Applications of Multi-variable calculus

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Introduction

Calculus with many variables, often known as multivariate calculus, is an area of mathematics that can be used to determine how input and output variables are related. Calculus with many variables is an extension of calculus in mathematics known as multivariable calculus.

In addition, multivariable calculus includes vector fields, partial differentiation, multiple integrations, limits and derivatives, and fundamental theorems with multivariable dimensions. In addition, multivariable calculus is crucial in a wide range of disciplines, including engineering, astronomy, natural and social sciences, weather forecasting, etc. Multivariable calculus is used to study the two- and three-dimensional models.

We will be discussing three domain-based applications of multi-variable calculus.

1 Calculus in Pharmacy

Pharmacy is the science or practice of preparing and dispensing medicinal drugs.
General uses of pharmacy in calculus are:

- To determine rates of drug elimination from the body.

- To determine rates of absorption in the body.
- To determine what medication can be taken with orders and in what doses.
- To make dosages according to a person's height and weight.
- To research how the body metabolizes drugs and other substances.

Every drug is metabolized in the body, and the available dose in the body is calculated as a function of time.

Pharmacokinetics: the study of the time of drug absorption, distribution, metabolism, and excretion.

In order for doctors to prescribe the correct dosage of a drug and provide a regimen for treatment (i.e., "take 2 capsules twice a day"), the drug's concentration over time must be tracked. This prevents under and over-dosing.

The way that a drug's concentration over time is calculated is by using calculus. In fact, the course of a drug over time can be calculated using a differential equation.

In applications of differential equations, the functions represent physical quantities, and the derivatives, as we know, represent the rates of change of these qualities. Therefore, a differential equation describes the relationship between these physical quantities and their rates of change.

Calculating the rate of absorption:
In order to calculate the rate of absorption, the equation must include the drug dosage, the absorption constant, and the drug's bioavailability.

Medical professionals need calculus!
Without drug specialists in the pharmaceutical industry testing drug concentrations over time and modeling them using calculus, we would not have labels on medication that provide instructions for dosage use.

Although the equation for each drug looks unique depending on its properties and the patient's anatomy, calculus is necessary for medical professionals to have the ability to map the relationship between drug concentration in the body over time.

In pharmacy, calculus is used to determine dosage and administration of medication. It also plays a role in understanding pharmacokinetics and pharmacodynamics, which are important concepts in the field of pharmacology. Additionally, calculus can be used in the analysis and interpretation of data in pharmaceutical research and development. Overall, calculus is an essential tool for pharmacists and other professionals in the field of pharmacy.

2 Calculus in Baseball

Baseball is a game where the ball hits the bat, and the batter or the hitter places it far in the ground. So there are many forces included in this game. We can estimate the force of impact on the bat, kinetic energy of the baseball hitting the bat, the energy of the spin of the bat, and so on. There is a sweet spot on the bat where the batter feels less impact than other batter sites. This spot occurs because the vibration of the bat at that point is not excited at all.

The impact of baseball on the bat is very short; it is about 1 msec. Due to the impact, both fundamental and second vibration modes get excited with almost the same amplitude.A strange phenomenon occurs that the impact on the fundamental mode does not excite that mode, but it excites the second mode. Similarly, the second mode impact doesn't excite the second mode, but it excites the fundamental mode.

In baseball, calculus can be used to optimize the pitcher's throw to achieve maximum efficiency. Also, calculus can be used to calculate the projectile motion of baseball's trajectory and to predict if runners can make it to the next base on time given their running speed and the speed of a hit ball.

Finding the work required to throw the baseball

The work done W on a moving ball from a position s0 to s1 is equal to the change in ball's kinetic energy. The kinectic energy K of a baseball of mass m and velocity v is given by,

$$k = 1/2mv^2$$

$$\text{W}{=}\int_{s_0}^{s_1} F(s)\,ds = (1/2\text{mv}_1^2) - (1/2mv_0^2)$$

where v0 and v1 are initial and final velocities. Using this, baseball players can figure out how much force they need to exert on the ball to reach the place where they want the ball to go.

Finding the Average force on the bat during the collision
The collision of ball and bat, are quite complex.
We can calculate the average force on the bat during this collision by first calculating the change in the ball's momentum.

