

Logic, First Course, Winter 2020. Week 7, Practice Problems. [Back to course website](#)

Week 7, Practice Problems

The practice problems fall into four groups:

- [Memorizing and recognizing negation introduction and elimination](#)
- [Proofs with negation introduction and elimination](#)
- [Memorizing and recognizing EFSQ](#)
- [Proofs with EFSQ](#)

Before we begin, [recall](#) that the falsum symbol \perp can be typed in one of three ways. First, you can just cut and paste from this screen or anywhere else. Second, you can alternatively type `!?`. Third, you can program your computer to accept a shortcut like `\bot` or anything else you want. (Here are external sites which contain instructions on how to do this on a [Mac](#) or [Windows](#)).

Memorizing and recognizing negation introduction and elimination

The first set of five problems is just practice in memorizing and recognizing negation introduction and elimination. The proofs involving negation elimination are just 1-2 extra lines beyond the assumptions. The proofs involving negation introduction are just 3-5 extra lines beyond the assumptions.

Example 1.

exercise

$a, \neg a \vdash \perp$

1. a :assumption
2. $\neg a$:assumption

Example 2.

exercise

$(a \wedge b), \neg(a \wedge b) \vdash \perp$

1. $(a \wedge b)$:assumption
2. $\neg(a \wedge b)$:assumption

Example 3.

exercise

$$((a \wedge b) \wedge c), \neg(a \wedge b) \vdash \perp$$

1. $(a \wedge b) \wedge c$:assumption
2. $\neg(a \wedge b)$:assumption

Example 4.

exercise

$$(p \rightarrow \neg q) \vdash \neg(p \wedge q)$$

1. $p \rightarrow \neg q$:assumption

Example 5.

exercise

$(p \rightarrow \neg q), p, q \vdash \perp$

1. $p \rightarrow \neg q$:assumption
2. p :assumption
3. q :assumption

Proofs with negation introduction and elimination

These next five proofs are simple proofs involving negation introduction and elimination, along with the other rules which we learned earlier.

Example 6.

exercise

$(a \rightarrow c), (b \rightarrow d), \neg(c \wedge d) \vdash \neg(a \wedge b)$

1. $a \rightarrow c$:assumption
2. $b \rightarrow d$:assumption
3. $\neg(c \wedge d)$:assumption

Example 7.

exercise

$(p \rightarrow r), (q \rightarrow r), \neg r \vdash \neg(p \vee q)$

1. $p \rightarrow r$:assumption
2. $q \rightarrow r$:assumption
3. $\neg r$:assumption

Example 8.

exercise

$c, (c \rightarrow (d \wedge a)), (b \rightarrow \neg d) \vdash \neg(a \rightarrow b)$

1. c :assumption
2. $c \rightarrow (d \wedge a)$:assumption
3. $b \rightarrow \neg d$:assumption

Example 9.

This example shows that we can derive modus tollens in our proof system.

exercise

$(p \rightarrow q), \neg q \vdash \neg p$

1. $p \rightarrow q$:assumption
2. $\neg q$:assumption

Example 10.

In this example, be sure to remember to think of $\neg\neg a$ as "a negation applies to $\neg a$ ".

exercise

$(\neg a \rightarrow \neg b), b \vdash \neg\neg a$

1. $\neg a \rightarrow \neg b$:assumption
2. b :assumption

Memorizing and recognizing EFSQ

These next proofs are designed to help you do EFSQ, which we cover in the second lecture from this week. These proofs are intended to be very short, all under 5 lines beyond the assumptions.

Example 11.

exercise

$a, (a \rightarrow \perp) \vdash b$

1. a :assumption
2. $a \rightarrow \perp$:assumption

Example 12.

exercise

$a, b, (b \rightarrow \neg a) \vdash c$

1. a :assumption
2. b :assumption
3. $b \rightarrow \neg a$:assumption

Example 13.

exercise

$(a \wedge b), (c \wedge \neg b) \vdash d$

1. $a \wedge b$:assumption
2. $c \wedge \neg b$:assumption

Example 14.

On this one, resist the inclination to do an arrow introduction, and focus rather on finding a contradiction among the assumptions and then doing ex falso.

exercise

$(a \rightarrow b), \neg(a \rightarrow b) \vdash (a \rightarrow c)$

1. $a \rightarrow b$:assumption
2. $\neg(a \rightarrow b)$:assumption

Example 15.

This one is just practice with the repeat rule. We did one like it in the second lecture.

exercise

$\top \vdash (b \rightarrow b)$

1.

Proofs with EFSQ

These are simple proofs with EFSQ and the other rules which we have available.

Example 15.

exercise

$(a \rightarrow b), \neg b \vdash \neg a$

1. $a \rightarrow b$:assumption
2. $\neg b$:assumption

Example 16.

exercise

$$\neg a \vdash (a \rightarrow b)$$

1. $\neg a$:assumption

Example 17.

This next one does not involve ex falso, but rather involves the repeat rule.

exercise

$$b \vdash (a \rightarrow b)$$

1. b :assumption

Example 18.

On this one, the last line is justified via disjunction elimination. To complete the disjunction elimination, one first needs to prove $\neg a \rightarrow (a \rightarrow b)$, but this just amounts to doing Example 16 inside a bracket. Finally, one needs to get $b \rightarrow (a \rightarrow b)$, but this just amounts to doing Example

17 inside a bracket. Note that this problem is one-half of the equivalence of material conditional and a certain kind of disjunction [which we learned about earlier](#).

exercise

$$(\neg a \vee b) \vdash (a \rightarrow b)$$

1. $\neg a \vee b$:assumption

Example 19.

This one shows the converse, under the assumption of law of excluded middle for the consequent. One proceeds by doing a disjunction elimination on the instance of the law of the excluded middle. One line one needs is $\neg b \rightarrow (\neg a \vee b)$, but this will be like Example 15 inside a bracket (since one has the assumption $a \rightarrow b$). Another line one needs is $b \rightarrow (\neg a \vee b)$ but this can be done in a familiar way using disjunction introduction.

exercise

$$(a \rightarrow b), (\neg b \vee b) \vdash (\neg a \vee b)$$

1. $a \rightarrow b$:assumption
2. $\neg b \vee b$:assumption

Example 20.

exercise

$$(a \wedge (b \wedge c)), (a \rightarrow d), (b \rightarrow e), (c \rightarrow f), \neg(d \wedge f) \vdash ((a \wedge c) \rightarrow e)$$

1. $a \wedge (b \wedge c)$:assumption
2. $a \rightarrow d$:assumption
3. $b \rightarrow e$:assumption
4. $c \rightarrow f$:assumption
5. $\neg(d \wedge f)$:assumption