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① $\begin{pmatrix} \overset{x}{2}, & \overset{y}{-2\pi/3} \\ r & \theta \end{pmatrix}$ $x = r.\cos\phi$

$y = r.\sin\phi$

$r = 2$

$\phi = -2\pi/3 = \frac{-2(180)}{3} = -120°$

$\cos\left(-\frac{2\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right) = \boxed{-\frac{1}{2}}$

$\boxed{x = 2.\left(-\frac{1}{2}\right) = -1}$

② $r = 2\cos\phi$ $\sin\phi = \frac{\pi}{3}$

$r = 2\cos\phi\,(r)$ $\sin\left(\frac{\pi}{3}\right) = \frac{\pi}{3}$

$r^2 = 2r\cos\phi$

$r\cos\phi = x$

$x^2 + y^2 = 2x$

$x^2 - 2x + y^2 = 0$

$x^2 - 2x + 1 + y^2 = 1$

$\boxed{(x-1)^2 + y^2 = 1}$

③ $r = e^{-\theta/4}$. $\pi/2 \leq \phi \leq \pi$

$f(\alpha) = e^{-\theta/4}$ $A = \frac{1}{2}\int_{\pi/2}^{\pi}\left(e^{-\theta/4}\right)^2 d\theta = \frac{1}{2}\int_{\pi/2}^{\pi} e^{-\theta/2}\,d\theta$

$\alpha = \frac{\pi}{2}$

$B = \pi$

$e^{-\theta/2} = -2e^{-\theta/2}$

$\lim = \frac{1}{2}\left[-2e^{-\theta/2}\right]_{\pi/2}^{\pi} = \frac{1}{2}\left(-2e^{-\pi/2} + 2e^{-\pi/4}\right) = -e^{-\pi/2} + e^{-\pi/4} = e^{-\pi/4} - e^{-\pi/2}$

$\boxed{e^{-\pi/4} - e^{-\pi/2}}$

④ $a_N = \dfrac{3^{N+2}}{5^N}$ $\qquad 0 < \frac{3}{5} < 1$

$a_N = \dfrac{3^N + 3^2}{5^N} = 9\left(\frac{3}{5}\right)^N$

$\lim_{N\to\infty} 9\cdot\left(\frac{3}{5}\right)^N = 9.0 = \boxed{0}$

Converge

⑤ $10 - 2 + 0.4 - 0.8 + \ldots$

$\sum_{N=0}^{\infty} 10\left(-\frac{1}{5}\right)^N \qquad \left|-\frac{1}{5}\right| = \frac{1}{5} < 1$ Converge

$S = \dfrac{a}{1-r}$

$S = \dfrac{10}{1-(-\frac{1}{5})} = \dfrac{10}{1+\frac{1}{5}} = \dfrac{10}{\frac{6}{5}} = 10\cdot\frac{5}{6} = \frac{50}{6} = \boxed{\frac{25}{3}}$

⑥ $f(x) = \dfrac{-3}{1-2x^5}$

$|u| < 1 = \dfrac{1}{1-u} = \sum_{N=0}^{\infty} u^N$

$f(x) = -3 \cdot \dfrac{1}{1-2x^5} =$

$u = 2x^5$

$|2x^5| < 1 = |x| < \left(\frac{1}{2}\right)^{1/5}$

$\dfrac{1}{1-2x^5} = \sum_{N=0}^{\infty} (2x^5)^N = \sum_{N=0}^{\infty} 2^N x^{5N}$

$f(x) = -3\sum_{N=0}^{\infty} 2^N x^{5N} = \sum_{N=0}^{\infty} -3\cdot 2^N x^{5N}$

$\boxed{\sum_{N=0}^{\infty} -3\cdot 2^N x^{5N}}$

$$\sum_{N=1}^{\infty} \frac{1}{\sqrt{N+4}} \qquad N \geq 1$$

$$F(x) = \frac{1}{\sqrt{x+4}} \qquad \int_1^{\infty} \frac{1}{\sqrt{x+4}}\,dx = \lim_{b \to \infty} \int_1^b (x+4)^{-1/2}\,dx$$

$$\int (x+4)^{1/2}\,dx = 2(x+4)^{1/2} + C$$

diverge

$$\lim_{b \to \infty} \Big[2\sqrt{x+4}\Big]_1^b = \lim_{b \to \infty} \left(2\sqrt{b+4} - 2\sqrt{5}\right) = \infty$$

3) $$\sum_{N=0}^{\infty} \frac{(-1)^N}{5N+1} \qquad a_{N+1} = \frac{(-1)^{N+1}}{5(N+1)+1} = \frac{(-1)^{N+1}}{5N+6}$$

$$\lim_{N \to \infty} \left|\frac{a_{N+1}}{a_N}\right| \qquad \left|\frac{a_{N+1}}{a_N}\right| = \left|\frac{\frac{(-1)^{N+1}}{5N+6}}{\frac{(-1)^N}{5N+1}}\right| = \left|\frac{5N+1}{5N+6}\right|$$

$$a_N = \frac{(-1)^N}{5N+1}$$

$$\lim_{N \to \infty} \left|\frac{5N+1}{5N+6}\right| = \lim_{N \to \infty} \frac{5+\frac{1}{N}}{5+\frac{6}{N}} = \frac{5}{5} = \boxed{1}$$

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$$b_{N+1} = \frac{1}{5(N+1)+1} = \frac{1}{5N+6} < \frac{1}{5N+1} = b_N$$

$$\lim_{N \to \infty} b_N = 0 \qquad \lim_{N \to \infty} \frac{1}{5N+1} = 0$$

converge

4) $$1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \dots$$

$$\sum_{N=1}^{\infty} \frac{1}{2N-1} \qquad \text{para } N \geq 1$$

$$\frac{1}{2N-1} > \frac{1}{2N} \qquad \frac{1}{2N-1} > \frac{1}{2N} \text{ y } \sum_{N=1}^{\infty} \frac{1}{2N} \text{ diverge}$$

$$\sum_{N=1}^{\infty} \frac{1}{2N} \qquad \frac{1}{2} \sum_{N=1}^{\infty} \frac{1}{N}$$

(10)

$$\sum_{N=0}^{\infty} (-1)^{N+3} \frac{x^{2N+5}}{(2N+1)!}$$

$$(-1)^{N+3}$$

$$(-1)^{N+3} = (-1)^N \cdot (-1)^3 = -(-1)^N$$

$$-\sum_{N=0}^{\infty} (-1)^N \frac{x^{2N+5}}{(2N+1)!}$$

$$-x^5 \sum_{N=0}^{\infty} (-1)^N \frac{x^{2N}}{(2N+1)!}$$

$$\sum_{N=0}^{\infty} (-1)^N \frac{x^{2N}}{(2N+1)!} = \boxed{\frac{\sin(x)}{x}}$$

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$$-x^5 \cdot \frac{\sin(x)}{x} = \boxed{-x^4 \sin(x)}$$