Logic, First Course, Winter 2020. Week 4, Homework (due Tuesday February 18 by 11:59pm).

Week 6, Homework

The homework problems in this set fall into four groups:

- Memorizing rules
- Simple proofs with just conjunction and arrow
- Simple proofs involving disjunction
- More challenging proofs

There are 20 problems total, each equally weighted. Submitted problems received on time receive 5 points each. Hence, there are 100 possible points. Late submissions receive 2 points each. The cutoff for submitting late homework is one week after the homework is due. At that point, the points recorded in carnap.io will be transferred to the ccle grading system and no further late work will be accepted.

Please remember to **press the "Submit" button** next to each problem after you are done. If you do not do this, you will **not** get credit for the problem. Once you have submitted your answer, your points are recorded. You can always check your points by going to the "user home" at the top right. You must be signed in in order to submit your answers. *Please keep in mind that you cannot submit a proof until it has been successfully completed and the top turns green, which indicates that the proof-checker has verified that it is indeed a proof.*

If you want to do some problems at one point in time and other problems at another point in time, you can do that by just coming back to this assignment page. Once submitted, your points are permanently recorded. If you return to this page later or refresh this page, it won't display your previously recorded answers, but again your points are permanently recorded. If you have forgotten which ones you still need to do, you can check your points by going to the "user home" at the top right.

Before you begin the homework, *please remember* that one should be sketching out the proofs by hand prior to typing them into the proof-checker. A nice pdf of this page is here, although some of the more complex proofs towards the end might require more space than what is displayed here initially. Hence, it might be better to sketch out the proofs on paper or a tablet.

Memorizing rules

The first set of five problems is just practice in memorizing the rules, with substitution instances. In the first three, you need to only add one line beyond the assumptions. In the last two, it involves arrow introduction, and you have to add 3-4 further lines beyond the assumptions.

exercise

((a ∧ b) ∧ (c ∧ d)) ⊢ (c ∧ d)

1. (a∧b)∧(c∧d) :assumption

Submit

exercise

(¬a ∨ ¬b), (a ∧ c) ⊢ ((¬a ∨ ¬b) ∨ ¬c)

1. ~a⋁~b :assumption
2. a⋀c :assumption

Submit

exercise

(¬a ∨ ¬b), (¬a → (c ∧ d)), (¬b → (c ∧ d)) ⊢
(c ∧ d)

1. ~a⋁~b :assumption
2. ~a→(c⋀d) :assumption
3. ~b→(c⋀d) :assumption

Submit

exercise

```
b ⊢ (a → (b ∧ a))
```

```
1. b :assumption
```

Submit

exercise

```
(p → q), p ⊢ (r → (r ∧ q))
```

```
1. p→q :assumption
2. p :assumption
```

Submit

Simple proofs with just conjunction and arrow

These next five problems can be done using only the rules for conjunction and arrow.

exercise

(a ∧ (b ∧ c)) ⊢ ((a ∧ b) ∧ c)

1. a∧(b∧c) :assumption

Submit

exercise

(a → (b ∧ c)), (b → d), (c → e) ⊢ (a → (d ∧ e))

1. a→(b∧c) :assumption
2. b→d :assumption
3. c→e :assumption

Submit

exercise

```
(a → b), (b → c), (c → d) ⊢ (a → d)
```

1. a→b :assumption
2. b→c :assumption
3. c→d :assumption

Submit

Hint on this one (6.09), after you do some conjunction elimination, it is just like the previous one (6.08).

exercise

```
((a → b) ∧ ((b → c) ∧ (c → d))) ⊢ (a → d)
```

1. (a→b)∧((b→c)∧(c→d)) :assumption

Submit

Hint: while this one (6.10) is conceptually easy, it might be a bit longer than previous proofs, and might be in the vicinity of 15 lines total. Hence, don't get dissuaded if you proof ends up being a little long: eliminating all the conjunctions and then putting them back together again, so to speak, just takes awhile.

