A Monte Carlo Simulation Of Clinic Waiting Time

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Introduction

Waiting is a pain no matter if you are waiting at a restaurant feeling hungry or sitting in the traffic feeling stuck. The waiting problem becomes more frustrating when someone is ill. During this COVID-19 pandemic, the waiting time can become extremely long when COVID-19 cases surge. In a report from 2018, it is said that the average waiting time for primary care is 18 minutes, 13 seconds. In my research, I created a hypothetical scenario that simulates a clinic with a waiting time of 18 minutes. I changed the frequency of different treatment times and the frequency of customer arrivals to observe how these two values affect the average waiting time at this hypothetical clinic. This research makes suggestions to doctors on how to manage the duration of each treatment on one customer and helps patients decide when they should arrive at a clinic to avoid waiting.

Details

In my simulation, the hypothetical clinic has 10 patient appointments every day. The dataset includes the data that describes each patient's visit for 10 years which is equal to 3650 days. There are 36,500 records in the dataset for simulation. The variables in the dataset are day (1-36,500), patient (1-10 for each day), arrival_time (customer arrival time in minutes), start_time (treatment start time in minutes), finish_time (treatment finish time in minutes), wait_time (patient wait time in minutes). There are two random values. One is arrival intervals which describes the interval in minutes between two customer arrivals or between the minute clinic opens and the first customer arrival. Another random value is treatment time which is the duration of each patient's treatment. Arrival_interval and treat_time are both random values from uniform distributions. There are three levels of arrival interval, which are 10, 20, and 30 minutes. Treat time also has three levels, which are also 10, 20, and 30 minutes. Table 1 presents the probability distribution of our baseline scenario. The probability of having 10, 20, and 30 minutes arrival intervals are respectively 50%, 20%, 30%. The probability of having 10, 20, and 30 minutes treat time are respectively 40%, 20%, 40%. This distribution of arrival interval and treat time gives an average mean waiting time of 18.31 minutes, which is equivalent to 18 minutes and 20 seconds. I got this mean waiting time by trying out different proportion splits of 10, 20, 30 minutes for the arrival interval and treat time.

Table 1. Probability Distribution of Arrival Interval and Treat Time for Baseline Scenario

	10 minutes	20 minutes	30 minutes	Mean Wait Time
Arrival Interval	50%	20%	30%	18.31 minutes (18 minutes, 20 seconds)
Treat Time	40%	20%	40%	

Table 2 shows the mapping between random values to arrival interval and treat time. The probability of arrival interval values is managed by a rand_arrival which is a uniformly distributed random value ranging from 0-1. Since the probability of 10 minutes is designed to be 50% for the arrival interval, rand_arrival with values that are less than 0.5 are mapped to 10-minute arrival_interval. In the same fashion, rand_arrival of values between 0.5 and 0.7 are mapped to an arrival_interval of 20 minutes. The rand_arrival between 0.7-1 are mapped to 30-minute arrival_interval. The mapping of treat time follows the same logic.

Figure 1. Use Random Values to Generate arrival_interval and treat_time

```
data customer_arrivals (keep= day patient arrival_interval treat_time);
    num_of_patients = 10;
    num_of_days = 10*365;

    call streaminit(123);

    do day = 1 to num_of_days;
        do patient = 1 to num_of_patients;
           /* more customers, shorter treatement time */
            rand_arrival = rand("Uniform");
            if rand_arrival lt 0.5 then arrival_interval = 10;
            else if rand_arrival lt 0.7 then arrival_interval = 20;
            else if rand_arrival lt 1 then arrival_interval = 30;

            rand_treat = rand("Uniform");
            if rand_treat lt 0.4 then treat_time = 10;
            else if rand_treat lt 0.6 then treat_time = 20;
            else if rand_treat lt 1 then treat_time = 30;

           output;
        end;
    end;
run;
```

Table 2. Mapping Table for Arrival Interval and Treat Time

rand_arrival	arrival_interval	rand_treat	treat_time
Rand_arrival < 0.5	10 minutes	rand_treat < 0.4	10 minutes
0.5 <= Rand_arrival < 0.7	20 minutes	0.4 <= rand_treat < 0.6	20 minutes
Rand_arrival >= 0.7	30 minutes	rand_treat >= 0.6	30 minutes

Table 3 shows the frequencies of arrival_interval and treat_time values generated by the program. The frequency distributions for both values are exactly what we expected.

