Modeling Population Growth with Stochastic Shocks

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1 Introduction

Population growth is influenced by numerous environmental and external factors. Classical deterministic models, such as the logistic growth equation, describe population dynamics under stable conditions. However, real-world populations experience random fluctuations due to environmental shocks, including natural disasters and resource shortages. These stochastic events introduce variability not captured by deterministic models.

In this project, we extend the standard logistic growth model to incorporate stochastic shocks by modeling them as a Poisson process, capturing sudden population declines. Additionally, environmental noise is introduced using a Wiener process to represent continuous fluctuations. Our goal is to investigate the impact of random catastrophic events on long-term population stability and extinction risk.

While stochastic models have been applied to various species, we specifically focus on human population dynamics. Human populations are influenced not only by environmental factors but also by global pandemics, wars, and socio-economic shifts. By modeling these shocks realistically, we aim to gain insights into the long-term sustainability of human populations and their resilience to crises.

2 Methods

Before we started building the model, we laid out a few key assumptions to keep things manageable. First, we assumed that in the absence of any disruptions, the population would follow logistic growth. This means it grows quickly at first but slows down as it gets closer to the Earth's carrying capacity. We treated shocks, such as pandemics or disasters, as random events that occur over time and assumed they follow a Poisson process with a constant rate. In other words, the chance of a major shock occurring remains the same throughout the simulation.

Each time a shock occurs, it reduces the population by a certain fraction, which could either be a fixed percentage or a randomly chosen one. We assumed that the carrying capacity, K, stays constant and does not change over time. To represent smaller, ongoing fluctuations such as changes in climate or economic conditions, we used Brownian motion. We also ensured that the population could never drop below zero.

The model starts with an initial population of 1.6 billion, which was approximately the global population in 1900 [5]. For the size of each shock, we assumed the impact follows a normal distribution, meaning some shocks are mild while others are more severe. Finally, we assumed that the intrinsic growth rate, r, stays the same over the entire time period.

Now that we have some assumptions in place, we can start building our model. To model human population growth, we use the logistic growth equation, which accounts for limited resources:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) \tag{1}$$

where: N is the population size, r is the intrinsic growth rate, and K is the carrying capacity. This formulation is commonly used in population dynamics [8].

This equation describes how populations grow rapidly when small but slow down as they approach the carrying capacity K. However, real populations do not grow in a perfectly smooth manner due to various environmental factors that introduce random fluctuations. These include climate variability, natural disasters, resource availability (e.g., food or water), economic instability, and human activities like deforestation or pollution.

To account for these unpredictable influences, we introduced a Wiener process W_t into the model. This captures continuous, small-scale random fluctuations in population growth. The equation becomes:

$$dN_t = rN_t\left(1 - \frac{N_t}{K}\right)dt + \beta N_t dW_t \tag{2}$$

where: β is the diffusion coefficient representing environmental variability, and $dW_t \sim \mathcal{N}(0, dt)$. This formulation follows standard stochastic modeling practices [6].

We then introduce a term to model major population shocks such as pandemics. These are represented as discrete, sudden events that occur randomly, modeled using a Poisson process. The corresponding equation is:

$$dN_t = -J_t N_t dP_t \tag{3}$$

where: dP_t is the increment of a Poisson process with rate λ, indicating whether a shock occurs in the time interval dt. The value of dP_t is either 0 (no shock) or 1 (shock occurred). J_t is the fractional loss in population per event, drawn from a normal distribution with mean J_{mean} and standard deviation J_{std}.

Combining both environmental noise and discrete shocks, we obtain the final stochastic differential equation:

$$dN_t = rN_t\left(1 - \frac{N_t}{K}\right)dt + \beta N_t dW_t - J_t N_t dP_t \tag{4}$$

where: N_t is the population at time t, r is the intrinsic growth rate, K is the carrying capacity, β controls environmental noise, W_t is the Wiener process, dP_t is the Poisson increment, and J_t is the fractional loss per shock.

