

# Lec 18 - patsy + statsmodels

**Statistical Computing and Computation**

**Sta 663 | Spring 2022**

**Dr. Colin Rundel**

**patsy**

# patsy

patsy is a Python package for describing statistical models (especially linear models, or models that have a linear component) and building design matrices. It is closely inspired by and compatible with the formula mini-language used in R and S.

...

Patsy's goal is to become the standard high-level interface to describing statistical models in Python, regardless of what particular model or library is being used underneath.

# Formulas

```
from patsy import ModelDesc

ModelDesc.from_formula("y ~ a + a:b + np.log(x)")

## ModelDesc(lhs_termlist=[Term([EvalFactor('y')])],
##           rhs_termlist=[Term([]),
##                         Term([EvalFactor('a')]),
##                         Term([EvalFactor('a'), EvalFactor('b')]),
##                         Term([EvalFactor('np.log(x)')])])

ModelDesc.from_formula("y ~ a*b + np.log(x) - 1")

## ModelDesc(lhs_termlist=[Term([EvalFactor('y')])],
##           rhs_termlist=[Term([EvalFactor('a')]),
##                         Term([EvalFactor('b')]),
##                         Term([EvalFactor('a'), EvalFactor('b')]),
##                         Term([EvalFactor('np.log(x)')])])
```

# Model matrix

```
from patsy import demo_data, dmatrix, dmatrices
```

```
data = demo_data("y", "a", "b", "x1", "x2")
data
```

```
## {'a': ['a1', 'a1', 'a2', 'a2', 'a1', 'a1', 'a2']
pd.DataFrame(data)
```

```
##      a      b      x1      x2      y
## 0  a1  b1  1.764052 -0.103219  1.494079
## 1  a1  b2  0.400157  0.410599 -0.205158
## 2  a2  b1  0.978738  0.144044  0.313068
## 3  a2  b2  2.240893  1.454274 -0.854096
## 4  a1  b1  1.867558  0.761038 -2.552990
## 5  a1  b2 -0.977278  0.121675  0.653619
## 6  a2  b1  0.950088  0.443863  0.864436
## 7  a2  b2 -0.151357  0.333674 -0.742165
```

```
dmatrix("a + a:b + np.exp(x1)", data)
```

```
## DesignMatrix with shape (8, 5)
##   Intercept  a[T.a2]  a[a1]:b[T.b2]  a[a2]:b[T.b1]
##   1          0          0          0
##   1          0          0          1
##   1          1          0          0
##   1          1          1          0
##   1          1          0          0
##   1          0          0          1
##   1          0          1          0
##   1          1          1          0
##   Terms:
##   'Intercept' (column 0)
##   'a' (column 1)
##   'a:b' (columns 2:4)
##   'np.exp(x1)' (column 4)
```

Note the  $T.$  in  $a[T.a2]$  is there to indicate treatment coding (i.e. typical dummy coding)

# Model matrices

```
y, x = dmatrices("y ~ a + a:b + np.exp(x1)", data)
```

y

```
## DesignMatrix with shape (8, 1)
##      y
## 1.49408
## -0.20516
## 0.31307
## -0.85410
## -2.55299
## 0.65362
## 0.86444
## -0.74217
## Terms:
## 'y' (column 0)
```

x

```
## DesignMatrix with shape (8, 5)
## Intercept a[T.a2] a[a1]:b[T.b2] a[a2]:b[T.
## 1       0       0
## 1       0       1
## 1       1       0
## 1       1       0
## 1       0       0
## 1       0       1
## 1       1       0
## 1       1       0
## Terms:
## 'Intercept' (column 0)
## 'a' (column 1)
## 'a:b' (columns 2:4)
## 'np.exp(x1)' (column 4)
```

# as DataFrames

```
dmatrix("a + a:b + np.exp(x1)", data, return_type='dataframe')
```

```
##      Intercept  a[T.a2]  a[a1]:b[T.b2]  a[a2]:b[T.b2]  np.exp(x1)
## 0        1.0     0.0          0.0          0.0    5.836039
## 1        1.0     0.0          1.0          0.0    1.492059
## 2        1.0     1.0          0.0          0.0    2.661096
## 3        1.0     1.0          0.0          1.0    9.401725
## 4        1.0     0.0          0.0          0.0    6.472471
## 5        1.0     0.0          1.0          0.0    0.376334
## 6        1.0     1.0          0.0          0.0    2.585938
## 7        1.0     1.0          0.0          1.0    0.859541
```

# Formula Syntax

Code	Description	Example
+	unions terms on the left and right	$a+a \Rightarrow a$
-	removes terms on the right from terms on the left	$a+b-a \Rightarrow b$
:	constructs interactions between each term on the left and right	$(a+b):c \Rightarrow a:c + b:c$
*	short-hand for terms and their interactions	$a*b \Rightarrow a + b + a:b$
/	short-hand for left terms and their interactions with right terms	$a/b \Rightarrow a + a:b$
I()	used for calculating arithmetic calculations	$I(x1 + x2)$
Q()	used to quote column names, e.g. columns with spaces or symbols	$Q('bad name!')$