Table 3: Frequency Table for arrival_interval and treat_time

The FREQ Procedure

arrival_interval	Frequency	Percent	Cumulative Frequency	Cumulative Percent
10	18112	49.62	18112	49.62
20	7228	19.80	25340	69.42
30	11160	30.58	36500	100.00

treat_time	Frequency	Percent	Cumulative Frequency	Cumulative Percent
10	14606	40.02	14606	40.02
20	7268	19.91	21874	59.93
30	14626	40.07	36500	100.00

Table 4 displays the mean and standard deviation of arrival_interval and treat_time. The mean of arrival_interval is 18.09 minutes and the mean of treat_time is 20.01 minutes. Both values are quite close to the patient's average waiting time.

Table 4. The MEANS of arrival_interval and treat_time

The MEANS Procedure

Variable	N	Mean	Std Dev	Minimum	Maximum
arrival_interval	36500	18.0953425	8.7505221	10.0000000	30.0000000
treat_time	36500	20.0054795	8.9492925	10.0000000	30.0000000

Arrival time is the time in minutes when each patient arrives. For example, if arrival time for a patient is 10 minutes, it means this patient arrives 10 minutes after the clinic starts its business on that day. Each patient arrival time is the sum of the last patient's arrival time and the current patient's arrival_interval. Start time is the time that a patient starts the treatment. If the patient is the first one that arrives at the clinic on that day, then the start time for this patient is the same as the arrival time. If the patient is not the first customer that arrives on that day, then the treatment start time depends on whether this customer arrives before or after the treatment finish time of the last patient. If this patient arrives before the last customer finishes treatment, then the start time of this patient equals the finish time of the last patient. However, if the current patient arrives after the last patient finishes his/her treatment, then the current patient can start his/her treatment without waiting which means start time will be equal to arrival time. Finish time is the sum of treatment start time and treatment duration which is treat time. Wait time is the difference between a patient's arrival time and treatment start time.

Figure 2. Calculate the arrival_time, start_time, finish_time, and wait_time

```
data customer_arrivals (keep= day patient arrival_time start_time treat_time finish_time wait_time);
    retain  day patient arrival_time start_time treat_time finish_time wait_time;
    set customer_arrivals;

    by day;
    if first.day then do;
        arrival_time = arrival_interval;
        start_time = arrival_time;

    end;
        retain arrival_time;
    else do;
        arrival_time = arrival_interval + arrival_time;
        if finish_time lt arrival_time then start_time = arrival_time;
        else start_time = finish_time;
    end;

    finish_time = start_time + treat_time;
    retain finish_time;
    wait_time = start_time - arrival_time;

run;
```

The frequency distribution of clinic waiting time generated by our baseline model is presented in table 5. From the table, we can see that the longest waiting time is 150 minutes which is 2 hours, 30 minutes. Luckily, this situation only occurred once in our 36,500 patient visits within 10 years. 80% of the customers have a waiting time of fewer than 30 minutes. 38% of patients have 0 wait time which means 30% of patients either arrive before the last patient finishes the treatment or are the first customer of that given day.

Table 5. Frequency Table of Clinic Waiting Time for Baseline

The FREQ Procedure

wait_time	Frequency	Percent	Cumulative Frequency	Cumulative Percent
0	13867	37.99	13867	37.99
10	5368	14.71	19235	52.70
20	6465	17.71	25700	70.41
30	3676	10.07	29376	80.48
40	2947	8.07	32323	88.56
50	1718	4.71	34041	93.26
60	1198	3.28	35239	96.55
70	629	1.72	35868	98.27
80	341	0.93	36209	99.20
90	157	0.43	36366	99.63
100	85	0.23	36451	99.87
110	27	0.07	36478	99.94
120	15	0.04	36493	99.98
130	3	0.01	36496	99.99
140	3	0.01	36499	100.00
150	1	0.00	36500	100.00