Since this model involves a stochastic differential equation (SDE), we use a numerical approximation to simulate its evolution over time. Specifically, we use the Euler-Maruyama method:

$$N_{t+1} = N_t + \left[rN_t \left(1 - \frac{N_t}{K} \right) dt \right] + [\beta N_t dW_t] - [J_t N_t dP_t] \tag{5}$$

This method allows us to simulate the model by stepping forward in time using small increments dt, and is commonly used in discrete-time simulations of SDEs [7].

We set the intrinsic growth rate to $r = 0.015$, which is the average growth rate given by the World Bank [1]. Since every population has an upper limit based on environmental and resource constraints, we defined the carrying capacity as 8 billion, which is the estimated human carrying capacity of Earth according to 20 different studies [2]. This prevents the model from allowing unbounded growth.

To incorporate environmental fluctuations, we introduced a diffusion coefficient, β, which was estimated using historical world population data from the World Bank [3]. By computing the standard deviation of annual human population growth rates, we obtained $\beta = 0.00363$. This allows the model to account for small, random variations in yearly population growth. However, real-world population trends are also affected by major catastrophic events such as pandemics. To simulate these events, we modeled them as a Poisson process with a rate of $\lambda = 1/15 = 0.0667$, meaning that a major event is expected approximately once every 15 years [4]. This ensures that large-scale disturbances occur at realistic intervals.

Each catastrophic event in the model reduces the population by a fractional amount. Based on historical pandemic data, we determined that the mean fractional drop per event is $J_{\text{mean}} = 0.0293$, with a standard deviation of $J_{\text{std}} = 0.0587$ [4]. This accounts for the fact that some pandemics, like the 1918 flu, caused significant population declines, while others, like more recent pandemics, had a smaller impact. To ensure the model captures long-term trends in population growth and decline, we ran the simulation for $T = 500$ years, allowing us to analyze multiple cycles of growth and catastrophic events. We chose the Euler-Maruyama method in order to numerically solve the stochastic differential equation, as it is well-suited for models incorporating stochastic processes. We set the time step to $dt = 0.1$.

3 Simulated Results

We coded our model in Python to see how the population changes over 500 years when influenced by random fluctuations as well as major shocks.

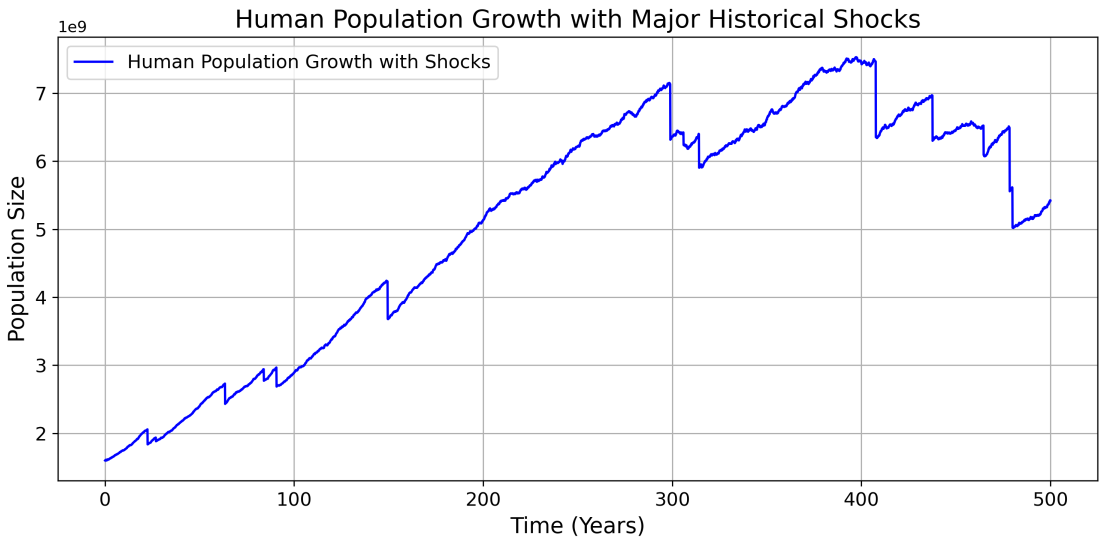


Figure 1: One simulation of population growth over 500 years using the full stochastic model.

