Medical Image Analysis, handin 4

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1 Introduction

Image registration is the method of superimposing the features points or pixels intensities from an image, namely the template, over another image of the same scene, namely the reference. It can be approached as an optimization problem in which an optimal geometric transformation is applied to the template image, such that the differences between the transformed image and the reference are minimized. In order to do so, different similarity or dissimilarity measures can be used (Section 3), such as sum of squared errors (SSE), cross correlation (CC) and mutual information (MI). In this regard, there are two main types of image registrations: feature based, in which the difference of features points between the reference and the template is minimized, and area based, in which the pixels intensity similarity is used for the alignment. Furthermore, image registration algorithms can use different geometrical transformations such as rigid, affine, and nonrigid transformations. Rigid and affine transformations (Section 2) have only few degrees of freedom and are mainly used for the registration of bone structures, in fact they often don't obtain satisfactory results in the registration of elastic tissues. For the latter, nonrigid registration algorithms are usually used. In fact, they provide a transformation field with high dimensionality, which enable a satisfactory registration of structures that can undergo free-form deformations. However, due to high dimensional space, the solutions obtained by these algorithms can be unstable and often leading to local minima. In order to alleviate these problems it is possible to apply regularization methods, which can incorporate prior physiological knowledge into the problem. Typically, in a regularized nonrigid medical registration algorithm, the loss function to be minimized consist of a similarity measure and an additional penalty term, which is applied to discourage unreasonable transformations.

The following sections contain a brief description of five registration problems. The first two tasks involved the alignment of two set of points using sum of squared errors (SSE) loss function and rigid and affine transformations. The last three problems involved the intensity based registration of a brain image using the same transformations but different similarity measures.

2 Set of points registration

In the first two problems we were asked to perform the registration of a set of points using both rigid and affine transformations. Homogeneous coordinates were used in order to apply a single matrix R to compute the geometrical transformations. The matrix performed both rotation and translation in the case of

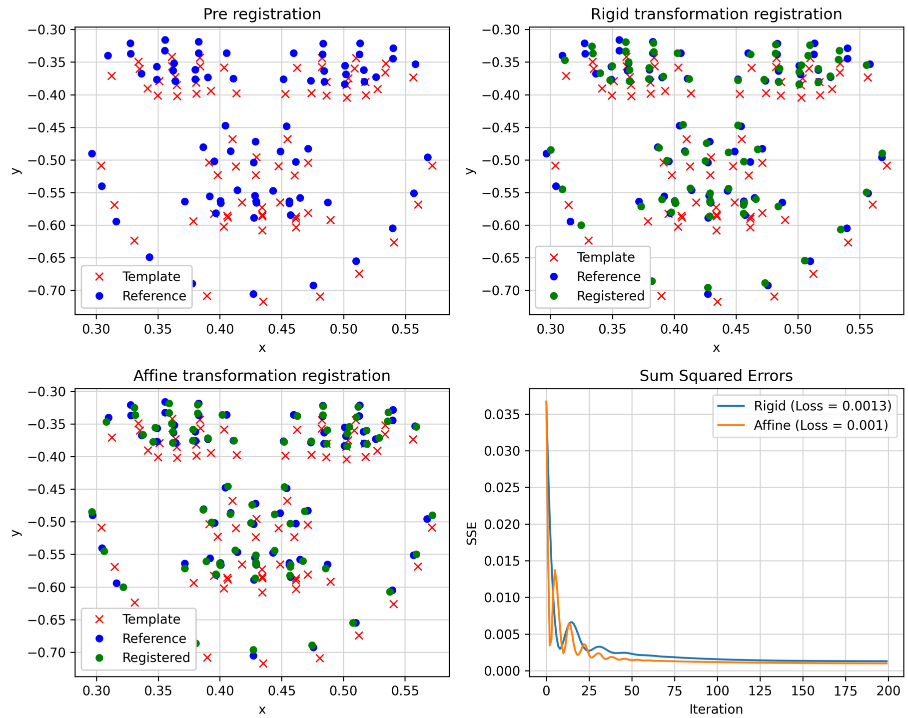

Fig. 1. Face points registration. The first three plots of the image (top and bottom left) show the template and the reference set of points as well as the result of the registration using rigid and affine transformations. The last plot (bottom right) shows the progress of the loss (SSE) during the parameters optimization of both affine and rigid registrations.

the rigid transformation, and a combination of rotation, translation, dilations, and shears for the affine one. Both tasks were solved as an optimization problem, where we had to find the optimal parameters of the transformation matrix R, such that a sum of squared errors (SSE) loss function was minimized. The main difference between the two registration problems was in the number of parameters that were optimized. Since in the rigid transformation the distance between any two object points is preserved, it has only three degrees of freedom and the parameters to be optimized are θ for rotation, and x and y for translation. Therefore, using homogeneous coordinates the transformation matrix R can be defined as

