Lec 20 - More PyMC3

Statistical Computing and Computation

Sta 663 | Spring 2022

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Demo 1 - Bayesian Lasso

```
n = 50
k = 100

np.random.seed(1234)
X = np.random.normal(size=(n, k))

beta = np.zeros(shape=k)
beta[[10,30,50,70]] =  10
beta[[20,40,60,80]] = -10

y = X @ beta + np.random.normal(size=n)
```

Based on Bayesian Sparse Regression and Lasso regression with block updating

Naive Model

```
with pm.Model() as bayes_lasso:
  b = pm.Laplace("beta", 0, 1, shape=k)#lam*tau, shape=k)
  y_est = X @ b
  s = pm.HalfNormal('sigma', sd=1)

  likelihood = pm.Normal("y", mu=y_est, sigma=s, observed=y)

  trace = pm.sample(return_inferencedata=True, random_seed=1234)
```

```
## █
## Auto-assigning NUTS sampler...
## Initializing NUTS using jitter+adapt_diag...
## Multiprocess sampling (4 chains in 4 jobs)
## NUTS: [sigma, beta]
## Sampling 4 chains for 1_000 tune and 1_000 draw iterations (4_000 + 4_000 draws total) took 19 seconds.
## There were 2 divergences after tuning. Increase `target_accept` or reparameterize.
## The acceptance probability does not match the target. It is 0.878942077718847, but should be close to 0.8. T
   increase the number of tuning steps.
## The estimated number of effective samples is smaller than 200 for some parameters.
```

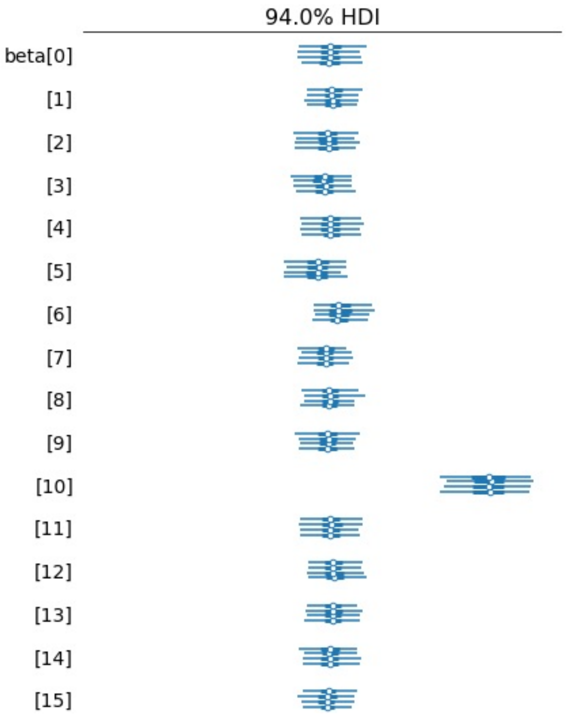
```
az.summary(trace)
```

```
##             mean     sd  hdi_3%  hdi_97%  mcse_mean  mcse_sd  ess_bulk  ess_tail  r_hat
## beta[0]    0.067  0.861  -1.650    1.681      0.015    0.015    3234.0    1938.0   1.00
## beta[1]    0.215  0.729  -1.133    1.693      0.012    0.013    3632.0    2284.0   1.00
## beta[2]   -0.080  0.852  -1.789    1.501      0.014    0.015    3866.0    2652.0   1.00
## beta[3]   -0.290  0.814  -1.926    1.193      0.016    0.015    2870.0    1729.0   1.00
## beta[4]    0.079  0.809  -1.479    1.691      0.014    0.014    3577.0    2158.0   1.00
## ...          ...    ...     ...      ...        ...      ...       ...       ...    ...
## beta[96]   0.106  0.726  -1.271    1.542      0.013    0.013    3471.0    2487.0   1.00
## beta[97]  -0.156  0.716  -1.591    1.160      0.013    0.013    3188.0    1798.0   1.00
## beta[98]   0.289  0.763  -1.076    1.827      0.014    0.015    3107.0    2408.0   1.00
## beta[99]  -0.278  0.768  -1.747    1.205      0.013    0.013    3575.0    2568.0   1.00
## sigma      0.980  0.478   0.275    1.859      0.046    0.032     102.0     211.0   1.05
##
## [101 rows x 9 columns]
```

```
az.summary(trace).iloc[[0,10,20,30,40,50,60,70,80,100]]
```

```
##             mean     sd  hdi_3%  hdi_97%  mcse_mean  mcse_sd  ess_bulk  ess_tail  r_hat
## beta[0]    0.067  0.861  -1.650    1.681      0.015    0.015    3234.0    1938.0   1.00
## beta[10]   8.327  1.242   5.945   10.622      0.027    0.019    2075.0    2710.0   1.00
## beta[20]  -8.288  1.335 -10.697   -5.733      0.030    0.021    2003.0    1746.0   1.00
## beta[30]   8.610  1.023   6.678   10.447      0.023    0.017    2011.0    1702.0   1.00
## beta[40]  -8.765  1.507 -11.485   -5.929      0.030    0.022    2461.0    2531.0   1.00
## beta[50]   8.966  1.016   6.995   10.860      0.023    0.016    2035.0    1842.0   1.00
## beta[60]  -9.248  1.121 -11.381   -7.162      0.022    0.015    2708.0    2371.0   1.00
```

