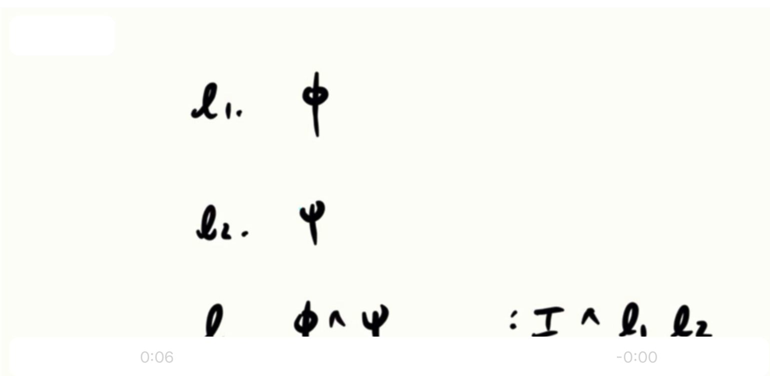
Logic, First Course, Winter 2020. Week 6, Lecture 1, Handout.

Introduction rule for conjunction

The rule is: if you have ϕ on a line ℓ_1, and you have ψ on line ℓ_2, then you may write $\phi \wedge \psi$ on any subsequent line $\ell > \ell_1, \ell_2$.

This rule is abbreviated as $I\wedge$, where the 'I' is for *introduction*.

In terms of a picture, the rule is:



In the rule, it does not matter whether ℓ_1 or ℓ_2 comes first.

Here's a worked-out example which contains two applications of $I\wedge$:

exercise

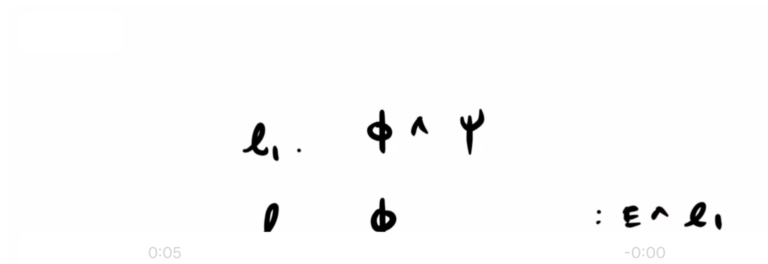
```
p, q, r ⊢ (p ∧ (r ∧ q))

  1. p          :assumption
  2. q          :assumption
  3. r          :assumption
```

Elimination rule for conjunction

The rule is: if you have $\phi \wedge \psi$ on line ℓ_1, then you may write ϕ on any subsequent line $\ell > \ell_1$, and likewise you may write ψ on any subsequent line $\ell > \ell_1$.

In terms of a picture, the rule is either of the following:



Here's a worked-out example:

exercise

```
(p ∧ (r ∧ q)) ⊢ r

  1. p∧(r∧q)    :assumption
```

Elimination rule for implication

The rule is: if you have ϕ on line ℓ_1, and you have $\phi \to \psi$ on line ℓ_2, then you may write ψ on any subsequent line $\ell > \ell_1, \ell_2$.

In terms of a picture, the rule is:

$$\begin{array}{lll} \ell_1. & \varphi & \\ \ell_2. & \varphi \to \psi & \\ \ell & \psi & : E\to \ell_1, \ell_2 \end{array}$$

In the rule, it does not matter whether ℓ_1 or ℓ_2 comes first.

Here's a worked-out example:

exercise

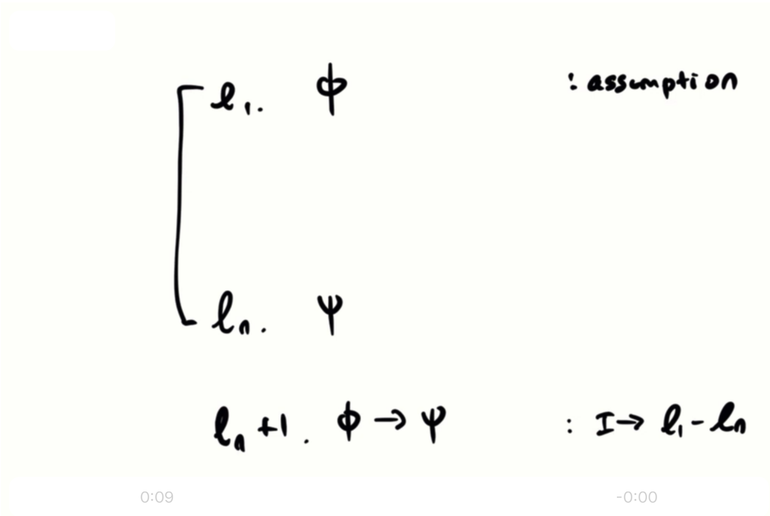
```
((p → q) ∧ (p → r)), p ⊢ (q ∧ r)

  1.
```

The introduction rule for implication

The rule is: suppose that consecutive lines $\ell_1 - \ell_n$ constitute a proof with premise ϕ and conclusion ψ. Then one may introduce $\phi \to \psi$ at line $\ell_n + 1$, so long as one brackets off $\ell_1 - \ell_n$ and never appeals to them again.

In a picture, it is:



Here is a worked out example:

exercise

$(p \to (q \wedge r)) \vdash (p \to (r \wedge q))$

1.

Another example and proof-sketches by hand

exercise

$(p \to q) \vdash (p \to (p \wedge q))$

1.

Yet another example

exercise

$(p \wedge q) \vdash (p \rightarrow q)$

1.

Nested example of implication introduction

exercise

$(p \rightarrow (q \rightarrow r)) \vdash (q \rightarrow (p \rightarrow r))$ ⚠

1. p→(q→r) :assumption +

These is a handout for this course.[1]

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1. It is run on the Carnap software, which is↩