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习题1.2

1. $\because P(AB)=P(A)+P(B)-P(A\cup B)=0.6+0.5-0.7=0.4$

$\therefore P(A\bar{B})=P(A(S-B))=P(A-AB)=P(A)-P(AB)=0.6-0.4=0.$

$P(\overline{AB})=1-P(AB)=1-0.4=0.2$

$P(\bar{A}\bar{B})=P(\overline{A\cup B})=1-P(A\cup B)=1-0.7=0.3$

2. $\frac{1}{10}$

3. $P_{10}^{4}/10^{4}=0.504$.

4. (1) 思路：10张票中取3张作全排列，而抽到的3张中最小号码为5的

$\frac{C_3^1 P_5^2}{P_{10}^3}=\frac{1}{12}$

(2) $\frac{C_3^1 P_4^2}{P_{10}^3}=\frac{1}{20}$

5. $2/99=49.5 \quad 49-4=45$

$3/99=33 \quad 33-3=30$

$6/99=16.5 \quad 16-1=15$

$\frac{45+30-15}{90}=\frac{60}{90}=\frac{2}{3}$

$= \frac{15}{60} = \frac{1}{4}$

0.2116

$= \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$

习题 1.3

1. $P(B|A\cup\bar{B}) = \frac{P(B(A\cup\bar{B}))}{P(A\cup\bar{B})} = \frac{P(AB)}{P(A\cup\bar{B})} = \frac{P(A)-P(A\bar{B})}{P(A)+P(\bar{B})-P(A\bar{B})} = \frac{1}{4}$

2. $P(B|A) = \frac{P(AB)}{P(A)} = \frac{1}{3} \Rightarrow P(AB) = \frac{1}{3}\cdot\frac{1}{4} = \frac{1}{12}$

$P(A|B) = \frac{P(AB)}{P(B)} = \frac{1}{2} \Rightarrow P(B) = \frac{1}{12}/\frac{1}{2} = \frac{1}{6}$

$P(A\cup B) = P(A)+P(B)-P(AB) = \frac{1}{4}+\frac{1}{6}-\frac{1}{12} = \frac{1}{3}$

3. $A=\{$甲在乙前面$\}$ $B=\{$甲乙相邻$\}$

$P(B|A) = \frac{P(AB)}{P(A)} = \frac{P_{n-1}^{n-1}/P_n^n}{\frac{1}{2}} = \frac{2}{n}$

4. $\frac{P_{90}^1 P_{10}^2}{P_{100}^3} = 0.0083.$

5. (1) $P(B|\bar{A}) = 0.85 \Rightarrow P(\bar{A}B) = P(B|\bar{A})P(\bar{A}) = 0.85\times0.08 = 0.0$

$P(AB) = P(B-\bar{A}B) = P(B)-P(\bar{A}B) = 0.93-0.068 = 0.862$

$\therefore P(A\cup B) = P(A)+P(B)-P(AB) = 0.92+0.93-0.862 = 0.988.$

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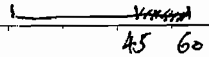
6.  $P(A) = \frac{60-45}{60} = \frac{15}{60} = \frac{1}{4}$

7. (1) $\frac{60^3}{10^3} = 0.216$ (2) $\frac{C_{60}^3}{C_{100}^3} = 0.2116$

8. (1) $\frac{1}{2}$

(2) $\frac{1}{6}$

(3) $P(B\bar{A}) = P(B-AB) = P(B) - P(AB) = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$

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习题 1.3

1. $P(B|A\cup\bar{B}) = \frac{P}{(}$

2. $P(B|A) = \frac{P(AB)}{P(A)}$

$P(A|B) = \frac{P(AB}{P(B}$

$P(A\cup B) = P(A$

3. $A = \{$甲在乙前

$P(B|A) = \frac{P}{}$

4. $\frac{P_{90}^1 P_{10}^2}{P_{100}^3} = 0$

5. (1) $P(B|\bar{A}) =$

$P(AB) = P$

$\therefore P(A\cup B) = $

$(\bar{A} \cup B)$

$P(\bar{A}B))$

$8. = 0.058$

$= 0.829.$

$\cdots\}$ $C = \{$父亲$\cdots\}$

$P(C|AB) = 0.4$

$(B|A)]$

$= 0.18.$

习题1.4.

1. $A_i = \{$售出 i 台次品$\}$, $i = 0,1,2$, $B = \{$取出一台为正品$\}$

$P(B) = \sum_{i=0}^{2} \cancel{P(B|A_i)}\, P(A_i)\, P(B|A_i)$

$= \frac{C_7^2}{C_{10}^2} \cdot \frac{C_5^1}{C_8^1} + \frac{C_7^1 C_3^1}{C_{10}^2} \cdot \frac{C_6^1}{C_8^1} + \frac{C_3^2}{C_{10}^2} \cdot \frac{C_7^1}{C_8^1} = 0.7$

2. $A_i = \{$从甲盒取出的2个球中有 i 个红球$\}$

$B = \{$从乙盒中取出的是红球$\}$

$P(B) = \sum_{i=0}^{2} P(A_i)\, P(B|A_i)$

$= \frac{C_2^2}{C_5^2} \cdot \frac{C_2^1}{C_{10}^1} + \frac{C_2^1 C_3^1}{C_5^2} \cdot \frac{C_3^1}{C_{10}^1} + \frac{C_3^2}{C_5^2} \cdot \frac{C_4^1}{C_{10}^1} = 0.32$

3. $A = \{$机器调整好$\}$, $B = \{$产品合格$\}$

$P(A) = 0.95$, $P(\bar{A}) = 0.05$, $P(B|A) = 0.98$, $P(B|\bar{A}) = 0.55$

$P(A|B) = \frac{P(B|A)\,P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})} = \frac{0.98 \times 0.95}{0.98 \times 0.95 + 0.55 \times 0.05}$

$= 0.9713$

4. $A = \{$枪校准过$\}$, $B = \{$中靶$\}$

$P(A) = \frac{5}{8}$, $P(\bar{A}) = \frac{3}{8}$, $P(B|A) = 0.8$, $P(B|\bar{A}) = 0.3$

$P(A|B) = \frac{P(B|A)\,P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})} = \frac{0.8 \times \frac{5}{8}}{0.8 \times \frac{5}{8} + 0.3 \times \frac{3}{8}} = 0.8163$

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5. $A_i = \{$到甲/乙/丙地拉菜$\}$

$B = \{$拉到1级菜$\}$

(1) $P(B) = \sum_{i=1}^{3} P(BA_i) = \sum_{i=1}^{3} P(B|A_i)P(A_i)$

$= 0.10\times0.20 + 0.30\times0.45 + 0.70\times0.30 = 0.365$

(2) $P(A_2|B) = \dfrac{P(B|A_2)P(A_2)}{\sum_{i=1}^{3} P(B|A_i)P(A_i)} = \dfrac{0.30\times0.45}{0.365} = 0.3699$

习题1.5.

1. (1) $P(AB) = P($

(2) $P(A\bar{B}) = $

2. $P(AB\bar{C}) = $

3. $P(A\cup B\cup C)$

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$\}$. $B=\{1,2,3,4\}$, $C=\{2,4\}$

, $B-A=\{2,4\}$

$=\{1,2,3,4,6\}$

A_3

$\le 2\}$

$2\}$

仅有一件次品$\}$

$\cup B)$ $\therefore A\subset B$ ~~且B⊂A~~

6. (1) $\dfrac{C_3^1 C_{45}^2}{C_{50}^3} = \dfrac{5\times45\times44}{2} \cdot \dfrac{3\times2}{50\times49\times48} = 0.25$

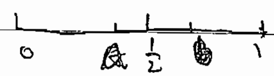
(2) $\dfrac{C_{45}^3}{C_{50}^3} = \dfrac{45\times44\times43}{50\times49\times48} = 0.72$

7. $\dfrac{C_3^2 C_3^1}{4^3} = \dfrac{9}{64}$

8. (1) $\dfrac{P_{365}^{30}}{365^{30}}$ (2) $\dfrac{364^{30}}{365^{30}}$

9. $\dfrac{C_5^1 C_8^2 - C_5^2}{C_{10}^4} = \dfrac{13}{21}$

10. $p(A) = 1 - p(\bar{A}) = 1 - \dfrac{C_{95}^3}{C_{100}^3} \doteq 0.144$.

11. 

$A=\{a\in(0,\tfrac{1}{2})\}$

$B=\{b\in(\tfrac{1}{2},1)\}$

$P(A\bar{B}\bar{C}) = \frac{1}{2}\times\frac{1}{2} - \frac{1}{4}\times\frac{1}{4} = \frac{3}{16}$

$C=\{a\in(0,\tfrac{1}{4}),\ b\in(\tfrac{3}{4},1)\}$

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12. (1) $P(AB\overline{C}) = P(AB - ABC) = 0.1 - 0.03 = 0.07$

(2) $P(\overline{A}\,\overline{B}\,C) = P((\overline{A \cup B})\cdot C)$

$= P(C) - P(AC) - P(BC) + P(ABC)$

$= 0.3 - 0.08 - 0.05 + 0.03 = 0.2$

13. $P(A\cup B\cup C\cup D) = 1 - P(\overline{A\cup B\cup C\cup D})$

$= 1 - P(\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}) = 1 - P(\overline{A})P(\overline{B})P(\overline{C})P(\overline{D})$

$= 1 - (1-x)^2(1-2x)^2 = \frac{481}{625}$

$[(1-x)(1-2x)]^2 = \frac{144}{625}$

$(1-x)(1-2x) = 12/25$

$x = \frac{1}{5}$

14. (1) $\frac{3}{10}$ (2) $\frac{3}{5}$

15. (1) $\frac{C_8^2}{C_{10}^2} = \frac{56}{90}$

(2) $A=\{$第一次取出正品$\}$, $B=\{$第二次取出正品$\}$

$P(B) = P(AB + \overline{A}B) = P(AB) + P(\overline{A}B)$

$= P(A)\cdot P(B|A) + P(\overline{A})P(B|\overline{A})$

$= \frac{8}{10}\cdot\frac{7}{9} + \frac{2}{10}\cdot\frac{8}{9} = 0.8$

16. $A=\{$发出

$P(A|B) =$

$=$

$P(\overline{A}|B) =$

$=$

$\because P(A|B$

17. $A=\{$男生

(1) $P(A|B)$

(2) $P(A|B) =$

$=$

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$, i=1,2,3$

$P(A_1A_2)+P(A_1A_2A_3)$

r^3

$, i=1,\cdots,6.$

$-P(A_1\cdots A_6)$

$6))$

$A_3+A_6)$

$(A_2)-P(A_1A_2)]^3$

$.97.$

[illegible]目}

$\bar{A}_2A_3)$

$)+P(\bar{A}_1\bar{A}_2A_3)$

$+\frac{4}{5}\times\frac{2}{3}\times\frac{1}{4}=\frac{26}{60}=0.43$

(2) $P(A_1+A_2+A_3)$

$=P(A_1)+P(A_2)+P(A_3)-P(A_1A_2)-P(A_1A_3)-P(A_3A_1)+P(A_1A_2A_3)$

$=\frac{1}{5}+\frac{1}{3}+\frac{1}{4}-\frac{1}{5}\times\frac{1}{3}-\frac{1}{3}\times\frac{1}{4}-\frac{1}{4}\times\frac{1}{5}+\frac{1}{5}\times\frac{1}{3}\times\frac{1}{4}$

$=0.6$

22. $A=\{$正品$\}$, $B=\{$[illegible]$\}$

$$P(A|B)=\frac{P(A)P(B|A)}{P(A)P(B|A)+P(\bar{A})P(B|\bar{A})}$$

$$=\frac{\frac{m}{m+n}\cdot\frac{1}{2^r}}{\frac{m}{m+n}\frac{1}{2^r}+\frac{n}{m+n}}=\frac{m}{m+n\cdot 2^r}$$

23. (1) $P(ABC)=P(A)P(B)P(C)=0.9\times0.8\times0.6=0.432$

(2) $P(A+B+C)=P(A)+P(B)+P(C)-P(AB)-P(BC)-P(CA)+P($

$=0.9+0.8+0.6-0.9\times0.8-0.8\times0.6-0.6\times0.9+0.9\times0.8\times0.6$

$=0.992$

(3) $P(A\bar{B}\bar{C}+\bar{A}B\bar{C}+\bar{A}\bar{B}C)$

$=P(A\bar{B}\bar{C})+P(\bar{A}B\bar{C})+P(\bar{A}\bar{B}C)$

$=0.9\times0.2\times0.4+0.1\times0.8\times0.4+0.1\times0.2\times0.6.$

$=0.072+0.032+0.012=0.116$

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24. ~~由~~ $A=\{$新药有效$\}$, $B=\{$被治愈4人$\}$

(1) $P(\bar{B}|A)=\sum_{i=0}^{3}C_{10}^{i}(0.35)^{i}(0.65)^{10-i}$

(2) $P(B|\bar{A})=\sum_{i=4}^{10}C_{10}^{i}(0.25)^{i}(0.75)^{10-i}$

$=1-\sum_{i=0}^{3}C_{10}^{i}(0.25)^{i}(0.75)^{10-i}$

25. $A_i=\{$表抽自第i个地区$\}$, $i=1,2,3$

$B_j=\{$第j次抽到是女生$\}$, $j=1,2$

(1) $P(B_1)=\sum_{i=1}^{3}P(A_i)P(B_1|A_i)$

$=\frac{1}{3}\left(\frac{3}{10}+\frac{7}{15}+\frac{5}{25}\right)=\frac{29}{90}$

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习2.2.

1. $X=0,1,2,$

$P(X=0)=\frac{C}{C}$

2. $X=3,4,5$

$P(X=3)=\frac{1}{C_5^3}$

3. $X=1,2,3,4$

$\sum_{k=1}^{4}P(X=k)=$

$\therefore a=\frac{1}{10}.$

$X\sim\begin{bmatrix}1\\0.1\end{bmatrix}$

4. $X=0,1,\cdots,5$

$P(X=k)=($

5. $X\sim P(0.8)$

$P(X\geq 3)=1-$

$=1-$

$=1-$

合出数

$^{-2} = (0\times0.01\times 0.729) = 0.0729$

$1-0.99144=0.00856$

$=0.99144+0.0081=0.99954$

$,\ P(\bar{A})=0.06$

$94)^{n}=(0.94)^{n}$

$94)^{n-2}$

$=1-P(X=0)-P(X=1)$

$94)^{n-1}$

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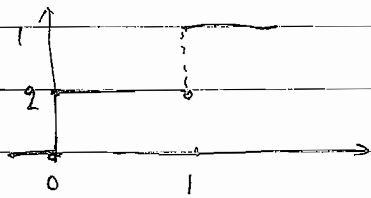
习题2.3

1.



$$F(x)=\begin{cases}0 & x<0\\ 1-p & 0\le x<1\\ 1 & x\ge 1\end{cases}$$

2. $x<-1,\quad F(x)=0$

$-1\le x<2,\quad F(x)=\frac{1}{4}$

$2\le x<3,\quad F(x)=\frac{3}{4}$

$3\le x,\quad F(x)=1$

$$F(x)=\begin{cases}0 & x<-1\\ \frac{1}{4} & -1\le x<2\\ \frac{3}{4} & 2\le x<3\\ 1 & 3\le x\end{cases}$$

(1) $P(X\le\frac{1}{2})=F(\frac{1}{2})=\frac{1}{4}$

(2) $P(\frac{3}{2}<X\le\frac{5}{2})=F(\frac{5}{2})-F(\frac{3}{2})=\frac{3}{4}-\frac{1}{4}=\frac{1}{2}$

(3) $P(2\le X\le 3)=F(3)-F(2^-)=1-\frac{1}{4}=\frac{3}{4}$

3. (1) $X\sim\begin{bmatrix}0 & 2 & 4 & 5\\ 0.2 & 0.3 & 0.1 & 0.4\end{bmatrix}$

(2) $P(1<X\le 2)=F(2)-F(1)=0.5-0.2=0.3$

$P(2\le X\le 4)=F(4)-F(2^-)=0.6-0.2=0.4$

$P(X>3)=P(X=4)+P(X=5)=0.5$

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习2.4.

1. (1) $[0, \frac{\pi}{2}]$ ✓ (2) $[0, \pi]$ ✗ (3) $[0, \frac{3\pi}{2}]$ ✗

2. $P(X>1) = \frac{a-1}{2a} = \frac{1}{3}$, $\therefore a = 3$

3. $F(\infty) = 1$. $A + \frac{\pi}{2}B = 1$ $A = \frac{1}{2}$

$F(-\infty) = 0$ $A - \frac{\pi}{2}B = 0$ $B = \frac{1}{\pi}$

$P(|X|<1) = F(1) - F(-1) = (A+\frac{\pi}{4}B) - (A - \frac{\pi}{4}B) = \frac{\pi}{2}B = \frac{1}{2}$.

