Logic, First Course, Winter 2020. Week 2, Lecture 2, Handout.

Definitions and law of excluded middle

A *tautology* is a well-formed formula that is always true. That is, when you look at its truth-table, you see all true's in the column under the main connective.

Law of excluded middle: $p \vee \neg p$ is a tautology.

(p ∨ ¬p)

p	(	p	∨	¬	p	)
T		T			T	
F		F			F	

Check

Law of non-contradiction: $\neg(p \wedge \neg p)$ is a tautology.

¬(p ∧ ¬p)

p	¬	(	p	∧	¬	p	)
T			T			T	
F			F			F	

Check

An *equivalence* is just two formulas ϕ and ψ such that their biconditional $\phi \leftrightarrow \psi$ is a tautology.

The law of double-negation: p is equivalent to $\neg\neg p$.

p, ¬¬p

p	p	,	¬	¬	p
T	T				T
F	F				F

Check

(p ⇔ ¬¬p)

p	(	p	⇔	¬	¬	p	)
T		T				T	
F		F				F	

Check

(p → q), (¬p ∨ q)

p	q	(	p	→	q	),(	¬	p	∨	q	)
T	T		T		T			T		T	
T	F		T		F			T		F	
F	T		F		T			F		T	
F	F		F		F			F		F	

Check

Commutativity and associativity and distributivity

Law of commutativity for conjunction: $p \wedge q$ is equivalent to $q \wedge p$.

Law of commutativity for disjunction: $p \vee q$ is equivalent to $q \vee p$

Law of associativity for conjunction: $(p \wedge q) \wedge r$ is equivalent to $p \wedge (q \wedge r)$.

Law of associativity for disjunction: $(p \vee q) \vee r$ is equivalent to $p \vee (q \vee r)$.

Law of distribution, part 1: $p \wedge (q \vee r)$ is equivalent to $(p \wedge q) \vee (p \wedge r)$

Law of distribution, part 2: $p \vee (q \wedge r)$ is equivalent to $(p \vee q) \wedge (p \vee r)$

DeMorgan Law, part 1: $\neg(p \wedge q)$ is equivalent to $\neg p \vee \neg q$.

DeMorgan Law, part 2: $\neg(p \vee q)$ is equivalent to $\neg p \wedge \neg q$.

Recognizing tautologies and equivalences quickly

Substitution. Given any tautology, if we uniformly substitute other formulas for the basic propositional letters, we still get a tautology.

(p ∨ ¬p)

p	(	p	∨	¬	p	)
T		T	T	F	T	
F		F	T	T	F	

Check

((q ∨ r) ∨ ¬(q ∨ r))

q	r	((	q	∨	r	)	∨	¬	(	q	∨	r	))
T	T		T	T	T		T	F		T	T	T	
T	F		T	T	F		T	F		T	T	F	
F	T		F	T	T		T	F		F	T	T	
F	F		F	F	F		T	T		F	F	F	

Check

Replacement of equivalents with equivalents. If two formulas are equivalent, then you can replace the one with the other in any formula and not change the truth-value.

Chaining. If a first formula ϕ is the same as a second formula ψ in terms of truth-values and the second formula ψ is the same as a third formula ξ in terms of truth-values, then the first formula ϕ is the same as the third formula ξ in terms of truth-values.

Boole and the laws of thought

(p ∧ p), p

p	(p	∧	p),	p
T	T		T	T
F	F		F	F

Check

$(p \vee p), p$

p	(p	$\vee$	p),	p
T	T		T	T
F	F		F	F

Check

$(p \wedge (p \vee q)), p$

p	q	(p	$\wedge$	(p	$\vee$	q)),	p
T	T	T		T		T	T
T	F	T		T		F	T
F	T	F		F		T	F
F	F	F		F		F	F

Check

$(p \vee (p \wedge q)), p$

p	q	(p	$\vee$	(p	$\wedge$	q)),	p
T	T	T		T		T	T
T	F	T		T		F	T
F	T	F		F		T	F
F	F	F		F		F	F

Check

Spotting non-tautologies and spotting non-equivalences

(p ∨ (q ∨ r))

p	q	r	(p	∨	(q	∨	r))
T	T	T	T		T		T
T	T	F	T		T		F
T	F	T	T		F		T
T	F	F	T		F		F
F	T	T	F		T		T
F	T	F	F		T		F
F	F	T	F		F		T
F	F	F	F		F		F

Non-Tautology Check

(p ∧ q), (p ∨ q)

p	q	(p	∧	q),	(p	∨	q)
T	T	T		T	T		T
T	F	T		F	T		F
F	T	F		T	F		T
F	F	F		F	F		F

Inequivalent Check

These is a handout written for Logic, First Course, Winter 2020. It is run on the Carnap software, which is an: