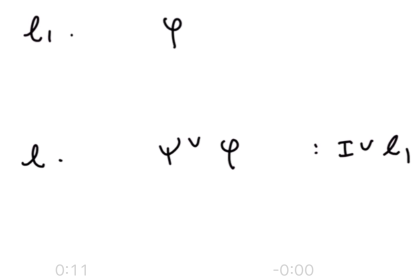
Logic, First Course, Winter 2020. Week 6, Lecture 2, Handout.

Introduction rule for disjunction

The rule is: if you have ϕ on line ℓ_1, then you may write $\phi \vee \psi$ on any subsequent line $\ell > \ell_1$. Likewise, if you have ϕ on line ℓ_1, then you may write $\psi \vee \phi$ on any subsequent line $\ell > \ell_1$.

This rule is abbreviated as $I\vee$, where the 'I' is for *introduction*.

In terms of a picture, the rule is either of the following:



Note that the rule does **not** require that ψ appear on any previous line. In many ways, this is what gives $I\vee$ its strength.

Example of disjunction introduction

exercise

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p, ((p ∨ q) → r) ⊢ r

1.
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Another example of disjunction introduction

exercise

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a, c, (((a ∨ b) ∧ (b ∨ c)) → d) ⊢ d

    1.
```

Elimination rule for disjunction

The rule is: if you have $\phi \vee \psi$ on line ℓ_1, and you have $\phi \to \xi$ on line ℓ_2, and you have $\psi \to \xi$ on line ℓ_3, then you may write ξ on any subsequent line $\ell > \ell_1, \ell_2, \ell_3$.

Again, the order in which ℓ_1, ℓ_2, ℓ_3 occurs does not matter. All that matters is the all of three of these come before the ℓ, where we apply the rule.

This rule is abbreviated as $E\vee$, where the 'E' is for *elimination*.

In terms of a picture, the rule is the following:

$\ell_1.\quad \varphi \vee \psi$

$\ell_2.\quad \varphi \to \xi$

$\ell_3.\quad \psi \to \xi$

$\ell.\quad \xi \qquad : E\vee\ \ell_1, \ell_2, \ell_3$

0:21 -0:00

Example of disjunction elimination

exercise

((c ∨ d) ∧ a), (a ∧ (c → e)), ((d → e) ∧ b) ⊢ e

1.

exercise

⊤ ⊢ ((p ∨ q) → (q ∨ p))

1.

Another example of disjunction elimination

exercise

((a ∧ b) ∨ (a ∧ c)) ⊢ a

1.

Deriving commutativity of disjunction

These is a handout written for this course.[1]

1. It is run on the Carnap software, which is↩