$f(x) = F'(x) = (\frac{1}{2} + \frac{1}{\pi}\arctan x)' = \frac{1}{\pi(1+x^2)}$

4. (1) $1 = \int_{-\infty}^{\infty} f(x)\,dx = \int_{-1}^{1} \frac{A}{\sqrt{1-x^2}}\,dx = A\arcsin x\Big|_{-1}^{1} = \pi A$

$\therefore A = \frac{1}{\pi}$

(2) $P(-\frac{1}{2} < x \le \frac{1}{2}) = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\pi\sqrt{1-x^2}}\,dx = \frac{1}{\pi}\arcsin x\Big|_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{1}{3}$

(3)
$$F(x) = \begin{cases} 0 & x \le -1 \\ \int_{-1}^{x} \frac{1}{\pi}\arcsin x\,dx & -1<x<1 \\ 1 & x \ge 1 \end{cases}$$

5. (1) $f(x) = F'(x) = \begin{cases} 0 & x \le 0 \\ e^{-x} & x > 0 \end{cases}$

(2) $P(-1 \le X \le 2) = P(X \le 2) = F(2) = 1 - e^{-2}$.

6. $q = F_X(10) =$

$Y \sim B(5,$

$P(Y \ge 1) =$

7. $P(X \le -1.4)$

~~$P(|X| > 2.1$~~

~~$= 2\Phi(2$~~

8. $P(120 < X \le 20$

$= P(-\frac{40}{\sigma} < X$

$= 2\Phi(\frac{40}{\sigma}) -$

$\Phi(\frac{40}{\sigma}) \ge$

9. $\gamma = P(X \le 0.$

$Y_n \sim B(n,$

$P(Y_n = m) =$

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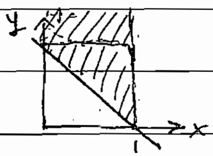
6

$\frac{1}{3}$

3

$\frac{1}{8}$

$-y \le X)$



<1

1

≤ 0

≥ 1

<1

≤ 0

$y<1$

它

$Y,\ (\ln Y)' = \frac{1}{Y}$

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(2) $(-2\ln X)' = -\frac{2}{X} < 0.\quad X = e^{-\frac{Y}{2}},\ (e^{-\frac{Y}{2}})' = -\frac{1}{2}e^{-\frac{Y}{2}}.$

$$f_Y(y) = \begin{cases} \frac{1}{2}e^{-\frac{y}{2}} & 0<y<+\infty \\ 0 & \text{其它} \end{cases}$$

4. $(e^x)' = e^x > 0,\quad X = \ln Y\ \ (\ln Y)' = \frac{1}{Y}$

$$f_Y(y) = \begin{cases} 0 & y<1 \\ e^{-\ln y}\cdot\frac{1}{y} & y\ge 1 \end{cases} = \begin{cases} 0 & y<1 \\ \frac{1}{y^2} & y\ge 1 \end{cases}$$

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习题2.

1. (1) $X=1,2,3,4$

$P(X=1)=\frac{10}{13}$

$P(X=2)=\frac{P_3^1 P_{10}^1}{P_{13}^2}=\frac{5}{26}$

$P(X=3)=\frac{P_3^2 P_{10}^1}{P_{13}^3}=\frac{5}{143}$

$P(X=4)=\frac{P_3^3 \cdot P_{10}^1}{P_{13}^4}=\frac{1}{286}$

X	1	2	3	4
P	$\frac{10}{13}$	$\frac{5}{26}$	$\frac{5}{143}$	$\frac{1}{286}$

(2) $X=1,2,3,4$

$P(X=1)=\frac{10}{13}$

$P(X=2)=P(\bar{A}_1 A_2)=\frac{3}{13}\cdot\frac{11}{13}=P(\bar{A}_1)P(A_2|\bar{A}_1)=\frac{33}{169}$

$P(X=3)=P(\bar{A}_1\bar{A}_2A_3)=\frac{3}{13}\cdot\frac{2}{13}\cdot\frac{12}{13}=\frac{72}{2197}$

$P(X=4)=P(\bar{A}_1\bar{A}_2\bar{A}_3A_4)=\frac{3}{13}\cdot\frac{2}{13}\cdot\frac{1}{13}\cdot\frac{13}{13}=\frac{6}{2197}$

X	1	2	3	4
P	$\frac{10}{13}$	$\frac{33}{169}$	$\frac{72}{2197}$	$\frac{6}{2197}$

2.

X	1	2	3	4	5	0
P	0.08	0.23	0.34	0.26	0.08	0.01

$P(X=k)=C_5^k p^k (1-p)^{5-k}$

3. $P(X\le 3)=P(X=0)+P(X=1)+P(X=2)+P(X=3)$

$=0.98^{100}+0.02\times 0.98^{99}+0.02^2\times 0.98^{98}+0.02^3\times 0.98^{97}$

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4. (1) $P(X=Y)=$

$=\sum_{k=0}^{8} C_3^k (0.6)$

$=\sum_{k=0}^{3}(C_3^k)^2($

$=0.001728+$

$=0.32076$

(2) $P(Y<X)=$

$=C_3^0(0.7)^0(0.3)$

$+C_3^1(0.7)^1($

$+C_3^2(0.7)^2($

$=0.025272+$

$=0.243$

5. (1) $\frac{1}{C_8^4}=\frac{1}{70}$

(2) X: 10次

~~$P(X\ge 3)$~~ $P($

$P(X\ge 3)=1-$

$=1-$

$=3.$

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$= 2$

-2

$e^{-\frac{3}{2}}$

$= 1 - e^{-\frac{5}{2}}$.

0

()

$x \le 4$

x

(3) $P(1 \le X < \frac{7}{2}) = F(\frac{7}{2}) - F(1) = -3 + 2 \times \frac{7}{2} - \frac{1}{4} \times (\frac{7}{2})^2 - \frac{1}{12} = \frac{42}{48} = \frac{7}{8}$

10. (1) $\alpha = P(X < 200 \mid X > 150)$

$= \dfrac{P(150 < X < 200)}{P(X > 150)} = \dfrac{\int_{150}^{200} \frac{100}{x^2} dx}{\int_{150}^{\infty} \frac{100}{x^2} dx} = \dfrac{-\frac{100}{x}\Big|_{150}^{200}}{-\frac{100}{x}\Big|_{150}^{\infty}} = \frac{1}{4}$

(2) $p = \int_{0}^{150} \frac{100}{x^2} dx = -\frac{100}{x}\Big|_{100}^{150} = 1 - \frac{2}{3} = \frac{1}{3}$

X：使用150h应坏的个数．$X \sim B(3, \frac{1}{3})$

$\beta = P(X = 1) = C_3^1 \frac{1}{3} \cdot (\frac{2}{3})^2 = \frac{4}{9}$.

11. $\Delta = (4k)^2 - 4 \times 4 \times (k+2)$

$= 16(k-2)(k+1) \ge 0$.

$\therefore k \ge 2$.

$P(K \ge 2) = \frac{3}{5}$.

12. $P(|X| < \lambda) = \Phi(\lambda) - \Phi(-\lambda) = 2\Phi(\lambda) - 1 = 0.9$

$\Phi(\lambda) = 0.95 \therefore \lambda = 1.65$.

13. (1) ① $P(2 < X \le 5) = P(-\frac{1}{2} < \frac{X-3}{2} \le 1) = \Phi(1) - \Phi(-\frac{1}{2})$

$= \Phi(1) + \Phi(\frac{1}{2}) - 1 = 0.8413 + 0.6915 - 1 = 0.5328$

② $P(-4 < X \le 10) = P(|\frac{X-3}{2}| < \frac{7}{2}) = 2\Phi(\frac{7}{2}) - 1 = 1$

$\cancel{P(|X| > 2) = P(\frac{X-3}{2}}$

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③ $P(|X|>2) = 1-P(-2<X<2) = 1-P\left(-\frac{5}{2}<\frac{X-3}{2}<-\frac{1}{2}\right)$

$= 1-\left[1-\Phi\left(\frac{1}{2}\right)-\left(1-\Phi\left(\frac{5}{2}\right)\right)\right] = 1-\left[\Phi\left(\frac{5}{2}\right)-\Phi\left(\frac{1}{2}\right)\right]$

$= 1-[0.9938-0.6915] = 0.6977$

④ $P(X>3) = P\left(\frac{X-3}{2}>0\right) = 0.5$

(2) $c = \mu = 3$.

14. ~~$P\left(\left|\frac{X-20}{40}\right|\le 30\right)$~~ ~~$p = 2\int_0^{30} \frac{1}{40\sqrt{2\pi}} e^{-\frac{(X-20)^2}{3200}} dx =$~~

$p = P(-30<X<30) = P\left(-\frac{5}{4}<\frac{X-20}{40}<\frac{1}{4}\right)$

$= \Phi\left(\frac{1}{4}\right)-\left[1-\Phi\left(\frac{5}{4}\right)\right] = \Phi\left(\frac{1}{4}\right)+\Phi\left(\frac{5}{4}\right)-1$

$= 0.5987+0.8944-1 = 0.4931$

X：误差绝对值不超过30m的次数 $\sim B(3,p)$

$P(X\ge 1) = 1-P(X=0)$

$= 1-C_3^0(0.4931)^0(1-0.4931)^3$

$= 1-0.13 = 0.87$.

15. 求 $F_Y(y)$： $y\le 0$, $F_Y(y)=0$.

$y>0$: $F_Y(y) = P(Y\le y) = P(|X|\le y^2)$

$= 2\Phi(y^2)-1$

$f_Y(y) = F_Y'(y) = \begin{cases} 4y\varphi(y^2) & y>0 \\ 0 & y\le 0 \end{cases} = \begin{cases} \frac{4y}{\sqrt{2\pi}} e^{-\frac{y^4}{2}} & y>0 \\ 0 & y\le 0 \end{cases}$

16. $F_Y(y)$: ...

∴ $f_Y(y) = F_Y'(y)$ ...

17. $A_1 = \{X\le 20$ ...

$p_1 = P(A_1) =$ ...

$= P\left(\frac{X-22}{\ldots}\right.$ ...

$p_3 = 0.2119$

$p_2 = 1-p_1-$ ...

(1) B：元件 ...

$P(B) = \sum_{i=1}^{3}$ ...

$=$ ...

$=$ ...

(2) $P(A_2|B) =$ ...

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$b \le X \le 19.6)$

5

$1 - P(X=0) - P(X=1) - P(X=2)$

$- 18.5 \cdot e^{-t}$

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习题 3.1

1. (1) X：红球； Y：白球.

X \ Y	0	1	2
0	0	$\frac{2}{15}$	$\frac{3}{15}$
1	$\frac{1}{15}$	$\frac{6}{15}$	$\frac{3}{15}$

$P(X=0, Y=1) = \frac{C_2^1 C_3^3}{C_6^4} = \frac{2}{15}$.

(2) $P(|X-Y|=1) = P_{01} + P_{12} + P_{10} = \frac{2}{15} + \frac{3}{15} + \frac{1}{15} = \frac{6}{15}$

2. (1) $\int_{0}^{\infty} \left(\int_0^{\infty} c e^{-(3x+4y)} dx \right) dy$

$= c \int_0^{\infty} e^{-3x} dx \int_0^{\infty} e^{-4y} dy$

$= c \cdot \left(-\frac{1}{3} e^{-3x} \Big|_0^{\infty}\right)\left(-\frac{1}{4} e^{-4y} \Big|_0^{\infty}\right) = \frac{c}{12} = 1$

$\therefore c = 12$

(2) $P(0 \le X < 1, 0 \le Y < 2)$

$= 12 \int_0^1 e^{-3x} dx \int_0^2 e^{-4y} dy$

$= 12\left(-\frac{1}{3} e^{-3x} \Big|_0^1\right) \cdot \left(-\frac{1}{4} e^{-4y} \Big|_0^2\right) = (1-e^{-3})(1-e^{-8})$

(3) $F(x,y) = \begin{cases} (1-e^{-3x})(1-e^{-4y}) & x, y > 0 \\ 0 & \text{其它} \end{cases}$

3. $F(-1, 0) = P(X \le -1, Y \le 0) = 0$

$F(0.2, 1.5) = P(X \le 0.2, Y \le 1.5) = P_{01} = \frac{2}{15}$

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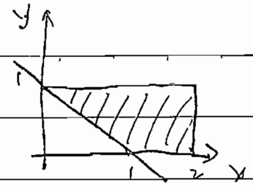
4. $P(X+Y>1)$

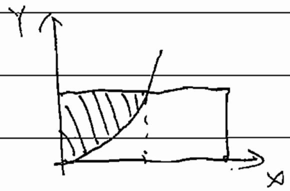


$= \int_0^1 dy \int_{1-y}^{2} \frac{1}{2}\, dx$

$= \int_0^1 \frac{1}{2}(1+y)\, dy$

$= \frac{1}{2}(y+\frac{y^2}{2})\Big|_0^1 = \frac{3}{4}$

$P(X^2<Y)$



$= \int_0^1 dx \int_{x^2}^{1} \frac{1}{2}\, dy$

$= \int_0^1 \frac{1}{2}(1-x^2)\, dx$

$= \frac{1}{2}(x-\frac{x^3}{3})\Big|_0^1 = \frac{1}{3}$

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习题3.2

1. X | 0

$p_{i\cdot}$ | $\frac{1}{3}$

2. $f_X(x) = \int_{-\infty}^{\infty}$

$= 2.$

$f_Y(y) = \int$

$= 4.$

3. $f_X(x) = \int$

$= e$

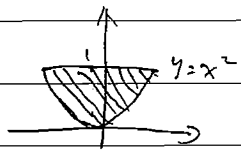
$f_Y(y) = \int$

$= e$

$/P(Y=2) = \frac{3}{15} / (\frac{3}{15}+\frac{3}{15}) = \frac{1}{2}$

$(X=0) = \frac{3}{15} / (\frac{2}{15}+\frac{3}{15}) = \frac{2}{5}$.

$X=1$



$-\leq x \leq 1$

$0 < y \leq 1$

$f_{X|Y}(x|y) = \begin{cases} \frac{3}{2}x^2 y^{-\frac{3}{2}}, & -\sqrt{y} \leq x \leq \sqrt{y}\ (0 < y \leq 1) \\ 0 & 其它 \end{cases}$

$)$

$f_{Y|X}(y|x) = \begin{cases} \frac{2y}{1-x^4} & x^2 \leq y \leq 1\ (-1 < x < 1) \\ 0 & 其它 \end{cases}$

$- = \frac{2y}{1-x^4},\quad x^2 \leq y \leq 1,\ -1 \leq x \leq 1$

$y\,dy$

$\Big|_{\frac{3}{4}}^{1} = \frac{7}{15}$

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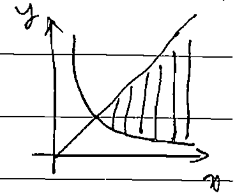
3.

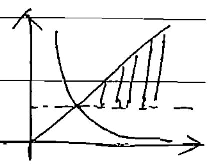
$f_X(x) = \begin{cases} \int_{\frac{1}{x}}^{x} \frac{1}{2x^2 y} dy & x > 1 \\ 0 & \text{else} \end{cases}$



$= \begin{cases} \frac{\ln x}{x^2} & x > 1 \\ 0 & \text{else} \end{cases}$

$f_Y(y) = \begin{cases} \int_{y}^{\infty} \frac{1}{2x^2 y} dx & y \geq 1 \\ \int_{\frac{1}{y}}^{\infty} \frac{1}{2x^2 y} dx & 0 < y < 1 \\ 0 & \text{else} \end{cases}$

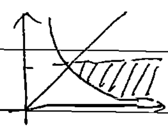
$= \begin{cases} \frac{1}{2y^2} & y \geq 1 \\ \frac{1}{2} & 0 < y < 1 \\ 0 & \text{else} \end{cases}$



(1) $y \geq 1$

$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} = \begin{cases} \frac{y}{x^2}, & y < x < +\infty \\ 0, & \text{else}. \end{cases}$

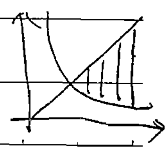
(2) $0 < y < y$



$f_{X|Y}(x|y) = \begin{cases} \frac{1}{x^2 y} & \frac{1}{y} < x < +\infty \\ 0 & \text{else}. \end{cases}$

(3) $x > 1$

$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \begin{cases} \frac{1}{2y\ln x} & \frac{1}{x} < y < x \\ 0 & \text{else} \end{cases}$



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习题3.4

1. $f(x,y)=\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2-2\rho\left(\frac{x-\mu_1}{\sigma_1}\right)\left(\frac{y-\mu_2}{\sigma_2}\right)+\left(\frac{y-\mu_2}{\sigma_2}\right)^2\right]\right\}$

$f_X(x)=\frac{1}{\sqrt{2\pi}\sigma_1}\exp\left\{-\frac{1}{2}\left(\frac{x-\mu_1}{\sigma_1}\right)^2\right\}$, $f_Y(y)=\frac{1}{\sqrt{2\pi}\sigma_2}\exp\left\{-\frac{1}{2}\left(\frac{y-\mu_2}{\sigma_2}\right)^2\right\}$

(1) 充分性.

