

Logic, First Course, Winter 2020. Week 3, Practice Problems.

# Week 3, Practice problems

The practice problems in this set fall into two groups:

- [Assessing validity](#)
- [Assessing soundness](#)

For the **solutions**, click [here](#).

## Assessing validity

### Problem 1

$(\neg p \rightarrow \neg q), q \vdash p$

p	q	(	$\neg$	p	$\rightarrow$	$\neg$	q	)	,	q	$\vdash$	p
T	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
T	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

Counterexample

Check

Is it valid?

☐ Yes.

☐ No.

Check

It is a substitution instance of which of the following (perhaps together with use of double-negation)?

- ☐ modus ponens.
- ☐ modus tollens.
- ☐ affirming the consequent.
- ☐ denying the antecedent.

Check

## Problem 2

$(\neg p \rightarrow \neg q), \neg q \vdash \neg p$

$p$	$q$	$(\neg p \rightarrow \neg q), \neg q \vdash \neg p$									
T	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
T	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

Counterexample

Check

Is it valid?

- ☐ Yes.
- ☐ No.

Check

It is a substitution instance of which of the following (perhaps together with use of double-negation)?

- ☐ modus ponens.
- ☐ modus tollens.
- ☐ affirming the consequent.
- ☐ denying the antecedent.

Check

## Problem 3

$(\neg p \rightarrow \neg q), p \vdash q$

$p$	$q$	$(\neg p \rightarrow \neg q), p \vdash q$									
T	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
T	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

Counterexample

Check

Is it valid?

☐ Yes.

☐ No.

Check

It is a substitution instance of which of the following (perhaps together with use of double-negation)?

☐ modus ponens.

☐ modus tollens.

☐ affirming the consequent.

☐ denying the antecedent.

Check

#### Problem 4

$(\neg p \rightarrow \neg q), q \vdash p$

$p$	$q$	$(\neg p \rightarrow \neg q), q \vdash p$									
T	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
T	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

Counterexample

Check

Is it valid?

☐ Yes.

☐ No.

Check

It is a substitution instance of which of the following (perhaps together with use of double-negation)?

☐ modus ponens.

☐ modus tollens.

☐ affirming the consequent.

☐ denying the antecedent.

Check

### Problem 5

$(a \vee (\neg b \vee \neg c)), \neg a \vdash \neg(b \wedge c)$

a	b	c	$(a \vee (\neg b \vee \neg c)), \neg a \vdash \neg(b \wedge c)$											
T	T	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
T	T	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
T	F	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
T	F	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	T	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	T	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	F	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	F	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

Counterexample

Check

Is it valid?

☐ Yes.

☐ No.

Check

What laws and paradigm examples can you use to quickly assess for validity, in tandem with DeMorgan?

- ☐ commutativity.
- ☐ distribution.
- ☐ disjunctive syllogism.
- ☐ reasoning by cases.

Check

### Problem 6

$(a \rightarrow b), (b \vee c), (a \rightarrow c) \vdash a$

a	b	c	( a → b )	( b ∨ c )	( a → c )	⊢	a
T	T	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
T	T	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
T	F	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
T	F	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	T	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	T	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	F	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	F	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

Counterexample

Check

Is it valid?

- ☐ Yes.
- ☐ No.

Check

Each of the following is true. Which of these could you use to quickly assess for validity in this case?

- ☐ conditionals with false antecedents are true.
- ☐ conditionals with true antecedents and false consequents are false (and this is the only circumstance in which they are false).
- ☐ conditionals with true consequents are true.

Check

### Problem 7

$(\neg a \rightarrow \neg c), (\neg a \vee b), (b \rightarrow \neg c) \vdash \neg c$

a	b	c	$(\neg a \rightarrow \neg c)$	$(\neg a \vee b)$	$(b \rightarrow \neg c)$	$\vdash \neg c$
T	T	T	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
T	T	F	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
T	F	T	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
T	F	F	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
F	T	T	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
F	T	F	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
F	F	T	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
F	F	F	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Counterexample

Check

Is it valid?

- ☐ Yes.
- ☐ No.

Check

Which of the following is this a substitution instance of?

- ☐ commutativity.
- ☐ distribution.
- ☐ disjunctive syllogism.
- ☐ reasoning by cases.

