On the Forms of Logical Proposition
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Source: *Mind*, Vol. 5, No. 19 (Jul., 1880), pp. 336-349
Published by: Oxford University Press on behalf of the Mind Association
Stable URL: https://www.jstor.org/stable/2246393
Accessed: 09-04-2020 03:44 UTC

tion of any molecule can maintain itself amidst its colliding atoms in a multifariously resisting medium, is one of those mechanical puzzles as yet neglected by science or noticed only when materiality is altogether dissolved into modes of motions. Then we are at the end of all our scientific jugglery, and face to face with nothing but a mystic world of woven motions, gorgeously spread in phenomenal repose, and safely poised on that one mathematical point, our present consciousness.

EDMUND MONTGOMERY.

(To be continued.)

III.—ON THE FORMS OF LOGICAL PROPOSITION.

LOGICIANS have been much exercised in the attempt to determine the number and arrangement of the simple forms of proposition, and hardly any two who have reconsidered the question for themselves seem to have agreed in their decision. If we were constructing a complete theory of Logic we should have to ask what is the *true* account, by which we should understand the most fundamental account, of the nature and import of a proposition, and on this point different accounts would be in direct hostility to one another. But when we are discussing methods rather than theories, this is not necessary. The question then becomes, which is the most convenient account rather than which is the most fundamental; and convenience is dependent upon circumstances, varying according to the particular purpose we have in view. For the present purposes of inquiry, there seem to be three different accounts of the import of a proposition; the ordinary or *predication* view, the *class inclusion and exclusion* view, and that which may be called the *compartmental* view. It may fairly be maintained that one of these views must be more fundamental than the others, or possess a better psychological warrant, but it cannot be denied that they are all three tenable views; that is, that we may, if we please, interpret a proposition in accordance with any one of the three. We propose to inquire what are the prominent characteristics of each of these distinct, but not hostile, views. What are their relative advantages and disadvantages; to what arrangement and division of propositional forms do they respectively lead; and which of them must be adopted if we wish to carry out the design of securing the widest extension possible of our logical processes by the aid of symbols?

The neglect of some such inquiry as this seems to me to have led to error and confusion. Logicians have been too much in the habit of considering that there could be only one account given of the import of propositions. Consequently, instead of discussing the number of forms of proposition demanded by one or the other view, they have attempted to decide absolutely *the* number of forms. And the very useful question as to the fittest view for this or that purpose has been lost in the too summary decision that one view was right and the others wrong.

Let us first look at the traditional four forms, *A, E, I, O;* in reference to which a very few words will here suffice. The light in which a proposition has to be consistently interpreted on this view is that of *predication.* We distinguish between subject and attribute here, and we assert that a given subject does or does not possess certain attributes. These forms appear to be naturally determined by the ordinary needs of mankind, and the ordinary pre-logical modes of expressing those needs; all that Logic has done being to make them somewhat more precise in their signification than they conventionally are. They adopt, as just remarked, the natural and simple method of asserting or denying attributes of a subject, that is, of the whole or part of a subject; whence they naturally yield four forms,—the universal and particular, affirmative and negative. For all ordinary purposes they answer admirably as they are, and by a little management they can be made to express nearly all the simple forms of assertion or denial which the human mind can well want to express.

With regard to these forms it must be very decidedly maintained that, as they generally and primarily regard the predicate in the light of an attribute and the subject in that of a class (whole or part), they do not naturally quantify this predicate: that is, they do not tell us whether any other things besides the whole or partial class in question possess the assigned attribute. No doubt they sometimes decide this point indirectly. Thus in the case of a universal negative proposition we can easily see that anything which possesses the attributes in the predicate cannot possess the attributes distinctive of the subject; that is, that the proposition can be simply converted. But this does not seem to be any part of the primary meaning of the proposition, which thinks of nothing but asserting or denying an attribute, and does not directly inquire about the extent of that attribute, or where else it is or is not to be found.