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & x \\ \sin\theta & \cos\theta & y \\ 0 & 0 & 1 \end{bmatrix}. \tag{1}$$

While, in the affine transformation only collinearity and ratio of distances are preserved. It has six degrees of freedom that allows to perform a combination of rotations, translations, dilations, and shears. With homogeneous coordinates, the transformation matrix R for the affine transformation can be defined as

$$R = \begin{bmatrix} p1 & p2 & p3 \\ p4 & p5 & p6 \\ 0 & 0 & 1 \end{bmatrix}. \tag{2}$$

The implementation of both registration tasks were performed using PyTorch machine learning library. The transformation matrix was initialized as the identity matrix, and the gradient of the loss function was computed in order to update the trainable parameters using a certain learning rate. This was possible using PyTorch autograd, which records a graph of all operations performed on a trainable tensor, and then computes the gradient by applying the chain rule from the root to the leaf of the graph (backpropagation). The Adam optimizer and 200 iterations were used in both registration problems, and the learning rate was respectively 0.002 and 0.005 for the rigid and affine registrations. The result of both tasks, as well as the progress of the SSE loss during the optimization process, is shown in Fig 1. Even if the difference is subtle, it is possible to observe (bottom right plot) that, as expected, the affine registration aligned the template set of points to the reference with slightly more accuracy than the rigid one.

3 Brain image registration

In the last three tasks we were asked to performed the registration of a brain image using both, rigid and affine transformations, as well as different metrics as loss functions: sum of squared errors (SSE), cross correlation (CC), mutual information (MI), and normalized mutual information (NMI).

As in the previous exercises, these registration tasks were approached as optimization problems. This time we used the pixel intensities of the images instead of the set of points and this offered additional challenges. In particular, we had to apply the geometric transformation to the coordinates of the points and then obtain the intensity of the pixels in the transformed coordinates. In this regard, a non-trivial problem was that, once applied the transformation, there was high chance that the new pixels were off-grid in respect to the original coordinates. Therefore, to obtain the off-grid values we needed to perform a bilinear interpolation (linear interpolation in x and y directions) between pixels values of the original image. The implementation was obtained using TensorFlow machine learning library. In particular, a spatial transformer network was used to perform the transformation and the interpolation in a differentiable manner. This allowed the spatial manipulation of the image within the network in such a way that we could compute the gradient of the loss with respect to the parameters. As in the set of points registrations, three trainable parameters were initialized for the rigid transformation and six for the affine one, also, the transformation matrix R was initialized as an identity matrix. Lastly, for all optimization processes we used the Adam optimizer with 0.005 learning rate and 100 iterations.

As mentioned, different loss functions were minimized or maximized to obtain the optimal geometric transformation. We succeeded to perform both rigid and affine registrations using sum squared errors (SSE) and cross correlation (CC) but we were not able to implement registrations with mutual information (MU) and normalized mutual information (NMI). CC and MI are two measures of similarity used in area based registration methods, where larger CC and MI values result for corresponding points. CC was implemented as zero-normalized cross-correlation (ZNCC), in which at each step the image is normalized by subtracting the mean and by diving for the standard deviation of the pixels intensities:

$$ZNCC = \frac{1}{n} \sum_{x,y} \frac{1}{\sigma_f \sigma_t} \left(f(x,y) - \mu_f\right) \left(t(x,y) - \mu_t\right). \tag{3}$$

