

Lec 20 - More PyMC3

Statistical Computing and Computation

Sta 663 | Spring 2022

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Demo 1 - Bayesian Lasso

```
n = 50
k = 100

np.random.seed(1234)
X = np.random.normal(size=(n, k))

beta = np.zeros(shape=k)
beta[[10,30,50,70]] = 10
beta[[20,40,60,80]] = -10

y = X @ beta + np.random.normal(size=n)
```

Naive Model

```
with pm.Model() as bayes_lasso:
    b = pm.Laplace("beta", 0, 1, shape=k)#lam*tau, shape=k)
    y_est = X @ b
    s = pm.HalfNormal('sigma', sd=1)

    likelihood = pm.Normal("y", mu=y_est, sigma=s, observed=y)

    trace = pm.sample(return_inferencedata=True, random_seed=1234)
```

```
## ■
## Auto-assigning NUTS sampler...
## Initializing NUTS using jitter+adapt_diag...
## Multiprocess sampling (4 chains in 4 jobs)
## NUTS: [sigma, beta]
## Sampling 4 chains for 1_000 tune and 1_000 draw iterations (4_000 + 4_000 draws total) took 19 seconds.
## There were 2 divergences after tuning. Increase `target_accept` or reparameterize.
## The acceptance probability does not match the target. It is 0.878942077718847, but should be close to 0.8. T
    increase the number of tuning steps.
## The estimated number of effective samples is smaller than 200 for some parameters.
```

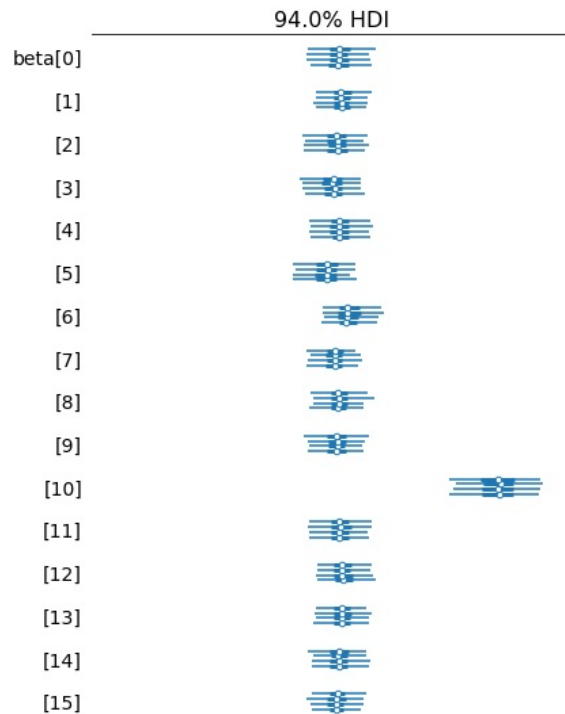
```
az.summary(trace)
```

```
##          mean      sd  hdi_3%  hdi_97%  mcse_mean  mcse_sd  ess_bulk  ess_tail  r_hat
## beta[0]  0.067  0.861 -1.650   1.681     0.015   0.015   3234.0   1938.0   1.00
## beta[1]  0.215  0.729 -1.133   1.693     0.012   0.013   3632.0   2284.0   1.00
## beta[2] -0.080  0.852 -1.789   1.501     0.014   0.015   3866.0   2652.0   1.00
## beta[3] -0.290  0.814 -1.926   1.193     0.016   0.015   2870.0   1729.0   1.00
## beta[4]  0.079  0.809 -1.479   1.691     0.014   0.014   3577.0   2158.0   1.00
## ...      ...      ...      ...      ...      ...      ...      ...      ...      ...
## beta[96] 0.106  0.726 -1.271   1.542     0.013   0.013   3471.0   2487.0   1.00
## beta[97] -0.156 0.716 -1.591   1.160     0.013   0.013   3188.0   1798.0   1.00
## beta[98] 0.289  0.763 -1.076   1.827     0.014   0.015   3107.0   2408.0   1.00
## beta[99] -0.278 0.768 -1.747   1.205     0.013   0.013   3575.0   2568.0   1.00
## sigma    0.980  0.478  0.275   1.859     0.046   0.032   102.0    211.0   1.05
##
## [101 rows x 9 columns]
```

```
az.summary(trace).iloc[[0,10,20,30,40,50,60,70,80,100]]
```

```
##          mean      sd  hdi_3%  hdi_97%  mcse_mean  mcse_sd  ess_bulk  ess_tail  r_hat
## beta[0]  0.067  0.861 -1.650   1.681     0.015   0.015   3234.0   1938.0   1.00
## beta[10] 8.327  1.242  5.945   10.622     0.027   0.019   2075.0   2710.0   1.00
## beta[20] -8.288 1.335 -10.697  -5.733     0.030   0.021   2003.0   1746.0   1.00
## beta[30]  8.610  1.023  6.678   10.447     0.023   0.017   2011.0   1702.0   1.00
## beta[40] -8.765 1.507 -11.485  -5.929     0.030   0.022   2461.0   2531.0   1.00
## beta[50]  8.966  1.016  6.995   10.860     0.023   0.016   2035.0   1842.0   1.00
## beta[60] -9.248 1.121 -11.381  -7.162     0.022   0.015   2708.0   2371.0   1.00
```

