Logic, First Course, Winter 2020. Week 7, Lecture 1, Handout.

Elimination rule for negation

$$l_1. \qquad \varphi$$

$$l_2. \qquad {\sim}\varphi$$

$$l. \qquad \underline{\perp} \qquad\qquad : E {\wedge} \, l_1, l_2$$
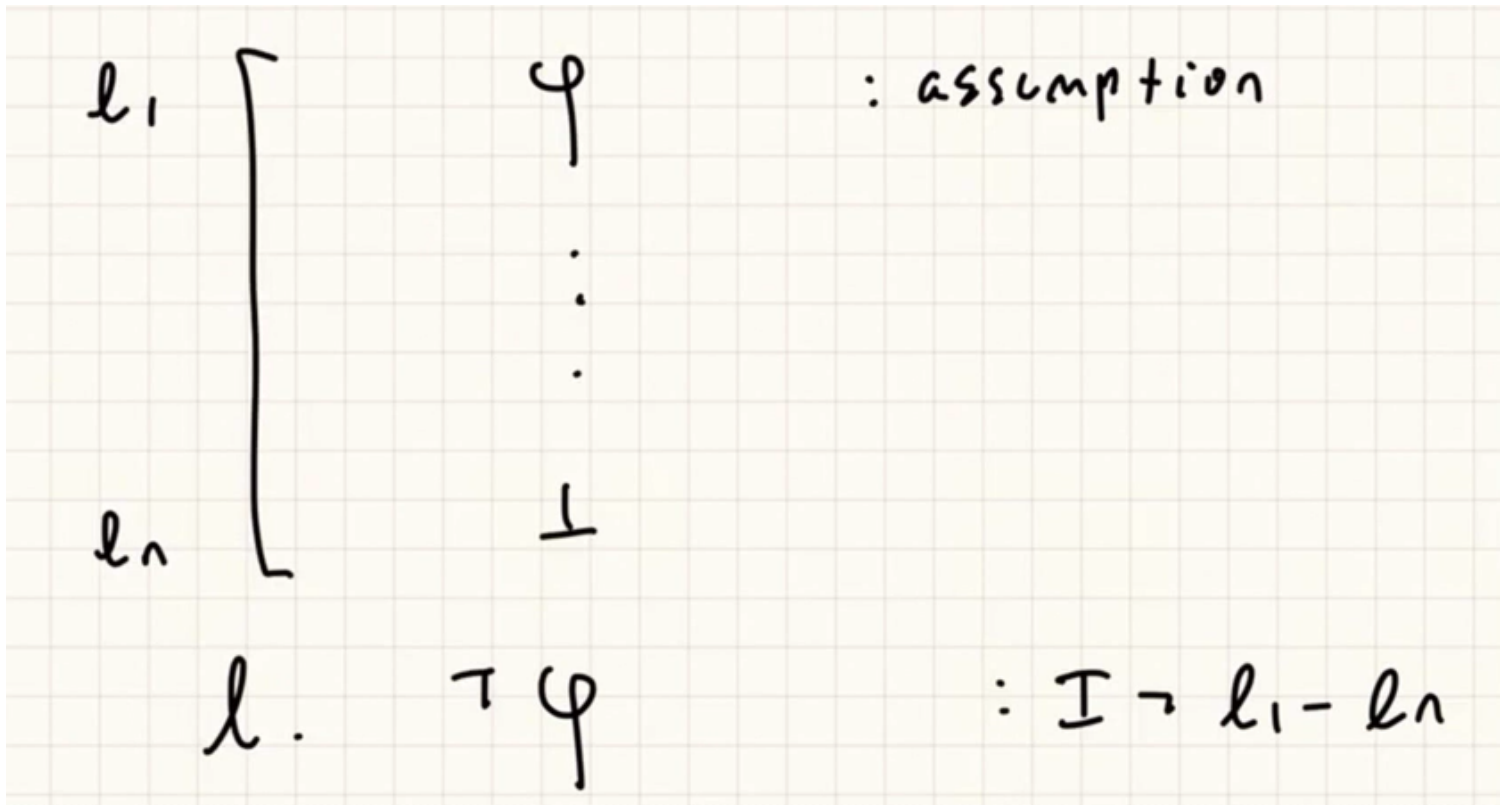
Example 1. $p \rightarrow \neg q, q, p \vdash \perp$.

exercise

$(p \rightarrow \neg q), q, p \vdash \bot$

1. p→~q :assumption
2. q :assumption
3. p :assumption

Introduction rule for negation

Here is a picture of the rule:

$$
l_1 \left[\begin{array}{c} \varphi \\ \vdots \\ \bot \end{array} \right. \quad : \text{assumption}
$$

$$
l_n
$$

$$
l. \quad \neg \varphi \qquad : I \neg \; l_1 - l_n
$$

Example 2. $p \vdash \neg\neg p$.

exercise

$p \vdash \neg\neg p$

```
  1. p  :assumption
```

A very common pattern

A very common pattern to get used to and to anticipate in these proofs is this one:

$$1. \quad \varphi \qquad\qquad\qquad\qquad\qquad\qquad : \text{assumption}$$

$$\left[\begin{array}{l} 2. \quad \psi \qquad\qquad\qquad\qquad : \text{assumption} \\ \\ \quad\quad\quad\quad \vdots \\ \\ \quad\quad\quad\quad \vdots \\ \\ 50 \quad\quad \varphi \\ \\ 51. \quad\quad \bot \qquad\qquad\qquad : E \neg 1, 50 \end{array}\right.$$

$$52 \quad\quad \neg \psi \qquad\qquad\qquad\qquad : I \neg 2 - 51$$

Illustrating the common pattern

Example 3. $q \vdash \neg(p \wedge \neg q)$.

```
exercise
┌─────────────────────────────────────────┐
│ q ⊢ ¬(p ∧ ¬q)                            │
├─────────────────────────────────────────┤
│                                          │
│    1. q  :assumption                     │
│                                          │
│                                          │
│                                          │
│                                          │
│                                          │
└─────────────────────────────────────────┘
```

Illustrating the common pattern again

Example 4. $q, p \rightarrow \neg q \vdash \neg p$.

Second, you can try to input it into the proof-checker yourself, or come back later and practice:

<div>

exercise

```
q, (p → ¬q) ⊢ ¬p

    1. q :assumption
    2. p→~q :assumption
```

</div>

A more challenging example

Example 5. ¬¬¬*p* ⊢ ¬*p*.

```
┌─ exercise ─────────────────────────────────────────
│  ┌──────────────────────────────────────────┐
│  │ ¬¬¬p ⊢ ¬p                                 │
│  ├──────────────────────────────────────────┤
│  │                                           │
│  │    1. ~~~p :assumption                    │
│  │                                           │
│  │                                           │
│  │                                           │
│  │                                           │
│  └──────────────────────────────────────────┘
│
│
└────────────────────────────────────────────────────
```

This example is important because it shows that triple negation implies single negation. Next time, we will add rules which allow us to prove the converse as well.

These is a handout for [this course](.).[1]

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1. It is run on the Carnap software, which is↩