# Mathematical Modeling of Heat Transfer in Steak Cooking

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## 1 Introduction

Steaks can be cooked in various ways, depending on the cut and desired doneness level. But knowing how rapidly technology is developing these days, there's no doubt that robots are also being actively worked on too. If it ever comes that restaurants need robots to help run their business, how exactly could we tell a robot to mathematically cook steak?

The goal of this study is to mathematically model heat transfer using the heat equation and simulate the cooking process to determine the time required to reach different doneness levels. We utilize partial differential equations (PDEs), specifically the heat equation, to model heat diffusion through different steak cuts, determining optimal cooking times for various doneness levels. We implement the finite difference method to analyze temperature distributions over time and assess the effects of flipping the steak during cooking. Our results give us a framework for accurate steak cooking and insights that can be applied to robotic cooking automation.

## 2 Methods

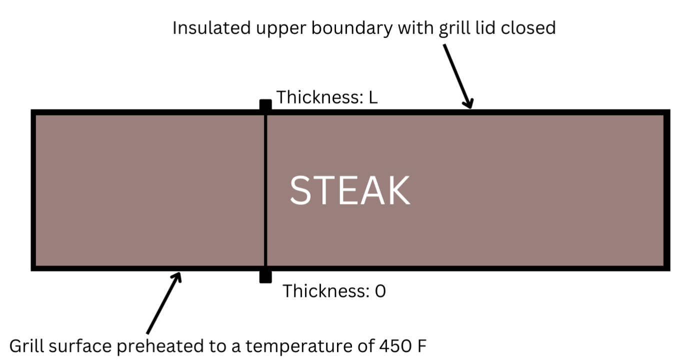


Figure 1: Model Setup

| Description | Symbol | Dimensions | Type |
|---|---|---|---|
| Amount of heat diffused from the grill surface to the steak | $Q$ | F | Dependent |
| Depth of the steak at a point | $z$ | cm | Independent |
| Time elapsed during cooking | $t$ | s | Independent |
| Temperature inside the steak at depth $z$ and time $t$ | $u(z,t)$ | F | Dependent |
| Thickness of the steak | $L$ | cm | Independent |
| Thermal diffusivity of the steak | $\kappa$ | $\text{cm}^2/\text{s}$ | Independent |
| Temperature of the bottom surface of the steak | $T_{\text{bottom}}$ | F | Independent |
| Temperature of the top surface of the steak | $T_{\text{top}}$ | F | Dependent |
| Initial temperature of the steak | $T_{\text{initial}}$ | F | Independent |
| Steak doneness level | $D$ | 1 (categorical) | Dependent |

Table 1: Variables and Parameters

Table 1 presents some of the variables and parameters used in the modeling of steak cooking as seen in the setup given in Figure 1. We use these to define the heat transfer process and to determine the conditions under which the steak reaches different levels of doneness and to come up with the following assumptions in order to accurately model the cooking process.

- Assume that for a steak to be cooked in different styles, the temperature reached is within the following ranges:
  - Rare: 120°F – 125°F [4]
  - Medium Rare: 130°F – 135°F [4]
  - Medium: 140°F – 145°F [4]
  - Medium Well: 150°F – 155°F [4]
  - Well Done: 160°F – 165°F [4]
- The restaurant serves only one cut of steak.
- Any steak cooked below the given temperature ranges is considered undercooked, while any steak cooked above the given ranges is considered overcooked.
- All steaks start at the fixed initial temperature of room temperature, which is assumed to be 70°F.
- Assume that heat diffuses uniformly from the surface of the steak through its core.
- Only one steak is cooked at a time to avoid interference from heat transfer between multiple steaks.
- The cooking process is governed solely by temperature and effects from chemical reactions are ignored.
- The heat source (grill) maintains a constant surface temperature of 450°F, ensuring a continuous heat flux at the steak's contact surface.
- Heat loss is neglected during the cooking process.
- After cooking, the steak is rested for 5 minutes to allow heat diffusion to reach an optimal core temperature.

- The grill is preheated and is at 450°F.
- The crust of the steak does not undergo burning.
- We assume there is only one dimensional heat transfer and the heat conduction within the steak occurs in the vertical direction (perpendicular to the grill surface).

