Lec 5

Model

$$Y_i = \alpha_{j(i)} + \beta\, days + \varepsilon_i$$

$$\alpha_{j(i)} \sim N(\beta_\alpha, \sigma_\alpha^2)$$

$$\varepsilon_i \sim N(0, \sigma^2)$$

$$\underline{Y} \sim MVN(\underline{\mu}, \underline{\Sigma}) \qquad \theta = \{\beta, \beta_\alpha, \sigma^2, \sigma_\alpha^2\}$$

$$\underline{\mu} = ?$$

$$\underline{\Sigma} = ?$$

$$\underline{\mu} = E(\underline{Y}|\theta) \quad \{E(Y_i|\theta)\} = \beta_\alpha + \beta\, \underline{days}$$

$$E(Y_i|\theta) = E(\alpha_{j(i)} + \beta\, days + \varepsilon_i \mid \theta)$$

$$= E(\alpha_{j(i)}|\theta) + \beta\, days = \beta_\alpha + \beta\, days$$

$$\underline{\Sigma} = \{ \Sigma_{mn} \}$$

$$\Sigma_{mn} = \mathrm{Cov}(Y_m | \theta, Y_n | \theta)$$

$$= \mathrm{Cov}(\alpha_{j(m)} + \beta d + \epsilon_m, \alpha_{j(n)} + \beta d + \epsilon_n)$$

$$= \mathrm{Cov}(\alpha_{j(m)} + \epsilon_m, \alpha_{j(n)} + \epsilon_n)$$

$$= \mathrm{Cov}(\alpha_{j(m)}, \alpha_{j(n)}) + \mathrm{Cov}(\cancelto{0}{\alpha_{j(m)}}, \epsilon_n)$$

$$+ \mathrm{Cov}(\epsilon_m, \cancelto{0}{\alpha_{j(n)}}) + \mathrm{Cov}(\epsilon_m, \epsilon_n)$$

$$\mathrm{Cov}(\alpha_{j(m)}, \alpha_{j(n)}) = \begin{cases} \sigma_\alpha^2 & \text{if } j(m) = j(n) \\ 0 & \text{if } j(m) \neq j(n) \end{cases}$$

$$\mathrm{Cov}(\epsilon_m, \epsilon_n) = \begin{cases} \sigma^2 & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}$$

$$\Sigma_{mn} = \begin{cases} \sigma^2 + \sigma_\alpha^2 & \text{if } m = n \\ \sigma_\alpha^2 & \text{if } j(m) = j(n) \text{ \& } m \neq n \\ 0 & \text{otherwise} \end{cases}$$

$$
\Sigma = \begin{array}{c} \\ m \end{array}
\begin{bmatrix}
\begin{array}{c|cccc}
n & 1 & 2 & \cdots & 10 \\
\hline
1 & \sigma^2 + \sigma_\alpha^2 & \sigma_\alpha^2 & \cdots & \sigma_\alpha^2 \\
2 & \sigma_\alpha^2 & \sigma^2 + \sigma_\alpha^2 & \cdots & \sigma_\alpha^2 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
10 & \sigma_\alpha^2 & \sigma_\alpha^2 & \cdots & \sigma^2 + \sigma_\alpha^2
\end{array} & 0 & \cdots \\
0 & \begin{array}{cccc}
\sigma^2 + \sigma_\alpha^2 & \sigma_\alpha^2 & \cdots & \sigma_\alpha^2 \\
\sigma_\alpha^2 & \sigma^2 + \sigma_\alpha^2 & \cdots & \sigma_\alpha^2 \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_\alpha^2 & \sigma_\alpha^2 & \cdots & \sigma^2 + \sigma_\alpha^2
\end{array} & \cdots \\
\vdots & \vdots & \ddots
\end{bmatrix}
$$

$$
= \begin{bmatrix}
\boxed{\;} & 0 & 0 & 0 & 0 & \\
0 & \boxed{\;} & 0 & 0 & 0 & \\
0 & 0 & \boxed{\;} & 0 & 0 & \cdots \\
0 & 0 & 0 & \boxed{\;} & 0 & \\
0 & 0 & 0 & 0 & \boxed{\;} & \\
 & & \vdots & & & \ddots
\end{bmatrix}
$$