Exponential Distribution vs. Central Limit Theorem

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Overview

In this project, we will investigate the exponential distribution in R and compare it with the Central Limit Theorem.

In order to illustrate the properties of the distribution, we will:

- Show the sample mean and compare it to the theoretical mean of the distribution.
- Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution.
- Show that the distribution is approximately normal.

Simulations

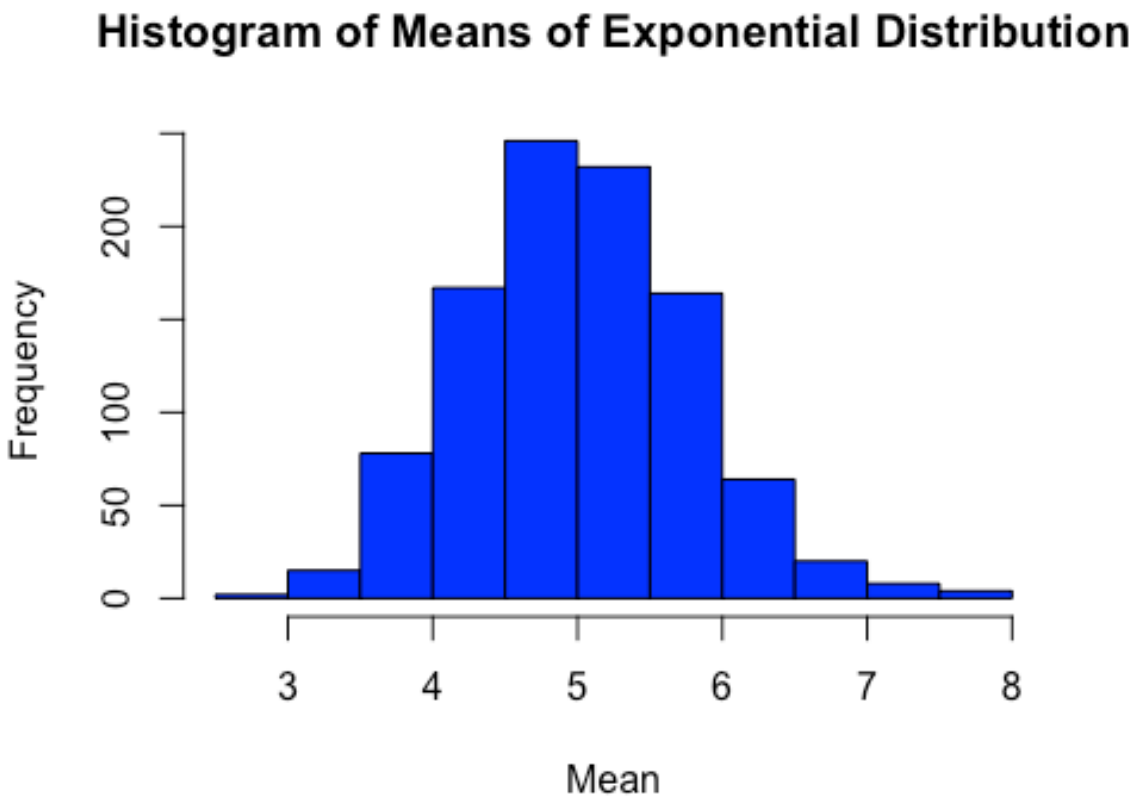
The exponential distribution can be simulated in R with rexp(n, lambda), where lambda is the rate parameter. For all simulations, lambda = 0.2. We will investigate the distribution of averages of 40 exponentials, and perform 1,000 simulations.

```
## Load ggplot2 package (for generating plots)
library(ggplot2)

## Initialize variables to control simulation
sim.count <- 1000
exp.count <- 40
lambda <- 0.2
set.seed(123)

## Perform simulation
sim <- matrix(rexp(sim.count * exp.count, rate = lambda), sim.count,
exp.count)
```

Sample Mean vs. Theoretical Mean

1. Plot the sample data.

```
sim.mean <- rowMeans(sim)
hist(sim.mean, col = "blue",
     main = "Histogram of Means of Exponential Distribution",
     xlab = "Mean")
```



The figure above shows the frequency of means in our simulated exponential distribution.