In our baseline scenario, I set the probability of 10, 20, and 30-minute arrival_interval to be 50%, 20%, and 30%. The probability of 10, 20, and 30-minute treat_time is set to be 40%, 20%, and 40%. However, this probability distribution only captured one of the thousands of possible situations in reality. In our baseline scenario, there are four varying values that can change the distribution of probabilities (shown in Figure 3). Figure 4 is a visualization of the two boundary values that change the proportion of arrival_interval. These two boundaries correspond to rand_arrival in the code in Figure 3. Treat_time also has two boundaries which correspond to rand_treat in the code (Figure 5).

Figure 3. Set Probabilities to Create Baseline Scenario

```
rand_arrival = rand("Uniform");
if rand_arrival lt 0.5 then arrival_interval = 10;
else if rand_arrival lt 0.7 then arrival_interval = 20;
else if rand_arrival lt 1 then arrival_interval = 30;

rand_treat = rand("Uniform");
if rand_treat lt 0.4 then treat_time = 10;
else if rand_treat lt 0.6 then treat_time = 20;
else if rand_treat lt 1 then treat_time = 30;
```

Figure 4. Stacked Bar Chart of Proportion Distribution of Arrival_Interval

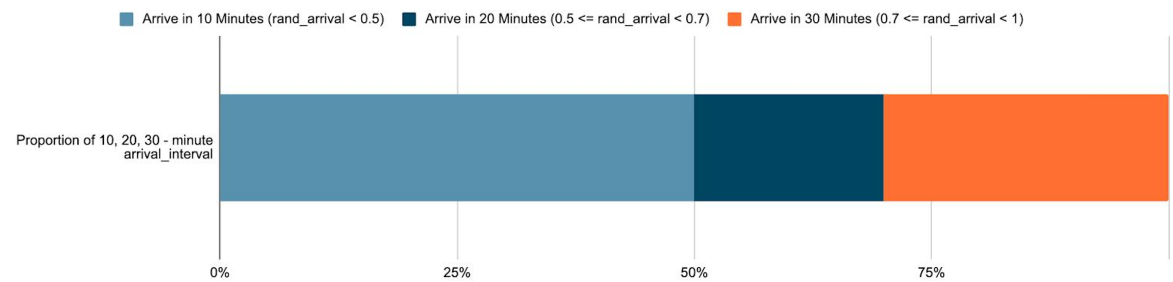
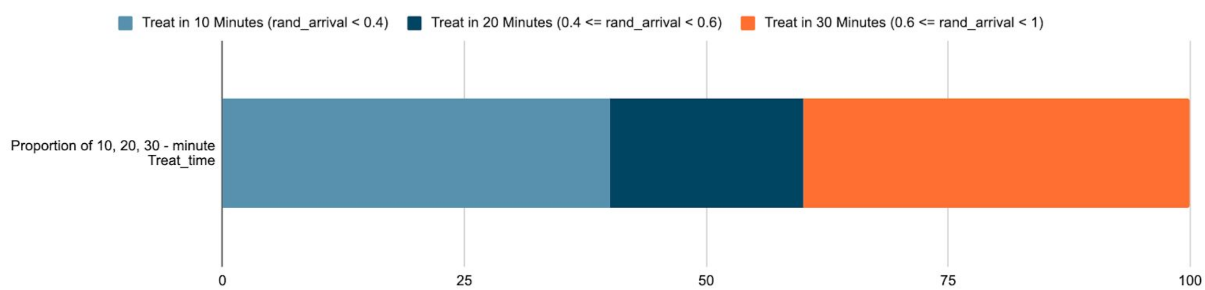


Figure 5. Pie Chart of Proportion Distribution of Treat_Time



I used four do-loops to create all possible proportions of arrival_interval and treat_time (Figure 4). The four varying values are arrive_1, arrive_2, treat_1, and treat_2. Each value has 11 possible values ranging from 0%-100% incrementing by 10%. For the two boundaries for arrival_interval, there are 66 possibilities. For the two boundaries for treat_time, there also are 66 possibilities. The combination of all possible arrival_interval and treat_time creates 66*66 = 4356 different possibilities.

