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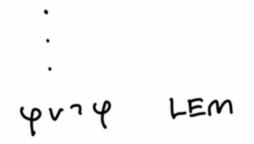
Logic, First Course, Winter 2020. Week 8, Lecture 1, Handout.

Double negation rule



Law of the excluded middle

Since we will often want to appeal to double-negation without having to redo this proof over and over, we simply include a new rule for law of the excluded middle which says that one can always put $\phi \lor \neg \phi$ on a line, and justify it as LEM . Note that no line number is put down as part of the justification.



Other derived rules

- Law of excluded middle: p ∨ ¬p is a tautology. Abbreviation: LEM
- Law of non-contradiction: $\neg(p \land \neg p)$ is a tautology. Abbreviation: LNC
- The law of double-negation: p is equivalent to ¬¬p. Abbreviation: DN
- Law of commutativity for conjunction: $p \land q$ is equivalent to $q \land p$. Abbreviation: LCC
- Law of commutativity for disjunction: $p \lor q$ is equivalent to $q \lor p$. Abbreviation LCD.
- Law of associativity for conjunction: (p ∧ q) ∧ r is equivalent to p ∧ (q ∧ r). Abbreviation:

 LAC
- Law of associativity for disjunction: $(p \lor q) \lor r$ is equivalent to $p \lor (q \lor r)$. Abbreviation: LAD
- Law of distribution, part 1: p ∧ (q ∨ r) is equivalent to (p ∧ q) ∨ (p ∧ r). Abbreviation: LDC (where the final "C" is short for the initial conjunction)
- Law of distribution, part 2: p ∨ (q ∧ r) is equivalent to (p ∨ q) ∧ (p ∨ r). Abbreviation: LDD (where the final "D" is short for the initial disjunction)

- DeMorgan Law, part 1: $\neg(p \land q)$ is equivalent to $\neg p \lor \neg q$. Abbreviation: DMOR (since it ends in an or statement)
- DeMorgan Law, part 2: ¬(p ∨ q) is equivalent to ¬p ∧ ¬q. Abbreviation: DMAND (since it ends in an and statement)
- Modus ponens. $p \rightarrow q, p \vdash q$. Abbreviation: E->
- Modus tollens. $p \rightarrow q, \neg q \vdash \neg p$. Abbreviation: MT
- Disjunctive syllogism, p ∨ q, ¬p ⊢ q. Abbreviation: PDS (for positive disjunctive syllogism, since it starts with an initial unnegated "positive" disjunction).
- Disjunctive syllogism. ¬(p ∧ q), p ⊢ ¬q. Abbreviation: NDS (for negative disjunctive syllogism, since it starts with a negated conjunction).
- Reasoning by cases. $p \lor q, p \to r, q \to r \vdash r$. Abbreviation: $E \lor r$

Some examples

Example 1. $\vdash \neg p \lor p$.

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exercise

⊤ ⊢ (¬p v p)

1.
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Example 2. $p \lor q, \neg q \vdash p$.

exercise (p v q), ¬q ⊢ p 1. p∨q :assumption 2. ~q :assumption

Example 3. $(\neg c \land \neg d) \rightarrow e, \neg e \vdash c \lor d$.

exercise $((\neg c \land \neg d) \rightarrow e), \neg e \vdash (c \lor d)$ $1. (\neg c \land \neg d) \rightarrow e : assumption$ $2. \neg e : assumption$

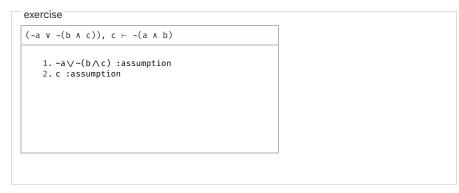
Example 4. $a, b \lor c, \neg(a \land c) \vdash a \land b$.

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exercise

a, (b \ v \ c), \neg(a \ \wedge c) \vdash (a \ \wedge b)

1. a :assumption
2. b \lor c :assumption
3. \neg(a \land c) :assumption
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Example 5. $\neg a \lor \neg (b \land c), c \vdash \neg (a \land b)$.



This is a handout written for this course.¹

1. It is run on the Carnap software, which is ←

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