若 $\rho=0$, 则 $f(x,y)=f_X(x)\cdot f_Y(y)$, 显然 X和Y独立.

(2) 必要性

若X和Y独立, 即有 $f(x,y)=f_X(x)\cdot f_Y(y)$, (*)

令 $x=\mu_1$, $y=\mu_2$, 则由(*)式, 有 $\frac{1}{\sqrt{1-\rho^2}}=1$, $\therefore \rho=0$.

2.

Y\X	~~x_1~~	~~x_2~~	
y_1	① $\frac{1}{6}-\frac{1}{8}=\frac{1}{24}$	$\frac{1}{8}$	$\frac{1}{6}$
y_2	$\frac{1}{8}$	⑦ $\frac{7}{40}$	⑤ $\frac{1}{8}\div\frac{5}{12}=\frac{3}{10}$
y_3	② $\frac{1}{24}\div\frac{1}{6}=\frac{1}{4}$	⑧ $\frac{17}{60}$	⑥ $\frac{8}{15}$
	③ $\frac{5}{12}$	④ $\frac{7}{12}$	1

Y\X	x_1	x_2	
y_1	① $\frac{1}{6}-\frac{1}{8}=\frac{1}{24}$	$\frac{1}{8}$	$\frac{1}{6}$
y_2	$\frac{1}{8}$	⑦ $\frac{1}{2}-\frac{1}{8}=\frac{3}{8}$	⑤ $\frac{1}{8}\div\frac{1}{4}=\frac{1}{2}$
y_3	⑧ $\frac{1}{4}-\frac{1}{24}-\frac{1}{8}=\frac{1}{12}$	⑧ $\frac{1}{3}-\frac{1}{12}=\frac{1}{4}$	⑥ $1-\frac{1}{2}-\frac{1}{6}=\frac{1}{3}$
	② $\frac{1}{24}\div\frac{1}{6}=\frac{1}{4}$	④ $1-\frac{1}{4}=\frac{3}{4}$	1

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3. $F(x,y)=$

$=($

$=($

$f(x,y)=$

$\therefore$ X和Y

4.

Y\X	
0	
1	
2	

$\because P_{1\cdot}\cdot P_{\cdot 1}$

5. $f_X(x)=\begin{cases}\int_x^{2-x} \\ 0\end{cases}$

$f_Y(y)=\begin{cases}\int_0^{2-y} \\ \int_0^{y} \\ 0\end{cases}$

$\therefore f_X(x)\cdot f_Y$

习题3.5

1. $X=-1,2;\quad Y=0,1,2.$

(1) $2X+Y=-2,-1,0;4,5,6.$

$2X+Y$	$\overset{(-1,0)}{-2}$	$\overset{(-1,1)}{-1}$	$\overset{(-1,2)}{0}$	$\overset{(2,0)}{4}$	$\overset{(2,1)}{5}$	$\overset{(2,2)}{6}$
p_k	0.1	0.2	0.1	0.2	0.1	0.3

(2) $XY+1=-1,0,1,3,5$

$XY+1$	$\overset{(-1,2)}{-1}$	$\overset{(-1,1)}{0}$	$\overset{(-1,0)(2,0)}{1}$	$\overset{(2,1)}{3}$	$\overset{(2,2)}{5}$
p_k	0.1	0.2	0.3	0.1	0.3

(3) $\max\{X,Y\}=0,1,2$

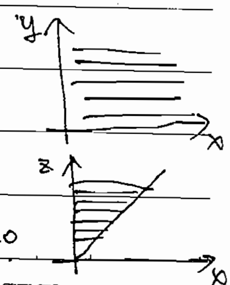
$\max(X,Y)$	$\overset{(-1,0)}{0}$	$\overset{(-1,1)}{1}$	$\overset{(-1,2)(2,0),(2,1),(2,2)}{2}$
p_k	0.1	0.2	0.7

2. 证：$X\sim P(\lambda_1)$，$Y\sim P(\lambda_2)$，$Z=X+Y$

$$P(Z=k)=\sum_{i=0}^{k}P(X=i,Y=k-i)=\sum_{i=0}^{k}P(X=i)P(Y=k-i)$$
$$=\sum_{i=0}^{k}e^{-\lambda_1}\frac{\lambda_1^i}{i!}\cdot e^{-\lambda_2}\cdot\frac{\lambda_2^{k-i}}{(k-i)!}=\frac{e^{-(\lambda_1+\lambda_2)}}{k!}\sum_{i=0}^{k}\frac{k!}{i!(k-i)!}\lambda_1^i\lambda_2^{k-i}$$
$$=\frac{e^{-(\lambda_1+\lambda_2)}}{k!}(\lambda_1+\lambda_2)^k\sim P(\lambda_1+\lambda_2).$$

3. 由卷积公式 $\begin{cases}x\geq 0\\ z-x\geq 0\end{cases}\Rightarrow\begin{cases}z\geq 0\\ z\geq x\end{cases}$

$$f_Z(z)=\int_{-\infty}^{\infty}f_X(x)f_Y(z-x)dx=\int_0^z e^{-x}\cdot\frac12 e^{-\frac{z-x}{2}}dx$$
$$=\int_0^z\frac12 e^{-\frac{z+x}{2}}dx=-e^{-\frac{z+x}{2}}\Big|_0^z=e^{-\frac{z}{2}}-e^{-z},\quad z\geq 0$$



$X_1=-1)\cdot P(X_2=0)=\frac14\cdot\frac12=\frac18$

	$\frac12$
0	$\frac12$
	1

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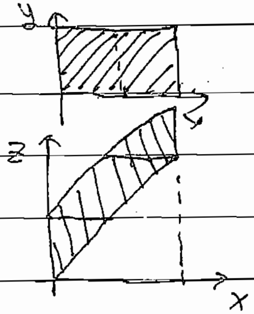
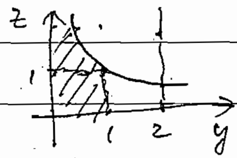
4. $\begin{cases} 0 \le x \le 2 \\ 0 \le z-x \le 1 \end{cases} \Rightarrow \begin{cases} 0 \le x \le 2 \\ z-1 \le x \le z \end{cases}$



$$f_Z(z) = \begin{cases} \int_0^z \frac{1}{3}(x+(z-x))\,dx & 0 \le z \le 1 \\ \int_{z-1}^z \frac{1}{3}(x+(z-x))\,dx & 1 \le z \le 2 \\ \int_{z-1}^{2} \frac{1}{3}(x+(z-x))\,dx & 2 \le z \le 3 \end{cases}$$

$$= \begin{cases} \frac{1}{3}z^2 & 0 \le z < 1 \\ \frac{1}{3}z & 1 \le z \le 2 \\ \frac{1}{3}z(3-z) & 2 \le z \le 3 \\ 0 & \text{else} \end{cases}$$

5. $\begin{cases} 0 \le y \le 1 \\ 0 \le yz \le 1 \end{cases} \Rightarrow \begin{cases} 0 \le y \le 1 \\ 0 \le y \le \frac{1}{z} \end{cases} z > 0$ 或 $\begin{cases} 0 \le y \le 1 \\ \frac{1}{z} \le y \le 0 \end{cases} z < 0$ (舍去)



$$f_Z(z) = \int_{-\infty}^{\infty} |y|\, f_X(yz) \cdot f_Y(y)\,dy$$

$$= \begin{cases} \int_0^{\frac{1}{z}} y \cdot dy & z \ge 1 \\ \int_0^1 y\,dy & 0 < z < 1 \\ 0 & \text{else} \end{cases}$$

$$= \begin{cases} \frac{1}{2z^2} & z \ge 1 \\ \frac{1}{2} & 0 < z < 1 \\ 0 & \text{else} \end{cases}$$

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6. $F_T(t) = \begin{cases} 1- \\ 0 \end{cases}$

(1) 并联: M

$F_M(t) =$

$f_M(t) =$

$P(M > 1.2)$

(2) 串联: $N =$

$F_N(t) = 1-$

$f_N(t) =$

$P(M > 1.2)$

7. $F_Z(z) = P($

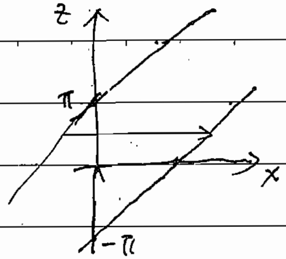
$= P($

$= P($

$= \int_0^z$

0

$x \le z+\pi$



dx

$\Phi\left(\frac{z-\pi-\mu}{\sigma}\right)\Big]$.

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习题3

1.

$Y \backslash X$	0	1	2	3	
1	0	$\frac{3}{8}$	$\frac{3}{8}$	0	$\frac{6}{8}$
3	$\frac{1}{8}$	0	0	$\frac{1}{8}$	$\frac{2}{8}$
	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	

2. $f(x,y)=\frac{\partial F(x,y)}{\partial x\,\partial y}=\begin{cases}\cos x\cos y & 0\le x\le\frac{\pi}{2},\ 0\le y\le\frac{\pi}{2}\\ 0 & \text{else}\end{cases}$

$P(0<x\le\frac{\pi}{4},\frac{\pi}{6}<y\le\frac{\pi}{3})=\int_0^{\frac{\pi}{4}}dx\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}f(x,y)\,dy=\int_0^{\frac{\pi}{4}}\cos x\,dx\cdot\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\cos y\,dy$

$=\sin x\Big|_0^{\frac{\pi}{4}}\cdot\sin y\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}=\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right).$

3. (1)

$Y \backslash X$	2	5	8	
0.4	0.15	0.30	0.35	0.8
0.8	0.05	0.12	0.03	0.2
	0.20	0.42	0.38	1

(2) $\because P_{11}=0.15 \ne P_{1\cdot}\cdot P_{\cdot 1}=0.8\times0.20=0.16$.

$\therefore$ X与Y不独立.

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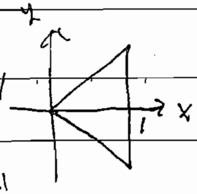
4. (1) $f_X(x)=\int_{-\infty}^{\infty} f(x,y)\,dy=\int_{-x}^{x} dy=2x,\quad 0\leq x\leq 1$



$$f_Y(y)=\int_{-\infty}^{\infty} f(x,y)\,dx=\begin{cases}\int_y^1 dx=1-y, & 0\leq y\leq 1\\ \int_{-y}^1 =1+y & -1\leq y\leq 0\end{cases}$$

① 当 $0\leq x\leq 1$ 时,

$$f_{Y|X}(y|x)=\frac{f(x,y)}{f_X(x)}=\begin{cases}\frac{1}{2x} & -x<y<x\\ 0 & \text{else}\end{cases}$$

② 当 $-1\leq y\leq 1$ 时

$$f_{X|Y}(x|y)=\frac{f(x,y)}{f_Y(y)}=\begin{cases}\frac{1}{1-|y|} & |y|<x<1\\ 0 & \text{else}\end{cases}$$

(2) $\because f(x,y)\neq f_X(x)\cdot f_Y(y)$ $\therefore X$ 与 Y 不独立

5. (1) $f_X(x)=\int_0^1 6xy(2-x-y)\,dy=3x(2-x)y^2\Big|_0^1-2xy^3\Big|_0^1$

$=3x(2-x)-2x=4x-3x^2,\quad 0\leq x\leq 1.$

$f_Y(y)=\int_0^1 6xy(2-x-y)\,dx=4y-3y^2,\quad 0\leq y\leq 1$

$\because f(x,y)\neq f_X(x)\cdot f_Y(y)$ $\therefore X$ 与 Y 不独立

6. (1) $f(x,y)=f_X(x)\cdot f_Y(y)=\begin{cases}\frac{1}{5}e^{-\frac{y}{5}}, & 0\leq x\leq 1,\ y>0\\ 0 & \text{else}\end{cases}$

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(2) $P(y\leq x)=$

$=\int$

$=$

7. (1) $f(x,y)=\{$

$f_X(x)=\int$

$=\{$

$=\{$

0

对 $-2<x<$

$f_{Y|X}(y|x)$

(2) $P(Y>0|X=$

$=\int_0^{min}$

2	
$\frac{1}{16}$	$\frac{9}{16}$
0	$\frac{6}{16}$
0	$\frac{1}{16}$
$\frac{1}{16}$	

C_2^1 —→ 剩下两部留取一个

从封信取一封

$= \frac{6}{11}$

$= \frac{36}{49}$

$\frac{396}{539}$
$\frac{81}{539}$
$\frac{44}{539}$
1

X+Y:	0	-1	-2
	1	0	-1
	2	1	0

2

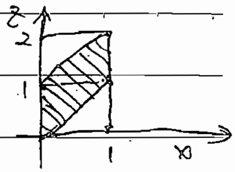
$\frac{72}{539}$

斜线上的数之和.

10. 由卷积公式.

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x)\cdot f_Y(z-x)\,dx$$

$\begin{cases} 0 \le x \le 1 \\ 0 \le z-x \le 1 \end{cases} \Rightarrow \begin{cases} 0 \le x \le 1 \\ z-1 \le x \le z \end{cases}$



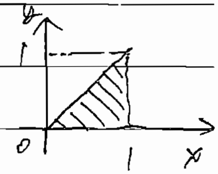
$$= \begin{cases} \int_0^z 6x(1-x)\,dx & 0 \le z \le 1 \\ \int_{z-1}^1 6x(1-x)\,dx & 1 \le z \le 2 \\ 0 & \text{else} \end{cases}$$

$$= \begin{cases} 3z^2 - 2z^3 & 0 \le z < 1 \\ 1-3(z-1)^2+2(z-1)^3 & 1 \le z \le 2 \\ 0 & \text{else} \end{cases}$$

11. (1). $f_X(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & \text{else} \end{cases}$ 已知 $f_X(x)$, $f_{Y|X}(y|x)$.

$$f_{Y|X}(y|x) = \begin{cases} \frac{1}{x} & 0<y<x \\ 0 & \text{else} \end{cases}$$ 对于 $x \in (0,1)$.

$$f(x,y) = f_X(x)\cdot f_{Y|X}(y|x)$$
$$= \begin{cases} \frac{1}{x} & 0<y<x<1 \\ 0 & \text{else} \end{cases}$$



(2) $f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \begin{cases} \int_y^1 \frac{1}{x}\,dx & 0<y<1 \\ 0 & \text{else.} \end{cases}$

$$= \begin{cases} -\ln y & 0<y<1 \\ 0 & \text{else.} \end{cases}$$

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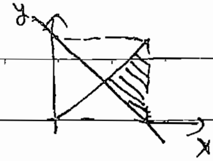
(3) $P(X+Y>1)$



$= \int_{\frac{1}{2}}^{1} dx \int_{1-x}^{x} \frac{1}{x} dy$

$= \int_{\frac{1}{2}}^{1} \frac{1}{x}(2x-1) dx = 1 - \ln 2$.

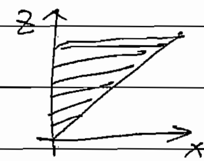
12. (1) $f_X(x) = \int_0^{\infty} \frac{1}{2}(x+y) e^{-(x+y)} dy \overset{t=x+y}{=} \int_x^{\infty} \frac{t}{2} e^{-t} dt$

$= \begin{cases} \frac{1}{2}(x-1)e^{-x}, & x>0 \\ 0 & \text{else} \end{cases}$

$f_Y(y) = \begin{cases} \frac{1}{2}(y-1)e^{-y}, & y>0 \\ 0 & \text{else} \end{cases}$

$\because f_X(x) \cdot f_Y(y) \neq f(x,y)$, $\therefore$ X与Y不独立.

(2) $f_Z(z) = \int_{-\infty}^{\infty} f(x, z-x) dx = \int_0^{z} \frac{1}{2}(x+z-x) e^{-(x+z-x)} dx$

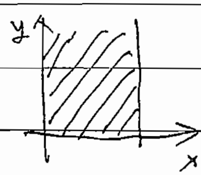


$= \begin{cases} \frac{1}{2} z^2 e^{-z} & z>0 \\ 0 & z \le 0 \end{cases}$

13. (1) $1 = \int_0^{\infty} dy \int_0^1 b e^{-(x+y)} dx$

$= b e^{-x}\Big|_0^1 \cdot e^{-y}\Big|_0^{\infty} = b(1-\frac{1}{e})$ $\therefore b = \frac{e}{e-1}$

(2). $f_X(x) = \int_0^{\infty} \frac{e}{e-1} e^{-x} \cdot e^{-y} dy$



$= \begin{cases} \frac{e}{e-1} e^{-x} & x>0 \\ 0 & \text{else} \end{cases}$

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$f_Y(y) = \int_0^1 \frac{e}{e-}$

(3) $\because$ X与Y独立

$\therefore F_U(u) =$

$=$

14. $\cancel{Y = g(X)} =$

$X = h(Y) =$

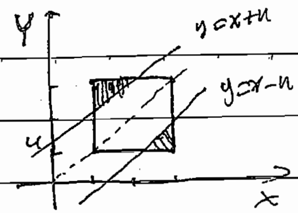
$f_Y(y) = f_Y($

X, Y 独

$f_Z(z) = \int_{-\infty}^{\infty}$

$= e^{-}$

$\therefore f_Z(z)$



$\frac{1}{4}dx \left(-\int_1^{3-u} dy \int_{y+u}^{3} \frac{1}{4} dx\right)$

$u-1)\,dy \left(-\int_1^{3-u} \frac{1}{4}(3-y-u)\,dy\right)$

$u)$

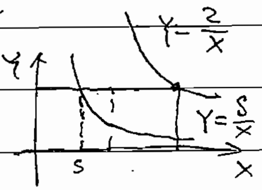
$u \le 0$

$0 \le u \le 2$

$u \ge 2$

$u+1 \quad 0<u<2$

else



$\int_{s}^{2} dx \int_0^{\frac{s}{x}} \frac{1}{2}\, dy$

$$F_S(s) = \begin{cases} 0 & s<0 \\ \frac{s}{2}\left(1-\ln\frac{s}{2}\right) & 0<s<2 \\ 1 & s \ge 2 \end{cases}$$

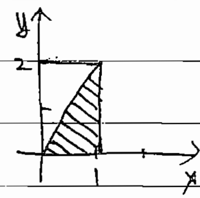
$$\therefore f_S(s) = \begin{cases} \frac{1}{2}\left(\ln\frac{2}{s}\right) & 0<s<2 \\ 0 & \text{else.} \end{cases}$$

17.(1) $P(Y=m \mid X=n) = C_n^m p^m (1-p)^{n-m}$, $m=0,1,\cdots,n;\ n=1,2,\cdots$

(2) $P(Y=m, X=n) = P(Y=m \mid X=n)\cdot P(X=n)$

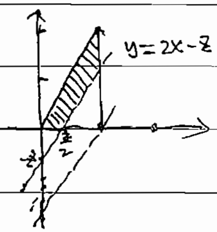
$= C_n^m p^m (1-p)^{n-m} \cdot e^{-\lambda}\frac{\lambda^n}{n!}$, $m=0,1,\cdots,n;\ n=1,2,\cdots$

18. (1) $f_X(x) = \begin{cases} \int_0^{2x} dy = 2x. & 0<x<1 \\ 0 & \text{else} \end{cases}$



$f_Y(y) = \begin{cases} \int_{\frac{y}{2}}^{1} dx = 1-\frac{y}{2} & 0<y<2 \\ 0 & \text{else} \end{cases}$

(2) ① $z \le 0$, $P(2X-Y \le z) = 0$



② $0<z<2$, $P(2X-Y \le z)$

$= 1 - P(2X-Y > z)$

$= 1 - \int_{\frac{z}{2}}^{1} dx \int_0^{2x-z} dy$

$= 1 - \left(1-z+\frac{z^2}{4}\right) = z - \frac{z^2}{4}$

③ $z \ge 2$. $P(2X-Y \le z) = 1$

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$$\therefore F_Z(z)=\begin{cases}0 & z<0\\ z-\frac{z^2}{4} & 0<z<2\\ 1 & z\geq 2\end{cases}$$

$$f_Z(z)=\begin{cases}1-\frac{z}{2} & 0<z<2\\ 0 & \text{else}\end{cases}$$

19. (1) $y\leq 0,\ F_Y(y)=0,\quad y\geq 4,\ F_Y(y)=1$

$0<y<4.$

$$F_Y(y)=P(X^2\leq y)=P(-\sqrt{y}\leq X\leq\sqrt{y})$$

$$=\int_{\max(-1,-\sqrt{y})}^{0}\frac{1}{2}dx+\int_0^{\min(\sqrt{y},2)}\frac{1}{4}dx$$

$$=\begin{cases}\int_{-\sqrt{y}}^{0}\frac{1}{2}dx+\int_0^{\sqrt{y}}\frac{1}{4}dx & 0<y<1\\ \int_{-1}^{0}\frac{1}{2}dx+\int_0^{\sqrt{y}}\frac{1}{4}dx & 1\leq y<4\end{cases}$$

$$=\begin{cases}\frac{3}{4}\sqrt{y} & 0<y<1\\ \frac{1}{2}+\frac{1}{4}\sqrt{y} & 1\leq y<4.\end{cases}$$

$$\therefore f_Y(y)=\begin{cases}\frac{3}{8\sqrt{y}} & 0<y<1\\ \frac{1}{8\sqrt{y}} & 1\leq y<4\\ 0 & \text{else}\end{cases}$$

(2) $F(-\frac{1}{2},4)=F(-\frac{1}{2},+\infty)=F_X(-\frac{1}{2})=\int_{-1}^{-\frac{1}{2}}\frac{1}{2}dx$

$=\frac{1}{4}$

20. (1) $P(X>2Y$

(2) $\begin{cases}0<x<\\ 0<z-x\end{cases}$

$f_Z(z)=\int_{-\infty}^{\infty}$

$=\begin{cases}\\ \\ \end{cases}$

$=\begin{cases}(\\ z\\ \end{cases}$

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$\cdot 3) = -0.2$

8.

$\cdot 3) = 13.4$

数. $\sim B(10, 0.1)$

.k

$. \sim B(4, p)$

$\leq 1) = 1 - P(Y>0) - P(Y=1)$

$_{10}(0.1)^1(0.9)^9$

$= 0.2639$

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3. X：~~每人命~~ 每人 射击的发数 ~ 几何分布.

$X = 1, 2, 3, 4$. ~~用4表示其它情形~~

X	1	2	3	4	~~4~~
p	0.8	0.16	0.032	~~0.008~~ 0.008	~~0.002~~

$EX = 1\times0.8 + 2\times0.16 + 3\times0.032 + 4\times0.008 = 1.25$

∴ 应准备 $100\times(EX) = 100\times1.25 = 125$ 发子弹.

4. $E(X) = \int_{-\infty}^{\infty} x \frac{1}{2\lambda} e^{-\frac{|x-\mu|}{\lambda}} dx$ $\quad y = x - \mu$

$x = y + \mu$

$= \int_{-\infty}^{\infty} (y+\mu) \frac{1}{2\lambda} e^{-\frac{|y|}{\lambda}} dy$

$= \underbrace{\int_{-\infty}^{\infty} y \frac{1}{2\lambda} e^{-\frac{|y|}{\lambda}} dy}_{=0} + \int_{-\infty}^{\infty} \frac{\mu}{2\lambda} e^{-\frac{|y|}{\lambda}} dy$

$= 2\int_{0}^{\infty} \frac{\mu}{2\lambda} e^{-\frac{|y|}{\lambda}} dy = \mu\left(-e^{-\frac{|y|}{\lambda}}\right)\Big|_0^{\infty} = \mu$

5. (1) $EY = E(2X) = \int_0^{\infty} 2x e^{-x} dx = -2xe^{-x}\Big|_0^{\infty} + \int_0^{\infty} 2e^{-x} dx$

$= -2e^{-x}\Big|_0^{\infty} = 2$.

(2) $EY = E(e^{-2X}) = \int_0^{\infty} e^{-2x} e^{-x} dx = \int_0^{\infty} e^{-3x} dx$

$= -\frac{1}{3}e^{-3x}\Big|_0^{\infty} = \frac{1}{3}$.

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6. $p = P(X \le 1) = \int_0^1 f(x)\,dx = \int_0^1 \frac{1}{4} e^{-\frac{x}{4}}\,dx = 1 - e^{-\frac{1}{4}}$

Y：工厂售出一个产品的净赢利.

Y	100	-300
p_k	$e^{-\frac{1}{4}}$	$1-e^{-\frac{1}{4}}$

$EY = 100 \times e^{-\frac{1}{4}} - 300(1-e^{-\frac{1}{4}}) = 400 \times e^{-\frac{1}{4}} - 300$

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习题4.2

1. (1) $EX = -2$

$E(X^2) = 4$

$\therefore DX = E$

(2) $D(\sqrt{10}X - 5$

2. $EX = 1 \times 0.4$

$DX = 5.3 -$

$EY = 1 \times 0.2$

$DY = 4.9 - 2$

$EX = EY$.

但 $DX \ne DY$

3. $EX = \int_{-\infty}^{\infty} \frac{1}{4}x$

$E(X^2) = \int_{-\infty}^{\infty}$

$DX = E($

(1) $D(2X-3) =$

(2) $D(X_1 + 2X_2$

4. 证明：$E[(X-a)^2]$

等号成立当且

10个全排列：P_{10}^{10} 分母

$=3'628'800$

0

00

$=453'600+30'240=483'840$

$P_2^2=635'040+211'680=846'720$

$\times 846'720=1693'440$

$P_{10}^{10}=7.875.$

$-62.0156=1.9761$

$=2EX+3EXEYEZ-EZ+5$

$-1.5+5=25.5$

$9DX+4DY+DZ$

$=52\frac{1}{12}$

$+cx)\Big|_0^1=\frac{a}{3}+\frac{b}{2}+c=1$

$x^3+\frac{cx^2}{2})\Big|_0^1=\frac{a}{4}+\frac{b}{3}+\frac{c}{2}=\frac{1}{2}$

$=0.15+(0.5)^2=\frac{2}{5}$

$c=3$

习题4.5.

1.

Y\X	1	2	3	
1	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$
2	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{3}$
3	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{3}$
	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1

$E(X)=E(Y)=(1+2+3)\frac{1}{3}=2$

XY	2	3	6
p_k	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

$E(XY)=(2+3+6)\frac{1}{3}=\frac{11}{3}$

$Cov(X,Y)=E(XY)-E(X)E(Y)=\frac{11}{3}-4=-\frac{1}{3}$

2. (1)

XY	-1	0	1
p_k	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$E(XY)=(-1)\times\frac{1}{4}+0\times\frac{1}{2}+1\times\frac{1}{4}=0$

又∵ $E(X)=E(Y)=0$.

∴ $Cov(X,Y)=E(XY)-E(X)E(Y)=0$.

∴ $\rho_{XY}=\frac{Cov(X,Y)}{\sqrt{DX}\cdot\sqrt{DY}}=0$.

(2) ∵ $P_{1\cdot}\cdot P_{\cdot 1}=\frac{3}{8}\times\frac{3}{8}=\frac{9}{64}\neq P_{11}=\frac{1}{8}$

∴ X与Y不独立

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3. 证明: $f_X(x) = \begin{cases} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{1}{\pi} dy = \frac{2}{\pi}\sqrt{1-x^2} & -1 \le x \le 1 \\ 0 & else \end{cases}$

$f_Y(y) = \begin{cases} \frac{2}{\pi}\sqrt{1-y^2} & -1 \le y \le 1 \\ 0 & else \end{cases}$

$\therefore E(X) = \int_{-1}^{1} x \frac{2}{\pi}\sqrt{1-x^2}\, dx = 0$. 同理 $E(Y) = 0$

另一方面, $E(XY) = \iint xy \frac{1}{\pi} dx\, dy$

$= \frac{1}{\pi}\int_{-1}^{1} x\, dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} y\, dy = 0$

$\therefore \rho_{XY} = 0$, X与Y不相关.

但 $f_X(x) \cdot f_Y(y) \ne f(x,y)$. $\therefore$ X与Y不独立.

4. 当 $x \in [0, \frac{\pi}{2}]$ 时.

$f_X(x) = \int_0^{\frac{\pi}{2}} \frac{1}{2}\sin(x+y)\, dy = -\frac{1}{2}\cos(x+y)\Big|_0^{\frac{\pi}{2}}$

$= \frac{1}{2}\left[\cos x - \cos(x+\frac{\pi}{2})\right] = \frac{1}{2}(\cos x + \sin x)$

$E(X) = \int_0^{\frac{\pi}{2}} x \cdot \frac{1}{2}(\cos x + \sin x)\, dx = \frac{\pi}{4}$

其中: $\int_0^{\frac{\pi}{2}} x\cos x\, dx = x\sin x\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x\, dx = \frac{\pi}{2} - 1$

$\int_0^{\frac{\pi}{2}} x\sin x\, dx = -x\cos x\Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x\, dx = 1$

$E(X^2) = \int_0^{\frac{\pi}{2}} x^2 \cdot \frac{1}{2}(\cos x + \sin x)\, dx = \frac{\pi^2}{8} + \frac{\pi}{2} - 2$

其中: $\int_0^{\frac{\pi}{2}} x^2\cos x\, dx = x^2\sin x\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2x\sin x\, dx = \frac{\pi^2}{4} - 2$

$\int_0^{\frac{\pi}{2}} x^2\sin x\, dx = -x^2\cos x\Big|_0^{\frac{\pi}{2}} + 2\int_0^{\frac{\pi}{2}} x\cos x\, dx = \pi - 2$

$\therefore DX = E(X^2) - (EX)^2 = \frac{\pi^2}{16} + \frac{\pi}{2} - 2$

同理 $DY = \frac{\pi^2}{16} + \frac{\pi}{2} - 2$

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$Cov(X,Y) = \iint (x - \ldots$

$= \int_0^{\frac{\pi}{2}} \frac{1}{2}(\ldots$

$= \int_0^{\frac{\pi}{2}} \frac{1}{2}(\ldots$

$= \frac{1}{2}(\frac{\pi}{4} \ldots$

$\therefore$ 协方差矩阵

$C = \begin{bmatrix} \frac{\pi}{16} \ldots \\ \frac{\pi}{2} \ldots \end{bmatrix}$

5. $DX_1 = D(\ldots$

$DY_1 = D(\ldots$

$Cov(X_1, Y\ldots$

$\therefore Cov(X_1 \ldots$

$(X$: 100次A发生频数$)$

$)$ ，(切比雪夫不等式)

$=0.75 \qquad P(A)\cdot(1-P(A)) \leq \frac{1}{4}$

$\cancel{(|X-100P(A)|<10)}$

$< \frac{10}{\sqrt{100P(A)(1-P(A))}})$

$\frac{1}{\sqrt{P(A)(1-P(A))}})$

$2-1 = 0.9544$

$\sim B(2000, \frac{1}{2})$

$\leq \frac{N-1000}{\sqrt{500}})$

$) \geq 0.99.$

$\geq 1052.$

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3. $P\left(\left|\frac{X}{6000} - \frac{1}{6}\right| < \varepsilon\right) = P\left(\left|X - 6000 \times \frac{1}{6}\right| \leq 6000\varepsilon\right)$

$= P\left(\left|\frac{X - 6000\times\frac{1}{6}}{\sqrt{6000\times\frac{1}{6}\times\frac{5}{6}}}\right| \leq \frac{6000\varepsilon}{\sqrt{6000\times\frac{1}{6}\times\frac{5}{6}}}\right)$

$\simeq 2\Phi\left(\frac{6000\varepsilon}{\sqrt{6000\times\frac{1}{6}\times\frac{5}{6}}}\right) - 1 \geq 0.99$

$\Phi\left(\frac{6000\varepsilon}{\sqrt{6000\times\frac{1}{6}\times\frac{5}{6}}}\right) \geq 0.995, \quad \frac{6000\varepsilon}{\sqrt{6000\times\frac{1}{6}\times\frac{5}{6}}} \geq 2.58$

$\therefore \varepsilon \geq \frac{2.58}{120\sqrt{3}} \simeq 0.00124$

4. $P(|\bar{X} - D| < 0.25) = P(|\sum X_i - nD| < 0.25n)$

$= P\left(\left|\frac{\sum X_i - nD}{\sqrt{D(\sum X_i)}}\right| < \frac{0.25n}{\sqrt{D(\sum X_i)}}\right) \simeq 2\Phi\left(\frac{0.25n}{2\sqrt{n}}\right) - 1 \geq 0.98.$

$\therefore \Phi(0.125\sqrt{n}) \geq 0.99, \quad 0.125\sqrt{n} \geq 2.34$

$\therefore n \geq 351$

5. X_i：第i页错误个数 $\sim \pi(0.2)$，$i = 1, \cdots, 400$

$P(0 \leq \sum X_i \leq 90) \simeq P\left(\frac{\sum X_i - n\lambda}{\sqrt{n\lambda}} \leq \frac{90 - n\lambda}{\sqrt{n\lambda}}\right) \qquad EX_i = DX_i = \lambda$

$\simeq \Phi\left(\frac{90 - n\lambda}{\sqrt{n\lambda}}\right) = \Phi\left(\frac{\sqrt{5}}{2}\right) = \Phi(1.12) = 0.8686.$

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