

Logic, First Course, Winter 2020. Week 2, Lecture 1. [Back to course website](#)

# Translating between natural language and propositional logic

In this lecture, we describe how to translate between natural language and propositional logic.

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## Translating and a simple example

Whenever we are translating, something is preserved and something is lost. What are we trying to preserve in translating between natural language and propositional logic? We are trying to preserve the structure of the sentence that shows the way in which the truth of the whole depends on the truth of the parts.

*Example 1:* "If the economy prospers, then there will be low unemployment. Therefore, if the economy prospers, then people will be happy."

We take each sentence and translate it into propositional logic as follows:

"If  $p$ , then  $q$ . Therefore, if  $p$ , then  $r$ ."

If we replace "if . . . then . . ." by the arrow connective, we have

" $p \rightarrow q$ . Therefore,  $p \rightarrow r$ ."

Consider the following example:

*Example 2:* "If the weather is worse, then there will be low unemployment. Therefore, if the weather is worse, then people will be happy."

We would end up translating this one exactly the same way. For, the two examples have a common structure: they are "if . . . then . . ." statements organized in the same way: the premise says that an antecedent implies a consequent, and it is concluded from this that the very same antecedent implies a different consequent. This sameness of structure holds even though Example 2 has a false premise whereas Example 1 has a true premise. Hence, the relevant kind of structure is different from the truth-value of the premises; and it is likewise different than the truth-value of the conclusions.

## Another simple example

*Example 3:* "It is raining or it's not raining. If it is raining, then I am staying inside. If it's not raining, then I'm going to the beach."

We translate this as:

" $p$  or not  $p$ . If  $p$ , then  $r$ . If not  $p$ , then  $s$ ."

In symbols, the translation is:

" $p \vee \neg p. p \rightarrow r. \neg p \rightarrow s$ ."

This translation preserves structure in the relevant sense. But suppose alternatively, you had translated as follows:

" $p$  or  $n$ . If  $p$ , then  $r$ . If  $n$ , then  $s$ ."

On this alternative translation, the truth-value of "If it's not raining, then I'm going to the beach" does not depend on the truth-value of "it is raining." However, pre-theoretically, there is such a dependence.

# A recipe for translation

Here is an easy three-step recipe for translating:

1. Identify the smallest sub-sentences of the example which can be true or false.
2. Assign arbitrary propositional letters to these, and rewrite using them in place of originals.
3. Look for propositional letters that are connected by "and", "or", as well as "if . . . then. . ." and "if and only if." Then translate via the symbols  $\wedge$ ,  $\vee$ ,  $\rightarrow$ ,  $\leftrightarrow$ . Do the same for negation.

## An example following the recipe

*Example 4.* Anthony attends the meeting if and only if Briana stays at the office. If Briana stays at the office, then Cynthia attends the meeting and Dylan checks his email.

First we identify the smallest sub-sentences of the example which can be true or false. It is the various claims about the actions of the individual agents, taken one by one.

Second we assign arbitrary propositional letters to these:

$a$  = Anthony attends the meeting

$b$  = Briana stays at the office

$c$  = Cynthia attends the meeting

$d$  = Dylan checks his email

and then rewrite the sentence using them in place of originals:

$a$  if and only if  $b$ . If  $b$ , then  $c$  and  $d$ .

Finally, we for propositional letters that are connected by "and", "or", as well as "if . . . then. . ." and "iff." Then translate via the symbols  $\wedge$ ,  $\vee$ ,  $\rightarrow$ ,  $\leftrightarrow$ :

$a \leftrightarrow b. b \rightarrow (c \wedge d)$

The translation-checker presumes that the assignment of propositional letters has already been done; and in the problems, the assignment will be given. Hence, the translation-checker only asks for the third and final step. Further, the translation-checker only works sentence by

sentence. Since Example 4 involved two sentences, it would be split into two parts in the translation-checker:

Anthony attends the meeting if and only if Br

Anthony attends the meeting if and only if Briana stays at the office.

If Briana stays at the office, then Cynthia a

If Briana stays at the office, then Cynthia attends the meeting and Dylan checks his email.

Three quick notes about the translation-checker:

1. It assumes that you remember how to type the logical symbols. If you need a refreshed on this, consider reviewing [how to type the connectives on the keyboard](#).
2. It recopies the English sentence in question into the place where you are supposed to type your translation into propositional logic. Hence, you will first have to delete the English sentence before inputting your answer. Doing a "select all" (usually a command A), and then hitting "delete" is fast.
3. Press **return** to check your answer.

## The recipe applied to negation

Applying the recipe to negation is tricky because of the variability of where "not" can occur within the sentence. For instance, consider:

- Dylan does not drive.
- It's not the case that Dylan drives
- It's not true that Dylan drives

Intuitively, these are all true precisely when "Dylan drives" is false. Hence we would translate each by  $\neg p$ , where  $p$  = Dylan drives.

