Forecasting

ARIMA(3,1,1)

$$\phi_p(L)(1-L)^d Y_t = \delta + \theta_q(L) w_t$$

$$(1-\phi_1 L - \phi_2 L^2 - \phi_3 L^3)(1-L) Y_t = \delta + (1+\theta L) w_t$$

$$(1-\phi_1 L - \phi_2 L^2 - \phi_3 L^3)(Y_t - Y_{t-1}) = \delta + w_t + \theta w_{t-1}$$

$$Y_t - \phi_1 Y_{t-1} - \phi_2 Y_{t-2} - \phi_3 Y_{t-3}$$
$$- Y_{t-1} + \phi_1 Y_{t-2} + \phi_2 Y_{t-3} + \phi_3 Y_{t-4} = \delta + w_t + \theta w_{t-1}$$

$$Y_t = (1+\phi_1) Y_{t-1} - (\phi_1 - \phi_2) Y_{t-2} - (\phi_2 - \phi_3) Y_{t-3}$$
$$- \phi_3 Y_{t-4} + \delta + w_t + \theta w_{t-1}$$

$$Y_{t+1} = (1+\phi_1) Y_t - (\phi_1 - \phi_2) Y_{t-1} - (\phi_2 - \phi_3) Y_{t-2}$$
$$- \phi_3 Y_{t-3} + \delta + \overset{0}{\cancel{w_{t+1}}} + \theta \overset{Y_t - \hat{Y}_t}{\cancel{w_t}}$$

$$Y_{t+2} = (1+\phi_1) \overset{\hat{Y}_{t+1}}{\cancel{Y_{t+1}}} - (\phi_1 - \phi_2) Y_t - (\phi_2 - \phi_3) Y_{t-1}$$
$$- \phi_3 Y_{t-2} + \delta + \overset{0}{\cancel{w_{t+2}}} \; \theta \overset{0}{\cancel{w_{t+1}}}$$