Figure 6. Vary Probabilities to Create Average Waiting Time for All Possible Scenarios

```
data customer_arrivals(keep= avg_wait_time arrive_10 arrive_20 arrive_30
                        treat_10 treat_20 treat_30);
    retain avg_wait_time arrive_10 arrive_20 arrive_30
                        treat_10 treat_20 treat_30;
    do arrive_1 = 0 to 10;
        do arrive_2 = 0 to (10-arrive_1);
            do treat_1 = 0 to 10;
                do treat_2 = 0 to (10-treat_1);
                    %get_results(arrive_1,arrive_2,treat_1,treat_2);
                end;
            end;
        end;
    end;

run;
```

Results

Figure 7 presents the dataset of all possible average waiting times. The first row represents a scenario that the probabilities of the next patient arrival in 10, 20, 30 minutes are respectively 0%, 0%, and 100%. In the same row, the probabilities of the treatment time for this patient to be 10, 20, 30 minutes are respectively 0%, 0%, and 100%.

Figure 7. Dataset of All Possible Average Wait Time

Total rows: 4356 Total columns: 7 Rows 1-100

	avg_wait_time	arrive_10	arrive_20	arrive_30	treat_10	treat_20	treat_30
1	0	0	0	100	0	0	100
2	0	0	0	100	0	10	90
3	0	0	0	100	0	20	80
4	0	0	0	100	0	30	70
5	0	0	0	100	0	40	60
6	0	0	0	100	0	50	50
7	0	0	0	100	0	60	40
8	0	0	0	100	0	70	30

The highlighted row in Figure 8 is our baseline scenario. Since this dataset average waiting time is generated from a much larger dataset, the average waiting time is slightly different from the previous baseline value (18.31 minutes). Here the baseline average waiting time is 18.21 minutes. It ranks 2879 among 4356 records. The adjacent rows with similar average waiting time have very different arrival_interval and treat_time proportions. For example, the arrival_interval for record 2878 has its proportions split at 10%, 80%, and 10%, which is very different from the 50%, 20% , 30% split of our baseline scenario, but their average waiting times only differ by 0.002 minutes. This difference is less than 1 second.

Figure 8. Our Baseline Scenario (Record 2879)

Total rows: 4356 Total columns: 7

	avg_wait_time	arrive_10	arrive_20	arrive_30	treat_10	treat_20	treat_30
2870	[illegible]	30	40	30	30	20	30
2871	18.106027397	60	10	30	40	30	30
2872	18.108767123	0	90	10	20	20	60
2873	18.134246575	40	60	0	0	100	0
2874	18.150136986	20	70	10	20	40	40
2875	18.163561644	90	0	10	70	10	20
2876	18.195616438	70	0	30	60	0	40
2877	18.196986301	100	0	0	70	20	10
2878	18.210684932	10	80	10	20	30	50
2879	18.212328767	50	20	30	40	20	40
2880	18.223287671	10	60	30	10	30	60
2881	18.230410959	10	50	40	10	20	70
2882	18.242465753	50	30	20	10	80	10
2883	18.272876712	50	30	20	50	10	40
2884	18.288767123	70	0	30	40	40	20
2885	18.290410959	50	0	50	10	60	30
2886	18.307123288	40	0	60	20	20	60
2887	18.356986301	60	0	40	20	60	20
2888	18.375342466	40	40	20	10	70	20

The frequency distribution of average waiting time is shown in figure 9. The longest average waiting time is 90 minutes, which only happened once. The frequency table shows that 66.09% of the average waiting times are shorter than our baseline value 18.2123 minutes.

Figure 9. Frequency Distribution of Average Waiting Time at the Clinic in 10 Years

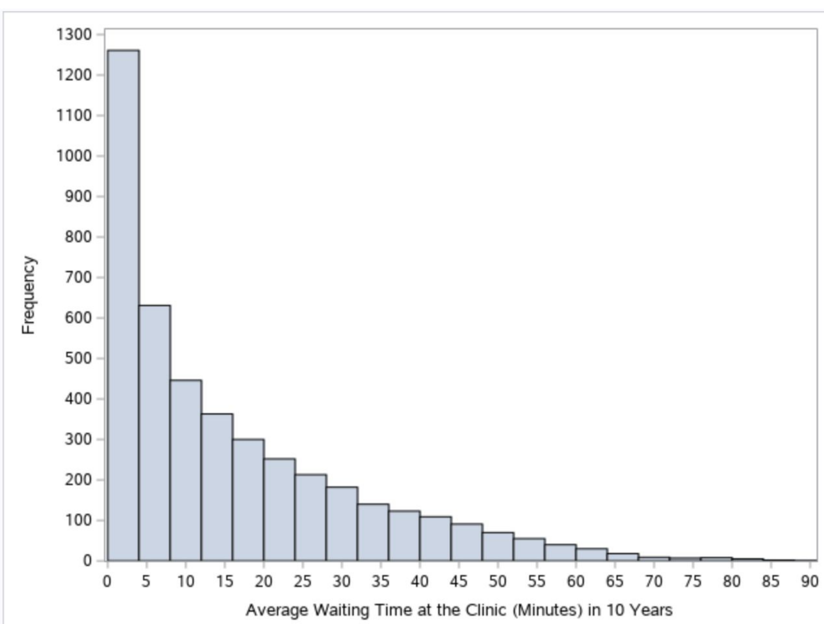


Table 5 presents that the mean value of the all the average waiting time is 15.96 minutes, which is slightly shorter than average waiting time in the US (18 minutes).

Table 5. Mean Value of Average Waiting Time

The MEANS Procedure

Analysis Variable : avg_wait_time				
N	Mean	Std Dev	Minimum	Maximum
4356	15.9557978	16.1579833	0	90.0000000

There is a positive relationship between the 10-minute arrival interval and average waiting time. The relationship between 20- and 30- minute arrival interval and average waiting time is negative. The shorter the arrival interval, the longer the waiting time. This trend shows us that we should avoid visiting the clinic when other patients are arriving.

Figure 10. Scatter Plot of Average Wait Time, and 10-, 20-, 30-Minute Arrival Intervals

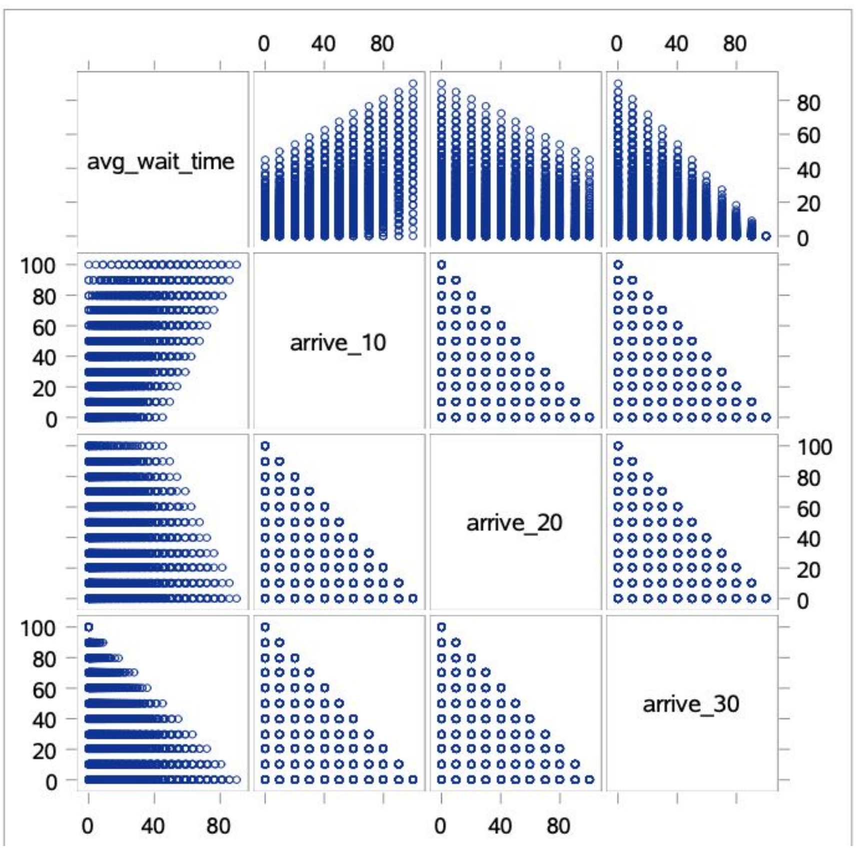
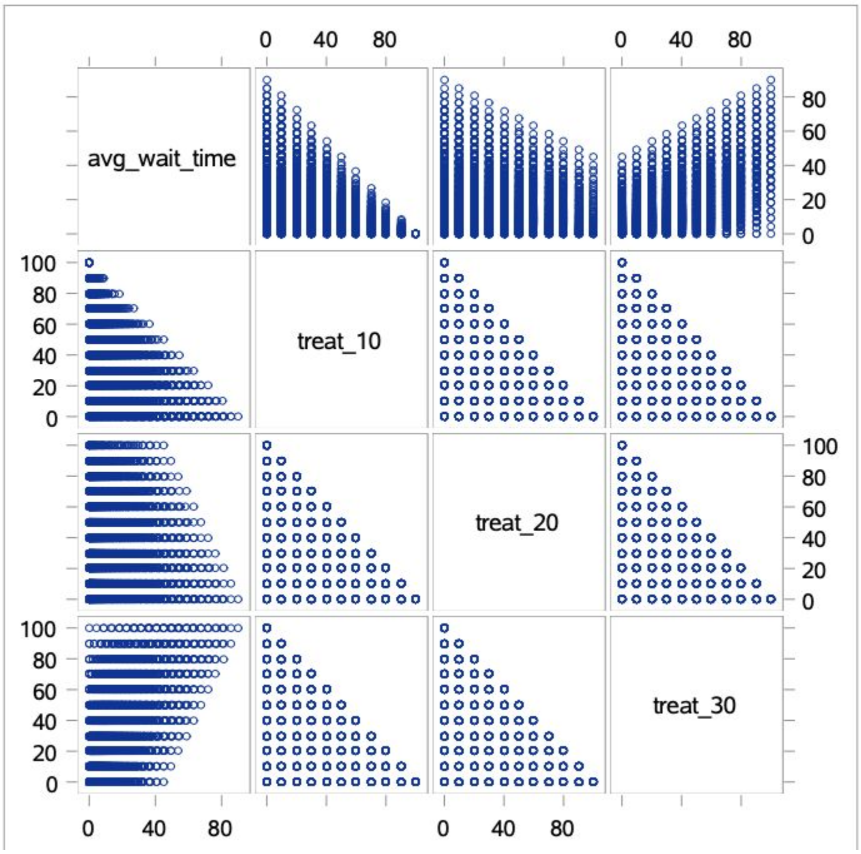


Figure 11 shows the trend between treatment duration and average waiting time. We can see that when there are more 10-minute treat times, the relationship between treat time and wait time is negative. However, when there are more 30-minute treat times, the relationship turns positive. This result is quite intuitive that the longer each treatment takes, the longer time the next patient has to wait.

Figure 11. Scatter Plot of Average Wait Time, and 10-, 20-, 30-Minute Arrival Intervals



Conclusions

This result of this experiment shows us that there are two easy ways to reduce waiting time. One is to arrive at a time when there are fewer patients. Another way is to arrive when the doctor is less busy. The clinic also can try to space out the arrival time for every patient. One solution is to arrange the longer appointments at a less busy time, so there will be fewer patients waiting. The limitation of this research is that there are only three types of values for arrival intervals and treatment time - 10, 20, 30 minutes. Further research can be conducted by using more random arrival intervals and treatment time.

References

https://www.fiercehealthcare.com/practices/ppatients-switched-doctors-long-wait-times-vitals#:~:text=Across%20specialties%2C%20the%20average%20wait,consecutive%20decrease%20in%20wait%20times.