Hence, when you're applying the recipe to negation, you have to look at the various "not" 's occurring in the sentence and see them as negating some other sentence.

## Example with negation

*Example 5.* If Anthony attends the meeting, then Briana stays at the office. If Briana does not stay at the office, then Anthony does not attend the meeting.

Again, the idea is that you have to look at the various *not*'s occurring in the sentence and see them as negating some other sentence.

Consider "Briana does not stay at the office." Looks like this negates "Briana stays at the office." Similarly, consider "Anthony does not attend the meeting." Looks like this negates "Anthony attends the meeting." Hence we should translate as:

"if  $a$ , then  $b$ . If not  $b$ , then not  $a$ ."

where we still use the abbreviations:

$a$  = Anthony attends the meeting

$b$  = Briana stays at the office

Finally, we translate the propositional connectives:

$a \rightarrow b. \neg b \rightarrow \neg a.$

Again, the translation-checker does this sentence by sentence. Try it out:

If Anthony attends the meeting, then Briana s

If Anthony attends the meeting, then Briana stays at the office

If Briana does not stay at the office, then Al

If Briana does not stay at the office, then Anthony does not attend the meeting.

# Further examples

In the following examples, we use the abbreviations:

$a$  = Anthony attends

$b$  = Briana attends

$c$  = Cynthia attends

$d$  = Dylan attends

If Anthony attends or Briana attends then Cyn

If Anthony attends or Briana attends then Cynthia attends and Dylan attends.

If Anthony attends and Briana attends then Cyn

If Anthony attends and Briana attends then Cynthia attends or Dylan attends.

If Anthony attends then Briana attends, and i

If Anthony attends then Briana attends, and if Claire attends then Dylan attends.

If Anthony does not attend then Briana attend

If Anthony does not attend then Briana attends.

It is not the case that if Anthony attends th

It is not the case that if Anthony attends then Briana attends.

Anthony attends if and only if Briana does no

Anthony attends if and only if Briana does not attend.

Try these out for yourself. If you cannot get the answer, the solutions are down at the bottom under "[Solutions to further examples.](#)"

## Translations and ambiguity

We had previously spoken about [ambiguity](#), and we learned that in our formal system we handle ambiguity by the use of parentheses.

Take the following example. When we say a list of three conjunctions in English, we cannot translate it as e.g.  $a \wedge b \wedge c$  since this is ambiguous between the well-formed formula  $(a \wedge b) \wedge c$  and the well-formed formula  $a \wedge (b \wedge c)$ . The translation checker only accepts well-formed formulas, and so it will not accept  $a \wedge b \wedge c$ , but it will accept either  $(a \wedge b) \wedge c$  or  $a \wedge (b \wedge c)$ , as you can see for yourself:

Anthony attends and Briana attends and Cynthia

Anthony attends and Briana attends and Cynthia attends.

This raises a natural question about our translations in general: since the translations into propositional logic make more distinctions than we make in natural language, aren't the translations inaccurate in a certain respect? One way we can handle this is with a notion of logical equivalence which we will study soon. Logical equivalence roughly means: has the same truth-conditions. In short, while  $(a \wedge b) \wedge c$  and  $a \wedge (b \wedge c)$  are different well-formed formulas, it turns out that they are logically equivalent.

But other types of ambiguities do not involve such equivalence. For instance, take  $a \wedge b \vee c$ . This is ambiguous between  $(a \wedge b) \vee c$  and  $a \wedge (b \vee c)$ , and it turns out that these have different truth-conditions. For instance, only from the second can you infer  $a$ . As we mentioned in our earlier discussion of [ambiguity](#), in natural language we typically handle this by adding extra words or emphases or commas. Based on this earlier discussion, you should have no problem solving the following:

Either both Anthony attends and Briana attend:

Either both Anthony attends and Briana attends, or Cynthia attends

Anthony attends, and either Briana attends or

Anthony attends, and either Briana attends or Cynthia attends

Again, the parentheses in our translations serve many of the same roles that extra words like "either" in the phrase "either . . . or" or the grammatical commas.

## More practice with translation and ambiguity

In this set of examples, we use the following abbreviations:

$f$ = The *foreign* minister supports the bill

$o$ = The *opposition* leader supports the bill

$p$ = The *prime* minister supports the bill

$c$ = The bill is contentious

$a$ = The bill passes



The prime minister does not support the bill .

The prime minister does not support the bill or the opposition leader supports the bill or the foreign minister supports the bill.

The following is tricky only because you have to think to use a propositional connective to handle the "and in addition":

If the prime minister supports the bill then .

If the prime minister supports the bill then the opposition leader supports the bill; and in addition the foreign minister supports the bill.

If the prime minister supports the bill then |

If the prime minister supports the bill then both the opposition leader supports the bill and the foreign minister supports the bill.

The bill is contentious and either the prime |

The bill is contentious and either the prime minister supports the bill or the foreign minister supports the bill.

Either the bill is contentious and the prime |

Either the bill is contentious and the prime minister supports the bill, or the foreign minister supports the bill.

Before you try this next one, remember that the key was " $a$  = The bill passes" (we had already used  $p$  for the claim about the prime minister).

If the bill passes, then either the opposition

If the bill passes, then either the opposition leader supports the bill or the prime minister supports the bill.

Either the opposition leader supports the bill

Either the opposition leader supports the bill if it the bill passes, or the prime minister supports the bill.

This last example is tricky because of the placement of the "if". We will practice more with such placement later.

Try these out for yourself. If you cannot get the answer, the solutions are down at the bottom under [Solutions to more practice with translation and ambiguity](#).

## Principle of compositionality

Earlier we said that in translating, we are trying to preserve the structure of the sentence that illustrates the way in which the truth of the whole depends on the truth of the parts. Truth is one aspect of meaning, and the more general point about meaning (or "sense") was described by Carnap in his 1947 book *Meaning and Necessity* (this quotation is [section 28, p. 121](#)):

**In other words, the sense of the whole expression is a function of the senses of the names occurring in it.**

Applied to our setting, one is thinking of the little  $p$ 's and  $q$ 's as names of English sentences and of the relevant facet of meaning being just being the truth-value of these smaller atomic propositions. The more general principle is sometimes called the *principle of compositionality*: the meaning of the larger linguistic units is a function of the meaning of the smaller linguistic units out of which it is composed. In short, the meaning of the larger units is determined by the meaning of the smaller units out of which it is structurally composed.<sup>1</sup>

However, in some cases, there is a more pronounced dependence in the opposite direction, so that the meaning of the smaller linguistic units is determined by the meaning of the larger linguistic units. For instance, take a non-empirical and abstract concept like "number." One

natural thought is that number-claims like " $7+5=12$ " and "there are infinitely many prime numbers" are not true just taken one by one, but that their truth is somehow dependent on either the basic axioms which pin down their use, or on the larger practice of our usage with number words. Examples like this prompted Wittgenstein (who [we mentioned in Week 1](#)) and his predecessors to say things like the following:

**Only propositions have sense; only in the nexus of a proposition does a name have meaning.**

This kind of principle is sometimes called "the context principle."

## Solutions to further examples

This section contains the solution to the examples in ["Further Examples"](#).

Recall that we used the abbreviations:

$a$  = Anthony attends

$b$  = Briana attends

$c$  = Cynthia attends

$d$  = Dylan attends

$$(a \vee b) \rightarrow (c \wedge d)$$

If Anthony attends or Briana attends then Cynthia attends and Dylan attends.

$$(a \wedge b) \rightarrow (c \vee d)$$

If Anthony attends and Briana attends then Cynthia attends or Dylan attends.

$(a \rightarrow b) \wedge (c \rightarrow d)$

If Anthony attends then Briana attends, and if Claire attends then Dylan attends.

$(\sim a \rightarrow b)$

If Anthony does not attend then Briana attends.

$\sim(a \rightarrow b)$

It is not the case that if Anthony attends then Briana attends.

$(a \leftrightarrow \sim b)$

Anthony attends if and only if Briana does not attend.

## Solutions to more practice with translation and ambiguity

This section contains the solution to the examples in [More practice with translation and ambiguity](#). Recall we used the abbreviations:

$f$  = The foreign minister supports the bill

$o$  = The opposition leader supports the bill

$p$  = The prime minister supports the bill

$c$  = The bill is contentious

$a$  = The bill passes

$$\sim p \vee (o \vee f)$$

The prime minister does not support the bill or the opposition leader supports the bill or the foreign minister supports the bill.

This one has another solution, namely:

$$(\sim p \vee o) \vee f$$

The prime minister does not support the bill or the opposition leader supports the bill or the foreign minister supports the bill.

$$(p \rightarrow o) \wedge f$$

If the prime minister supports the bill then the opposition leader supports the bill; and in addition the foreign minister supports the bill.

$$(p \rightarrow (o \wedge f))$$

If the prime minister supports the bill then both the opposition leader supports the bill and the foreign minister supports the bill.

$$c \wedge (p \vee f)$$

The bill is contentious and either the prime minister supports the bill or the foreign minister supports the bill.

$$(c \wedge p) \vee f$$

Either the bill is contentious and the prime minister supports the bill, or the foreign minister supports the bill.

$a \rightarrow (o \vee p)$

If the bill passes, then either the opposition leader supports the bill or the prime minister supports the bill.

$((a \rightarrow o) \vee p)$

Either the opposition leader supports the bill if it the bill passes, or the prime minister supports the bill.

These are lecture notes written for [this course](#).<sup>2</sup>

1. The history of the principle of compositionality is studied by [Janssen, Theo M. V. 2012. "Compositionality: Its Historic Context." In Oxford Handbook of Compositionality, edited by Wolfram Hinzen, Edouard Machery, and Markus Werning, 19–46.](#)↵
2. It is run on the Carnap software, which is↵

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