As we see from the graph, the population grows rapidly within the first 280 years, then consistently fluctuates in the last 220 years despite slightly sharp periodic declines caused by catastrophic events. These drops occur at irregular intervals as they represent major disruptions like pandemics, modeled using the Poisson process. However, the overall trend remains strongly positive, with the intrinsic growth rate high enough to drive continuous population expansion, even in the presence of shocks.

Between years 280 and 500, we see periods of slightly sharp fluctuations, where the population first stabilizes, then drops, then rises again. This pattern suggests that the population is reaching its carrying capacity. The fluctuations could be due to resource constraints, technological advancements as well as large-scale disruptions that prevent sustained increases beyond a certain threshold.

After year 300, we notice that while the shocks remain relatively frequent within the last 200 years, the population fluctuates within a more defined range rather than the continuing unchecked growth. By year 400, the population oscillates between 6 and 7 billion, suggesting a stabilization effect where growth is limited by external pressures. The results show that under these conditions, the population can reach a long-term equilibrium despite recurring shocks. However, as the total population increases, each decline becomes more pronounced in absolute numbers, meaning that while relative impacts remain stable, the number of people affected by each event becomes significantly larger.

Time Period (years)	**Population Start** ($\times 10^9$)	**Population End** ($\times 10^9$)	**Growth (%)**
0–100	1.6	3	87.5
100–200	3	5.8	93.3333
200–300	5.8	5.5	-5.1724
300–400	5.5	6	9.090909
400–500	5.5	6	10

Table 1: Population Growth Percentage per Century

Table 1 represents the difference in growth between every century, and tells us whether said growth increases or decreases, as well as how little or how huge these changes really are. Overall, we see that the population increases more than not, although the growth between centuries is quite unstable, with drastic changes happening between 0-200 years and more steady increases happening between 300-500 years.

4 Analysis

In this section, we analyze the key findings from our model, focusing on the impact of stochasticity, catastrophic shocks, and parameter variability on long-term population dynamics. Figure 2 provides a comparative analysis of different model components, while Figure 3 presents a sensitivity analysis for varying model parameters, illustrating their effects on population stability and extinction risk. Finally, to account for uncertainty introduced by stochastic processes, Figure 4 uses a Monte Carlo simulation to evaluate the ensemble behavior of the system, offering deeper insights into typical outcomes and potential extreme scenarios.

4.1 Comparison of Model Components

To better understand the behavior of our full stochastic model (blue), we compare it against three other models that isolate individual effects: the deterministic baseline model (red), the shocks-only model (green), and the stochasticity-only model (magenta).

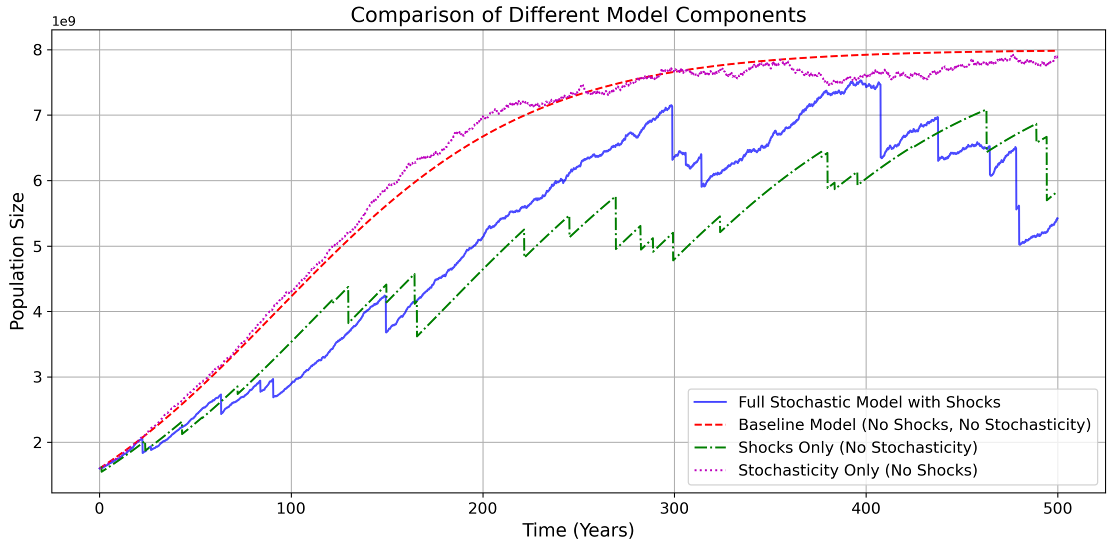


Figure 2: Comparison of four models to isolate the effects of different factors. The deterministic model shows smooth growth, while adding stochasticity or shocks changes recovery patterns and long-term outcomes. The full model combines both and results in more realistic fluctuations and sharper declines.

The **deterministic baseline model** (red-dashed line) follows the classical logistic growth equation, assuming no randomness in population dynamics. The population grows smoothly until it stabilizes at the carrying capacity ($K = 8$ billion). This model provides a theoretical upper limit for population size under ideal conditions where external disruptions are absent. When compared to the full stochastic model, we see that real-world is significantly different from this ideal growth, primarily due to the influence of catastrophic events.

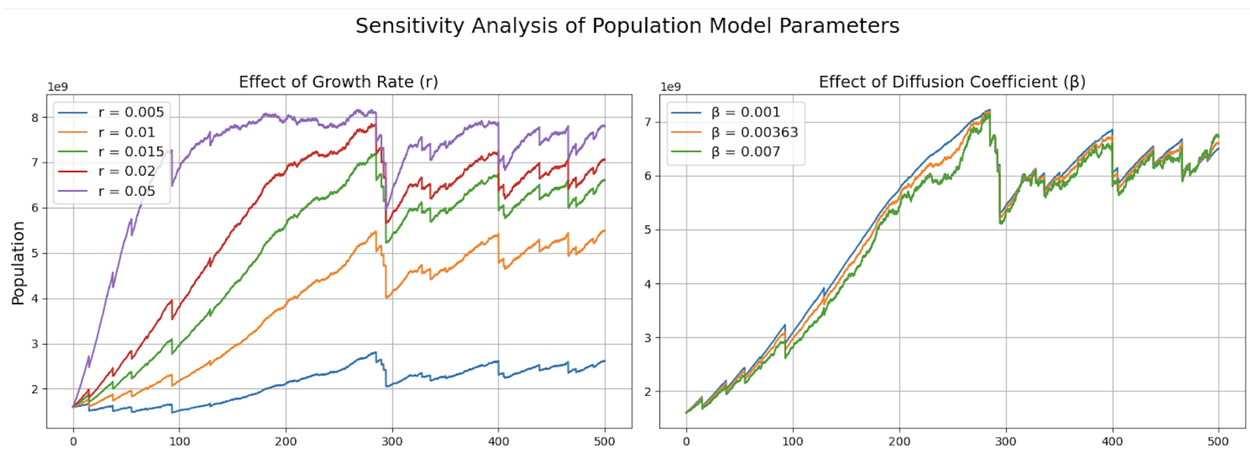
The **shocks-only model** (green dash-dotted line) introduces discrete catastrophic events in a deterministic logistic growth trajectory. This model exhibits periodic, stepwise drops in population size corresponding to external shocks such as pandemics, modeled using a Poisson process. Unlike the full stochastic model, the green model lacks continuous environmental fluctuations, resulting in more predictable recovery patterns. The difference between the green and blue models highlights that without environmental noise, the population follows a relatively stable trend between shocks. However, in the full stochastic model, background fluctuations slow recovery at times or amplify population losses due to compounding effects.

The **stochasticity-only model** (magenta dotted line) captures the effect of continuous environmental variability but excludes catastrophic shocks by following an overall logistic growth trend with persistent, small-scale fluctuations caused by environmental noise. When compared to the full stochastic model, we see that the magenta model explains the non-catastrophic fluctuations present in the blue model. This model generally overshoots our model due to the lack of sudden drops in population caused by external shocks.