We know that the momentum p of an object is the product of its mass m and its velocity v, that is, p=mv. Suppose an object, moving along a straight line, is acted on by a force F=F(t) which is a continuous function of time t.
The change in ball's momentum over a time interval [t0t1] is equal to the integral of force F from time t0 to t1.

$$p(t_1)p(t_0) = \int_{t_0}^{t_1} F(t)\,dtt1t0F(t)dt$$

Using the above formula, one can find the average force on the bat during the collision F=ma where a= vt.

Calculus can be used in baseball to analyze and predict various aspects of the game, such as the trajectory of a pitched ball and the movements of players on the field. By using mathematical models and equations, coaches

and analysts can gain a deeper understanding of the game and make more informed decisions. Additionally, calculus can be used to improve player performance by analyzing their movements and helping them optimize their techniques. Overall, calculus plays a significant role in the analysis and improvement of baseball performance.
The application of calculus in sports does not end with baseball, it includes running, basketball etc.

3 Calculus in Robotics

All mechanical robots use calculus for their functions.
Calculus allows robots to:

- be mobile
- have velocity and acceleration
- be controlled to change speed, pick something up, or move to specific locations
- process changes in color or intensity of images

Need for application:

- Increased technology led to the desire for robotics
- Robots need calculus to function

Motion: Mobile robots drive around, with a velocity (the derivative of position) and an acceleration (the derivative of velocity). Robotic arms move, each joint has an angular velocity and acceleration. This is probably your most basic application of calculus to robotics.

Controls: After calculus, many people will take differential equations. A differential equal is often used as a description of how some type of system will change over time. It says "if the system is in some state right now, then this is how much the state will change by."

For example, if you are on a roller coaster going down a hill. Your current "state" might be described by how fast your are moving (velocity) and how far down the hill you are. At that point in time, you can use a differential equation to describe how much your speed and position will change over the next instant of time. Let's say now that this roller coaster has brakes, and you don't want to move too fast, you need to control your speed. So you can look at your current state and use that to calculate how much to apply the brakes to make sure you stay at a constant speed. Perhaps you want to move as some speed back up the hill? Well you can calculate how much power to apply to make it happen.

You can actually do this with a ton of different systems: how do you think a segway stays upright? how does your cruise control make sure you keep going 60 on the highway? The answer is controls, and it's all based on calculus (and a ton of linear/matrix algebra). This is also how you make a robot move where you want it to, or pick up something where you want it to.

Computer Vision: This one may seem a bit odd at first, applying calculus to images, but hear me out. A digital image is a giant array of pixels, and each pixel has some numbers representing the color for that pixel. To keep things simple, let's assume you have a black and white image, so each pixel value is just the brightness of that cell. Now, what if you want to find edges of objects in the image. Well, what defines an edge? A change in color, or in our case, intensity (since we aren't using color). How do you calculate this change? Calculus! By subtracting the value of each pixel from the value of the pixel to the right, you can get a sense of how much change occurred between those two pixels. If there was enough change, (i.e. the derivative is high enough), then you predict that there is an edge located at that pixel. This isn't quite the same derivative that you would perform in a calculus class, but it is a derivative.

An important concept is the **Ripple Factor**, which says that Calculus has allowed robots to move more freely and become more humanoid.

Calculus is a fundamental tool in the field of robotics, as it allows for the analysis and modeling of motion and change. It is used to model the dynamics of robotic systems, to design control systems, and to plan trajectories. Calculus also plays a crucial role in the field of computer vision, which is used for perception and localization in robotics. In conclusion, calculus is a key component in the design and operation of robotic systems, and its applications are wide-ranging and essential for the continued development of the field.

Conclusion

Calculus is a branch of mathematics that deals with the study of rates of change and accumulation. It has a wide range of applications in fields such as physics, engineering, economics, and more. The two main branches of calculus are differential calculus and integral calculus, which are used to study how a function changes and how to calculate the area under or between curves. Calculus is a fundamental tool in many areas of science and engineering and is used to model and analyze real-world phenomena. In conclusion, calculus is an essential tool for understanding and solving problems in various fields, and its applications continue to expand as technology and science advance.