

**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with  $\mu = 45$  minutes and  $\sigma = 8$  minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
- A. 0.3875  
B. 0.2676  
C. 0.5  
D. 0.6987

**Ans: B**

**We have a normal distribution with  $\mu = 45$  and  $\sigma = 8.0$ . Let  $X$  be the amount of time it takes to complete the repair on a customer's car. To finish in one hour you must have  $X \leq 50$  so the question is to find  $\Pr(X > 50)$ .  $\Pr(X > 50) = 1 - \Pr(X \leq 50)$ .  $Z = (X - \mu) / \sigma = (X - 45) / 8.0$  Thus the question can be answered by using the normal table to find  $\Pr(X \leq 50) = \Pr(Z \leq (50 - 45) / 8.0) = \Pr(Z \leq 0.625) = 73.4\%$  Probability that the service manager will not meet his demand will be  $= 100 - 73.4 = 26.6\%$  or **0.2676****

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean  $\mu = 38$  and Standard deviation  $\sigma = 6$ . For each statement below, please specify True/False. If false, briefly explain why.
- A. More employees at the processing center are older than 44 than between 38 and 44.

**Ans - We have a normal distribution with  $\mu = 38$  and  $\sigma = 6$ . Let  $X$  be the number of employees. So according to question a) Probabilty of employees greater than age of 44=  $\Pr(X > 44)$   $\Pr(X > 44) = 1 - \Pr(X \leq 44)$ .  $Z = (X - \mu) / \sigma = (X - 38) / 6$  Thus the question can be answered by using the normal table to find  $\Pr(X \leq 44) = \Pr(Z \leq (44 - 38) / 6) = \Pr(Z \leq 1) = 84.1345\%$  Probabilty that the employee will be greater than age of 44 =  $100 - 84.1345 = 15.86\%$  So the probability of number of employees between 38-44 years of age =  $\Pr(X < 44) - 0.5 = 84.1345\%$**

**Therefore the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE.**

- B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans. Probabilty of employees less than age of 30 =  $\Pr(X < 30)$   
 $Z = (X - \mu) / \sigma = (30 - 38) / 6$**

Thus the question can be answered by using the normal table to find

$$\Pr(X \leq 30) = \Pr(Z \leq (30 - 38)/6) = \Pr(Z \leq -1.333) = 9.12\%$$

So the number of employees with probability 0.912 of them being under age 30 =  $0.0912 \times 400 = 36.48$  ( or 36 employees)

Therefore the statement B of the question is also TRUE

3. If  $X_1 \sim N(\mu, \sigma^2)$  and  $X_2 \sim N(\mu, \sigma^2)$  are *iid* normal random variables, then what is the difference between  $2X_1$  and  $X_1 + X_2$ ? Discuss both their distributions and parameters.

**Ans.** As we know that if  $X \sim N(\mu_1, \sigma_1^2)$ , and  $Y \sim N(\mu_2, \sigma_2^2)$  are two independent random variables then  $X + Y \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$ , and  $X - Y \sim N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$ .

Similarly if  $Z = aX + bY$ , where  $X$  and  $Y$  are as defined above, i.e  $Z$  is linear combination of  $X$  and  $Y$ , then  $Z \sim N(a\mu_1 + b\mu_2, a^2\sigma_1^2 + b^2\sigma_2^2)$ .

Therefore in the question

$$2X_1 \sim N(2\mu, 4\sigma^2) \text{ and } X_1 + X_2 \sim N(\mu + \mu, \sigma^2 + \sigma^2) \sim N(2\mu, 2\sigma^2)$$

$$2X_1 - (X_1 + X_2) \sim N(4\mu, 6\sigma^2)$$

4. Let  $X \sim N(100, 20^2)$ . Find two values,  $a$  and  $b$ , symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

- A. 90.5, 105.9
- B. 80.2, 119.8
- C. 22, 78
- D. 48.5, 151.5
- E. 90.1, 109.9

**Ans** Since we need to find out the values of  $a$  and  $b$ , which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting value between  $a$  and  $b$  should be 0.99

So the Probability of going wrong, or the Probability outside the  $a$  and  $b$  area is 0.01 (ie.  $1 - 0.99$ ). The Probability towards left from  $a = -0.005$  (ie.  $0.01/2$ ).

The Probability towards right from  $b = +0.005$  (ie.  $0.01/2$ ). So since we have the probabilities of  $a$  and  $b$ , we need to calculate  $X$ , the random variable at  $a$  and  $b$  which has got these probabilities. By finding the Standard Normal Variable  $Z$  ( $Z$  Value), we can calculate the  $X$  values.  $Z = (X - \mu) / \sigma$

For Probability 0.005 the  $Z$  Value is -2.57 (from  $Z$  Table).

$$Z * \sigma + \mu = X$$

$$Z(-0.005) * 20 + 100 = -(-2.57) * 20 + 100 = 151.4$$

$$Z(+0.005) * 20 + 100 = (-2.57) * 20 + 100 = 48.6$$

So it is D

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions  $\text{Profit}_1 \sim N(5, 3^2)$  and  $\text{Profit}_2 \sim N(7, 4^2)$  respectively. Both the profits are in \$ Million. Answer the following questions about the total profit of the company in Rupees. Assume that \$1 = Rs. 45
- A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
  - B. Specify the 5<sup>th</sup> percentile of profit (in Rupees) for the company
  - C. Which of the two divisions has a larger probability of making a loss in a given year?