2. Calculate the sample mean.

```
mean(sim.mean)

## [1] 5.011911
```

3. Calculate theoretical mean.

```
1/lambda

## [1] 5
```

CONCLUSION: The sample mean of 5.011911 is *very close* to the theoretical mean of 5.

Sample Variance vs. Theoretical Variance

1. Calculate the sample variance

```
var(sim.mean)
```

```
## [1] 0.6088292
```

2. Calculate theoretical variance

```
(1/lambda)^2 / exp.count
```

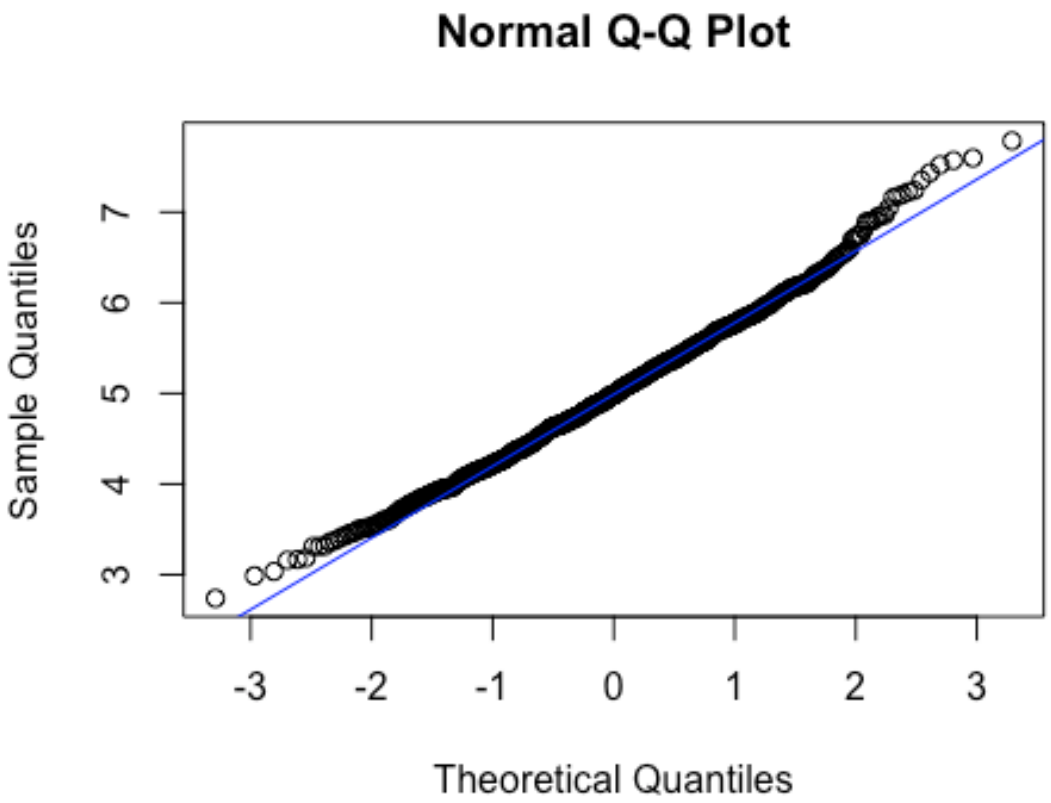
```
## [1] 0.625
```

CONCLUSION: The sample variance of 0.6088292 is again *very close* to the theoretical variance of 0.625.

Distribution

Generate a q-q plot comparing the sample quantiles to theoretically normal quantiles.

```
qqnorm(sim.mean, main = "Normal Q-Q Plot")
qqline(sim.mean, col="4")
```



CONCLUSION: The sample quantiles *closely resemble* the theoretical quantiles, suggesting that the distribution is **approximately normal**.
