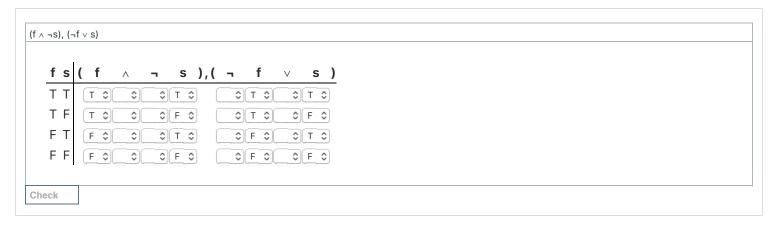
Logic, First Course, Winter 2020. Week 9, Lecture 2, Handout.

Assigning probabilities to rows of the truth table

f = the coin flips heads on the first flip

s = the coin flips heads on the second flip



It is natural to assign probabilities to each of these outcomes. If our coin was a fair coin, we would assign probabilities as follows:

Probability assignment for fair coin, for propositions f and s:

1/4 -- TT = Heads on first flip, heads on second flip.

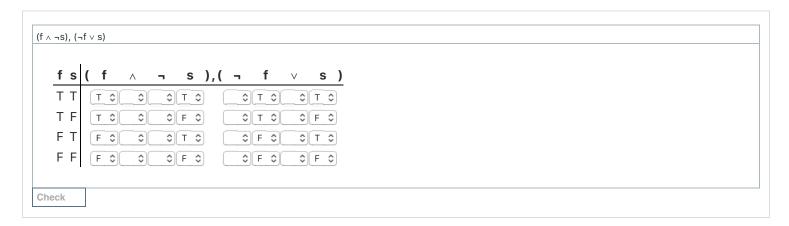
1/4 -- TF = Heads on first flip, tails on second flip.

1/4 -- FT = Tails on first flip, heads on second flip.

1/4 -- FF = Tails on first flip, tails on second flip.

What is the probability of $f \land \neg s$?

What is the probability of $\neg f \lor s$?



Probability assignment for biased coin, for propositions f and s:

1/9 -- TT = Heads on first flip and heads on second flip.

2/9 -- TF = Heads on first flip and tails on second flip.

2/9 -- FT = Tails on first flip and heads on second flip.

4/9 -- FF = Tails on first flip and tails on second flip.

What is the probability of $f \land \neg s$?

What is the probability of $\neg f \lor s$?

In general, we can summarize as follows:

- A probability assignment assigns numbers between 0 and 1 to each row of the truth-table, so that the numbers all add up to 1.
- One then assigns probabilities $Pr(\phi)$ to propositions ϕ by adding up the probabilities of all those rows at which the proposition ϕ is true.

To illustrate, let's note that it is possible to assign a row of a truth-table probability zero. For instance, suppose we use the key

p = it is sunny on Monday

q = it is sunny on Tuesday

Then we might assign probabilities as follows, since we live in a part of the world where it is very unlikely to have two days in a row that are not sunny:

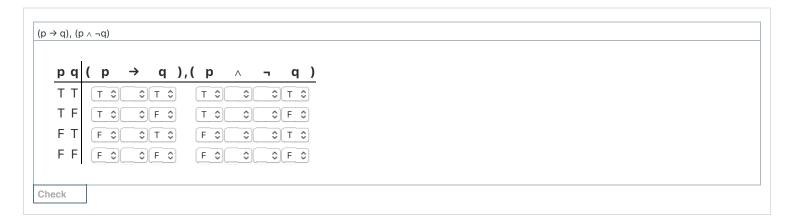
Probability assignment for an agent's degrees of belief about the weather, for propositions p and q:

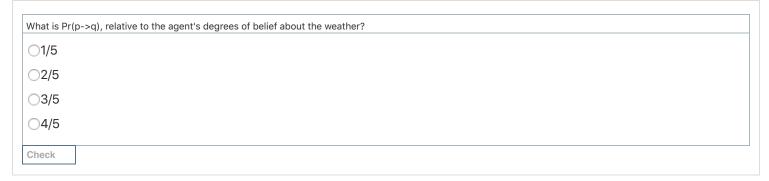
3/5 -- TT = It is sunny on Monday and it is sunny on Tuesday.

1/5 -- TF = It is sunny on Monday and it is not sunny on Tuesday.