Check

### Problem 8

$(\neg a \wedge (\neg b \vee \neg c)) \vdash (\neg(a \vee b) \vee \neg(a \vee c))$

abc	(	¬	a	∧	(	¬	b	∨	¬	c	)	⊢	(	¬	(	a	∨	b	)	∨	¬	(	a	∨	c	)	)	
TTT	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>
TTF	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>
TFT	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>
TFF	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>
FTT	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>
FTF	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>
FFT	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	T	<input type="text"/>
FFF	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>	<input type="text"/>	⇔	F	<input type="text"/>

Counterexample

Check

Is it valid?

- ☐ Yes.
- ☐ No.

Check

What laws and paradigm examples can you use to quickly assess for validity, in tandem with DeMorgan?

- ☐ commutativity.
- ☐ distribution.
- ☐ disjunctive syllogism.
- ☐ reasoning by cases.

Check

Problem 9

$(\neg a \wedge (\neg b \vee \neg c)) \vdash \neg((a \vee b) \wedge (a \vee c))$

a	b	c	(	¬	a	∧	(	¬	b	∨	¬	c	)	⊢	¬	((	a	∨	b	)	∧	(	a	∨	c	)	)
T	T	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	
T	T	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	
T	F	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	
T	F	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	
F	T	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	
F	T	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	
F	F	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	
F	F	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	

Counterexample

Check

Is it valid?

☐ Yes.

☐ No.

Check

What laws and paradigm examples can you use to quickly assess for validity, in tandem with DeMorgan?

☐ commutativity.

☐ distribution.

☐ disjunctive syllogism.

☐ reasoning by cases.

Check

Problem 10



$(a \vee (b \vee c)) \vdash (c \vee (a \vee b))$

<b>a b c</b>	<b>( a   ∨   ( b   ∨   c ) )   ⊢   ( c   ∨   ( a   ∨   b ) )</b>											
T T T	<input type="text" value="T"/>	<input type="text"/>	<input type="text" value="T"/>	<input type="text"/>	<input type="text" value="T"/>	<input type="text"/>	<input type="text"/>	<input type="text" value="T"/>	<input type="text"/>	<input type="text" value="T"/>	<input type="text"/>	<input type="text" value="T"/>
T T F	<input type="text" value="T"/>	<input type="text"/>	<input type="text" value="T"/>	<input type="text"/>	<input type="text" value="F"/>	<input type="text"/>	<input type="text"/>	<input type="text" value="F"/>	<input type="text"/>	<input type="text" value="T"/>	<input type="text"/>	<input type="text" value="T"/>
T F T	<input type="text" value="T"/>	<input type="text"/>	<input type="text" value="F"/>	<input type="text"/>	<input type="text" value="T"/>	<input type="text"/>	<input type="text"/>	<input type="text" value="T"/>	<input type="text"/>	<input type="text" value="T"/>	<input type="text"/>	<input type="text" value="F"/>
T F F	<input type="text" value="T"/>	<input type="text"/>	<input type="text" value="F"/>	<input type="text"/>	<input type="text" value="F"/>	<input type="text"/>	<input type="text"/>	<input type="text" value="F"/>	<input type="text"/>	<input type="text" value="T"/>	<input type="text"/>	<input type="text" value="F"/>
F T T	<input type="text" value="F"/>	<input type="text"/>	<input type="text" value="T"/>	<input type="text"/>	<input type="text" value="T"/>	<input type="text"/>	<input type="text"/>	<input type="text" value="T"/>	<input type="text"/>	<input type="text" value="F"/>	<input type="text"/>	<input type="text" value="T"/>
F T F	<input type="text" value="F"/>	<input type="text"/>	<input type="text" value="T"/>	<input type="text"/>	<input type="text" value="F"/>	<input type="text"/>	<input type="text"/>	<input type="text" value="F"/>	<input type="text"/>	<input type="text" value="F"/>	<input type="text"/>	<input type="text" value="T"/>
F F T	<input type="text" value="F"/>	<input type="text"/>	<input type="text" value="F"/>	<input type="text"/>	<input type="text" value="T"/>	<input type="text"/>	<input type="text"/>	<input type="text" value="T"/>	<input type="text"/>	<input type="text" value="F"/>	<input type="text"/>	<input type="text" value="F"/>
F F F	<input type="text" value="F"/>	<input type="text"/>	<input type="text" value="F"/>	<input type="text"/>	<input type="text" value="F"/>	<input type="text"/>	<input type="text"/>	<input type="text" value="F"/>	<input type="text"/>	<input type="text" value="F"/>	<input type="text"/>	<input type="text" value="F"/>

Counterexample

Check

Is it valid?

