

# Rules for Week 7

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# The Symbol $\perp$

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- ❖ To describe the introduction and elimination rules for negation, we need to introduce a special symbol.
- ❖ This symbol is  $\perp$  and is called *falsum*. Intuitively, it's a special symbol for a contradiction, for something that's always false.
- ❖ Initially it may seem strange to have one symbol which stands for all contradictions.
- ❖ However, recall when we studied validity that we said that contradictions imply anything. Hence, any two contradictions are equivalent.
- ❖ So if we wanted our deductive system to match up with our judgements of validity, perhaps it's plausible that we have one symbol  $\perp$  which stands for all contradictions.



# Elimination Rule for $\neg$

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- ❖ So here's the elimination rule for  $\neg$ , first in prose:

- ❖ If you have  $\varphi$  on line  $\ell_1$  and you have  $\neg\varphi$  on line  $\ell_2$ , then you may write  $\perp$  on any subsequent line  $\ell > \ell_1, \ell_2$ .

- ❖ Here's the picture:

- ❖  $\ell_1. \varphi$
- ❖  $\ell_2. \neg\varphi$
- ❖  $\ell. \perp$   $(E\neg, \ell_1, \ell_2).$

- ❖ In this rule, it doesn't matter whether the line with  $\varphi$  comes before or after the line with  $\neg\varphi$ .

- ❖ Example:

- ❖ 1.  $p \rightarrow \neg q$  (assumption)
- ❖ 2.  $q$  (assumption)
- ❖ 3.  $p$  (assumption)
- ❖ 4.  $\neg q$   $(E\rightarrow 1, 3)$
- ❖ 5.  $\perp$   $(I\neg 2, 4)$



# Introduction Rule for $\neg$

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- ❖ Here's introduction rule for  $\neg$ , first stated in prose:

- ❖ Suppose that consecutive lines  $\ell_1$ - $\ell_n$  constitute a proof with premise  $\varphi$  and conclusion  $\perp$ . Then one may introduce  $\neg\varphi$  at any subsequent line  $\ell > \ell_n$ , so long as one brackets off lines  $\ell_1$ - $\ell_n$  and never appeals to them again.

- ❖ Here's the picture version of the rule:

- ❖ 1.





# Repeat Rule

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- ❖ This doesn't come up so often, but you will see it occasionally.
- ❖ In prose, the rule says:
  - ❖ If you have  $\varphi$  on line  $\ell_1$ , then you can write  $\varphi$  again on any subsequent line  $\ell > \ell_1$ .
- ❖ In applying this, stay out of closed brackets. That is, don't use this rule to repeat things in closed brackets outside of them.
- ❖ In a picture, the rule is simple:
  - ❖  $\ell_1. \varphi$
  - ❖  $\ell. \varphi$  (repeat  $\ell$ ).
- ❖ Initially it's unclear why one would ever need / want to use this rule.
- ❖ This rule is mostly useful as an aid or helper to applying the introduction rule for  $\rightarrow$ .



# EFSQ Rule

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- ❖ The acronym EFSQ stands for the latin phrase “*ex falso sequitur quodlibet*” which translates as “from a contradiction, anything follows”.
- ❖ In prose, this rule says:
  - ❖ If you have falsum  $\perp$  on any line  $\ell_1$ , then you can write **any** formula  $\psi$  on any subsequent line  $\ell > \ell_1$ .
- ❖ In a picture, this rule looks like:
  - ❖  $\ell_1. \perp$
  - ❖  $\ell. \psi$  (EFSQ  $\ell_1$ ).
- ❖ In applying this rule, keep in mind that instances of  $\perp$  that appear in closed off brackets above line  $\ell$  are **off limits**: don’t appeal later on to anything in an already closed off brackets.