

Lec 3

$$\begin{aligned} D &= 2 \left(\ell(\theta_{best} | \mathcal{Y}) - \ell(\hat{\theta} | \mathcal{Y}) \right) \\ &= 2 \left(\sum_{i=1}^n \ell(\theta_{best} | y_i) - \sum_{i=1}^n \ell(\hat{\theta} | y_i) \right) \\ &= 2 \sum_{i=1}^n \left(\ell(\theta_{best} | y_i) - \ell(\hat{\theta} | y_i) \right) \end{aligned}$$

Poisson

$$\mathcal{L}(\lambda | y_i) = \frac{\lambda^{y_i} e^{-\lambda}}{y_i!}$$

$$\ell(\lambda | y_i) = y_i \log \lambda - \lambda - \log y_i!$$

$$E(y_i) = \lambda$$

$$Var(y_i) = \lambda$$

Best

$$E(Y_i) = \mu_i = \lambda_{best}$$

model

$$E(Y_i) = \hat{\lambda}$$

$$\ell(\lambda_{best} | Y_i) - \ell(\hat{\lambda} | Y_i)$$

$$= (Y_i \log Y_i - Y_i - \log Y_i!) - (Y_i \log \hat{\lambda} - \hat{\lambda} - \log Y_i!)$$

$$= Y_i \log \frac{Y_i}{\hat{\lambda}} - (Y_i - \hat{\lambda})$$

$$D = 2 \sum_{i=1}^n \left(Y_i \log \frac{Y_i}{\hat{\lambda}} - (Y_i - \hat{\lambda}) \right)$$

Normal

$$\ell(\mu, \sigma^2 | y_i) = -\frac{1}{2} \log 2\pi \sigma^2 - \frac{1}{2} \frac{(y_i - \mu)^2}{\sigma^2}$$

Best

$$E(y_i) = y_i = \mu$$

model

$$E(y_i) = \hat{\mu}$$

$$\ell(\theta_{best} | y_i) - \ell(\theta | y_i)$$

$$= -\frac{1}{2} \frac{(y_i - y_i)^2}{\sigma^2} + \frac{1}{2} \frac{(y_i - \mu)^2}{\sigma^2} = -\frac{1}{2} \frac{(y_i - \mu)^2}{\sigma^2}$$

$$D = \frac{(y_i - \mu)^2}{\sigma^2}$$