Logic, First Course, Winter 2020. Week 1, Section Meeting, Handout.

Definition of well-formed formula

We start with a basic list of basic propositional formulas $a, b, c, \ldots, p, q, r, \ldots$. They are basic in that they do not have any further structure.

Then we give some rules how to generate more complex well-formed formulas from simpler well-formed formulas, and we say what the *main connective* is of the complex well-formed formula:

- If ϕ is a well-formed formula, then $\neg\phi$ is a well-formed formula and its main connective is the negation symbol $\neg$.
- If ϕ and ψ are well-formed formulas, then $(\phi \wedge \psi)$ is a well-formed formula and its main connective is the conjunction symbol $\wedge$.
- If ϕ and ψ are well-formed formulas, then $(\phi \vee \psi)$ is a well-formed formula and its main connective is the disjunction symbol $\vee$.
- If ϕ and ψ are well-formed formulas, then $(\phi \rightarrow \psi)$ is a well-formed formula and its main connective is the implication symbol $\rightarrow$.
- If ϕ and ψ are well-formed formulas, then $(\phi \leftrightarrow \psi)$ is a well-formed formula and its main connective is the biconditional symbol $\leftrightarrow$.

Finally, we say that nothing else is a well-formed formula besides what can be generated in the way described above.

Practice finding the main connectives

((p ∧ q) → (¬q ∨ r))

$(q \to ((\neg p \lor r) \land s))$

$((p \land \neg q) \lor (q \leftrightarrow r))$

$((p \to (q \land \neg r)) \to (s \lor \neg t))$