Logic, First Course, Winter 2020. Week 8, Lecture 1, Handout.

Double negation rule

$l_1.$ $\neg\neg\varphi$

$\vdots$

$l.$ φ DN l_1

l_1 φ

$\vdots$

l $\neg\neg\varphi$ DN l_1

Law of the excluded middle

Since we will often want to appeal to double-negation without having to redo this proof over and over, we simply include a new rule for law of the excluded middle which says that one can always put $\phi \vee \neg\phi$ on a line, and justify it as `LEM` . Note that no line number is put down as part of the justification.

$$\vdots$$

$\varphi \vee \neg \varphi$ LEM

Other derived rules

- *Law of excluded middle*: $p \vee \neg p$ is a tautology. Abbreviation: `LEM`
- *Law of non-contradiction*: $\neg(p \wedge \neg p)$ is a tautology. Abbreviation: `LNC`
- *The law of double-negation*: p is equivalent to $\neg\neg p$. Abbreviation: `DN`
- *Law of commutativity for conjunction*: $p \wedge q$ is equivalent to $q \wedge p$. Abbreviation: `LCC`
- *Law of commutativity for disjunction*: $p \vee q$ is equivalent to $q \vee p$. Abbreviation `LCD` .
- *Law of associativity for conjunction*: $(p \wedge q) \wedge r$ is equivalent to $p \wedge (q \wedge r)$. Abbreviation: `LAC`
- *Law of associativity for disjunction*: $(p \vee q) \vee r$ is equivalent to $p \vee (q \vee r)$. Abbreviation: `LAD`
- *Law of distribution, part 1*: $p \wedge (q \vee r)$ is equivalent to $(p \wedge q) \vee (p \wedge r)$. Abbreviation: `LDC` (where the final "C" is short for the initial conjunction)
- *Law of distribution, part 2*: $p \vee (q \wedge r)$ is equivalent to $(p \vee q) \wedge (p \vee r)$. Abbreviation: `LDD` (where the final "D" is short for the initial disjunction)
- *DeMorgan Law, part 1*: $\neg(p \wedge q)$ is equivalent to $\neg p \vee \neg q$. Abbreviation: `DMOR` (since it ends in an or statement)
- *DeMorgan Law, part 2*: $\neg(p \vee q)$ is equivalent to $\neg p \wedge \neg q$. Abbreviation: `DMAND` (since it ends in an and statement)

- *Modus ponens*. $p \to q, p \vdash q$. Abbreviation: `E->`
- *Modus tollens*. $p \to q, \neg q \vdash \neg p$. Abbreviation: `MT`
- *Disjunctive syllogism*. $p \vee q, \neg p \vdash q$. Abbreviation: `PDS` (for positive disjunctive syllogism, since it starts with an initial unnegated "positive" disjunction).
- *Disjunctive syllogism*. $\neg(p \wedge q), p \vdash \neg q$. Abbreviation: `NDS` (for negative disjunctive syllogism, since it starts with a negated conjunction).
- *Reasoning by cases*. $p \vee q, p \to r, q \to r \vdash r$. Abbreviation: `E\/`

Some examples

Example 1. $\vdash \neg p \vee p$.

exercise

```
⊤ ⊢ (¬p ∨ p)

1.
```

Example 2. $p \vee q, \neg q \vdash p$.

exercise

```
(p ∨ q), ¬q ⊢ p

1. p⋁q :assumption
2. ~q :assumption
```

Example 3. $(\neg c \wedge \neg d) \to e, \neg e \vdash c \vee d$.

exercise

```
((¬c ∧ ¬d) → e), ¬e ⊢ (c ∨ d)

1. (~c⋀~d)→e :assumption
2. ~e :assumption
```

Example 4. $a, b \vee c, \neg(a \wedge c) \vdash a \wedge b$.

exercise

```
a, (b ∨ c), ¬(a ∧ c) ⊢ (a ∧ b)

1. a :assumption
2. b⋁c :assumption
3. ~(a⋀c) :assumption
```

Example 5. $\neg a \vee \neg(b \wedge c), c \vdash \neg(a \wedge b)$.

exercise

```
(¬a ∨ ¬(b ∧ c)), c ⊢ ¬(a ∧ b)

1. ~a⋁~(b⋀c) :assumption
2. c :assumption
```

This is a handout written for this course.[1]

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1. It is run on the Carnap software, which is↩