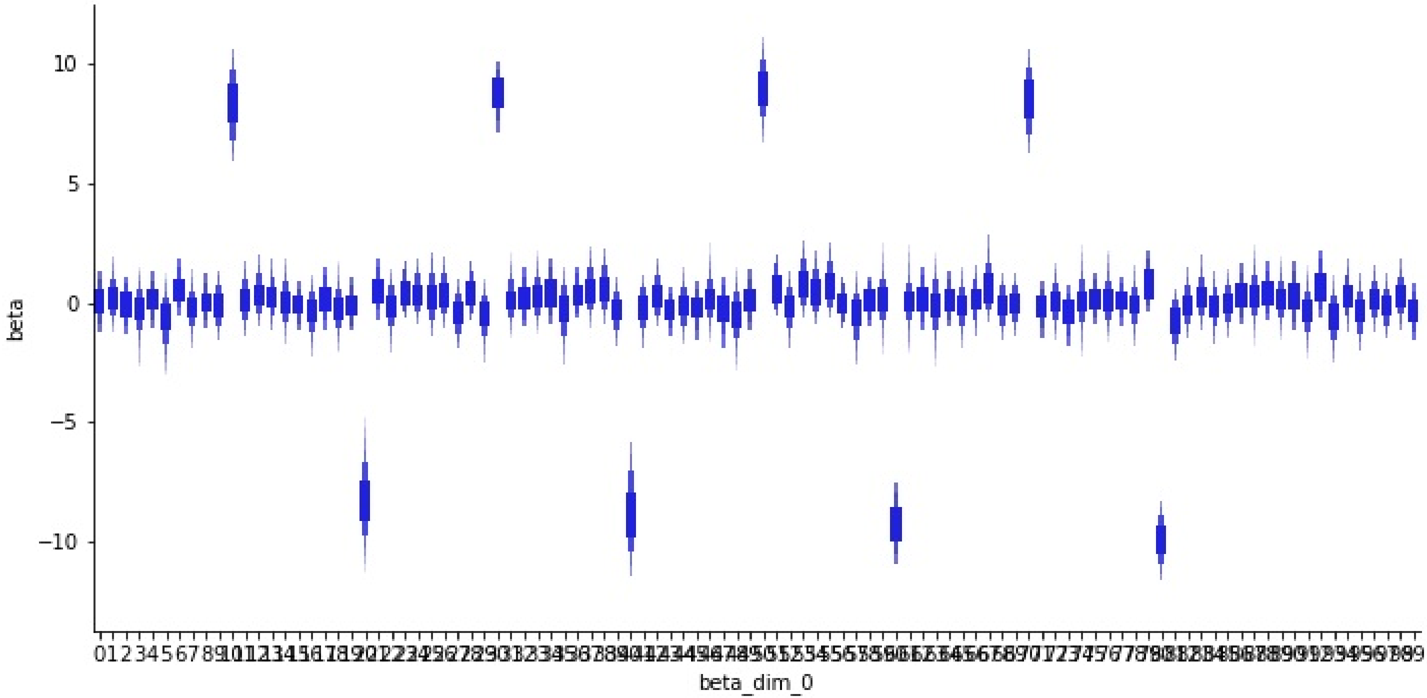
```
ax = az.plot_forest(trace)
plt.tight_layout()
plt.show()
```