By comparing these models, we gain insight into the behavior of the **full stochastic model**. The deterministic model sets an upper bound, demonstrating what would happen in the absence of randomness. The shocks-only model shows the impact of catastrophic events, which cause significant but discrete population losses. The stochasticity-only model reveals the effects of continuous variability, which introduces persistent fluctuations but does not cause drastic declines. The full stochastic model combines these effects, producing a more complex and realistic trajectory.

4.2 Sensitivity Analysis of Key Parameters

To assess the robustness of our model, we conducted a sensitivity analysis on key parameters. Figure 3 presents the effects of varying the intrinsic growth rate (r), diffusion coefficient (β), pandemic frequency (λ), mean and variability of pandemic severity (J_{mean} and J_{std}), and carrying capacity (K).



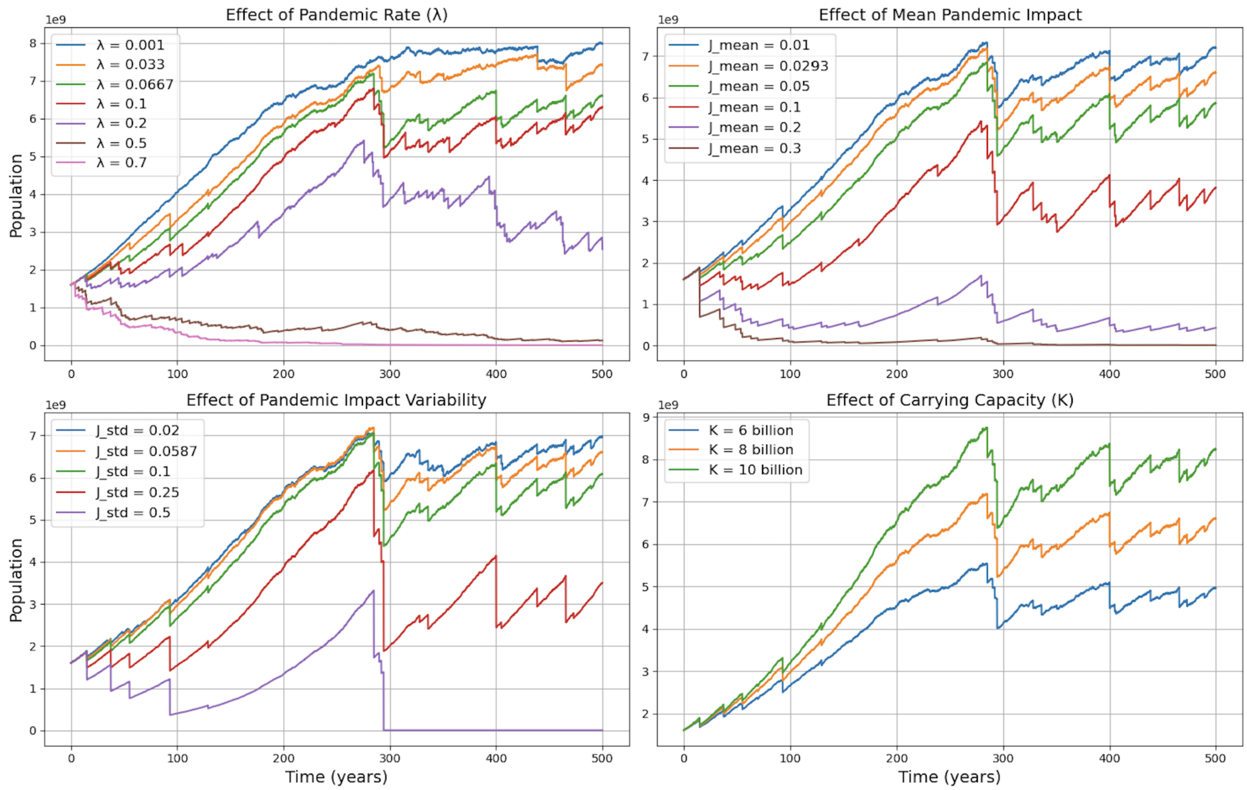


Figure 3: Sensitivity Analysis for Varying Population and Pandemic Parameters

The first subplot examines the effect of **intrinsic growth rate (r)** on long-term population trends. Lower growth rates ($r = 0.005$) lead to slower recovery from shocks, whereas higher values ($r = 0.05$) enable rapid population expansion. In scenarios with lower growth rates, populations remain vulnerable to extinction if multiple shocks occur in succession, preventing full recovery.