# Examples

```
dmatrix("x:y", demo_data("x", "y", "z"))
```

```
## DesignMatrix with shape (5, 2)
## Intercept      x:y
##      1 -1.72397
##      1  0.38018
##      1 -0.14814
##      1 -0.23130
##      1  0.76682
## Terms:
##   'Intercept' (column 0)
##   'x:y' (column 1)
```

```
dmatrix("x*y", demo_data("x", "y", "z"))
```

```
## DesignMatrix with shape (5, 4)
## Intercept      x      y      x:y
##      1  1.76405 -0.97728 -1.72397
##      1  0.40016  0.95009  0.38018
##      1  0.97874 -0.15136 -0.14814
##      1  2.24089 -0.10322 -0.23130
##      1  1.86756  0.41060  0.76682
## Terms:
##   'Intercept' (column 0)
##   'x' (column 1)
##   'y' (column 2)
##   'x:y' (column 3)
```

```
dmatrix("x/y", demo_data("x", "y", "z"))
```

```
## DesignMatrix with shape (5, 3)
## Intercept      x      x:y
##      1  1.76405 -1.72397
##      1  0.40016  0.38018
##      1  0.97874 -0.14814
##      1  2.24089 -0.23130
##      1  1.86756  0.76682
## Terms:
##   'Intercept' (column 0)
##   'x' (column 1)
##   'x:y' (column 2)
```

```
dmatrix("x*(y+z)", demo_data("x", "y", "z"))
```

```
## DesignMatrix with shape (5, 6)
## Intercept      x      y      z      x:y      x:z
##      1  1.76405 -0.97728  0.14404 -1.72397  0.25410
##      1  0.40016  0.95009  1.45427  0.38018  0.58194
##      1  0.97874 -0.15136  0.76104 -0.14814  0.74486
##      1  2.24089 -0.10322  0.12168 -0.23130  0.27266
##      1  1.86756  0.41060  0.44386  0.76682  0.82894
## Terms:
##   'Intercept' (column 0)
##   'x' (column 1)
##   'y' (column 2)
##   'z' (column 3)
##   'x:y' (column 4)
##   'x:z' (column 5)
```

# Intercept Examples

```
dmatrix("x", demo_data("x", "y", "z"))
```

```
## DesignMatrix with shape (5, 2)
## Intercept      x
##      1 1.76405
##      1 0.40016
##      1 0.97874
##      1 2.24089
##      1 1.86756
## Terms:
##   'Intercept' (column 0)
##   'x' (column 1)
```

```
dmatrix("x-1", demo_data("x", "y", "z"))
```

```
## DesignMatrix with shape (5, 1)
##      x
## 1.76405
## 0.40016
## 0.97874
## 2.24089
## 1.86756
## Terms:
##   'x' (column 0)
```

```
dmatrix("-1 + x", demo_data("x", "y", "z"))
```

```
## DesignMatrix with shape (5, 1)
##      x
## 1.76405
```

```
dmatrix("x+0", demo_data("x", "y", "z"))
```

```
## DesignMatrix with shape (5, 1)
##      x
## 1.76405
## 0.40016
## 0.97874
## 2.24089
## 1.86756
## Terms:
##   'x' (column 0)
```

```
dmatrix("x-0", demo_data("x", "y", "z"))
```

```
## DesignMatrix with shape (5, 2)
## Intercept      x
##      1 1.76405
##      1 0.40016
##      1 0.97874
##      1 2.24089
##      1 1.86756
## Terms:
##   'Intercept' (column 0)
##   'x' (column 1)
```

```
dmatrix("x - (-0)", demo_data("x", "y", "z"))
```

```
## DesignMatrix with shape (5, 1)
##      x
## 1.76405
```

# Design Info

One of the key features of the design matrix object is that it retains all the necessary details (including stateful transforms) that are necessary to apply to new data inputs (e.g. for prediction).

```
d = dmatrix("a + a:b + np.exp(x1)", data, return_type='dataframe')
d.design_info
```

```
## DesignInfo(['Intercept',
##              'a[T.a2]',
##              'a[a1]:b[T.b2]',
##              'a[a2]:b[T.b2]',
##              'np.exp(x1)'],
##             factor_infos={EvalFactor('a'): FactorInfo(factor=EvalFactor('a'),
##                                             type='categorical',
##                                             state=<factor state>,
##                                             categories=('a1', 'a2')),
##                           EvalFactor('b'): FactorInfo(factor=EvalFactor('b'),
##                                             type='categorical',
##                                             state=<factor state>,
##                                             categories=('b1', 'b2')),
##                           EvalFactor('np.exp(x1)': FactorInfo(factor=EvalFactor('np.exp(x1)'),
##                                                 type='numerical',
##                                                 state=<factor state>,
```

# Stateful transforms

```
data = {"x1": np.random.normal(size=10)}
new_data = {"x1": np.random.normal(size=10)}
```

```
d = dmatrix("scale(x1)", data)
d
```

```
## DesignMatrix with shape (10, 2)
##   Intercept  scale(x1)
##       1     -1.66439
##       1      1.31641
##       1      1.51247
##       1     -0.85749
##       1      0.40329
##       1     -1.27297
##       1     -0.50679
##       1      0.37778
##       1      0.18438
##       1      0.50732
## Terms:
##   'Intercept' (column 0)
##   'scale(x1)' (column 1)

np.mean(d, axis=0)
```

```
pred = dmatrix(d.design_info, new_data)
pred
```

```
## DesignMatrix with shape (10, 2)
##   Intercept  scale(x1)
##       1     -0.01379
##       1      0.64039
##       1      1.73684
##       1     -0.76110
##       1     -0.39034
##       1      0.90875
##       1      0.64408
##       1     -0.06190
##       1      0.08917
##       1     -0.06990
## Terms:
##   'Intercept' (column 0)
##   'scale(x1)' (column 1)

np.mean(pred, axis=0)
```

# scikit-lego PatsyTransformer

If you would like to use a Patsy formula in a scikitlearn pipeline, it is possible via the PatsyTransformer from the `scikit-lego` library.

```
from sklego.preprocessing import PatsyTransformer

df = pd.DataFrame({
    "y": [2, 2, 4, 4, 6],
    "x": [1, 2, 3, 4, 5],
    "a": ["yes", "yes", "no", "no", "yes"]
})

X, y = df[["x", "a"]], df[["y"]].values
```

```
pt = PatsyTransformer("x*a + np.log(x)")
pt.fit_transform(X)
```

```
## DesignMatrix with shape (5, 5)
##   Intercept  a[T.yes]  x  x:a[T.yes]  np.log(x)
##       1           1  1            1  0.00000
##       1           1  2            2  0.69315
##       1           0  3            0  1.09861
##       1           0  4            0  1.38629
```

```
make_pipeline(
    PatsyTransformer("x*a + np.log(x")),
    StandardScaler()
).fit_transform(X)

## array([[ 0.        ,  0.8165 , -1.41421, -0.3235 ,
##        [ 0.        ,  0.8165 , -0.70711,  0.21567,
##        [ 0.        , -1.22474,  0.        , -0.86266,
##        [ 0.        , -1.22474,  0.70711, -0.86266,
```