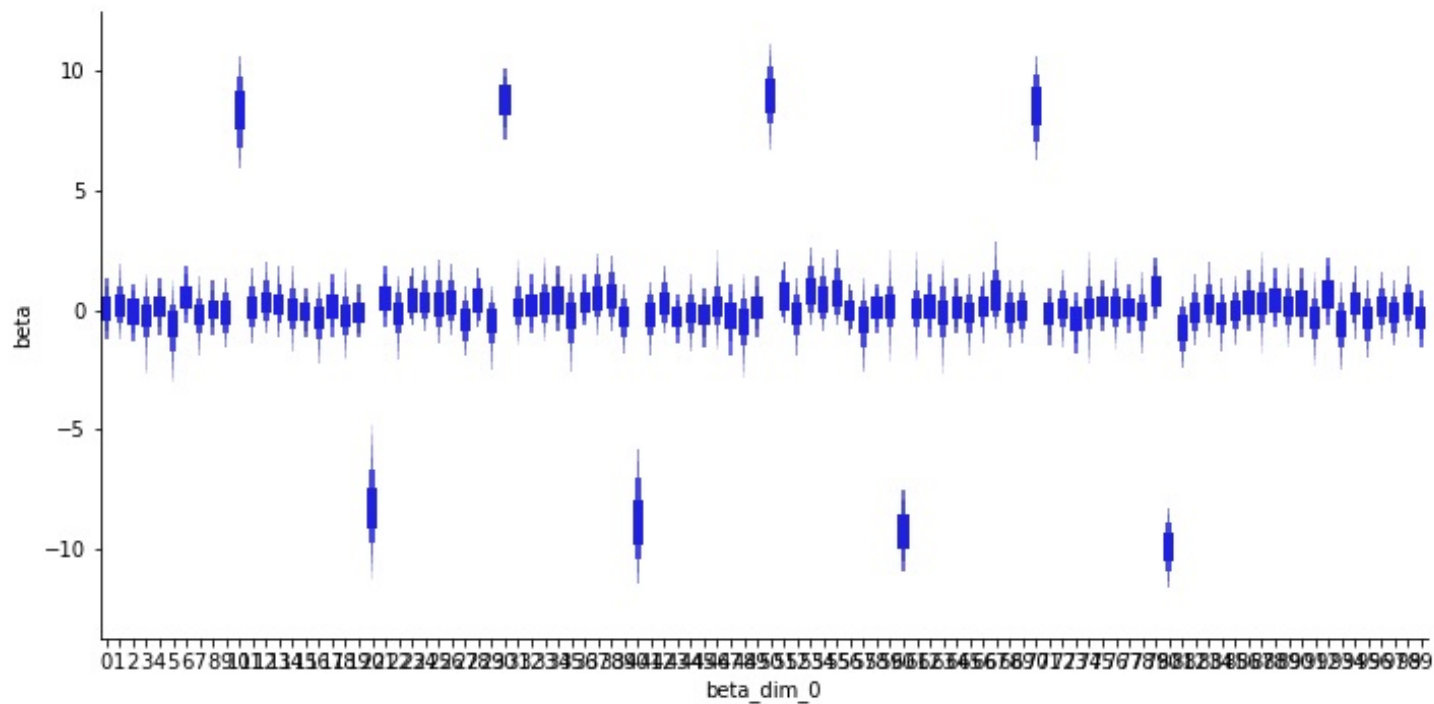
```
ax = az.plot_forest(trace)
plt.tight_layout()
plt.show()
```