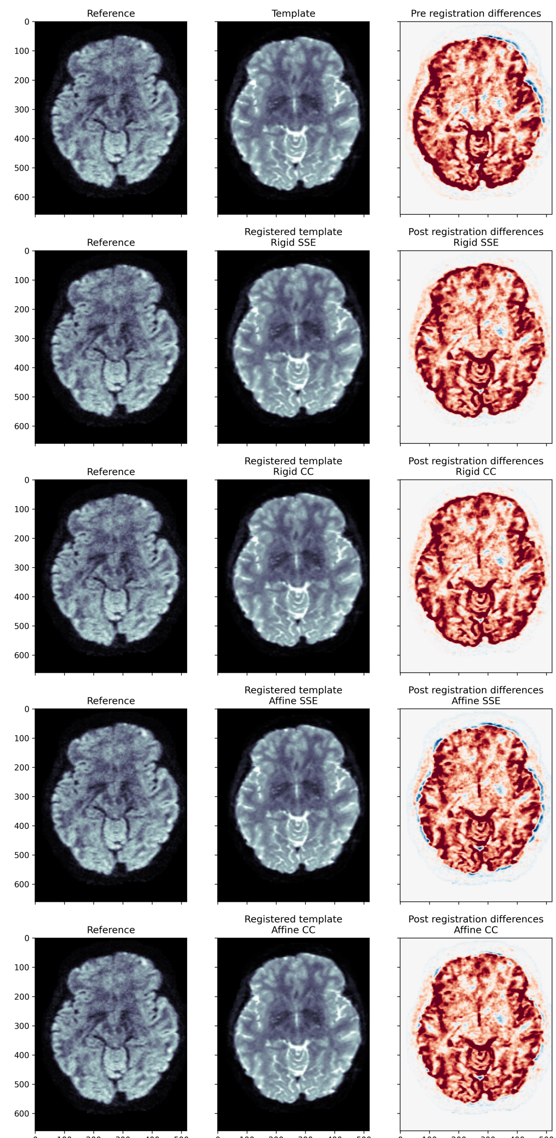

Fig. 2. Brain image registration. The image shows the reference and template images as well as the result of the different images registration tasks and their differences with respect to the reference image. The differences are obtained by subtracting the registered template image from the reference image. The red color indicate pixels whose intensity was larger in the registered image, while blue represents pixels whose intensity was larger in the reference image. White color indicates pixels where there is no difference between the two images.

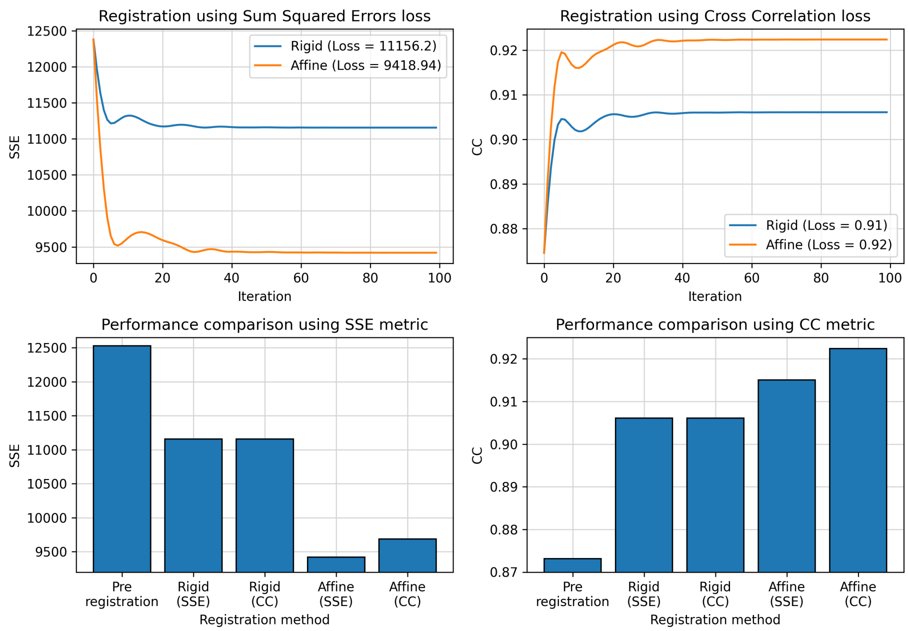

Fig. 3. Brain image loss progression and performances evaluation. The first row of plots of the image shows the loss progression of both, rigid and affine registrations, during parameters optimization performed using sum of squared errors (SSE, top left) and cross correlation (CC, top right) loss functions. The second row of plots shows the performance comparison of the different registration tasks using SSE (bottom left) and CC (bottom right) as metrics.

MI is a matching metric based on information-theoretic concepts, and it can be considered as the amount of information that can be extracted from one distribution regarding a second one. This similarity measure requires the computation of the joint probability distribution of the two images, which is estimated as a normalized joint histogram of the intensity values. We succeed in the implementation of MI but we were not able to implement it in such a way that we could correctly propagate the gradient through the computational graph, and therefore update the parameters of the geometric transformation. For the same reason we did not succeed in the implementation of the image registration using NMI, which is a normalized variant of MI.

The results of the performed images registration tasks is shown in Fig 2, while the progress of the loss function during parameter optimization, as well as the performances comparison of the different methods, is shown in Fig 3. We can observe (Fig 3) that, as expected, independently from the loss function used, the affine registrations obtained better results than the rigid ones. Also, it is interesting to note (Fig 3, bottom) that there is no difference between the results of the rigid registrations using different metrics (SSE and CC). In fact, since rigid transformations allow only for rotation and translation, the two registration methods (rigid registrations using SSE and CC) obtained almost identical parameters and ended up in the same minimum value of the loss function.