Lec 10

$$y_t = \delta + \phi y_{t-1} + \theta v_{t-1} + v_t$$

$$E(v_{t+1} | y_t \dots y_1 \dots) = 0$$

$$E(v_t | y_t \dots y_1 \dots) = v_t = y_t - (\delta + \phi y_{t-1} + \theta v_{t-1})$$

$$\hat{y}_{t+1} = E(y_{t+1} | y_t \dots y_1 \dots)$$

$$= \delta + \phi y_t + \theta w_t$$

$$\hat{y}_{t+2} = E(y_{t+2} | y_t \dots y_1 \dots)$$

$$= \delta + \phi E(y_{t+1} | y_t \dots y_1 \dots) + \theta E(v_{t+1} | y_t \dots y_1 \dots) +$$

$$E(v_{t+2} | y_t \dots y_1 \dots)$$

$$= \delta + \phi \hat{y}_{t+1} = \delta + \phi\delta + \phi^2 y_t + \phi\theta v_t$$

$$= \left(\sum_{i=0}^{2-1} \phi^i \delta \right) + \phi^2 y_t + \phi^{2-1} \theta v_t$$

$$\hat{y}_{t+h} = \sum_{i=0}^{h-1} \phi^i \delta + \phi^h y_t + \phi^{h-1} \theta v_t = \frac{\delta}{1-\phi}$$

ARIMA(3,1,1)

$$\phi_p(L)\,(1-L)^d\,y_t = \delta_t\,\theta_q(L)\,v_t$$

$$(1-\phi_1 L-\phi_2 L^2-\phi_3 L^3)\,(1-L)\,y_t = \delta_t\,(1+\theta L)\,v_t$$

$$(1-\phi_1 L-\phi_2 L^2-\phi_3 L^3)\,(y_t - y_{t-1}) = \delta_t\, v_t + \theta v_{t-1}$$

$$y_t - \phi_1 y_{t-1} - \phi_2 y_{t-2} - \phi_3 y_{t-3}$$

$$- y_{t-1} + \phi_1 y_{t-2} + \phi_2 y_{t-3} + \phi_3 y_{t-4} = \delta + v_t - \theta v_{t-1}$$

$$y_t = (1+\phi_1)\,y_{t-1} - (\phi_1-\phi_2)\,y_{t-2} - (\phi_2-\phi_3)\,y_{t-3} - \phi_3 y_{t-4} + \delta + v_t + \theta v_{t-1}$$

$$\hat{y}_{t+1} = (1+\phi_1)\,y_t - (\phi_1-\phi_2)\,y_{t-1} - (\phi_2-\phi_3)\,y_{t-2} - \phi_3 y_{t-3} + \delta + E(y_t)^{\wedge} + \theta v_t$$

$$\hat{y}_{t+2} = (1+\phi_1)\,\hat{y}_{t+1} - (\phi_1-\phi_2)\,y_t - (\phi_2-\phi_3)\,y_{t-1} - \phi_3 y_{t-2} + \delta + E(y_{t-2})^{\wedge} + E(\theta v_{t-1})^{\wedge}$$