Logic, First Course, Winter 2020. Week 8, Lecture 1, Handout.

Double negation rule

$\ell_1.$ $\neg\neg\varphi$

$\vdots$

$\ell.$ φ DN ℓ_1

ℓ_1 φ

$\vdots$

ℓ $\neg\neg\varphi$ DN ℓ_1

Law of the excluded middle

Since we will often want to appeal to double-negation without having to redo this proof over and over, we simply include a new rule for law of the excluded middle which says that one can always put $\phi \vee \neg\phi$ on a line, and justify it as `LEM` . Note that no line number is put down as part of the justification.

$$\vdots$$

$\varphi \vee \neg \varphi$ LEM

Other derived rules

- *Law of excluded middle*: $p \vee \neg p$ is a tautology. Abbreviation: `LEM`
- *Law of non-contradiction*: $\neg(p \wedge \neg p)$ is a tautology. Abbreviation: `LNC`
- *The law of double-negation*: p is equivalent to $\neg\neg p$. Abbreviation: `DN`
- *Law of commutativity for conjunction*: $p \wedge q$ is equivalent to $q \wedge p$. Abbreviation: `LCC`
- *Law of commutativity for disjunction*: $p \vee q$ is equivalent to $q \vee p$. Abbreviation `LCD` .
- *Law of associativity for conjunction*: $(p \wedge q) \wedge r$ is equivalent to $p \wedge (q \wedge r)$. Abbreviation: `LAC`
- *Law of associativity for disjunction*: $(p \vee q) \vee r$ is equivalent to $p \vee (q \vee r)$. Abbreviation: `LAD`
- *Law of distribution, part 1*: $p \wedge (q \vee r)$ is equivalent to $(p \wedge q) \vee (p \wedge r)$. Abbreviation: `LDC` (where the final "C" is short for the initial conjunction)
- *Law of distribution, part 2*: $p \vee (q \wedge r)$ is equivalent to $(p \vee q) \wedge (p \vee r)$. Abbreviation: `LDD` (where the final "D" is short for the initial disjunction)