The second subplot illustrates the impact of **environmental fluctuations (β)**. While different values of β introduce variability in short-term population dynamics, they do not significantly alter long-term trends. This suggests that environmental noise alone does not drive major demographic changes.

The third subplot explores the effect of **pandemic frequency (λ)**. At low values ($\lambda = 0.001$, corresponding to a catastrophic event every 1000 years), the population remains relatively stable. However, as the frequency increases ($\lambda = 0.5$, or one event every two years), the population undergoes persistent declines. In extreme cases, high λ values push the system toward extinction, where the population fails to recover between consecutive shocks.

The fourth subplot examines **mean pandemic impact ($J_{\mathbf{mean}}$)**. Higher values of J_{mean} indicate that each shock removes a greater fraction of the population. When $J_{\text{mean}} > 0.1$, representing highly severe events, the population is unable to sustain itself over time, leading to collapse.

The fifth subplot assesses the role of **pandemic impact variability ($J_{\mathbf{std}}$)**. Greater uncertainty in shock severity results in more unpredictable population dynamics. High variability increases the likelihood of extreme declines, which can drive the population toward extinction even if the mean impact remains moderate.

The final subplot investigates the effect of **carrying capacity (K)**. A lower K (e.g., 6 billion) restricts population growth, whereas a higher K (e.g., 10 billion) allows continued expansion. However, regardless of K, the population remains vulnerable to high-impact shocks, underscoring the importance of resilience in demographic models.

4.3 Extinction Risk and Population Resilience

The sensitivity analysis highlights key factors that contribute to population extinction. In particular, from Figure 3, we notice that the likelihood of extinction becomes very high when:

1. The pandemic rate (λ) is too high, causing frequent large declines.
2. The mean pandemic impact (J_{mean}) is too large, leading to massive losses.
3. The pandemic impact variability (J_{std}) is too large, leading to bigger losses less frequently.
4. The intrinsic growth rate (r) is too low, preventing effective recovery.

4.4 Monte Carlo Simulation for Aggregate Model

Our stochastic model is highly sensitive to individual trajectories due to the compounded effects of randomized fluctuations and high-impact events. Consequently, a single simulation cannot fully represent the system's behavior. To understand the broader dynamics and assess the long-term sustainability of human populations under stochastic influences, we employed a Monte Carlo simulation approach. By simulating the model multiple times, we can analyze not just individual outcomes but the statistical properties of the entire system — such as average behavior, variability, and risk of collapse — which is critical for drawing robust conclusions from probabilistic models.

