Logic, First Course, Winter 2020. Week 4, Section Meeting. Back to course website

Connecting multiple properties with some and all

Quantifiers are very good at capturing the truth-conditions of statements about "some" and "all" that connect two properties.

Translation schemas

Here are some common translation schemas connecting two properties:

Schema	Translation	Schema Negated	Translation of Negation
Some *F* are *G*	$\exists x\,(Fx \wedge Gx)$	No *F* are *G*	$\neg\exists x\,(Fx \wedge Gx)$
Some *F* are not *G*	$\exists x\,(Fx \wedge \neg Gx)$	No *F* are not *G*	$\neg\exists x\,(Fx \wedge \neg Gx)$
All *F* are *G*	$\forall x\,(Fx \rightarrow Gx)$	Not all *F* are *G*	$\neg\forall x\,(Fx \rightarrow Gx)$
All *F* are not *G*	$\forall x\,(Fx \rightarrow \neg Gx)$	Not all *F* are *G*	$\neg\forall x\,(Fx \rightarrow Gx)$

Note that in these schemas the "some"-statements are translated with an existential quantifier together with a conjunction, while the "all"-statements are translated with a universal quantifier together with an arrow statement.

Let us practice further with these translation schemas with respect to the following examples:

C = "is cheerful"
F = "is festive"
G = "is a geography major"
L = "is a law student"
R = "is relaxed"
T = "likes table-tennis"

Example 1

```
Every law student is cheerful and some geogra
```

Every law student is cheerful and some geography majors are cheerful.

Example 2

In the following, remember that "but" would just be translated by "and":

```
Some geography majors are relaxed but no law s
```

Some geography majors are relaxed but no law students are relaxed.

Example 3

```
Some geography majors are not festive, but no
```

Some geography majors are not festive, but no law students are not festive.

Example 4

```
Not all law students like table-tennis.
```

Not all law students like table-tennis.

Example 5

```
If all law students are relaxed, then all geo
```

If all law students are relaxed, then all geography majors are festive.

Example 6

```
If all law students are cheerful, then some l
```

If all law students are cheerful, then some law students are festive.

More practice

Let us practice with these translation schemas with respect to the following examples:

C = "is clever"
F = "is fearful"
H = "is a hobbit"
L = "is an elf"
T = "is tall"
W = "is a wizard"

Example 7

```
Some hobbits are fearful and no elves are fea
```

Some hobbits are fearful and no elves are fearful.

Example 8

```
All elves are clever and not all wizards are
```

All elves are clever and not all wizards are clever.

Example 9

On this one, be sure to use parentheses to separate the two conjunctions.

```
All elves are tall and some wizards are tall
```

All elves are tall and some wizards are tall and no hobbits are tall.

Example 10

```
If all elves are not fearful then some hobbit
```

If all elves are not fearful then some hobbit is not fearful.

Example 11

```
Not all wizards are tall and not all hobbits a
```

Not all wizards are tall and not all hobbits are fearful.

More than two properties

We can slightly expand on this by considering how to translate statements like "tall wizard" and "clever hobbit". Since to say that someone is a tall wizard is to say that they both tall *and* a wizard, it should not be too surprising that we translate this with a conjunction. We continue to use the same key as the previous section.

Example 12

```
All tall wizards are clever.
```

All tall wizards are clever.

Example 13

```
Not all clever wizards are tall.
```

Not all clever wizards are tall.

Example 14

```
Some clever hobbits are fearful.
```

Some clever hobbits are fearful.

Example 15

```
If some clever hobbits are fearful then some
```

If some clever hobbits are fearful then some fearful hobbits are clever.

Example 16

```
If not all tall wizards are fearful then not a
```

If not all tall wizards are fearful then not all fearful elves are clever.

Example 17

```
If some clever elves are wizards, then not al
```

If some clever elves are wizards, then not all clever elves are not tall.

More practice with more than two properties

Let us continue practicing with more than two properties using the key from the first section:

C = "is cheerful"
F = "is festive"
G = "is a geography major"
L = "is a law student"
R = "is relaxed"
T = "likes table-tennis"

Example 18

```
Every cheerful  law student is relaxed.
```

Every cheerful law student is relaxed.

Example 19

```
If some festive law student likes table-tenni
```

If some festive law student likes table-tennis, then some festive law student is relaxed.

Example 20

```
Not all relaxed geography majors like table-t
```

Not all relaxed geography majors like table-tennis.

These are section notes written for this course.[1]

1. It is run on the Carnap software, which is↩