Logic, First Course, Winter 2020. Week 10, Lecture 2. Back to course website

Review for exam

This is intended to be review for the exam, in the form of a practice exam. Like the exam, there are 20 problems which fall into four groups corresponding to Weeks 6-9.

- Week 6 problems
- Week 7 problems
- Week 8 problems
- Week 9 problems

Before you begin, you might consider printing a copy either to work out by hand as you go along, or to work with on a tablet. A nice pdf of this page is INSERT.

Please keep in mind that on the exam, there will be ample partial credit given for proofs. Hence, even if you don't get all the way done with the proof, try to get as much of it done as you can. Further, the proofs on the exam will be minor variations of the proofs which we are doing in this set and the Week 10 Practice Problems.

Week 6 problems

Each of these problems asks you to do a proof in the Week 6 deductive system. This is the system with just introduction and elimination rules for conjunction, arrow, and disjunction. Remember that the strategy in this system is: (1) try to do elimination rules at the top of your proof, and if that doesn't work, then (2) try to do an introduction rule at the bottom of your proof, and (3) repeat steps (1)-(2).

Example 1.

exercise

((a ∧ b) ∧ c) ⊢ (a ∨ c)

1. (a⋀b)⋀c :assumption

Example 2.

exercise

$(a \to (b \wedge c)), a, (c \to (d \wedge e)) \vdash e$

1. $a \to (b \wedge c)$:assumption
2. a :assumption
3. $c \to (d \wedge e)$:assumption

Example 3.

exercise

```
(a → c), (b → d), (d → e) ⊢ ((a ∧ b) → (d ∧
e))
```

1. a→c :assumption
2. b→d :assumption
3. d→e :assumption

Example 4.

exercise

$((a \vee b) \to c), ((a \vee b) \to d) \vdash ((a \wedge b) \to (c \wedge d))$

1. $(a \vee b) \to c$:assumption
2. $(a \vee b) \to d$:assumption

Example 5.

exercise

```
(a ∨ b), (c → e), (a → c), (d → e), (b → d)
⊢ e
```

1. a∨b :assumption
2. c→e :assumption
3. a→c :assumption
4. d→e :assumption
5. b→d :assumption

Week 7 problems

Each of these problems asks you to do a proof in the Week 7 deductive system. This system just adds onto the Week 6 deductive system with the introduction and elimination rule for negation, as well as EFSQ and the repeat rule. Remember that falsum ⊥ can be written in the proof-checker (and on exam) as `!?`. You can also just copy and paste ⊥ from here.

Example 6.

exercise

```
¬a ⊢ ¬(a ∧ b)
```

```
1. ~a :assumption
```

Example 7.

exercise

((a ∧ b) → c) ⊢ (¬c → ¬(a ∧ b))

1. (a∧b)→c :assumption

Example 8.

exercise

(a → b), (b → c), (c → ¬a) ⊢ ¬a

1. a→b :assumption
2. b→c :assumption
3. c→~a :assumption

Example 9.

On this one, keep in mind that we are proving it in the Week 7 system, and so we do not have disjunctive syllogism as a primitive rule. This proof is basically showing that a certain kind of disjunctive syllogism holds in the Week 7 system.

exercise

`(a ∨ (b ∧ c)), ¬a ⊢ (b ∧ c)`

1. a⋁(b⋀c) :assumption
2. ~a :assumption

Example 10.

exercise

```
q ⊢ ¬¬(p → q)
```

```
1. q :assumption
```

Week 8 problems

These problems pertain to the Week 8 deductive system. Remember that the Week 8 deductive system has rules built in for modus tollens (MT), law of excluded middle (LEM), law of non-contradiction (LNC), double-negation (DN) commutativity (LCC, LCD), associativity (LAC, LAD), distribution (LDC, LCDD), distribution (LDC, LDD), and disjunctive syllogism (PDS, NDS). Also keep in mind that the strategy for the Week 8 deductive system is to look for patterns associated to these rules, as opposed to following the main connectives like we did in Weeks 6-7.

Example 11.

exercise

```
((a → b) ∧ ((b → c) ∧ (c → d))) ⊢ ((a → b) ∧
((c → d) ∧ (b → c)))
```

```
1. (a→b)∧((b→c)∧(c→d)) :assumption
```

Example 12.

exercise

(¬(a ∨ b) → c) ⊢ (¬(b ∨ a) → c)

1. ~(a⋁b)→c :assumption

Example 13.

exercise

((a ∧ (b ∧ c)) ∨ (¬a ∧ (¬b ∧ ¬c))) ⊢ (((¬a ∧ ¬b) ∧ ¬c) ∨ (a ∧ (b ∧ c)))

1. (a⋀(b⋀c))⋁(~a⋀(~b⋀~c)) :assumption

Example 14.

exercise

(a ∨ (b ∧ c)), ((a ∨ b) → ¬d), ((a ∨ c) → ¬e)
⊢ ¬(d ∨ e)

1. a⋁(b⋀c) :assumption
2. (a⋁b)→~d :assumption
3. (a⋁c)→~e :assumption

Example 15.

exercise

(¬(a ∧ b) ∧ b) ⊢ ¬a

1. (~(a⋀b)⋀b) :assumption

Example 16.

By the Soundness and Completeness Theorems, we can assess the validity of this argument using truth-tables as well. Fill out the following truth-table, just like we did in Week 3:

(¬(a ∧ b) ∧ b) ⊢ ¬a

a	b	(	¬	(	a	∧	b	)	∧	b	)	⊢	¬	a
T	T	F	T	T	T	F	T	T	F	T	T	F	T	
T	F	T	T	F	F	F	F	T	F	T	T	F	T	
F	T	T	F	F	T	T	T	T	T	T	T	T	F	
F	F	T	F	F	F	F	F	T	T	T	T	T	F	

Counterexample

(We will need this truth-table in Example 20, and so perhaps consider taking a screen shot of it before you move on, for easy access later).

Week 9 problems

These problems pertain to the materials from Week 9 and the first lecture of Week 10.

Example 17

Consider the scale (always, often, sometimes). Suppose Laura and Sofia are talking about commuting, and Laura asks: "Do you drive to campus" and that Sofia responds by saying that "sometimes." Which of the following is a conversational implicature of Sofia's having said that?

Which one?

- It is not the case that Sofia always drives to campus.
- It is not the case that Sofia often drives to campus.
- It is not the case that Sofia sometimes drives to campus.
- It is not the case that Sofia believes that she sometimes drives to campus.

Example 18

Suppose that Lily meets Sara for the first time and asks Sara how long she has been working at her present job. Suppose that Sara responds by saying that she quit her last job because she did not like the hours. It seems that Sara has violated one of Grice's conversational maxims. Which one?

Which one?

- Quantity
- Quality
- Relation
- Manner

Example 19

Suppose that the probability of $p \vee q$ is 3/10 and the probability of $p \vee r$ is 4/10 and the probability of $p \vee (q \vee r)$ is 6/10. What is the probability of $p \vee (q \wedge r)$? *Hint*: use distribution.

What is it?

- 0/10
- 1/10
- 2/10
- 3/10

Example 20

Suppose that each of the rows in the truth-table in Example 16 is weighted equally, that is each has equal chance of happening, namely a 1/4 chance of happening. What is the probability of $\neg(a \wedge b)$ conditional on b?

What is it?

- 1/4
- 2/4
- 3/4
- 4/4

This is a practice problem set for this course. It is run on the Carnap software, which is an: