Logic, First Course, Winter 2020. Week 3, Section Meeting. Back to course website

Distinguishing valid from invalid arguments

In section, we focus on distinguishing valid from invalid arguments via a discussion of a series of examples.

Example 1

Use the table to assess the validity of the argument:

(p ∧ q) ⊢ q

p	q	(p	∧	q)	⊢	q
T	T	T		T		T
T	F	T		F		F
F	T	F		T		T
F	F	F		F		F

Counterexample Check

Hence, is the argument with premise $p \wedge q$ and conclusion q a valid argument?

Yes or no?

◯ Yes.

◯ No.

Check

Example 2

Use the table to assess the validity of the argument:

(¬e ∨ f) ⊢ ¬e

e	f	(¬	e	∨	f)	⊢	¬	e
T	T		T		T			T
T	F		T		F			T
F	T		F		T			F
F	F		F		F			F

Counterexample Check

Hence, is the argument with conclusion $\neg e$ and premise $\neg e \vee f$ a valid argument?

Yes or no?

◯ Yes.

◯ No.

Check

Example 3

Use the table to assess the validity of the argument:

¬j, ¬z ⊢ ¬(j ∨ z)

j	z	¬	j	,	¬	z	⊢	¬	(	j	∨	z	)
T	T		T			T				T		T	
T	F		T			F				T		F	
F	T		F			T				F		T	
F	F		F			F				F		F	

Counterexample Check

Hence, is the argument with conclusion ¬(j ∨ z) and premises ¬j and ¬z a valid argument?

Yes or no?

- Yes.
- No.

Check

One way to quickly assess validity is to see this argument as a valid argument obtained from an equivalence. Which equivalence can this be seen as an instance of? *Hint: the comma separating the premises acts like a conjunction.*

Which is it?

- law of double negation
- associativity
- DeMorgan
- distributivity

Check

Example 4

Use the table to assess the validity of the argument:

q ⊢ (p ∨ q)

p	q	q	⊢	(	p	∨	q	)
T	T	T			T		T	
T	F	F			T		F	
F	T	T			F		T	
F	F	F			F		F	

Counterexample Check

Hence, is the argument with conclusion p ∨ q and premise p a valid argument?

Yes or no?

◯Yes.

◯No.

Check

Example 5

Consider how large the table would be to assess the validity of the following:

$r, t \vdash s \vee (r \wedge t)$

It turns out that this is a valid argument. What substitution should you do to see this, on the basis of Example 4? Recall Example 4 was the valid argument $q \vdash p \vee q$.

◯substitute s for p and substitute r/\ t for q.

◯substitute t for p and substitute r/\ s for q.

◯substitute r/\ t for q and substitute s for p.

◯substitute r/\ s for q and substitute t for p.

Check

Example 6

Use the table to assess the validity of the argument, keeping in mind that the initial three columns are ordered alphabetically (i.e. r, s, t):

(t ∨ s) ⊢ ((t ∨ s) ∧ r)

r	s	t	(t	∨	s)	⊢	((t	∨	s)	∧	r)
T	T	T	T		T		T		T		T
T	T	F	F		T		F		T		T
T	F	T	T		F		T		F		T
T	F	F	F		F		F		F		T
F	T	T	T		T		T		T		F
F	T	F	F		T		F		T		F
F	F	T	T		F		T		F		F
F	F	F	F		F		F		F		F

Counterexample Check

Hence, is the argument with conclusion $(t \vee s) \wedge r$ and premise $t \vee s$ a valid argument?

Yes or no?

◯Yes.

◯No.

Check

Hint: think about t ∨ s as it appears in the premise and as part of the conclusion as a single proposition (like p).

Example 7

Use the table to assess the validity of the argument:

¬a, (¬a → b) ⊢ b

a	b	¬	a	,(¬	a	→	b)	⊢	b
T	T		T		T		T		T
T	F		T		T		F		F
F	T		F		F		T		T
F	F		F		F		F		F

Counterexample Check

Hence, is the argument with conclusion *b* and premises ¬*a* and ¬*a* → *b* a valid argument?

Yes or no?

- Yes.
- No.

Check

Which of the following is this example a substitution-instance of:

Which is it?

- modus ponens
- modus tollens
- affirming the consequent
- denying the antecedent

Check

Example 8

Use the table to assess the validity of the argument:

c, (¬d → c) ⊢ ¬d

c	d	c	,(¬	d	→	c)	⊢	¬	d
T	T	T		T		T			T
T	F	T		F		T			F
F	T	F		T		F			T
F	F	F		F		F			F

Counterexample Check

Hence, is the argument with conclusion ¬*d* and premises *c* and ¬*d* → *c* a valid argument?

Yes or no?

- Yes.
- No.

Check

Which of the following is this example a substitution-instance of:

Which is it?

- ◯ modus ponens.
- ◯ modus tollens.
- ◯ affirming the consequent.
- ◯ denying the antecedent.

Check

Example 9

Use the table to assess the validity of the argument:

(u ∨ v), (u → v) ⊢ u

u	v	(u	∨	v),	(u	→	v)	⊢	u
T	T	T		T	T		T		T
T	F	T		F	T		F		T
F	T	F		T	F		T		F
F	F	F		F	F		F		F

Counterexample Check

Hence, is the argument with premises $u \vee v$ and $u \to v$ and conclusion u a valid argument?

Yes or no?

- ◯ Yes.
- ◯ No.

Check

Example 10

Consider how large the table would be to assess the validity of the following:

$c \wedge (d \wedge e) \vdash e$

It turns out this argument is valid. One way to see it is to answer the following question:

What is the one circumstance in which $c \wedge (d \wedge e)$ is true?

Which is it?

- ◯ when exactly one of the propositional letters is true.
- ◯ when exactly two of the propositional letters are true.
- ◯ when all three of the propositional letters are true.

Check

Another way to discern the validity of the argument is to "chain together" the following two valid arguments:

$c \wedge (d \wedge e) \vdash d \wedge e$ and $d \wedge e \vdash e$

The second validity $d \wedge e \vdash e$ is clearly a substitution-instance of Example 1 (which recall said that $p \wedge q \vdash q$).

But what substitution do you have to do to see that the first validity $c \wedge (d \wedge e) \vdash d \wedge e$ is a substitution-instance of Example 1? Recall again Example 1 said that that $p \wedge q \vdash q$.

Which is it?

- ◯ substitute d for q, and substitute c/\ e for p.
- ◯ substitute c for p, and substitute d/\ e for q.
- ◯ substitute d for p, and substitute c/\ e for q.
- ◯ substitute c for q, and substitute d/\ e for p.

Check

Example 11

Use the table to assess the validity of the argument (this validity undergirds the "chaining" we mentioned in the previous example).

(p → q), (q → r) ⊢ (p → r)

p	q	r	(p	→	q),	(q	→	r)	⊢	(p	→	r)
T	T	T	T		T	T		T		T		T
T	T	F	T		T	T		F		T		F
T	F	T	T		F	F		T		T		T
T	F	F	T		F	F		F		T		F
F	T	T	F		T	T		T		F		T
F	T	F	F		T	T		F		F		F
F	F	T	F		F	F		T		F		T
F	F	F	F		F	F		F		F		F

Counterexample Check

Hence, is the argument with premises $p \to q$ and $q \to r$ and conclusion $p \to r$ a valid argument?

Yes or no?

- ◯ Yes.
- ◯ No.

Check

Example 12

Here is another way to think about the validity from the previous example:

$$p \to q, q \to r \vdash p \to r$$

What is the one circumstance in which the conclusion $p \to r$ is false?

Which is it?

- ◯ when p is true and r is true.
- ◯ when p is true and r is false.
- ◯ when p is false and r is true.
- ◯ when p is false and r is false.

Check

In this circumstance you just described, if q is true then premise $q \to r$ is false; and in this circumstance if q is false then $p \to q$ is false. Hence, in either case, one of the two premises is false. Hence, in the circumstance where the conclusion $p \to r$ is false, at least one of the premises is false.

The argument we just gave in the previous paragraph is an example of one of the valid arguments we saw in lecture.

Which is it?

- disjunctive syllogism.
- modus ponens.
- reasoning by cases.
- modus tollens.

Check

These are section notes written for this course.[1]

1. It is run on the Carnap software, which is↩