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Week 6, Practice Problems

The practice problems fall into two groups:

- Memorizing and recognizing negation introduction and elimination
- Proofs with negation introduction and elimination

Some more problems on ex falso will be added prior to the second lecture.

Memorizing and recognizing negation introduction and elimination

The first set of five problems is just practice in memorizing and recognizing negation introduction and elimination. The proofs involving negation elimination are just 1-2 extra lines beyond the assumptions. The proofs involving negation introduction are just 3-5 extra lines beyond the assumptions.

Example 1.

exercise

```
a, ¬a ⊢ ⊥
```

```
1. a :assumption
2. ~a :assumption
```

Example 2.

exercise

```
(a ∧ b), ¬(a ∧ b) ⊢ ⊥
```

```
1. (a∧b) :assumption
2. ~(a∧b) :assumption
```

Example 3.

exercise

((a ∧ b) ∧ c), ¬(a ∧ b) ⊢ ⊥

1. (a∧b)∧c :assumption
2. ~(a∧b) :assumption

Example 4.

exercise

(p → ¬q) ⊢ ¬(p ∧ q)

1. p→~q :assumption

Example 5.

exercise

(p → ¬q), p, q ⊢ ⊥

1. p→~q :assumption
2. p :assumption
3. q :assumption

Proofs with negation introduction and elimination

These next five proofs are simple proofs involving negation introduction and elimination, along with the other rules which we learned earlier.

Example 6.

exercise

(a → c), (b → d), ¬(c ∧ d) ⊢ ¬(a ∧ b)

1. a→c :assumption
2. b→d :assumption
3. ~(c∧d) :assumption

Example 7.

exercise

`(p → r), (q → r), ¬r ⊢ ¬(p ∨ q)`

1. p→r :assumption
2. q→r :assumption
3. ~r :assumption

Example 8.

exercise

`c, (c → (d ∧ a)), (b → ¬d) ⊢ ¬(a → b)`

1. c :assumption
2. c→(d∧a) :assumption
3. b→~d :assumption

Example 9.

This example shows that we can derive modus tollens in our proof system. INSERT ref

exercise

`(p → q), ¬q ⊢ ¬p`

1. p→q :assumption
2. ~q :assumption

Example 10.

In this example, be sure to remember to think of $\neg\neg a$ as "a negation applies to $\neg a$".

exercise

`(¬a → ¬b), b ⊢ ¬¬a`

1. ~a→~b :assumption
2. b :assumption

This is a practice problem set for this course. It is run on the Carnap software, which is an:

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