With these assumptions in place, we can now form our model. Our goal is to model how heat diffuses through a steak when placed on a grill that is preheated to the temperature of 450°F. The steak is in a closed grill environment, meaning heat loss is negligible, and the bottom surface is maintained at 450°F due to direct contact with the grill. The top surface is insulated because the grill lid is closed, preventing significant heat escape.

We can model this using the heat equation which governs this process:

$$u_t = \kappa u_{zz} \tag{1}$$

where $u(z,t)$ represents the temperature inside the steak at depth $z$ and time $t$, and $\kappa$ is the thermal diffusivity of the steak, given as 0.00133 $\text{cm}^2/\text{s}$ [3].

The boundary conditions define how heat interacts with the steak's surfaces. At the bottom surface ($z = 0$), the steak is in direct contact with the grill, which is maintained at a fixed temperature of 450°F. This is given by:

$$u(0,t) = 450 \tag{2}$$

At the top surface ($z = L$), the steak is not in direct contact with a heat source but is instead insulated by the closed grill lid. This means there is no heat flux at the upper boundary leading to the Neumann boundary condition:

$$\left.\frac{\partial u}{\partial z}\right|_{z=L} = 0 \tag{3}$$

Initially, the steak is assumed to be at room temperature, which we take as 70°F [5]. This provides the initial condition:

$$u(z,0) = 70 \tag{4}$$

Before we can solve the heat equation numerically, we need to figure out how the temperature inside the steak changes over time. To do this, we use the finite difference method, which breaks the steak into small layers and calculates how heat moves through them step by step.

The steak thickness is divided into small intervals of $dz$, and time is discretized using a step size $dt$. We choose our time step such that:

$$dt = \frac{dz^2}{4} \tag{5}$$

The formula we use updates the temperature at each layer based on the temperatures of the layers above and below it. It is given by:

$$u(z_i, t_{n+1}) = u(z_i, t_n) + \mu \left( u(z_{i+1}, t_n) - 2u(z_i, t_n) + u(z_{i-1}, t_n) \right) \tag{6}$$

Here, $u(z,t)$ represents the temperature at a certain depth $z$ and time $t$, and $\mu$ is a factor that controls how heat spreads. This equation helps us calculate how heat moves through the steak over time. We code this equation in python to simulate the cooking of the steak.

At $t = t_{\text{flip}}$ the steak is flipped, meaning the side that was previously on top is now touching the grill. This means the top boundary now becomes 450°F, just like the bottom boundary was before:

$$u(L, t) = 450 \tag{7}$$

At the same time, the bottom surface is no longer in contact with the grill, so instead of being at a fixed temperature, it now behaves like the previous top boundary (insulated), meaning heat does not escape. The new boundary condition is:

$$u(0, t_n) = u(0, t_n) + \mu \left( 2u(\Delta z, t_n) - 2u(0, t_n) \right) \tag{8}$$

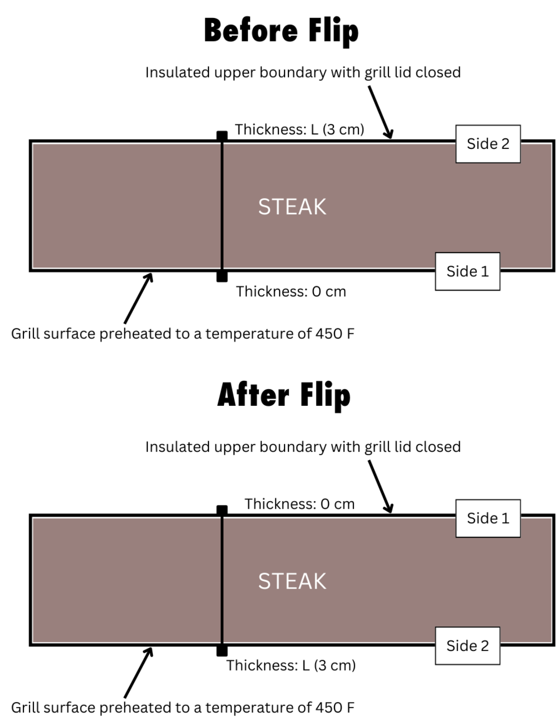


Figure 2: Simulating the Flip

By switching these boundary conditions at the flipping time, the model accurately reflects what happens in real life: heat is applied to one side at a time, and flipping ensures more even cooking. This flipping step is implemented in the numerical model by changing the boundary conditions after four minutes (240s) as that is the optimal time given in [1] to make sure the steak is cooked consistently throughout.