exercise

```
(a → (b ∧ (c ∧ d))), (b → e), (c → f), (d →
g) ⊢ (a → (e ∧ (f ∧ g)))
```

1. a→(b∧(c∧d)) :assumption
2. b→e :assumption
3. c→f :assumption
4. d→g :assumption

Submit

Simple proofs involving disjunction

These next proofs involve disjunction, along with conjunction and arrow.

exercise

```
((a ∨ b) → c) ⊢ (a → c)
```

1. (a∨b)→c :assumption

Submit

Hint: on this next one (6.12), just repeat the strategy from the previous one (6.11) twice over.

exercise

((a ∨ b) → c) ⊢ ((a → c) ∧ (b → c))

1. (a⋁b)→c :assumption

Submit

exercise

((a → c) ∧ (b → c)) ⊢ ((a ∨ b) → c)

1. (a→c)⋀(b→c) :assumption

Submit

Hint: on this one (6.14), the last step will be introducing the arrow in ($p \vee q$) → ($p \vee r$). This will result in a bracket whose top line is $p \vee q$ and whose ultimate bottom line is $p \vee r$. Do a disjunction elimination on the top line $p \vee q$ of the bracket, with associated arrow statements p → ($p \vee r$) and q → ($p \vee r$) which you build using-- among other things-- disjunction introduction.

exercise

```
(q → r) ⊢ ((p ∨ q) → (p ∨ r))
```

1. q→r :assumption

Submit

Hint: on this one (6.15), it will be a disjunction elimination on $(a \wedge b) \vee (c \wedge d)$. Try to build the arrows $(a \wedge b) \rightarrow e$ and $(c \wedge d) \rightarrow e$.

exercise

```
((a ∧ b) ∨ (c ∧ d)), (a → e), (b → f), (c →
f), (d → e) ⊢ e
```

1. (a∧b)∨(c∧d) :assumption
2. a→e :assumption
3. b→f :assumption
4. c→f :assumption
5. d→e :assumption

Submit

More challenging proofs

These problems are a little more challenging, and hence come prefaced with a hint.

On this one, try to think about how to build proofs of $a \to (a \wedge b)$ and $b \to (a \wedge b)$ using the premises.

exercise

```
((a → b) ∧ (b → a)) ⊢ ((a ∨ b) → (a ∧ b))
```

```
1. (a→b)∧(b→a) :assumption
```

Submit

This next one is commutativity of disjunction. Its key step is a disjunction elimination, where you have to build the two associated arrow statements. The proof of these arrow statements goes, in turn, through disjunction introduction.

exercise

```
(p ∨ q) ⊢ (q ∨ p)
```

```
1. p∨q :assumption
```

Submit

This next one is associativity of disjunction. Try to think about how to get the line $(q \vee r) \to ((p \vee q) \vee r)$. Doing this will involve first getting the lines $q \to ((p \vee q) \vee r)$ and $r \to ((p \vee q) \vee r)$.

exercise

(p ∨ (q ∨ r)) ⊢ ((p ∨ q) ∨ r)

1. p∨(q∨r) :assumption

Submit

This next one is one-half of one of the distribution laws. For this one, the premise is a disjunction, and so just try to do disjunction elimination on it, building the two associated arrow statements in the most obvious way.

exercise

((a ∧ b) ∨ (a ∧ c)) ⊢ (a ∧ (b ∨ c))

1. (a∧b)∨(a∧c) :assumption

Submit

This final one is the second-half of one of the distribution laws. Try to isolate the disjunction embedded in the premise and do disjunction elimination on it, building the two associated arrow statements in the most obvious way.

exercise

(a ∧ (b ∨ c)) ⊢ ((a ∧ b) ∨ (a ∧ c))

1. a∧(b∨c) :assumption

Submit

This is a homework set for this course. It is run on the Carnap software, which is an: