Logic, First Course, Winter 2020. Week 2, Lecture 2, Handout.

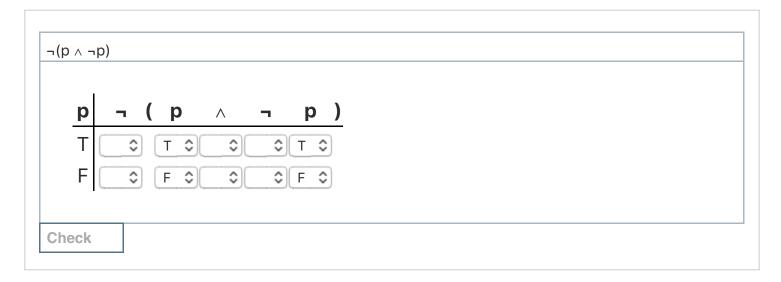
Definitions and law of excluded middle

A *tautology* is a well-formed formula that is always true. That is, when you look at its truth-table, you see all true's in the column under the main connective.

Law of excluded middle: $p \lor \neg p$ is a tautology.



Law of non-contradiction: $\neg(p \land \neg p)$ is a tautology.

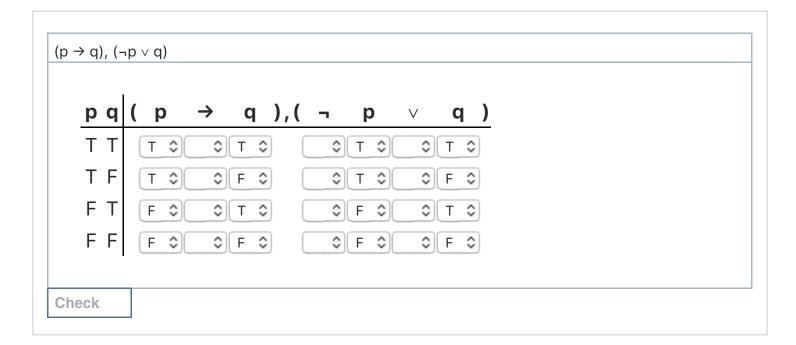


An equivalence is just two formulas ϕ and ψ such that their biconditional $\phi \leftrightarrow \psi$ is a tautology.

The law of double-negation: p is equivalent to $\neg \neg p$.

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Commutativity and associativity and distributivity

Law of commutativity for conjunction: $p \land q$ is equivalent to $q \land p$.

Law of commutativity for disjunction: $p \lor q$ is equivalent to $q \lor p$

Law of associativity for conjunction: $(p \land q) \land r$ is equivalent to $p \land (q \land r)$.

Law of associativity for disjunction: $(p \lor q) \lor r$ is equivalent to $p \lor (q \lor r)$.

Law of distribution, part 1: $p \land (q \lor r)$ is equivalent to $(p \land q) \lor (p \land r)$

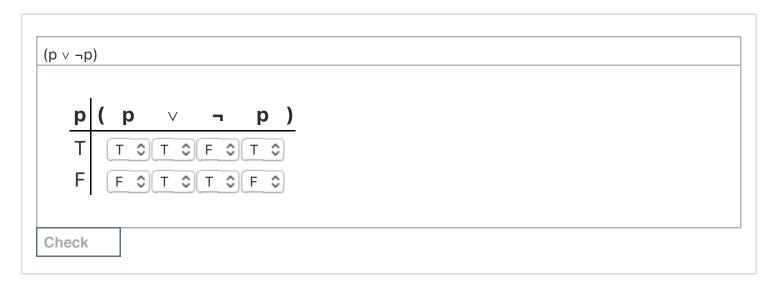
Law of distribution, part 2: $p \lor (q \land r)$ is equivalent to $(p \lor q) \land (p \lor r)$

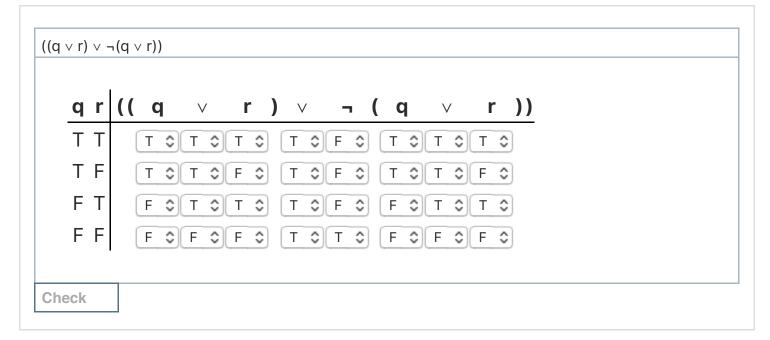
DeMorgan Law, part 1: $\neg(p \land q)$ is equivalent to $\neg p \lor \neg q$.

DeMorgan Law, part 2: $\neg(p \lor q)$ is equivalent to $\neg p \land \neg q$.

Recognizing tautologies and equivalences quickly

Substitution. Given any tautology, if we uniformly substitute other formulas for the basic propositional letters, we still get a tautology.





Replacement of equivalents with equivalents. If two formulas are equivalent, then you can replace the one with the other in any formula and not change the truth-value.

Chaining. If a first formula ϕ is the same as a second formula ψ in terms of truth-values and the second formula ψ is the same as a third formula ξ in terms of truth-values, then the first formula ϕ is the same as the third formula ξ in terms of truth-values.

Boole and the laws of thought

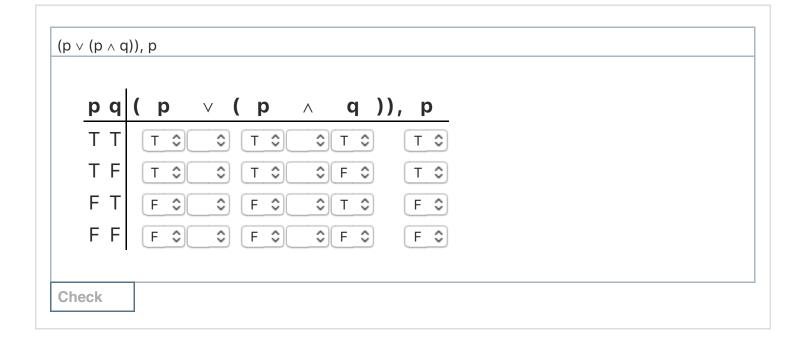


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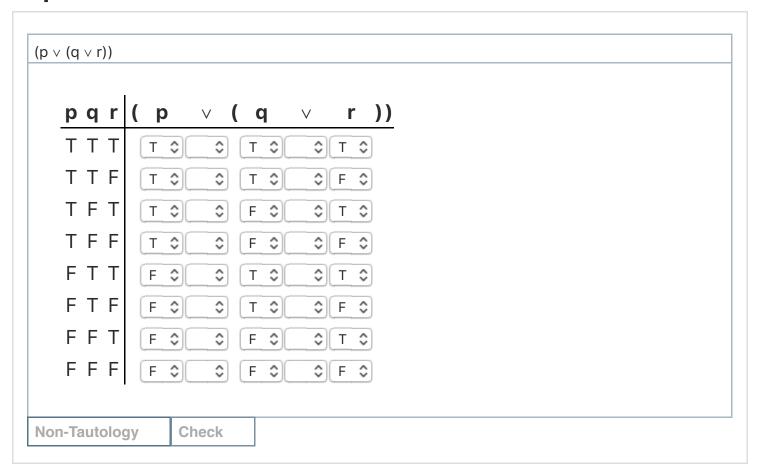
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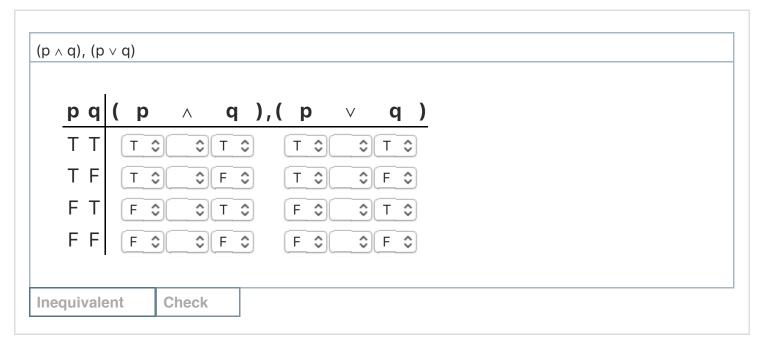
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Spotting non-tautologies and spotting non-equivalences





These is a handout written for Logic, First Course, Winter 2020. It is run on the Carnap software, which is an:

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