☐ Yes.

☐ No.

Check

What laws and paradigm examples can you use to quickly assess for validity, in tandem with associativity?

☐ commutativity.

☐ distribution.

☐ disjunctive syllogism.

☐ reasoning by cases.

Check

$(a \rightarrow (b \rightarrow c)), (a \rightarrow c) \vdash b$

a	b	c	( a → ( b → c ) )	( a → c )	⊢	b
T	T	T	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
T	T	F	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
T	F	T	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
T	F	F	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
F	T	T	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
F	T	F	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
F	F	T	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
F	F	F	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Counterexample

Check

Is it valid?

☐ Yes.

☐ No.

Check

Each of the following is true. Which of these could you use to quickly assess for validity in this case?

☐ conditionals with false antecedents are true.

☐

conditionals with true antecedents and false consequents are false (and this is the only circumstance in which they are false).

☐ conditionals with true consequents are true.

Check

$(a \wedge (b \wedge c)) \vdash (a \rightarrow (b \rightarrow c))$

<b>a b c</b>	<b>( a   ∧   ( b   ∧   c ) )   ⊢   ( a   →   ( b   →   c ) )</b>											
T T T	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
T T F	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
T F T	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
T F F	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
F T T	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
F T F	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
F F T	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
F F F	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Counterexample

Check

Is it valid?

☐ Yes.

☐ No.

Check

Each of the following is true. Which of these could you use to quickly assess for validity in this case?

☐ conditionals with false antecedents are true.

☐

conditionals with true antecedents and false consequents are false (and this is the only circumstance in which they are false).

☐ conditionals with true consequents are true.

Check

$(a \rightarrow b), a, (b \rightarrow c) \vdash ((a \rightarrow b) \rightarrow (a \rightarrow c))$

a	b	c	( a → b )	a	( b → c )	⊢	(( a → b ) → ( a → c ))
T	T	T	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
T	T	F	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
T	F	T	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
T	F	F	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
F	T	T	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
F	T	F	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
F	F	T	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
F	F	F	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Counterexample

Check

Is it valid?

☐ Yes.

☐ No.

Check

Each of the following is true. Which of these could you use to quickly assess for validity in this case?

☐ conditionals with false antecedents are true.

☐

conditionals with true antecedents and false consequents are false (and this is the only circumstance in which they are false).

☐ conditionals with true consequents are true.

Check

$\neg(a \rightarrow b) \vdash (a \wedge \neg b)$

<b>a</b>	<b>b</b>	$\neg ( a \rightarrow b ) \vdash ( a \wedge \neg b )$									
T	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
T	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

Counterexample

Check

Is it valid?

☐ Yes.

☐ No.

Check

Each of the following is true. Which of these could you use to quickly assess for validity in this case?

☐ conditionals with false antecedents are true.

☐

conditionals with true antecedents and false consequents are false (and this is the only circumstance in which they are false).

☐ conditionals with true consequents are true.

Check

### Problem 15

$(a \wedge \neg b) \vdash (a \rightarrow \neg b)$

<b>a</b>	<b>b</b>	$( a \wedge \neg b ) \vdash ( a \rightarrow \neg b )$									
T	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
T	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	T	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
F	F	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

Counterexample

Check

Is it valid?

☐ Yes.

☐ No.

Check

Each of the following is true. Which of these could you use to quickly assess for validity in this case?

☐ conditionals with false antecedents are true.

☐

conditionals with true antecedents and false consequents are false (and this is the only circumstance in which they are false).

☐ conditionals with true consequents are true.

Check

## Assessing soundness

As we will discuss in Week 3 Lecture 2, an argument is **sound** if it is both valid and all the premises are true. Note that sound arguments automatically have true conclusions.

To assess soundness we need to agree on what is true and what is false. While this is a hard thing to do in general, in the following five problems we simplify this by assuming that the following eight facts about colleges and their location and acceptance rates are all true (these were obtained by searching "US colleges" on google, which strangely leads to a google page that does not have an obvious link):

1. Harvard, Massachusetts, acceptance rate 5%
2. Princeton, New Jersey, acceptance rate 7%
3. MIT, Massachusetts, acceptance rate 8%
4. Columbia, New York, acceptance rate 7%
5. Yale, Connecticut, acceptance rate 6%
6. Cal Tech, California, acceptance rate 8%
7. UC Berkeley, California, acceptance rate 17%
8. Cornell, New York, acceptance rate 14%

### Problem 16

The following are four valid arguments which would be translated by substitution instances of modus ponens. But only one of the arguments is sound. Which one is it?

Which one is sound?

☐

If Harvard is in Connecticut then Harvard is in the same state as MIT. Harvard is in Connecticut. Therefore Harvard is in the same state as MIT.

☐

If Yale is in Massachusetts then Yale is in the same state as MIT. Yale is in Massachusetts. Therefore Yale is in the same state as MIT.

☐

If Yale is in Connecticut then Yale is in the same state as MIT. Yale is in Connecticut. Therefore Yale is in the same state as MIT.

☐

If Harvard is in Massachusetts then Harvard is in the same state as MIT. Harvard is in Massachusetts. Therefore Harvard is in the same state as MIT.

Check

#### Problem 17

The following are four valid arguments which would be translated by substitution instances of modus tollens (perhaps together with some replacement using double-negation). But only one of the arguments is sound. Which one is it?

Which one is sound?

☐

If Columbia is in New York then Columbia is not in the same state as Princeton. Columbia is in the same state as Princeton. Therefore Columbia is not in New York.

☐

If Columbia is in New Jersey then Columbia is in the same state as Princeton. Columbia is not in the same state as Princeton. Therefore Columbia is not in New Jersey.

☐

If Cal Tech is not in New Jersey then Cal Tech is in the same state as Princeton. Cal Tech is not in the same state as Princeton. Therefore Cal Tech is in New Jersey.

☐

If Cal Tech is in California then Cal Tech is in the same state as Cornell. Cal Tech is not in the same state as Cornell. Therefore Cal Tech is not in California.

Check

#### Problem 18

The following are four valid arguments which would be translated by substitution instances of disjunctive syllogism (perhaps together with some DeMorgan and double-negation). But only one of the arguments is sound. Which one is it? (In this problem, take "lowest acceptance rate in the state" to be shorthand for "lowest acceptance rate in the state of the colleges displayed in the list above." Also, in this problem, you'll probably have to actually look at the list of acceptance rates up above.)

Which one is sound?

☐

Not both Harvard and MIT have the lowest acceptance rate in Massachusetts. MIT has the lowest acceptance rate in Massachusetts. Therefore, Harvard does not have the lowest acceptance rate in Massachusetts.

☐

Harvard or MIT has the lowest acceptance rate in Massachusetts. Harvard does not have the lowest acceptance rate in Massachusetts. Therefore, MIT has the lowest acceptance rate in Massachusetts.

☐

MIT has the lowest acceptance rate in Massachusetts. Not both Harvard and MIT have the lowest acceptance rate in Massachusetts. Therefore, Harvard does not have the lowest acceptance rate in Massachusetts.

☐

Not both Harvard and MIT have the lowest acceptance rate in Massachusetts. Harvard has the lowest acceptance rate in Massachusetts. Therefore, MIT does not have the lowest acceptance rate in Massachusetts.

Check

#### Problem 19

Consider the argument: "If Cal Tech has a lower acceptance rate than Cornell, then Cal Tech has a lower acceptance rate than UC Berkeley. Cal Tech has a lower acceptance rate than UC Berkeley. Therefore, Cal Tech has a lower acceptance rate than Cornell."

Which of the following describes this argument:

Which best describes the argument?

☐

The argument is sound

☐

The argument is unsound but valid.

☐

The argument is invalid but has all true premises and a true conclusion.

☐

The argument is invalid and has a false premise or a false conclusion.

Check

#### Problem 20

Consider the argument: "If Cornell does not have a lower acceptance rate than Columbia, then Cornell has a lower acceptance rate than Princeton. Cornell does not have a lower acceptance rate than Columbia. Therefore, Cornell has a lower acceptance rate than Princeton."

Which of the following describes this argument:



Which best describes the argument?

- ☐ The argument is sound
- ☐ The argument is unsound but valid.
- ☐ The argument is invalid but has all true premises and a true conclusion.
- ☐ The argument is invalid and has a false premise or a false conclusion.

Check

This is a practice problem set for [this course](#). It is run on the Carnap software, which is an:

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