times \left(J \cdot \begin{bmatrix} {}_B\omega_x \\ {}_B\omega_y \\ {}_B\omega_z \end{bmatrix} \right) \right) + J^{-1} \cdot u$$

PS: here we use the ZYX Euler rotation

J is the inertia matrix, which is given by:

$$J = \begin{bmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & 0 \\ 0 & 0 & J_{zz} \end{bmatrix} = \begin{bmatrix} 0.672 & 0 & 0 \\ 0 & 0.804 & 0 \\ 0 & 0 & 1.428 \end{bmatrix} \cdot 10^{-2}\ kg \cdot m^2$$

Parameters range:

$\phi, \theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$

$\psi \in (-\pi, \pi)$

${}_B\omega_{x,y,z} \in [-1.25\pi, 1.25\pi]\ rad \cdot s^{-1}$

$\tau_{x,y} \in [-0.6, 0.6]\ N \cdot \mathrm{m}$

$\tau_z \in [-1, 1]\ N \cdot \mathrm{m}$