Sta 444 Sp 2017 2-13

AR(1)

$$y_t = \delta + \phi_1 y_{t-1} + w_t$$

$$y_t = \delta + \phi_1 L y_t + w_t$$

$$y_t - \phi_1 L y_t = \delta + w_t$$

$$(1 - \phi_1 L) y_t = \delta + w_t$$

$$1 - \phi_1 L = 0$$

$$L = 1/\phi$$

$$|1/\phi| > 1$$

$$1 > |\phi|$$

$$-1 < \phi < 1$$

$\boxed{AR(2)}$

$$Y_t = \delta + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + w_t$$

$$Y_t = \delta \quad \phi_1 L Y_t + \phi_2 L^2 Y_t + w_t$$

$$(1 - \phi_1 L - \phi_2 L^2) Y_t = \delta + w_t$$

$$1 - \phi_1 L - \phi_2 L^2 = 0 \qquad \boxed{\lambda = \frac{1}{L}}$$

$$\lambda^2 - \phi_1 \lambda - \phi_2 = 0$$

$$\lambda = \frac{\phi_1 \pm \sqrt{\phi_1^2 + 4\phi_2}}{2}$$

Assuming $\mathbb{R}$ roots

$$\frac{\phi_1 + \sqrt{\phi_1^2 + 4\phi_2}}{2} < 1 \qquad \frac{\phi_1 - \sqrt{\phi_1^2 + 4\phi_2}}{2} > -1$$

$$\phi_1^2 + 4\phi_2 < (2 - \phi_1)^2 \qquad \phi_1^2 + 4\theta_2 < (\phi_1 + 2)^2$$

$$\phi_1^2 + 4\phi_2 < 4 - 4\phi_1 + \phi_1^2 \qquad \phi_1^2 + 4\theta_2 < \phi_1^2 + 4\phi_1 + 4$$

$$\phi_1 + \phi_2 < 1 \qquad \phi_2 - \phi_1 < 1$$

Complex roots

$$\lambda = \frac{\phi_1}{2} \pm \left(\frac{\sqrt{\phi_1^2 + 4\phi_2}}{2}\right) i$$

$$\lambda^2 = \frac{\phi_1^2}{2^2} + \frac{\phi_1^2 + 4\phi_2}{2}(-1)$$

$$= -\phi_2 < 1$$

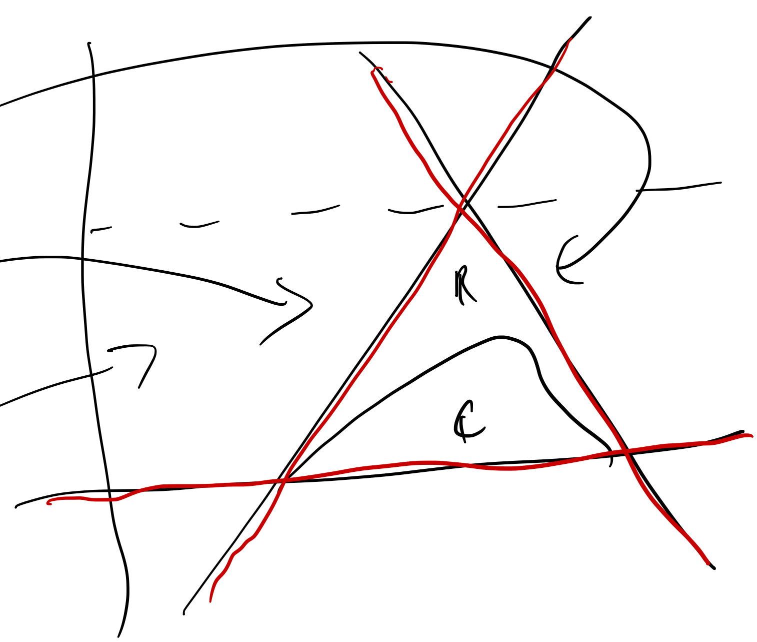
$$\Rightarrow \phi_2 > -1$$

$$\Rightarrow |\phi_2| < 1$$

$$\phi_1 + \phi_2 < 1$$

$$\phi_2 - \phi_1 < 1$$

$$|\phi_2| < 1$$



MA(1)

$$y_t = \delta + w_t + \theta w_{t-1}$$

$$E(y_t) = E(\delta + w_t + \theta w_{t-1})$$

$$= \delta$$

$$Var(y_t) = Var(\delta + w_t + \theta w_{t-1})$$

$$= \sigma_w^2 + \theta^2 \sigma_w^2 = (1+\theta^2)\sigma_w^2$$

$$Cov(y_t, y_{t+h}) = Cov(\delta + w_t + \phi w_{t-1}, \delta + w_{t+h} + \phi w_{t+h-1})$$

$$= Cov(w_t, w_{t+h}) + Cov(w_t, \phi w_{t+h-1})$$

$$+ Cov(\phi w_{t-1}, w_{t+h}) + Cov(\phi w_{t-1}, \phi w_{t+h-1})$$

$$= \begin{cases} (1+\theta^2)\sigma_w^2 & \text{if } h = 0 \\ \theta \sigma_w^2 & \text{if } h = \pm 1 \\ 0 & \text{if } |h| > 1 \end{cases}$$

$\text{Corr}(y_t, y_{t+h})$

$$= \begin{cases} 1 & \text{if } h = 0 \\ \dfrac{\theta}{1+\theta^2} & \text{if } h = \pm 1 \\ 0 & \text{if } |h| > 1 \end{cases}$$

MA(q) $\quad y_t = \delta + w_t + \theta_1 w_{t-1} + \theta_2 w_{t-2} + \dots + \theta_q w_{t-q}$

$E(y_t) = \delta$

$$\text{Var}(y_t) = \sigma_w^2 + \theta_1^2 \sigma_w^2 + \theta_2^2 \sigma_w^2 + \dots + \theta_q^2 \sigma_w^2$$
$$= (1 + \theta_1^2 + \theta_2^2 + \dots + \theta_q^2)\, \sigma_w^2$$

$\text{Cov}(y_t, y_{t+h})$

$$= \begin{cases} \theta_h + \theta_1 \theta_{h+1} + \dots + \theta_{q-h} \theta_q & \text{if } |h| < q \\ 0 & \text{if } |h| > q \end{cases}$$