# 3 Results

We code our model in Python to get the following results:

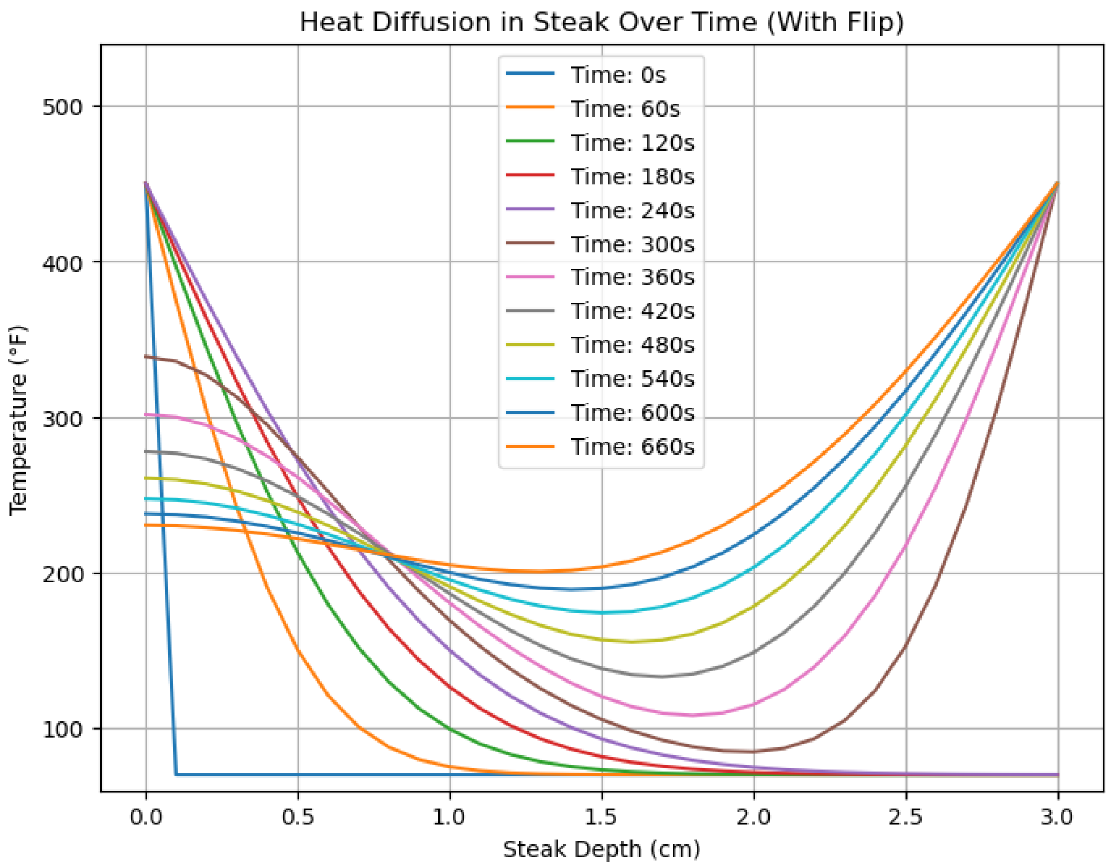


Figure 3: Heat Diffusion in Steak

Figure 3 shows that the steak's surface temperature starts at 450°F and the core heats more gradually due to its low thermal diffusivity. At t = 240s, the core remains below 120°F, indicating it has not yet reached rare doneness. By t = 360s, the core approaches 120–125°F, meaning it is just reaching rare. Around t = 420s, the temperature surpasses 130°F, marking the transition to medium-rare. If the steak is flipped too late, a strong temperature gradient develops, leading to uneven cooking. However, after flipping, the core temperature increases 15–20°F faster, showing how a more balanced heat exposure speeds up cooking and ensures a more uniform doneness.

One of the most important things to look at is the timing of the flip. Flipping too early means the steak hasn't had enough time to build up internal heat which resets the heating process and prolongs the total cooking time. Both sides remain underheated initially, and neither side reaches the optimal temperature efficiently. However, delaying the flip past approximately 300 seconds also leads to a greater temperature imbalance. The side initially in contact with the grill becomes excessively hot, leading to an unevenly cooked steak that is overcooked near one surface but still undercooked in the core.

