Other derived rules

Recall that we had a long list of tautologies and equivalences that we verified via truth-tables. With the inclusion of the double-negation to our deductive system, these are all now derivable. So that we do not have to be constantly replicating the long proofs over and over again, we include rules for them in our deductive system. Here is the list, along with the abbreviation used in the deductive system and proof-checker:

- *Law of excluded middle*: $p \vee \neg p$ is a tautology. Abbreviation: `LEM`
- *Law of non-contradiction*: $\neg(p \wedge \neg p)$ is a tautology. Abbreviation: `LNC`
- *The law of double-negation*: p is equivalent to $\neg\neg p$. Abbreviation: `DN`
- *Law of commutativity for conjunction*: $p \wedge q$ is equivalent to $q \wedge p$. Abbreviation: `LCC`
- *Law of commutativity for disjunction*: $p \vee q$ is equivalent to $q \vee p$. Abbreviation `LCD`.
- *Law of associativity for conjunction*: $(p \wedge q) \wedge r$ is equivalent to $p \wedge (q \wedge r)$. Abbreviation: `LAC`
- *Law of associativity for disjunction*: $(p \vee q) \vee r$ is equivalent to $p \vee (q \vee r)$. Abbreviation: `LAD`
- *Law of distribution, part 1*: $p \wedge (q \vee r)$ is equivalent to $(p \wedge q) \vee (p \wedge r)$. Abbreviation: `LDC` (where the final "C" is short for the initial conjunction)
- *Law of distribution, part 2*: $p \vee (q \wedge r)$ is equivalent to $(p \vee q) \wedge (p \vee r)$. Abbreviation: `LDD` (where the final "D" is short for the initial disjunction)
- *DeMorgan Law, part 1*: $\neg(p \wedge q)$ is equivalent to $\neg p \vee \neg q$. Abbreviation: `DMOR` (since it ends in an or statement)
- *DeMorgan Law, part 2*: $\neg(p \vee q)$ is equivalent to $\neg p \wedge \neg q$. Abbreviation: `DMAND` (since it ends in an and statement)

The truth-tables for all of these were done when we introduced them. We have done virtually all of the proofs. For instance, we just did the proof for law of the excluded middle, and we did one of the distribution rules last week. The remaining ones will be the focus of some practice problems and homeworks. As we will see when we put these into the proof-checker, the tautologies in this list do not require that you cite a line as a justification, and the equivalences in this list just require that you cite one previous line as a justification.

Recall that we also had a shorter list of validities from our lecture on validities. These are all now provable in our system, and we likewise add rules for them:

- *Modus ponens*. $p \rightarrow q, p \vdash q$. Abbreviation: `E->`
- *Modus tollens*. $p \rightarrow q, q \vdash \neg p$. Abbreviation: `MT`
- *Disjunctive syllogism*. $p \vee q, \neg p \vdash q$. Abbreviation: `PDS` (for positive disjunctive syllogism, since it starts with an initial unnegated "positive" disjunction).
- *Disjunctive syllogism*. $\neg(p \wedge q), p \vdash \neg q$. Abbreviation: `NDS` (for negative disjunctive syllogism, since it starts with a negated conjunction).
- *Reasoning by cases*. $p \vee q, p \rightarrow r, q \rightarrow r \vdash r$. Abbreviation: `E\/`

As is indicated, modus ponens and reasoning by cases are just elimination rules for two of the propositional connectives, namely arrow and disjunction. The first four of these require citing two lines for their justification separated by commas, and the last of these (as we know) requires citing three lines as the justification separated by commas.