Rules for Week 6

Introduction Rule for ∧

- Here's introduction rule for ∧:
  - If you have φ on line ℓ_1 and you have ψ on line ℓ_2, then you may write $\varphi\wedge\psi$ on any subsequent line $\ell>\ell_1, \ell_2$.
- In terms of a picture, rule is:
  - ℓ_1. φ
  - ℓ_2. ψ
  - ℓ. $\varphi\wedge\psi$ $(I\wedge,\ell_1,\ell_2)$
- In the rule, it does not matter whether ℓ_1 or ℓ_2 comes first. So you could have also written $\psi\wedge\varphi$ on line $\ell>\ell_1, \ell_2$.
- Here's an example:
- 1. p (assumption)
- 2. q (assumption)
- 3. r (assumption)
- 4. $q\wedge r$ (I∧ 2,3)
- 5. $p\wedge(q\wedge r)$ (I∧ 1,4)

Elimination Rule for ∧

- Here's elimination rule for ∧:
  - If you have $\varphi \wedge \psi$ on line ℓ_1, then you may write φ on any subsequent line $\ell > \ell_1$, and likewise you may write ψ on any subsequent line $\ell > \ell_1$.
- In terms of a picture, rule is:
  - ℓ_1. $\varphi \wedge \psi$
  - ℓ. φ $(E\wedge, \ell_1)$
- Or the following:
  - ℓ_1. $\varphi \wedge \psi$
  - ℓ. ψ $(E\wedge, \ell_1)$
- Here's an example:
- 1. $p \wedge (q \wedge r)$ (assumption)
- 2. $q \wedge r$ $(E\wedge, 1)$
- 3. r $(E\wedge, 2)$

Elimination Rule for $\to$

- Here's elimination rule for $\to$:
  - If you have φ on line ℓ_1, and you have $\varphi \to \psi$ on line ℓ_2, then you may write ψ on any subsequent line $\ell > \ell_1, \ell_2$.
- In terms of a picture, rule is:
  - ℓ_1. $\varphi \to \psi$
  - ℓ_2. φ
  - ℓ. ψ $(E\to, \ell_1, \ell_2)$
- In the rule, it does not matter whether ℓ_1 or ℓ_2 comes first.
- Here's an example:
- 1. $(p \to q) \wedge (p \to r)$ (assumption)
- 2. p (assumption)
- 3. $p \to q$ (E$\wedge$ 1)
- 4. $p \to r$ (E$\wedge$ 1)
- 5. q (E$\to$2,3)
- 6. r (E$\to$2,4)
- 7. $q \wedge r$ (I$\wedge$ 5,6)

Introduction Rule for →

- So without further ado, here's the introduction rule for →:
  - Suppose that consecutive lines ℓ_1-ℓ_n constitute a proof with premise φ and conclusion ψ. Then one may introduce $\varphi \rightarrow \psi$ at any subsequent line $\ell > \ell_n$, so long as one brackets off lines lines ℓ_1-ℓ_n and never appeals to them again.

- Here's an example (picture version of rule on next page):

- 1. $p \rightarrow (q \wedge r)$ (assumption)
  - 2. p (assumption)
  - 3. $(q \wedge r)$ (E→, 1,2)
  - 4. q (E∧, 3)
  - 5. r (E∧, 3)
  - 6. $r \wedge q$ (I∧ 4,5)
- 7. $p \rightarrow (r \wedge q)$ (I→ 2-6)

Introduction Rule for →

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- Here's the picture version of the rule:
- *1.*
  - $\ell_1.\ \varphi$ (assumption)
  - $\ell_n.\ \psi$
- $\ell.\ \varphi \rightarrow \psi$ (I→ ℓ_1-ℓ_n)

Introduction Rule for ∨

- Here's introduction rule for ∨:
  - If you have φ on line ℓ_1 then you may write $\varphi \vee \psi$ on any subsequent line $\ell > \ell_1$, and likewise you may write $\psi \vee \varphi$ on any subsequent line $\ell > \ell_1$.
- Note that the rule does **not** require that ψ appeared on any previous line. In many ways, this is what gives the ∨ rule its strength.
- Here's the picture of the rule:
  - ℓ_1. φ
  - ℓ. $\varphi \vee \psi$ $(I\vee, \ell_1)$
- Here's an example:
- 1. p (assumption)
- 2. $(p \vee q) \rightarrow r$ (assumption)
- 3. $p \vee q$ (I∨, 1)
- 4. r (E→, 2,3)

Elimination Rule for ∨

- Here's elimination rule for ∨:
  - If you have $\varphi \vee \psi$ on line ℓ_1, and you have $\varphi \rightarrow \xi$ on line ℓ_2, and you have $\psi \rightarrow \xi$ on line ℓ_3, then you may write ξ on any subsequent line $\ell > \ell_1, \ell_2, \ell_3$.
- Again, the order in which ℓ_1, ℓ_2, ℓ_3, doesn't matter. All that matters is that all three of these lines come before ℓ.
- Here's the picture of the rule:
- ℓ_1. $\varphi \vee \psi$
- ℓ_2. $\varphi \rightarrow \xi$
- ℓ_3. $\psi \rightarrow \xi$
- ℓ. ξ (*E*∨, ℓ_1, ℓ_2, ℓ_3)