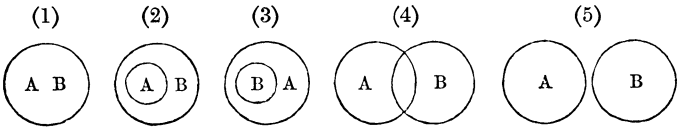
As just remarked, these forms of proposition certainly seem to represent the most primitive and natural modes in which thought begins to express itself with accuracy.[1] By combining two or more of them together they can readily be made equivalent to much more complicated forms. Thus by combining 'All X is Y' with 'All Y is X' we obtain the expression 'All X is all Y,' or 'X and Y are coextensive,' and so forth. As these familiar old forms have many centuries of possession in their favour, and the various rules for conversion and opposition, and for the syllogism, have been devised for them, there seem to be very strong reasons for not disturbing them from the position they have so long occupied. At least this should only be done if it could be shown either that they are actually insufficient to express what we require to express, or that they rest upon a wrong interpretation of the import of a proposition. The former is clearly not the case, for as was just remarked (and as no one would deny), a combination of two or more of these forms will express almost anything in the way of a definite statement. And as regards the latter, the point of this essay is to show that we are not necessarily tied down to one exclusive view as to the import of a proposition. I should say, therefore, that whatever other view we may find it convenient to adopt for special purposes, either of sensible illustration or with a view to solving intricate combinations of statements, there is no valid reason for not retaining the old forms as well. They may not be the most suitable materials for very complicated reasonings, but for the expression and improvement of ordinary thought and speech they are not likely to be surpassed.

So much for this view. Now suppose that, instead of regarding the proposition as made up of a subject determined by a predicate, we regard it as assigning the relations, in the way of mutual *inclusion* and *exclusion*, of two classes to one another. It will hardly be disputed that every proposition *can* be so interpreted. Of course, as already remarked, this interpretation may not be the most fundamental in a psychological sense; but when, as here, we are concerned with logical methods merely, this does not matter. For the justification of a method it is clearly not necessary that it should spring directly from an ultimate analysis of the phenomena; it is sufficient that the analysis should be a correct one.

Now how many possible relations are there, in this respect of mutual inclusion and exclusion, of two classes to one another? Clearly only five. For the question here, as I apprehend it, is this:—Given one class as known and determined in respect of

[1] At least this seems so in all the languages with which we need consider ourselves concerned. What might be the most natural arrangement of the forms of propositions in non-inflectional languages I must leave to philologists to determine.

its extent, in how many various relations can another class, also known and similarly determined, stand towards the first? Only in the following: It can coincide with the former, can include it, be included by it, partially include and partially exclude it, or entirely exclude it. In every recognised sense of the term these are distinct relations, and they seem to be the only such distinct relations which can possibly exist. These five possible arrangements would be represented diagrammatically as follows:—



Before comparing in detail the verbal statement of these five forms with that of the four old ones, it must be pointed out how entirely the distinction between subject and predicate is robbed of its significance on such a scheme as this. The terms of the proposition being thus classes of things of known extent, this distinction sinks down into one which is purely grammatical. It is the merest accident which of the two classes is represented first in our verbal statement; whether, for instance, in (2) we say that B lies outside A, or that A lies inside B. Certainly when the diagrammatic representation alone was shown to us, no one could give a guess as to which circle was intended to stand for a subject and which for a predicate. He could not, that is, read the diagram off in one way and one way only, with confidence.

A very little consideration will serve to convince us that this scheme of five forms, and the old one of four, will not by any means fit in with one another. Considering that they spring from different interpretations of the import of a proposition, it could not be expected that they should do so. (5) is the one unambiguous exception, corresponding precisely to the universal negative 'No A is B'. That is—given 'No A is B' we could only select this diagram; and conversely, given this diagram we could only utter it as 'No A is B'. But such correspondence does not exist in any other case. Given 'All A is B' we could not but hesitate between diagrams (1) and (2); and if diagram (4) were shown we should not know whether to describe it as 'Some A is B' or 'Some A is not B,' for it would fit either equally well.

Is there then no precise and unambiguous way of describing these five forms in ordinary speech? There is such a way, and to carry it out demands almost no violence to the established usage of language. It is merely necessary to say definitely that the word *some* shall signify 'some, not all'; a signification which on the whole seems more in accordance with popular usage than to say with most logicians that it signifies 'some, it may be all'. If we adopt this definition of the word our five diagrams will be completely, accurately and unambiguously expressed by the five following verbal statements:—

All A is all B.
All A is some B.
Some A is all B.
Some A is some B.
No A is any B.

That is—given one of these statements, only one diagram could be selected for it; and conversely, given any one diagram it could only be matched with one of these forms of words.

The tabular expression of these five forms will naturally recall to the reader's mind the well-known eight forms adopted by Hamilton, *viz.*:—

All A is all B.	No A is any B.
All A is some B.	No A is some B.
Some A is all B.	Some A is not any B.
Some A is some B.	Some A is not some B.

I might have termed the view as to the import of propositions now under discussion the Hamiltonian, instead of the *class inclusion and exclusion* view. It seemed better, however, at the outset to state the view independently in what seemed its most fundamental and characteristic form. Moreover, one is not so necessarily introduced into a well known and rather bitter controversy. At the same time I must state my own very decided opinion that the view in question is that which Hamilton, and those who have more or less implicitly followed him in his tabular scheme of propositions, are bound in consistency to adopt.

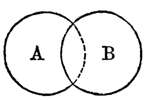
The logicians in question do not seem to me, indeed, to have at all adequately realised the importance of the innovation which they were thus engaged in introducing; nor, it must be added, the inadequacy of the means they were adopting for carrying it out. What they were really at work upon was not merely the rearrangement, or further subdivision, of old forms of proposition, but the introduction of another way of looking at and interpreting the function of propositions. The moment we insist upon 'quantifying' our predicate we have to interpret our propositions in respect of their extension; that is, to regard them as expressing something about the actual mutual relations of two classes of things to each other. The view of the proposition must be shifted from that of stating the relation of subject and predicate, or of object and attribute, to that of stating the relation of inclusion and exclusion of two classes to one another.

The question therefore at once arises, How do the eight verbal forms just quoted stand in relation to the five which we have seen to be, in their own way, exhaustive? As this is a very important inquiry towards a right understanding of the nature and functions of propositions, I shall make no apology for going somewhat into details respecting it. As regards the first five out of the eight, the correspondence is of course absolutely complete, if we understand that Hamilton's 'some' is to be understood, with ours, as distinctly excluding 'all'. But then, if so, what account is to be given of the remaining three out of the eight? But one account, I think, can be given. They are superfluous, or ambiguous equivalents for one or more of the first five. This may need a moment's explanation. By calling the first five complete and unambiguous we mean, as already remarked, that if one of these propositions was uttered, but one form of diagram could be selected in correspondence; and conversely, if a diagram were pointed out it could only be referred to one form of verbal expression. But if we were given one of the latter three to exhibit in a diagram we could not with certainty do so. Take, for instance, the proposition 'No A is some B'. If we had to exhibit this in a diagram we should find that diagrams (2) and (4) are equally appropriate for the purpose; whence this proposition is seen to be ambiguous and superfluous. Similarly, the proposition 'Some A is not any B,' is equally fitly exhibited in diagrams (3) and (4), and therefore appropriately in neither. Consequently it also must be regarded as needless in our scheme. The case of the remaining proposition, 'Some A is not some B,' is still worse. It is equally applicable to four of our five distinct possible cases, and therefore, as making no distinction whatever between one such proposition and another, this form is altogether useless to express the mutual relations of classes.

The ambiguity affecting these three last forms is, it need hardly be remarked, reciprocal. That is, so long as these three are retained in the scheme, we should not know, on a diagram being presented to us, which proposition was meant to be exhibited; any more than we can draw the diagram when a proposition is stated. Diagram (3), for instance, might under these circumstances be read off indifferently as 'Some A is all B,' 'Some A is not any B,' or 'Some A is not some B'.

It may perhaps be replied that there is still a use in retaining forms of proposition which thus refer ambiguously to two or more actual class relations, in addition to those forms that refer unambiguously to one only. It may be urged that if we do not know which of the two is really applicable, though one or the other must certainly be so, there is an opening for a form which covers both of them. I do not think that this will do. In the first place it may be objected that the employment of terms in their extensive signification implies that we are expressing their actual relation to one another in the way of inclusion and exclusion, and not our imperfect knowledge of that relation. At any rate this seems to be so when we make use of diagrams of this kind, for the circles must either cut one another or not do so; we cannot express a *doubt* whether they do or not. We may feel a doubt whether they should do so or not, but we must make them do one or the other.

An attempt is sometimes made in this way by the device of marking a part of one of the circles with a dotted line only. Thus 'Some A is not B' would be exhibited as follows:—



(as, for instance, is done, amongst others, by Thomson in his *Laws of Thought*). The dotted part here represents of course our ignorance or uncertainty as to whether the line should lie partly inside B, or should entirely include it. But surely, if we are thus ignorant, we have no business to prejudge the question by putting it inside, even as a row of dots. What we ought to do is to draw *two* lines, one intersecting B and the other including it. Doing this, there is no need to dot them; it is simpler to draw at once, in the ordinary way, the two figures (3) and (4) above, and to say frankly that the common 'Some A is not B' cannot distinguish between them. In other words this form cannot be adequately represented by one of these diagrams: it belongs to another propositional theory.

But there is a more conclusive objection than this last. If we were lenient enough to admit the three latter Hamiltonian forms on such a plea as the one in question, we should be bound in consistency to let in a good many more upon exactly the same grounds. Take, for instance, the first two, 'All A is all B,' 'All A is some B'. We often do practically want some common form of expression which shall cover them both, and this was excellently provided by the old *A* proposition 'All A is B,' which just left it uncertain whether the A was all B, or some B only. Perhaps this is indeed the very commonest of all the forms of assertion in ordinary use. Hence if once we come to expressing uncertainties or ambiguities we should have to insist upon retaining this old *A*, not as a substitute for one of the two first Hamiltonian forms, but in addition to them both. Similarly we should require a form to cover the first and the third. Or again, whilst we are about it, we might desire a form to cover all the first four; for we might merely know (as indeed is often the case) that A and B had some part, we did not know how much, in common. What we should want, in fact, would be a simple equivalent for 'Some or all A is some or all B'; or, otherwise expressed, a form for merely denying the truth of 'No A is B'.

The Hamiltonian scheme has, no doubt, a specious look of completeness and symmetry about it. Affirmative and denial, of some and of all, of the subject and predicate, gives clearly $2 \times 2 \times 2 = 8$ forms. But on subjecting them to criticism, by inquiring what they really say, we see that this completeness is illusory. Regard them as expressing the relations of class inclusion and exclusion (and this I strongly hold to be the right way of regarding them) and we only need, or can find place for, *five*. Regard them as expressing to some extent our uncertainty about these class relations, and we want more than eight. This exact group of eight seems merely the outcome of an exaggerated love of verbal symmetry.

If indeed our choice lay simply between the old group and the Hamiltonian, the old one seems to me the soundest and most useful. One or more of those four will express almost all that we can want to express for purely logical purposes, and as they have their root in the common needs and expressions of mankind, they have a knack of signifying just what we want to signify and nothing more. For instance, as just remarked, we may want to say that 'All A are B,' when we do not know whether or not the two terms are coextensive in their application. The old form just hit this off. An obvious imperfection in Hamilton's scheme is that with all his eight forms he cannot express this very common and very simple form of doubtful apprehension, by means of a single proposition. He can express the less common state of doubt between the two 'All A is some B,' and 'Some A is some B,' by one of what I have termed his superfluous forms, *viz.*, by his 'Some B is not any A,' for it exactly covers them both.[1]

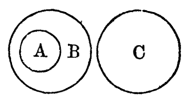
So long as we adhere to the five propositions which correspond to the five distinct diagrams, we are on clear ground. These rest on a tenable theory as to the import of propositions, sufficient to give them cohesion and make a scheme of them. That theory is of course that they are meant to express all the really distinct relations of actual class inclusion and exclusion of two logical terms, and none but these.

The advantages of this form of propositional statement, if few, are at any rate palpable and unmistakable. Each form has a corresponding diagram which illustrates its exact signification with the demonstrative power of an actual experiment. If any

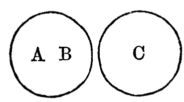
[1] Provided of course we define 'some' = 'some, not all'. This I understand Hamilton to do; but his opinion does not seem absolutely fixed here.

sluggish imagination did not at once realise that from 'All A is some B,' 'No B is any C,' we could infer that 'No A is any C,' he has only to trace the circles, and he sees it as clearly as any one sees the results of a physical experiment. And most imaginations, if the truth were told, are sluggish enough to avail themselves now and then of such a help with advantage.

But whilst this is said it ought clearly to be stated under what restrictions such an appeal may fairly be made. The common practice, adopted in so many manuals, of appealing to these diagrams—Eulerian diagrams as they are often called—seems to me very questionable. Indeed when it is done, as it generally is, without a word of caution as to the important distinction between the implied theories about the import of propositions, it seems to me that there can be no question as to its being wrong. The old four propositions *A, E, I, O,* do *not* correspond to the five diagrams, and consequently none of the figures in the syllogism can be properly represented thus.[1] We may sometimes see *Celarent* represented in the annexed form.



But this is too narrow. The affirmative represented here is not 'All A is B,' but 'All A is *some* B'. To represent *Celarent* adequately in this way we should have to append also the diagram, representing 'All A is all B,' 'No B is C,' and to say frankly that we can only know that one of these diagrams will represent our syllogism; we do not know which.[2] (The necessity of such an appeal to *alternative* diagrams is, I see, admitted by Ueberweg.)



Of course this inability to represent each syllogistic figure by one appropriate diagram will not always affect their cogency as illustrations. Any one can see in the above instance that one diagram will practically take the place of the other; and so it would in *Barbara*, but not in every instance, as we might easily show in detail. But none the less must we remember that the systems of propositions are really based on distinct theories, and we have consequently no right thus without warning to use the diagrams of one system to represent the propositions and syllogisms of another.

[1] The *rejected* figure 'No B is C,' 'No A is B,' with the consequent inability to draw any conclusion at all, *can* be thus exhibited; because the universal negative is the only form common to the two schemes.

[2] If it be urged that the upper diagram is the general one, including the lower as a special case, the answer is twofold. First, that this would be tantamount to a rejection of our scheme of five distinct propositions; and secondly, that even so we should not meet the case of syllogisms involving particular propositions, as the reader will see if he tries thus to exhibit, say, *Disamis.*

So much then for this second scheme of propositional import and arrangement. In spite of its merit of transparent clearness of illustration of a certain number of forms, it is far from answering our purpose as the basis of an extension of Logic. It soon becomes cumbrous and unsymmetrical, and has no flexibility or generality about it. Fortunately there is another mode of viewing the proposition, far more powerful in its applications than either of those hitherto mentioned. It is the basis of the system introduced by Boole, and could never have been realised by any one who had not a thorough grasp of those mathematical conceptions which Hamilton unfortunately both lacked and despised. The fact seems to be that when we quit the traditional arrangement and enumeration of propositions we must call for a far more thorough revision than that exhibited on the system just discussed. Any system which merely exhibits the mutual relations of two classes to one another is not extensive enough. We must provide a place and a notation for the various combinations which arise from considering three, four, or more classes; in fact we must be prepared for a complete generalisation. When we do this we shall soon see that the whole way of looking at the question which rests upon the mutual relation of classes, as regards exclusion and inclusion, will not suffice. There is a fatal cumbrousness and want of symmetry about it which renders it quite inappropriate for any but the simplest cases.

This third view of the interpretation of propositions which has to be substituted for both the preceding, for the purpose of an extended Symbolic Logic, is perhaps best described as indicating the *occupation or non-occupation of compartments.* What we here have to do is to conceive, and invent a notation for, all the possible combinations which any number of class terms can yield; and then find some mode of symbolic expression which shall indicate which of these various compartments are empty or occupied, by the implications involved in the given propositions. This is not so difficult as it might sound, since the resources of mathematical notation are quite competent to provide a simple and effective symbolic language for the purpose. Without entering fully into this scheme, enough may be said to bring out clearly its bearing on the particular subject which now occupies us, *viz.*, the number of distinct forms of proposition which ought to be recognised. The view which is here taken is still distinctly a *class*, rather than a predication, view; but instead of regarding the mutual relation of two or more classes in the way of inclusion and exclusion, it substitutes a complete classification of all the subdivisions which can be yielded by putting any number of classes together, and indicates whether any one or more of these classes is occupied; that is, whether things exist which possess the particular combination of attributes in question.

A fair idea of the meaning, scope, and power of this system will be gained if we begin with two class terms X and Y, and consider the simple cases yielded by their combination. It is clear that we are thus furnished with four possible cases, or *compartments*, as we shall often find it convenient to designate them; for everything which exists must certainly possess both the attributes marked by X and Y, or neither of them, or one and not the other. This is the range of possibilities, from which that of actualities may fall short; and the difference between these two is just what it is the function of the proposition to describe. It will be best to discuss here only intimations that such and such compartments are *empty*, since this happens to be the simplest case. Now how many distinct cases does this system naturally afford? We must approach it, let us remember, without any prepossessions derived from the customary divisions and arrangements.

We should naturally be led, I think, to distinguish fourteen different cases on such a system as this, which would fall into four groups. For there may be one compartment unoccupied, which yields four cases; or two unoccupied, which yields six cases; or three unoccupied, which yields again four cases; or none unoccupied, which yields but one case. *All* cannot be unoccupied, of course, for we cannot deny both the existence and the non-existence of a thing; or, to express it more appropriately on this scheme, given that a thing exists it must be put somewhere or other in our all-extensive scheme of possibilities.

I will only call attention here to the nature of the four simplest out of these 14 cases. Writing, for simplicity, $\bar{x}$ for not-x, and $\bar{y}$ for not-y, these four would be thus represented:—

$xy=0$ or No x is y.
$\bar{x}y=0$ or All y is x.
$x\bar{y}=0$ or All x is y.
$\bar{x}\bar{y}=0$ or Everything is either x or y.

On this plan of notation, of course, xy stands for the compartment, or class, of things which are both x and y; and the expression $xy=0$ states the fact that that compartment is unoccupied; that there is no such class of things. And similarly with the other sets of symbols.

A moment's glance will convince the reader how entirely distinct the group of elementary propositions thus obtained is from that yielded by either of the other two schemes; though, starting from its own grounds, it is just as simple and natural as either of them. Of the four simplest forms contained in it, as indicated above, one is of course the Universal Negative, which presents itself as fundamental on all the three schemes. Two others are Universal Affirmatives, but with the subject and predicate converted. But the fourth is significant, as reminding us how completely relative is the comparative simplicity of a propositional form. On the present scheme this is just as simple as any of the others; but in the traditional arrangement it would probably get in only as a disjunctive, since that arrangement dislikes the double negation 'No not-x is not-y'. Indeed on hardly any view could such a verbal statement as this last be considered as elementary, since almost every one would have to put it into other words before clearly understanding its import on a first acquaintance.

We might easily go through the ten remaining cases referred to above, but to do so would be unsuitable to a general essay like this. It may just be noticed, however, in passing, that the emptying out (as we may term it) of two compartments does not necessarily give a proposition demanding more of the common verbal statement, than that of one only does. For instance the combination, $x\bar{y}=0$, $\bar{x}y=0$, expresses the coincidence of the two classes x and y; it is 'All x is all y'. That of xy and $\bar{x}\bar{y}$ yields the statement that x and y are the contradictory opposites of one another; that x and not-y are the same thing, and consequently y and not-x.

The real merits of this way of regarding and expressing the logical propositions are not very obvious when only two terms are introduced, but it will readily be seen that some such method is indispensable if many terms are to be taken into account. Let us introduce three terms, x, y, and z; and suppose we want to express the fact that there is nothing in existence which combines the properties of all these three terms, that is, that there is no such thing as xyz. If we had to put this into the old forms we should find ourselves confronted with six alternative statements, all of them tainted with the flaw of unsymmetry, *viz.*, 'No x is yz,' 'No y is xz,' 'No z is xy,' as well as the three converse forms of these. No reason can be shown for selecting one rather than another of these; and if we attempted to work with the symmetrical form 'There is no xyz,' we should find that we had no rules at hand to connect it with propositions which had only x, y, or z for subject or predicate.

If we tried the second propositional theory we should only reduce the above six unsymmetrical alternatives to three; three being got rid of by our refusing to recognise that conversion makes any difference in the proposition. But the same inherent vice of unsymmetry in a choice of alternatives would still confront us. No reason could be given why we should rather say that the class x excludes the class yz, rather than that the class y excludes that of xz, or z that of xy. Common language may be perfectly right in tolerating such ambiguities; but a sound symbolic method ought to be naturally cast in a symmetrical form if it is not to break down under the strain imposed by having to work with three or more terms. This requires us to avoid such forms as 'The class x excludes that of yz,' and all the analogous statements of the common Logic, and to put what we have to express into the symbolic shape $xyz=0$. The verbal equivalents for this are, of course, that there is no such thing as xyz, or that the compartment which we denote by xyz is empty. There are theoretical reasons for regarding the latter form as the most rigidly accurate and consistent.

It deserves notice that ordinary language does occasionally recognise the advisability of using symmetrical expressions of this kind, though the common Logic shows no fondness for them. We should as naturally say, for example, that 'Cheapness, beauty, and durability never go together,' or that 'Nothing is at once cheap, beautiful, and durable,' as we should use one of the forms which divide these three terms between the subject and the predicate. But this latter plan is what would be adopted presumably by the strict logician, by his arranging it in some such form as 'No cheap things are beautiful and durable'. It need not be remarked that popular language, though occasionally making use of such symmetrical forms, has never hit upon any general scheme for their expression, and would be sadly at a loss to work upon more complicated materials. Especially would this be the case where negative predicates or attributes had to be taken into account as well as positive. However, what we are here concerned with is the insufficiency of the ordinary logical view rather than the occasional ingenuity of popular expression.

In these remarks it has been attempted to keep closely to the inquiry suggested at the outset—that, namely, of the number of fundamentally distinct logical forms, and the foundation on which each different arrangement must be understood to rest. It seems quite clear that no attempt can be made to answer this question until we have decided, in a preliminary way, what view we propose to take of the proposition and its nature. There is no occasion whatever to tie ourselves down to one view only, as if the import of propositions was fixed and invariable. Very likely other views might be introduced in addition to the three which have been thus examined, though these appear to me to be the only ones with which the student is likely to have to make much acquaintance.

Each of these three stands upon its own basis, yields its appropriate number of fundamentally distinct propositions, and possesses its own merits and defects. The old view has plenty to say for itself, and for ordinary educational purposes will probably never be superseded. It is very simple, it is in close relation with popular language, and it possesses a fine heritage of accurate technical terms and rules of application. Its defects seem to me to be principally these: that it does not yield itself readily to any accurately correspondent diagrammatic system of illustration; and that its want of symmetry forbids its successful extension and generalisation.

The great merit of the second plan—that of class inclusion and exclusion—is its transparent clearness of illustration. We may be said thus to *intuite* the proposition. This has indeed caused a most unwarranted amount of employment of its diagrams by those who do not realise that its five distinct forms of proposition cannot be properly fitted in with the four of the traditional scheme. This clearness is, however, almost its only merit. It possesses little more of symmetry or consequent adaptability of generalisation than the former, and it is considerably removed from popular forms of expression. Above all, as it insists upon exhibiting the *actual* relations of the two classes to one another, it has no power to express that degree of ignorance about these relations which many propositions are bound to do, if they are to state all that we know, and nothing but what we know, about the relation of the subject and predicate to one another. (Hamilton's eight propositions, as already remarked, seem an inconsistent and partial attempt to remedy this latter defect.)

The third scheme is, of course, in comparison with the others, an artificial one, and possesses the merits and defects which might be expected in consequence. It is couched in too technical form, and is too far removed from the language of common life, for it ever to become a serious rival of the traditional scheme on the ground appropriate to the latter. For symmetry, however, and the power which comes of symmetry, nothing can well be put into competition with it.

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NOTE.—Though the majority of logicians seem to be satisfied with the Eulerian mode of representing the ordinary logical propositions, there have always, I find, been some who have felt its insufficiency for the purpose. Ueberweg, for instance, offers an *alternative* of diagrams in some cases. But of all the writers whom I have been able to consult since this paper was in type, the two who lay most stress upon the point are Gergonne and F. A. Lange. The former of these (*Annales des Mathématiques*, vii., 189) has made a careful comparison between the five diagrammatical forms and the four propositional ones, showing in each case how many of the one set are included in any single form of the other. Lange (*Logische Studien*) has spoken even more strongly than I have here, though from a somewhat different point of view, on the radically distinct theories of the import of propositions involved in these two ways of stating or representing them. He has also worked out, as I had done, the number of distinct forms corresponding to the syllogistic figures which are demanded on the Eulerian scheme.