Based on Figure 3, flipping the steak at 240 seconds provides optimal results. It stops one side from overheating while allowing the center to reach the right temperature within a reasonable time. This results in a properly cooked steak without an overdone exterior or an undercooked core.

Before flipping, the heat transfer is one sided which causes the bottom of the steak to heat quickly while the top remains cooler. This leads to overcooking near the grill while the core takes longer to warm up. The temperature gradient is steep which means the steak is not cooking evenly.

Once flipped, heat is applied from both sides and so the core temperature to rises faster and this reduces the total cooking time. The model shows that flipping prevents overcooking on one side and helps achieve a more uniform doneness. This is seen from the temperature profiles where flipping at t = 240s results in a more balanced heat distribution across the steak compared to delaying the flip.

| **Doneness Level** | **Target Core Temperature (°F)** | **Estimated Cooking Time (s)** |
|---|---|---|
| Rare | 120–125 | 360-420 (6-7 min) |
| Medium Rare | 130–135 | 400-450 (6.5-7.5 min) |
| Medium | 140–145 | 420–480 (7–8 min) |
| Medium Well | 150–155 | 450–500 (7.5–8.5 min) |
| Well Done | 160–165 | 500+ (8.5+ min) |

Table 2: Estimated cooking times

Just by looking at the results that our model gave us, we notice that when flipping steak, the temperature sees a dramatic increase if we flip anywhere after 5 mins: this may deviate from what most optimal cooking times on the Internet state, so in order to have the most accurate measurements for our chosen steak, we have our own recommendations (Table 2) for cooking times, as they give us the best idea of how to achieve different types of doneness by using different optimal cooking times.

## 4 Analysis

To validate the model's predictions, we compare its cooking time estimates against recommendations from a reputable grilling guide at 450°F.

| **Steak Doneness** | **Traeger Guide @ 450°F Recommended Cook Time** | **Heat Transfer Model Predicted Cook Time** |
|---|---|---|
| Rare (120–125°F) | 3–4 min per side ( 6–8 min total) | 360–420 s ( 6–7 min total) |
| Medium-Rare (130–135°F) | 5–7 min per side ( 10–14 min total) | 400–450 s ( 6.5–7.5 min total) |
| Medium (140–145°F) | 7–8 min per side ( 14–16 min total) | 420–480 s ( 7–8 min total) |
| Medium-Well (150–155°F) | 8–9 min per side ( 16–18 min total) | 450–500 s ( 7.5–8.5 min total) |
| Well-Done (160–165°F) | ≈ 10 min per side ( 20 min total) | 500+ s ( 8.5+ min total) |

Table 3: Comparison of recommended grilling times vs. model predictions.

Table 3 compares the model's predicted times with Traeger's steak cooking guidelines for a 1-inch steak on a 450°F grill. The table highlights that both the model and the real-world source agree qualitatively that thicker cuts and higher doneness levels require longer cooking. However, the model consistently predicts shorter total times than the grilling guide. For instance, the model suggests reaching Medium-Rare in about 6.5–7.5 minutes, whereas Traeger's guide recommends about 10–14 minutes total at 450°F for the same doneness. This discrepancy suggests that the model may overestimate the rate of heat transfer.

The model's output graph (Figure 3) shows expected trends but also reveals some unrealistic behavior. When the steak is placed on the 450°F grill, the model instantly assigns the surface temperature as 450°F, creating a sharp temperature gradient. In reality, surface heating occurs over time due to contact resistance. The model's immediate temperature jump leads to faster predicted cooking times than seen in real-world guides.

The model also neglects moisture evaporation and protein denaturation, which slow real cooking by absorbing heat [6]. These physical processes cause temperature plateaus in real steaks but are absent in the model. Similarly, the Maillard reaction, which browns the steak's surface, alters heat penetration but is not included [7]. Chemical reactions occurring in the meat, such as collagen breakdown and protein transformations further impacting cooking times [8].

Overall, the model captures the general shape of steak heating but oversimplifies cooking dynamics. A more realistic approach could include contact heat transfer effects, evaporative cooling, and chemical reaction energy sinks. These additions would refine the model and bring its predictions closer to empirical data.

## References


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