# Exercise 1

Using patsy fit a linear regression model to the `books` data that includes an interaction term between `cover` and `volume`.

```
books = pd.read_csv("https://sta663-sp22.github.io/slides/data/daag_books.csv")
```

# B-splines

Patsy also has support for B-splines and other related models.

```
d = pd.read_csv("data/d1.csv")  
sns.relplot(x="x", y="y", data=d)
```



```
y, X = dmatrices("y ~ bs(x, df=6)", data=d)  
X  
## DesignMatrix with shape (75, 7)  
## Columns:  
##   ['Intercept',  
##    'bs(x, df=6)[0]',  
##    'bs(x, df=6)[1]',  
##    'bs(x, df=6)[2]',  
##    'bs(x, df=6)[3]',  
##    'bs(x, df=6)[4]',  
##    'bs(x, df=6)[5]']  
## Terms:  
##   'Intercept' (column 0), 'bs(x, df=6)' (colu  
## (to view full data, use np.asarray(this_obj))
```

# What is $\text{bs}(x)[i]$ ?

```
bs_df = (
    dmatrix("bs(x, df=6)", data=d, return_type="dataframe")
    .drop(["Intercept"], axis = 1)
    .assign(x = d["x"])
    .melt(id_vars="x")
)

sns.relplot(x="x", y="value", hue="variable", kind="line", data = bs_df, aspect=1.5)
```

# Fitting a model

```
from sklearn.linear_model import LinearRegression
lm = LinearRegression(fit_intercept=False).fit(X,y)
lm.coef_
## array([[ 1.28955, -1.69132,  3.17914, -5.3865 , -1.18284, -3.8488 , -2.42867]])

plt.figure(layout="constrained")
sns.lineplot(x=d["x"], y=lm.predict(X).ravel(), color="k")
sns.scatterplot(x="x", y="y", data=d)
plt.show()
```

# sklearn SplineTransformer

```
from sklearn.preprocessing import SplineTransformer

p = make_pipeline(
    SplineTransformer(
        n_knots=6,
        degree=3,
        include_bias=True
    ),
    LinearRegression(fit_intercept=False)
).fit(
    d[["x"]], d["y"]
)
```

```
plt.figure()
sns.lineplot(x=d["x"], y=p.predict(d[["x"]]).ravel())
sns.scatterplot(x="x", y="y", data=d)
plt.show()
```

# Why different?

For patsy the number of splines is determined by `df` while for sklearn this is determined by `n_knots + degree - 1`.

```
p = p.set_params(splinetransformer__n_knots = 5).fit(d[["x"]], d["y"])

plt.figure(layout="constrained")
sns.lineplot(x=d["x"], y=p.predict(d[["x"]]).ravel(), color="k")
sns.scatterplot(x="x", y="y", data=d)
plt.show()
```

but that is not the whole story, if we examine the bases we also see they differ slightly between implementations:

```
bs_df = pd.DataFrame(  
    SplineTransformer(n_knots=6, degree=3, include_bias=True).fit_transform(d[["x"]]),  
    columns = ["bs["+ str(i) +"]" for i in range(8)]  
).assign(  
    x = d.x  
).melt(  
    id_vars = "x"  
)  
  
sns.relplot(x="x", y="value", hue="variable", kind="line", data = bs_df, aspect=1.5)
```

# **statsmodels**

# statsmodels

statsmodels is a Python module that provides classes and functions for the estimation of many different statistical models, as well as for conducting statistical tests, and statistical data exploration. An extensive list of result statistics are available for each estimator. The results are tested against existing statistical packages to ensure that they are correct.

```
import statsmodels.api as sm
import statsmodels.formula.api as smf
import statsmodels.tsa.api as tsa
```

statsmodels uses slightly different terminology for referring to `y` / dependent / response and `x` / independent / explanatory variables. Specifically it uses `endog` to refer to the `y` and `exog` to refer to the `x` variable(s).

This is particularly important when using the main API, less so when using the formula API.

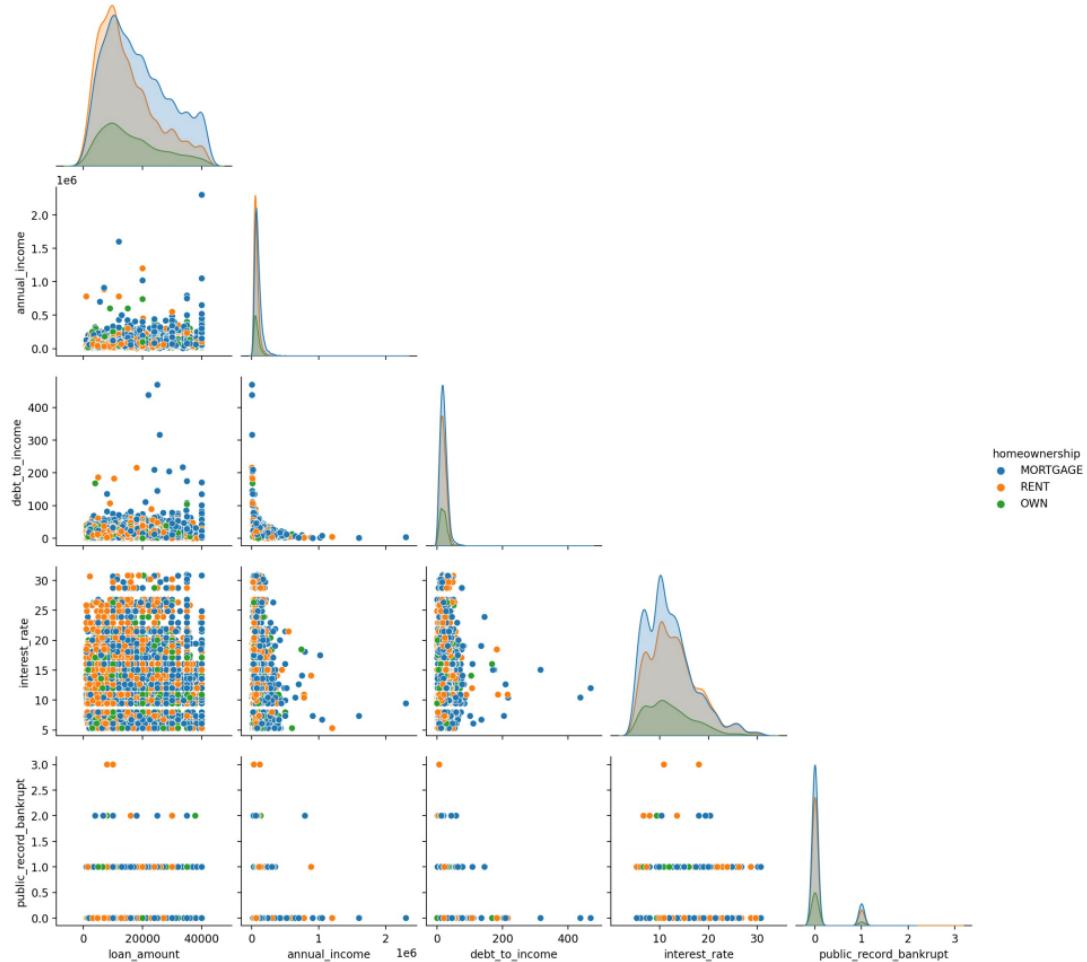
# OpenIntro Loans data

This data set represents thousands of loans made through the Lending Club platform, which is a platform that allows individuals to lend to other individuals. Of course, not all loans are created equal. Someone who is a essentially a sure bet to pay back a loan will have an easier time getting a loan with a low interest rate than someone who appears to be riskier. And for people who are very risky? They may not even get a loan offer, or they may not have accepted the loan offer due to a high interest rate. It is important to keep that last part in mind, since this data set only represents loans actually made, i.e. do not mistake this data for loan applications!

For the full data dictionary see [here](#). We have removed some of the columns to make the data set more reasonably sized and also dropped any rows with missing values.

```
loans = pd.read_csv("data/openintro_loans.csv")  
loans
```

```
##      state emp_length  term homeownership annual_income ... loan_amount grade interest_rate public_record_bankrupt loan_status  
## 0      NJ          3    60    MORTGAGE     90000.0 ...     28000   C       14.07           0        Current  
## 1      HI         10    36      RENT     40000.0 ...      5000   C       12.61           1        Current  
## 2      WI          3    36      RENT     40000.0 ...      2000   D       17.09           0        Current  
## 3      PA          1    36      RENT     30000.0 ...     21600   A        6.72           0        Current  
## 4      CA         10    36      RENT     35000.0 ...     23000   C       14.07           0        Current  
## ...     ...     ...     ...     ...     ...     ...     ...     ...     ...     ...     ...     ...  
## 9177    TX         10    36      RENT    108000.0 ...     24000   A        7.35           1        Current  
## 9178    PA          8    36    MORTGAGE    121000.0 ...     10000   D       19.03           0        Current  
## 9179    CT         10    36    MORTGAGE    67000.0 ...     30000   E       23.88           0        Current  
## 9180    WI          1    36    MORTGAGE    80000.0 ...     24000   A        5.32           0        Current  
## 9181    CT          3    36      RENT    66000.0 ...     12800   B       10.91           0        Current  
##  
## [9182 rows x 16 columns]
```



# OLS

```
y = loans["loan_amount"]
X = loans[["homeownership", "annual_income", "debt_to_income", "interest_rate", "public_record_bankrupt"]]

model = sm.OLS(endog=y, exog=X)

## ValueError: Pandas data cast to numpy dtype of object. Check input data with np.asarray(data).
```

What do you think the issue is here?

The error occurs because x contains mixed types - specifically we have categorical data columns which cannot be directly converted to a numeric dtype so we need to take care of the dummy coding for statsmodels (with this interface).

```
X_dc = pd.get_dummies(X)
model = sm.OLS(endog=y, exog=X_dc)
```

# Fitting and summary

```
res = model.fit()
print(res.summary())

##                                     OLS Regression Results
## =====
## Dep. Variable:      loan_amount    R-squared:           0.135
## Model:                 OLS    Adj. R-squared:         0.135
## Method:              Least Squares    F-statistic:        239.5
## Date:            Fri, 18 Mar 2022    Prob (F-statistic): 2.33e-285
## Time:                09:18:54    Log-Likelihood:     -97245.
## No. Observations:      9182    AIC:                  1.945e+05
## Df Residuals:          9175    BIC:                  1.946e+05
## Df Model:                   6
## Covariance Type:    nonrobust
## =====
##             coef    std err      t      P>|t|      [0.025      0.975]
## -----
## annual_income      0.0505     0.002   31.952      0.000      0.047      0.054
## debt_to_income     65.6641    7.310     8.982      0.000     51.334     79.994
## interest_rate      204.2480   20.448     9.989      0.000    164.166    244.330
## public_record_bankrupt -1362.3253  306.019    -4.452      0.000   -1962.191   -762.460
## homeownership_MORTGAGE  1.002e+04  357.245    28.048      0.000    9319.724   1.07e+04
## homeownership_own     8880.4144  422.296    21.029      0.000    8052.620    9708.209
## homeownership_rent     7446.5385  351.641    21.177      0.000    6757.243    8135.834
## =====
## Omnibus:             481.833    Durbin-Watson:       2.002
## Prob(Omnibus):        0.000    Jarque-Bera (JB):  916.542
## Skew:                  0.391    Prob(JB):        9.45e-200
## Kurtosis:                 4.336    Cond. No.       6.17e+05
## =====
```

# Formula interface

Most of the modeling interfaces are also provided by `smf` (`statsmodels.formula.api`) in which case `patsy` is used to construct the model matrices.

```
model = smf.ols(  
    "loan_amount ~ homeownership + annual_income + debt_to_income + interest_rate + public_record_bankrupt",  
    data = loans  
)  
res = model.fit()  
print(res.summary())
```

```
##                                     OLS Regression Results  
## ======  
## Dep. Variable:      loan_amount    R-squared:         0.135  
## Model:                 OLS            Adj. R-squared:      0.135  
## Method:                Least Squares    F-statistic:       239.5  
## Date:        Fri, 18 Mar 2022   Prob (F-statistic): 2.33e-285  
## Time:          09:18:55           Log-Likelihood:     -97245.  
## No. Observations:      9182            AIC:             1.945e+05  
## Df Residuals:          9175            BIC:             1.946e+05  
## Df Model:                   6  
## Covariance Type:    nonrobust  
## ======  
##              coef    std err        t     P>|t|      [0.025      0.975]  
## -----  
## Intercept      1.002e+04    357.245    28.048    0.000    9319.724    1.07e+04  
## homeownership[T.OWN] -1139.5893   322.361    -3.535    0.000   -1771.489    -507.690  
## homeownership[T.RENT] -2573.4652   221.101   -11.639    0.000   -3006.873   -2140.057  
## annual_income      0.0505     0.002     31.952    0.000      0.047      0.054
```

# Result values and model parameters

```
res.params
```

```
## Intercept          10020.003630
## homeownership[T.OWN] -1139.589268
## homeownership[T.RENT] -2573.465175
## annual_income        0.050505
## debt_to_income       65.664103
## interest_rate         204.247993
## public_record_bankrupt -1362.325291
## dtype: float64
```

```
res.bse
```

```
## Intercept          357.244896
## homeownership[T.OWN] 322.361151
## homeownership[T.RENT] 221.101300
## annual_income        0.001581
## debt_to_income       7.310428
## interest_rate         20.447644
## public_record_bankrupt 306.019080
## dtype: float64
```

```
res.rsquared
```

```
## 0.13542611095847512
```

```
res.aic
```

```
## 194503.99751598848
```

```
res.bic
```

```
## 194553.87251826216
```

```
res.predict()
```

```
## array([18621.86199, 11010.94015, 14346.14516, 1
##        18283.87492, 26719.61804, 18496.72337, 1
##        18845.26463, 20083.33976, ..., 19576.169
##        17161.96037, 18764.48833, 19252.9242 , 1
##        22144.19006, 21253.25932, 15934.34097, 1
```

# Diagnostic plots

## QQ Plot

```
plt.figure()  
sm.graphics.qqplot(res.resid, line="s")  
plt.show()
```

## Leverage plot

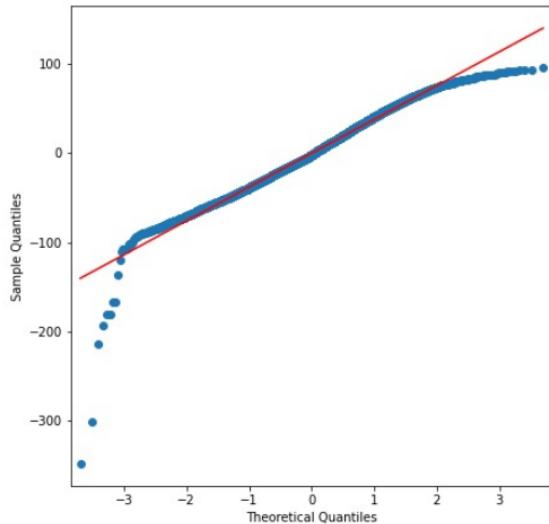
```
plt.figure()  
sm.graphics.plot_leverage_resid2(res)  
plt.show()
```

# Alternative model

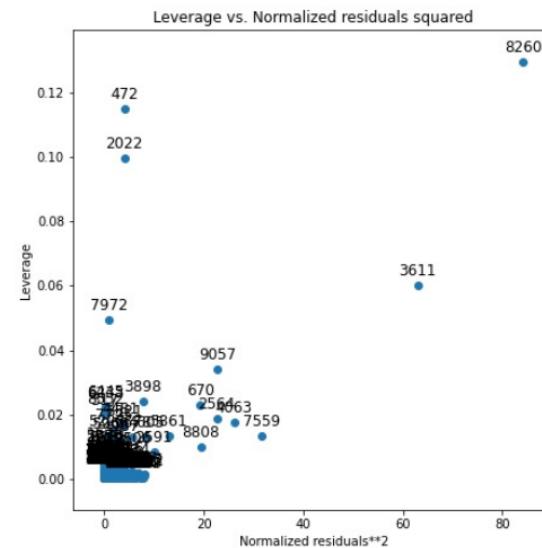
```
res = smf.ols(  
    "np.sqrt(loan_amount) ~ homeownership + annual_income + debt_to_income + interest_rate + public_record_bankrupt",  
    data = loans  
)  
res.fit()  
print(res.summary())
```

```
##                                     OLS Regression Results  
## ======  
## Dep. Variable: np.sqrt(loan_amount) R-squared:      0.132  
## Model:                 OLS   Adj. R-squared:      0.132  
## Method:                Least Squares   F-statistic:     232.7  
## Date:          Fri, 18 Mar 2022   Prob (F-statistic): 1.16e-277  
## Time:              09:18:58   Log-Likelihood:   -46429.  
## No. Observations:      9182   AIC:             9.287e+04  
## Df Residuals:         9175   BIC:             9.292e+04  
## Df Model:                   6  
## Covariance Type:    nonrobust  
## ======  
##            coef    std err       t   P>|t|    [0.025    0.975]  
## -----  
## Intercept      95.4915     1.411   67.687   0.000    92.726    98.257  
## homeownership[T.OWN] -4.4495     1.273   -3.495   0.000   -6.945   -1.954  
## homeownership[T.RENT] -10.4225     0.873  -11.937   0.000  -12.134   -8.711  
## annual_income      0.0002  6.24e-06   30.916   0.000    0.000    0.000  
## debt_to_income      0.2720     0.029    9.421   0.000    0.215    0.329  
## interest_rate       0.8911     0.081   11.035   0.000    0.733    1.049  
## public_record_bankrupt -4.6899     1.208   -3.881   0.000   -7.059   -2.321  
## ======  
## Omnibus:           178.498   Durbin-Watson:        2.011  
## Prob(Omnibus):     0.000   Jarque-Bera (JB):    333.598
```

```
plt.figure()  
sm.graphics.qqplot(res.resid, line="s")  
plt.show()
```



```
plt.figure()  
sm.graphics.plot_leverage_resid2(res)  
plt.show()
```



# Bushtail Possums

Data representing possums in Australia and New Guinea. This is a copy of the data set by the same name in the DAAG package, however, the data set included here includes fewer variables.

pop - Population, either `vic` (Victoria) or `other` (New South Wales or Queensland).

```
possum = pd.read_csv("data/possum.csv")
possum
```

```
##      site    pop sex   age head_l skull_w total_l tail_l
## 0       1     Vic   m  8.0   94.1    60.4    89.0   36.0
## 1       1     Vic   f  6.0   92.5    57.6    91.5   36.5
## 2       1     Vic   f  6.0   94.0    60.0    95.5   39.0
## 3       1     Vic   f  6.0   93.2    57.1    92.0   38.0
## 4       1     Vic   f  2.0   91.5    56.3    85.5   36.0
## ...
## 99      7 other   m  1.0   89.5    56.0    81.5   36.5
## 100     7 other   m  1.0   88.6    54.7    82.5   39.0
## 101     7 other   f  6.0   92.4    55.0    89.0   38.0
## 102     7 other   m  4.0   91.5    55.2    82.5   36.5
## 103     7 other   f  3.0   93.6    59.9    89.0   40.0
```

# Logistic regression models (GLM)

```
y = pd.get_dummies( possum["pop"] )
X = pd.get_dummies( possum.drop(["site","pop"], axis=1) )

model = sm.GLM(y, X, family = sm.families.Binomial())

## MissingDataError: exog contains inf or nans
```

Behavior for dealing with missing data can be handled via `missing`, possible values are "none", "drop", and "raise".

```
model = sm.GLM(y, X, family = sm.families.Binomial(), missing="drop")
```

# Fit and summary

```
res = model.fit()
print(res.summary())

##                                     Generalized Linear Model Regression Results
## =====
## Dep. Variable:      ['Vic', 'other']   No. Observations:          102
## Model:                  GLM    Df Residuals:                 95
## Model Family:           Binomial    Df Model:                      6
## Link Function:          Logit     Scale:                   1.0000
## Method:                  IRLS    Log-Likelihood:            -31.942
## Date:        Fri, 18 Mar 2022    Deviance:                63.885
## Time:             09:19:01    Pearson chi2:                  154.
## No. Iterations:          7    Pseudo R-squ. (CS):            0.5234
## Covariance Type:       nonrobust
## =====
##            coef    std err      z   P>|z|    [0.025    0.975]
## -----
## age         0.1373    0.183    0.751    0.453   -0.221    0.495
## head_l     -0.1972    0.158   -1.247    0.212   -0.507    0.113
## skull_w    -0.2001    0.139   -1.443    0.149   -0.472    0.072
## total_l     0.7569    0.176    4.290    0.000    0.411    1.103
## tail_l     -2.0698    0.429   -4.820    0.000   -2.912   -1.228
## sex_f      40.0148   13.077    3.060    0.002   14.385   65.645
## sex_m      38.5395   12.941    2.978    0.003   13.175   63.904
## =====
```

# Success vs failure

Note `endog` can be 1d or 2d for binomial models - in the case of the latter each row is interpreted as [success, failure]. s

```
y = pd.get_dummies( possum["pop"], drop_first = True)
X = pd.get_dummies( possum.drop(["site","pop"], axis=1) )

res = sm.GLM(y, X, family = sm.families.Binomial(), missing="drop").fit()
print(res.summary())
```

```
##                                     Generalized Linear Model Regression Results
## -----
## Dep. Variable:                  other    No. Observations:             102
## Model:                          GLM      Df Residuals:                  95
## Model Family:                 Binomial    Df Model:                      6
## Link Function:                  Logit     Scale:                   1.0000
## Method:                         IRLS     Log-Likelihood:            -31.942
## Date:              Fri, 18 Mar 2022   Deviance:                63.885
## Time:                  09:19:02     Pearson chi2:                  154.
## No. Iterations:                      7     Pseudo R-squ. (CS):        0.5234
## Covariance Type:                nonrobust
## -----
##          coef    std err         z      P>|z|      [0.025      0.975]
## -----
## age       -0.1373    0.183     -0.751     0.453     -0.495      0.221
## head_l      0.1972    0.158      1.247     0.212     -0.113      0.507
## skull_w      0.2001    0.139      1.443     0.149     -0.072      0.472
## total_l     -0.7569    0.176     -4.290     0.000     -1.103     -0.411
## tail_l       2.0698    0.429      4.820     0.000      1.228      2.912
## sex_f     -40.0148   13.077     -3.060     0.002     -65.645     -14.385
```

# Fit and summary

```
res = model.fit()
print(res.summary())

##                                     Generalized Linear Model Regression Results
## =====
## Dep. Variable:      ['Vic', 'other']   No. Observations:          102
## Model:                  GLM    Df Residuals:                 95
## Model Family:           Binomial   Df Model:                      6
## Link Function:          Logit     Scale:                   1.0000
## Method:                  IRLS    Log-Likelihood:        -31.942
## Date:  Fri, 18 Mar 2022   Deviance:            63.885
## Time: 09:19:03             Pearson chi2:          154.
## No. Iterations:          7    Pseudo R-squ. (CS):       0.5234
## Covariance Type:         nonrobust
## =====
##            coef    std err       z   P>|z|    [0.025    0.975]
## -----
## age        0.1373    0.183     0.751    0.453    -0.221    0.495
## head_l     -0.1972    0.158    -1.247    0.212    -0.507    0.113
## skull_w    -0.2001    0.139    -1.443    0.149    -0.472    0.072
## total_l     0.7569    0.176     4.290    0.000     0.411    1.103
## tail_l     -2.0698    0.429    -4.820    0.000    -2.912   -1.228
## sex_f      40.0148   13.077     3.060    0.002    14.385   65.645
## sex_m     38.5395   12.941     2.978    0.003    13.175   63.904
```