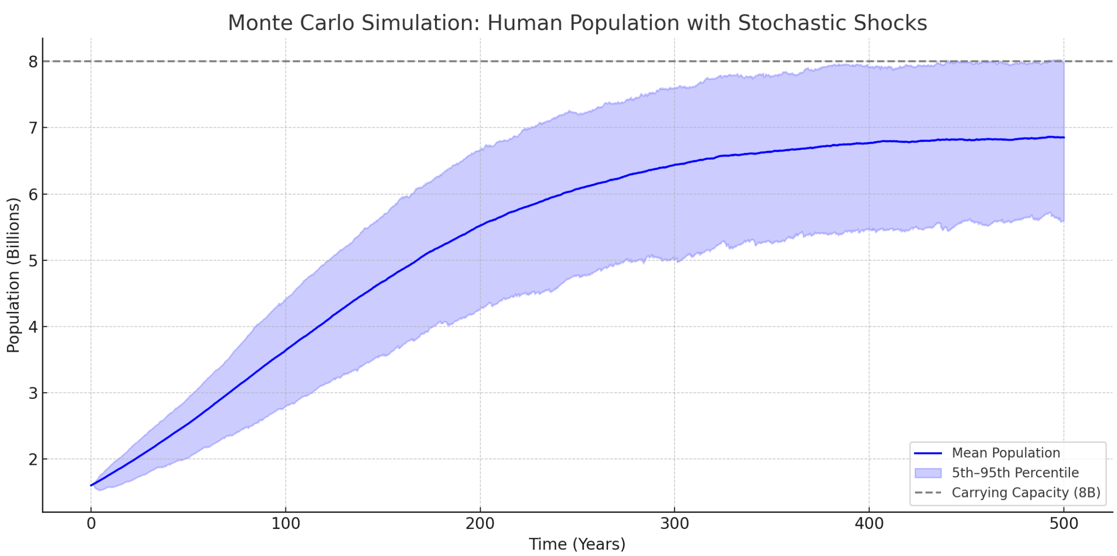


Figure 4: Monte Carlo simulation of 1000 runs. The average trajectory stabilizes, but the shaded region (5th–95th percentile) shows large variability.

To get a better sense of how the population behaves over time under randomness, we ran 1,000 simulations of our full model, each over a 500-year period. This Monte Carlo approach gave us a much clearer picture of what is typical and what is possible, rather than just focusing on a single outcome.

We plotted the average population across all simulations and added a shaded area to show where most simulations fall (between the 5th and 95th percentiles). The average population trajectory still follows the usual logistic curve and levels off near the carrying capacity. But the spread of the shaded region shows that there's a lot of variability in how things can play out, especially in the later years when the population is larger and more sensitive to shock events.

What's interesting is that even though shocks are fairly frequent (about one every 15 years), the population never dropped below 500 million or even 1 billion in any of the 1,000 runs. That suggests the current parameters make the population pretty resilient in the long run. In other words, even with all the randomness built into the model, most outcomes still hover around a stable range after year 300.

That said, this doesn't mean the population is completely safe. We didn't see any collapses with the settings we used, but things could look very different if major shocks happened more often or were more severe. A next step could be to test those more extreme scenarios and see when the population actually starts to crash. That would give us a better idea of how close we might be to situations where recovery isn't possible anymore.

Overall, our analysis demonstrates that while human populations exhibit resilience to moderate disruptions, frequent or extreme catastrophic events significantly increase extinction risk. The sensitivity plots show that when things start to go wrong, collapse happens fast. If shocks are too frequent or the growth rate is too low, the population doesn't have time to recover and just crashes. It's not a slow decline but rather it drops off quickly. This shows that small changes in the inputs can cause the system to fail, even if it seems stable at first. These insights underscore the necessity of incorporating stochastic elements in demographic modeling to capture realistic long-term population dynamics.

5 Conclusion

In this project, we built a stochastic model to look at how the human population might grow or decline over time when affected by both continuous environmental changes and sudden shocks like pandemics. We used a logistic growth base, added random fluctuations through Brownian motion, and included random shocks using a Poisson process. We ran 1000 simulations over 500 years using the Euler-Maruyama method.

Most of the simulations showed that the population tends to level off near the carrying capacity, and stays above any critical collapse point. This suggests the population is fairly stable under the parameters we used. But when we changed those parameters, especially the frequency or severity of shocks, the system became much more unstable. If recovery was also slow, the population could collapse or fail to recover properly.

This shows that even when things look stable, small changes in key inputs can have serious long-term effects. It's a reminder that stability depends heavily on the assumptions we make. One limitation of our model is that we kept growth rate, shock rate, and carrying capacity constant in the model, but in real life, those would probably change. For example, new technology could raise carrying capacity, or climate change could lower it. If we let those values change over time or depend on the population size, the results would probably look very different. It would be useful in future work to test what happens if these values shift or respond to population size. Comparing our model's results to real historical population data could also help check how realistic the outcomes are. Although the model is simplified, it helps break down how different factors affect long-term population trends and shows the kinds of risk that could threaten stability.

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