Plot helper

```
def plot_slope(trace, prior="beta", chain=0):
  post = (trace.posterior[prior]
          .to_dataframe()
          .reset_index()
          .query("chain == 0")
         )

  sns.catplot(x="beta_dim_0", y="beta", data=post, kind="boxen", linewidth=0, color='blue', aspect=2, sho
  plt.tight_layout()
  plt.show()
```

```
plot_slope(trace)
```

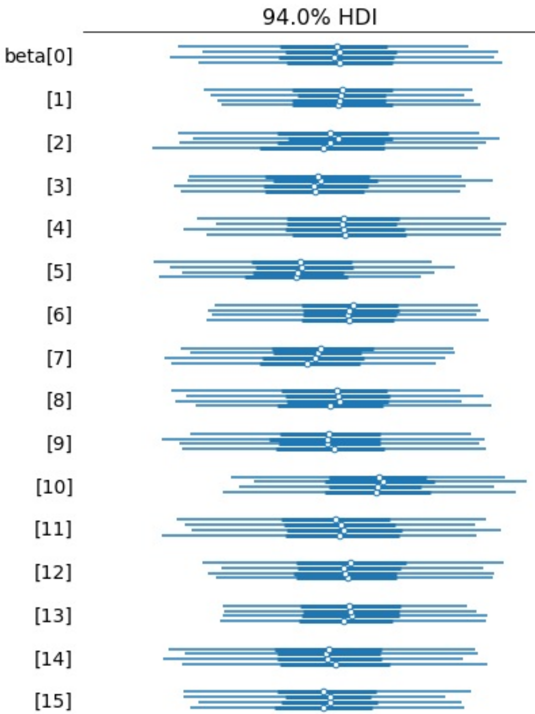


Weakly Informative Prior

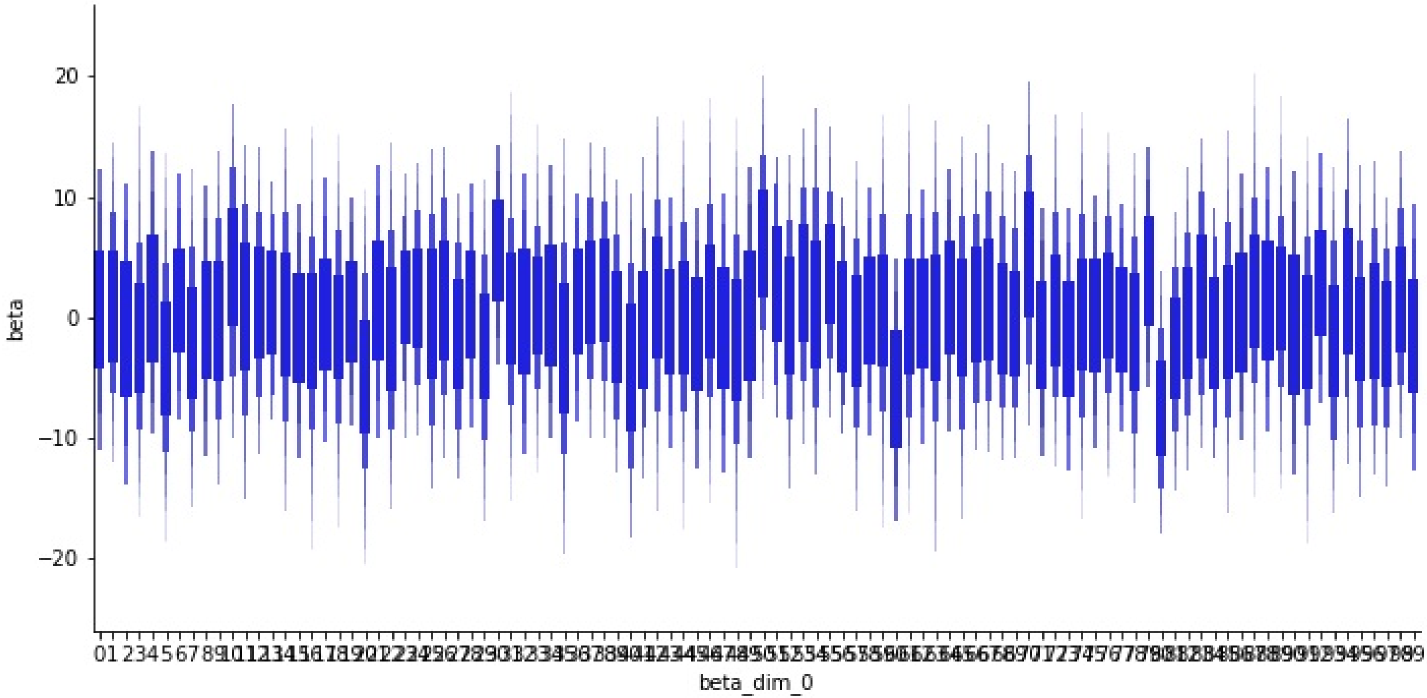
```
with pm.Model() as bayes_weak:
  b = pm.Normal("beta", 0, 10, shape=k)
  y_est = X @ b

  s = pm.HalfNormal('sigma', sd=2)

  likelihood = pm.Normal("y", mu=y_est, sigma=s, observed=y)

  trace = pm.sample(return_inferencedata=True, random_seed=12345)
```

```
## █
## Auto-assigning NUTS sampler...
## Initializing NUTS using jitter+adapt_diag...
## Multiprocess sampling (4 chains in 4 jobs)
## NUTS: [sigma, beta]
## Sampling 4 chains for 1_000 tune and 1_000 draw iterations (4_000 + 4_000 draws total) took 57 seconds.
## The acceptance probability does not match the target. It is 0.9760397075294559, but should be close to 0.8.
   increase the number of tuning steps.