# Formula interface

```
res = smf.glm(  
    "pop ~ sex + age + head_l + skull_w + total_l + tail_l-1",  
    data = possum,  
    family = sm.families.Binomial(),  
    missing="drop"  
).fit()  
print(res.summary())
```

```
##                                     Generalized Linear Model Regression Results  
## =====  
## Dep. Variable:  ['pop[Vic]', 'pop[other]'] No. Observations:      102  
## Model:          GLM             Df Residuals:                 95  
## Model Family:   Binomial        Df Model:                      6  
## Link Function:  Logit           Scale:                  1.0000  
## Method:         IRLS            Log-Likelihood:       -31.942  
## Date:           Fri, 18 Mar 2022 Deviance:                63.885  
## Time:            09:19:03      Pearson chi2:                154.  
## No. Iterations:                         7 Pseudo R-squ. (CS):     0.5234  
## Covariance Type:                       nonrobust  
## =====  
##            coef  std err      z   P>|z|    [0.025    0.975]  
## -----  
## sex[f]    40.0148   13.077   3.060   0.002    14.385   65.645  
## sex[m]    38.5395   12.941   2.978   0.003    13.175   63.904
```

# sleepstudy data

These data are from the study described in Belenky et al. (2003), for the most sleep-deprived group (3 hours time-in-bed) and for the first 10 days of the study, up to the recovery period. The original study analyzed speed ( $1/(reaction\ time)$ ) and treated day as a categorical rather than a continuous predictor.

The average reaction time per day (in milliseconds) for subjects in a sleep deprivation study. Days 0-1 were adaptation and training (T1/T2), day 2 was baseline (B); sleep deprivation started after day 2.

```
sleep = pd.read_csv("data/sleepstudy.csv")  
sleep
```

```
##      Reaction  Days Subject  
## 0    249.5600    0    308  
## 1    258.7047    1    308  
## 2    250.8006    2    308  
## 3    321.4398    3    308  
## 4    356.8519    4    308  
## ...
```

These data come from the `sleepstudy` dataset in the `lme4` R package

```
sns.relplot(x="Days", y="Reaction", col="Subject", col_wrap=6, data=sleep)
```



# Random intercept model

```
me_rand_int = smf.mixedlm(  
    "Reaction ~ Days", data=sleep, groups=sleep["Subject"],  
    subset=sleep.Days >= 2  
)  
res_rand_int = me_rand_int.fit(method=["lbfgs"])  
print(res_rand_int.summary())
```

```
##      Mixed Linear Model Regression Results  
## ======  
## Model:           MixedLM Dependent Variable: Reaction  
## No. Observations: 180     Method:             REML  
## No. Groups:       18      Scale:              960.4529  
## Min. group size: 10      Log-Likelihood:     -893.2325  
## Max. group size: 10      Converged:          Yes  
## Mean group size: 10.0  
## -----  
##      Coef.  Std.Err.   z   P>|z|  [0.025  0.975]  
## -----  
## Intercept 251.405    9.747 25.793 0.000  232.302 270.509  
## Days      10.467    0.804 13.015 0.000   8.891  12.044  
## Group Var 1378.232  17.157  
## ======
```

```
summary(  
    lmer(Reaction ~ Days + (1 | Subject), data=sleepstudy)  
)
```

```
## Linear mixed model fit by REML ['lmerMod']  
## Formula: Reaction ~ Days + (1 | Subject)  
##   Data: sleepstudy  
##  
## REML criterion at convergence: 1786.5  
##  
## Scaled residuals:  
##   Min     1Q Median     3Q    Max  
## -3.2257 -0.5529  0.0109  0.5188  4.2506  
##  
## Random effects:  
##   Groups   Name        Variance Std.Dev.  
##   Subject  (Intercept) 1378.2   37.12  
##   Residual            960.5   30.99  
## Number of obs: 180, groups: Subject, 18  
##  
## Fixed effects:  
##                   Estimate Std. Error t value  
## (Intercept) 251.4051    9.7467  25.79  
## Days        10.4673    0.8042  13.02  
##  
## Correlation of Fixed Effects:  
##   (Intr)  
## Days -0.371
```

# Predictions



The prediction is only taking into account the fixed effects here, not the group random effects.

# Recovering random effects for prediction

```
# Dictionary of random effects estimates
re = res_rand_int.random_effects

# Multiply each RE by the random effects design matrix for each group
rex = [np.dot(me_rand_int.exog_re_li[j], re[k]) for (j, k) in enumerate(me_rand_int.group_labels)]

# Add the fixed and random terms to get the overall prediction
rex = np.concatenate(rex)
y_hat = res_rand_int.predict() + rex
```