1/5 -- FT = It is not sunny on Monday and it is sunny on Tuesday.

0 -- FF = It is not sunny on Monday and it is not sunny on Tuesday.





What is Pr(p)	p/\~q), relative to the agent's degrees of belief about the weather?
<u></u> 1/5	
<u></u>	
○3/5	
04/5	
Check	

The rules of probability

- First rule: $0 \le Pr(\phi) \le 1$
- Second rule: if ϕ is a tautology, then $Pr(\phi) = 1$.
- Third rule: $Pr(\neg \phi) = 1 Pr(\phi)$.
- Fourth rule: $Pr(\phi \lor \psi) = Pr(\phi) + Pr(\psi) Pr(\phi \land \psi)$

Example 1. Suppose that Pr(p) = .4 and Pr(q) = .5 and $Pr(p \land q) = .2$. What is $Pr(p \lor q)$?

Example 2. Suppose that Pr(p) = .3 and Pr(q) = .2 and $Pr(p \lor q) = .45$. What is $Pr(p \land q)$?

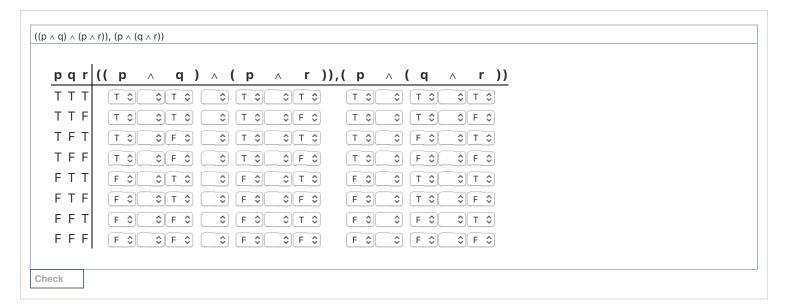
Example 3. Suppose that Pr(p) = .3 and Pr(q) = .2 and $Pr(\neg p \land \neg q) = .5$. What is $Pr(\neg p \lor \neg q)$?

Connections between probability and logic

- If $\phi \to \psi$ is a tautology, then $Pr(\phi) \le Pr(\psi)$.
- If ϕ and ψ are equivalent, then $Pr(\phi) = Pr(\psi)$.

Example 4. Show that $Pr(\phi \land (\psi \lor \xi)) = Pr(\phi \land \psi) + Pr(\phi \land \xi) - Pr(\phi \land (\psi \land \xi))$.

One can check the equivalence mentioned at the end of this video by doing a truth-table like follows:



One can calso check the equivalence by doing two simple proofs:

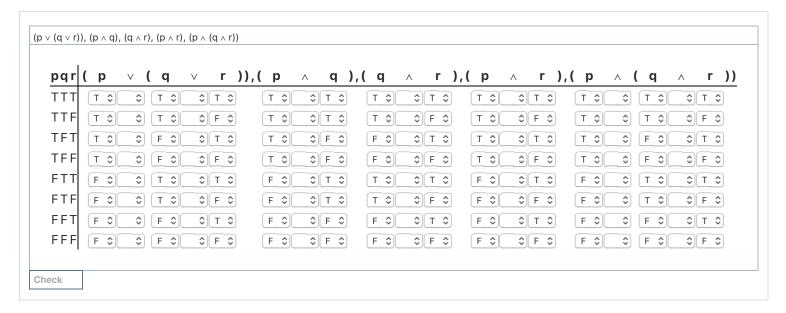
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exercise  \frac{((p \land q) \land (p \land r)) \vdash (p \land (q \land r))}{1. (p \land q) \land (p \land r) : assumption }
```



Example 5. Show that $Pr(\phi \lor (\psi \lor \xi)) = Pr(\phi) + Pr(\psi) + Pr(\xi) - Pr(\phi \land \psi) - Pr(\psi \land \xi) - Pr(\phi \land \xi) + Pr(\phi \land (\psi \land \xi))$.

This is a special case of the so-called 'Inclusion-Exclusion' formula.

Example 6. Let us verify this formula, in a specific case. Consider the following truth-table, which we complete just like in Week 1 (the commas separating the formulas is just a way for us to write many truth-tables in one box):



Suppose that each of the rows is weighted equally, that is each has equal chance of happening, namely a 1/8 chance of happening.

These are lecture notes written for this course.¹

1. It is run on the Carnap software, which is ←

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