Practical – 1 (Iterated limits. Two-path test, Limits and continuity of scalar fields and vector fields using "definition")

Objective

1. If $f(x,y) = x\left(\sin\frac{1}{x}\right)\left(\sin\frac{1}{y}\right)$. Find $\lim_{(x,y)\to(0,0)} f(x,y)$.

   (a) 0 **(b) 1**

   (c) -1 **(d) does not exist**

2. Let $f(x,y) = \frac{xy}{x^2+y^2}\,\forall(x,y) \neq (0,0); f(0,0) = 0$. What is $\lim_{(x,y)\to(0,0)} f(x,y)$ over the path $x = y$.

   (a) 0 **(b) $\frac{1}{2}$**

   (c) 1 **(d) none of these**

3. Define $f: \mathbb{R}^2 \to \mathbb{R}$ as $f(0,0) = 0; f(x,y) = \frac{x^2-y^2}{x^2+y^2}\,\forall(x,y) \neq (0,0)$. Find $\lim_{x\to 0}\lim_{y\to 0} f(x,y)$.

   (a) 0 **(b) $\frac{1}{2}$**

   (c) 1 **(d) none of these**

4. Evaluate $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^2+y^2}$

   (a) 0 **(b) 1**

   (c) -1 **(d) does not exist**

5. Fill in the blanks: $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6} =$ _____

6. State True/False: $f(x,y) = \frac{x^2-y^2}{x^2+y^2}$ is continuous on $\mathbb{R}^2$

Descriptive

1. Using $\mathcal{E} - \delta$ definition, show that $\lim_{(x,y)\to(1,1)} \frac{xy-y-2x+2}{x-1} = -1$.

2. Show that $\lim_{(x,y)\to(0,0)} \frac{x^3y}{2x^6+y^2}$ does not exist by considering different paths as $(x,y) \to (0,0)$. Also, find the iterated limits, if they exist.

3. Evaluate $\lim_{(x,y)\to(0,0)} y\ln(x^2+y^2)$, if it exists, by converting to polar coordinates.

4. Evaluate $\lim_{(x,y)\to(2,8)} 3x^2y + \sqrt{xy}$.