# Random intercept and slope model

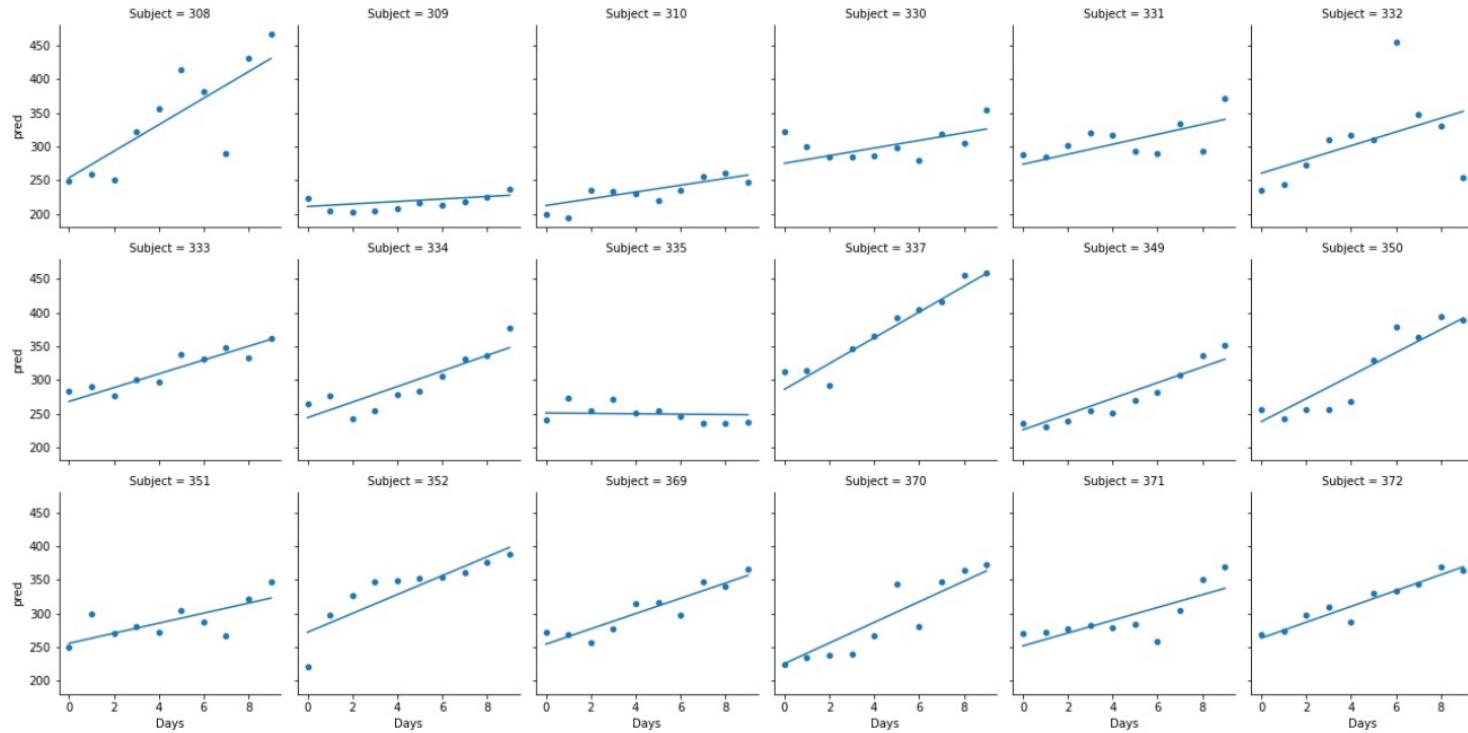
```
me_rand_sl= smf.mixedlm(  
    "Reaction ~ Days", data=sleep, groups=sleep["Subject"],  
    subset=sleep.Days >= 2,  
    re_formula=~Days  
)  
res_rand_sl = me_rand_sl.fit(method=["lbfgs"])  
print(res_rand_sl.summary())
```

```
## Mixed Linear Model Regression Results  
## =====  
## Model: MixedLM  Dependent Variable: Reaction  
## No. Observations: 180  Method: REML  
## No. Groups: 18  Scale: 654.9412  
## Min. group size: 10  Log-Likelihood: -871.814  
## Max. group size: 10  Converged: Yes  
## Mean group size: 10.0  
## -----  
##          Coef.  Std.Err.   z  P>|z|  [0.025  0.975]  
## -----  
## Intercept  251.405  6.825 36.838 0.000 238.029 264.78  
## Days       10.467  1.546  6.771 0.000   7.438 13.49  
## Group Var  612.089 11.881  
## Group x Days Cov  9.605  1.820  
## Days Var   35.072  0.610  
## =====
```

```
summary(  
    lmer(Reaction ~ Days + (Days|Subject), data=sleepstudy)  
)
```

```
## Linear mixed model fit by REML ['lmerMod']  
## Formula: Reaction ~ Days + (Days | Subject)  
##   Data: sleepstudy  
##  
## REML criterion at convergence: 1743.6  
##  
## Scaled residuals:  
##   Min     1Q  Median     3Q    Max  
## -3.9536 -0.4634  0.0231  0.4634  5.1793  
##  
## Random effects:  
##   Groups   Name        Variance Std.Dev. Corr  
##   Subject (Intercept) 612.10   24.741  
##             Days        35.07   5.922   0.07  
##   Residual    654.94   25.592  
## Number of obs: 180, groups: Subject, 18  
##  
## Fixed effects:  
##   Estimate Std. Error t value  
## (Intercept) 251.405      6.825 36.838  
## Days        10.467      1.546  6.771  
##  
## Correlation of Fixed Effects:  
##   (Intr)  
## Days -0.138
```

# Prediction



We are using the same approach described previously to obtain the RE estimates and use them in the predictions.

# t-test and z-test for equality of means

```
cm = sm.stats.CompareMeans(  
    sm.stats.DescrStatsW( books.weight[books.cover == "hb"] ),  
    sm.stats.DescrStatsW( books.weight[books.cover == "pb"] )  
)  
  
print(cm.summary())  
  
##                                     Test for equality of means  
## =====  
##      coef   std err       t     P>|t|    [0.025    0.975]  
## -----  
## subset #1  168.3036  136.636   1.232    0.240  -126.880  463.487  
## =====  
  
print(cm.summary(use_t=False))  
  
##                                     Test for equality of means  
## =====  
##      coef   std err       z     P>|z|    [0.025    0.975]  
## -----  
## subset #1  168.3036  136.636   1.232    0.218  -99.497  436.104  
## =====  
  
print(cm.summary(usevar="unequal"))  
  
##                                     Test for equality of means  
## =====  
##      coef   std err       t     P>|t|    [0.025    0.975]  
## -----  
## subset #1  168.3036  136.360   1.234    0.239  -126.686  463.293
```

# Contingency tables

Below are data from the GSS and a survey of Duke students in an intro stats class - the question asked about how concerned the respondent was about the effect of global warming on polar ice cap melt.

```
gss = pd.DataFrame({"US": [454, 226], "Duke": [56, 32]}, index=["A great deal", "Not a great deal"])
gss
```

```
##           US  Duke
## A great deal    454    56
## Not a great deal 226    32
```

```
tbl = sm.stats.Table2x2(gss.to_numpy())
print(tbl.summary())
```

```
##          Estimate   SE    LCB    UCB  p-value
## -----
## Odds ratio      1.148    0.723  1.823   0.559
## Log odds ratio  0.138  0.236 -0.325  0.601   0.559
## Risk ratio      1.016    0.962  1.074   0.567
## Log risk ratio  0.016  0.028 -0.039  0.071   0.567
## -----
```