Plot helper

```
def plot_slope(trace, prior="beta", chain=0):  
    post = (trace.posterior[prior]  
            .to_dataframe()  
            .reset_index()  
            .query("chain == 0"))  
    )  
  
    sns.catplot(x="beta_dim_0", y="beta", data=post, kind="boxen", linewidth=0, color='blue', aspect=2, show=False)  
    plt.tight_layout()  
    plt.show()
```

```
plot_slope(trace)
```




Weakly Informative Prior

```
with pm.Model() as bayes_weak:
    b = pm.Normal("beta", 0, 10, shape=k)
    y_est = X @ b

    s = pm.HalfNormal('sigma', sd=2)

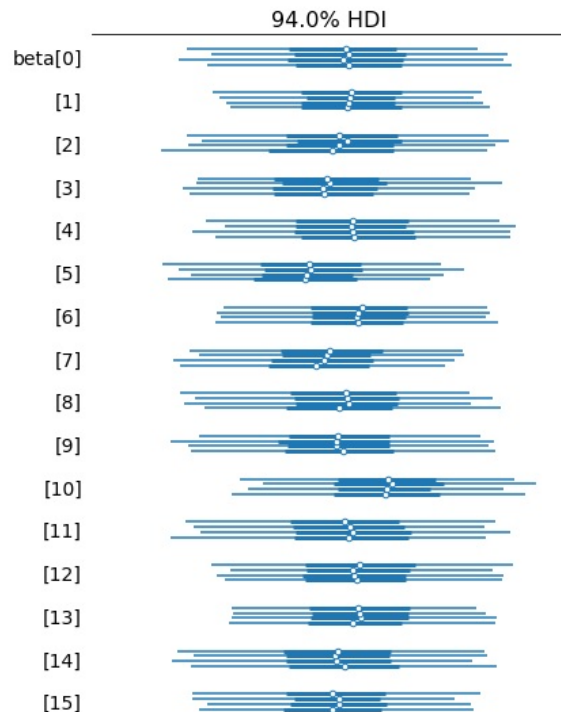
    likelihood = pm.Normal("y", mu=y_est, sigma=s, observed=y)

    trace = pm.sample(return_inferencedata=True, random_seed=12345)
```

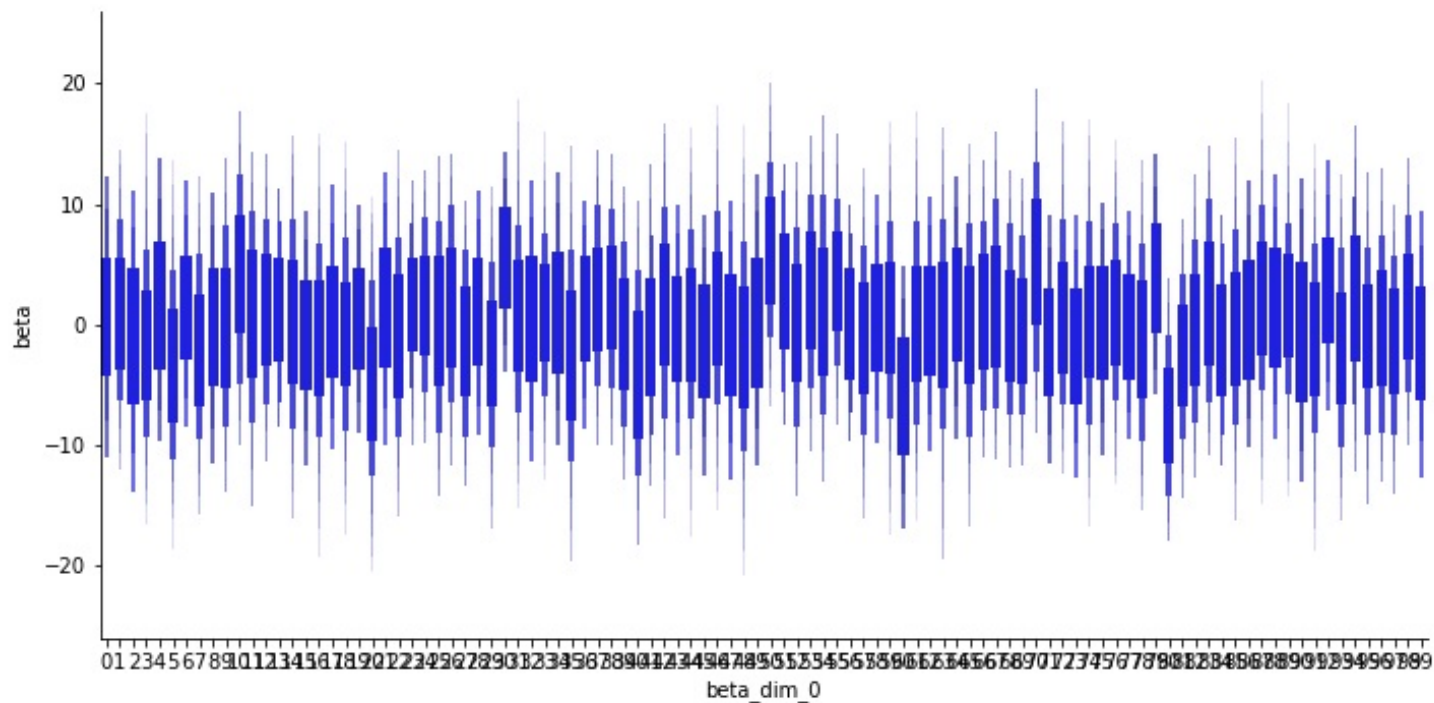
```
## 
## Auto-assigning NUTS sampler...
## Initializing NUTS using jitter+adapt_diag...
## Multiprocess sampling (4 chains in 4 jobs)
## NUTS: [sigma, beta]
## Sampling 4 chains for 1_000 tune and 1_000 draw iterations (4_000 + 4_000 draws total) took 57 seconds.
## The acceptance probability does not match the target. It is 0.9760397075294559, but should be close to 0.8.
    increase the number of tuning steps.
## The chain reached the maximum tree depth. Increase max_treedepth, increase target_accept or reparameterize.
## There was 1 divergence after tuning. Increase `target_accept` or reparameterize.
## There were 15 divergences after tuning. Increase `target_accept` or reparameterize.
## The acceptance probability does not match the target. It is 0.7066410867916934, but should be close to 0.8.
    increase the number of tuning steps.
## There was 1 divergence after tuning. Increase `target_accept` or reparameterize.
```



```
ax = az.plot_forest(trace)
plt.tight_layout()
plt.show()
```



```
plot_slope(trace)
```



Demo 2 - Gaussian Process

```
np.random.seed(12345)

n = 50
x = np.linspace(0, 1, n)
X = x.reshape(-1,1)

nugget = 0.75
sigma2_true = 4.0
l_true = 10

cov_func = sigma2_true * pm.gp.cov.ExpQuad(1, 1/l_true)
mean_func = pm.gp.mean.Zero()

y_true = np.random.multivariate_normal(
    mean_func(X).eval(), cov_func(X).eval(), 1
).flatten()

y = y_true + nugget * np.random.randn(n)
```

```
fig = plt.figure(figsize=(12, 5))  
plt.plot(X, y_true, "-b", lw=3)  
plt.plot(X, y, "ok", ".")  
plt.show()
```