## The chain reached the maximum tree depth. Increase max_treedepth, increase target_accept or reparameterize.
## There was 1 divergence after tuning. Increase `target_accept` or reparameterize.
## There were 15 divergences after tuning. Increase `target_accept` or reparameterize.
## The acceptance probability does not match the target. It is 0.7066410867916934, but should be close to 0.8.
   increase the number of tuning steps.
## There was 1 divergence after tuning. Increase `target_accept` or reparameterize.
```

```
ax = az.plot_forest(trace)
plt.tight_layout()
plt.show()
```



```
plot_slope(trace)
```



Demo 2 - Gaussian Process

```
np.random.seed(12345)

n = 50
x = np.linspace(0, 1, n)
X = x.reshape(-1,1)

nugget = 0.75
sigma2_true = 4.0
l_true = 10

cov_func = sigma2_true * pm.gp.cov.ExpQuad(1, 1/l_true)
mean_func = pm.gp.mean.Zero()

y_true = np.random.multivariate_normal(
    mean_func(X).eval(), cov_func(X).eval(), 1
).flatten()

y = y_true + nugget * np.random.randn(n)
```

```
fig = plt.figure(figsize=(12, 5))
plt.plot(X, y_true, "-b", lw=3)
plt.plot(X, y, "ok", ".")
plt.show()
```