Combinatorial Optimization for Holding/Selling Stocks

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March 2025

1 Introduction and Context

Financial markets are well-recognized to be highly unpredictable and influenced by numerous factors. While analyzing the time-series data for asset can provide helpful insights into predicting the behavior of the prices and other metrics associated with the asset, no model can make predictions about its future metrics with absolute certainty. A specific example, and an integral component of asset management is analyzing the performance of an asset and deciding whether to hold or sell it. Instead of basing our decision on predicting the future performance of an individual asset, we can make our choices based on the relative performance of all the assets in our portfolio, which is the approach we explore in this paper.

In our original project, we focused on constructing an optimal portfolio by selecting the best assets to invest in while balancing risk and return constraints. However, investment decisions do not stop at initial allocation. Investors must continuously evaluate whether to hold or sell assets to maximize future returns. To improve our model, we are extending it to collectively analyze the assets in our portfolio after a specific period, and determine whether each asset should be held for another period or sold. To achieve this, we treat asset selection as a knapsack problem, where:

- Each asset is an item with associated return (profit) and risk (cost).
- The total capital budget acts as a constraint.
- Risk is measured using Mean Absolute Deviation (MAD) of returns.
- The objective is to maximize expected future return while keeping total risk within acceptable levels.

The problem is framed as a combinatorial optimization task, where we decide whether to hold or sell a subset of assets based on their recent performance, as measured by the metrics mentioned earlier. The model does not account for the possibility of re-buying an asset once it is sold. As a result, each iteration of the model operates independently, without relying on any assumptions from previous iterations.

Additionally, the model does not address the reallocation of capital after selling assets. It focuses solely on providing a decision framework for whether to hold or sell based on a comparative analysis of asset performance. The reallocation of funds obtained from selling assets can be treated as a separate portfolio allocation problem, considering both new potential assets and assets that were previously held. This reallocation process was the focus of our first paper.

2 Assumptions and Constraints

Throughout this section, we will be using the following notation quite frequently.

Description	**Symbol**	**Dimensions**
Opening price of stock i	o_i	CAD$
Closing price of stock i	c_i	CAD$
Mean Absolute Deviation for stock i over n days	α_i	CAD$
Average Return over n days	R_i	CAD$
Exponential Moving Average for 12 days	$E_{i_{12}}$	CAD$
Exponential Moving Average for 26 days	$E_{i_{26}}$	CAD$
Difference in EMAs for 12 days and 26 days	E_i	CAD$
Risk parameter	γ	1
Score parameter	ϵ	1
Asset number	i	1
Final Deterministic Score	D_i	1
Number of days	n	$Days$

We start off with some assumptions to simplify our model:

1. The portfolio consists of a fixed number of assets at the beginning of the evaluation period.
2. Each asset's risk level is given using Mean Absolute Deviation (MAD) of returns over the past n days.
3. The future return and risk of each asset are considered independent of other assets, meaning no direct correlation between asset behaviors is accounted for in this model.
4. Assets can only be held or sold and so short positions are not allowed.
5. The model does not account for transaction fees, taxes, or liquidity constraints when selling assets.
6. The model evaluates asset performance at the end of a n-day period, and makes a binary decision at that point.
7. The model does not account for re-buying and asset, or re-allocating the amount obtained after selling an asset.

3 Building the model

The decision to hold or sell a stock is influenced by innumerable factors, and encompassing all of these factors into a single model is inefficient as well as infeasible. Instead, for our model, we focus on assessing three different aspect of stock data to calculate a score based on which we choose to hold or sell a stock from a collection of stocks:

- The recent performance of the stock
- The volatility/risk associated with the stock
- The overall trend of the stock

To quantify all of the above aspects, we consider the 60 most recent closing and opening prices of the stock. A 60-day window is considered to find a balance between capturing short-term trends and recognizing the overall long-term trend. A window shorter than 60 days might overfit noisy data and might not be able to recognize the general trend of the stock price, whereas a larger window might dilute the recent trends.

Performance of the stock

To quantify the performance of the stock, we calculated the average daily return over the past 60 days.

$$R_i = \frac{1}{60}\sum_{j=1}^{60} \frac{\text{Closing price on day } j - \text{Opening price on day } j}{\text{Opening price on day } j}$$

Using this metric to drive our decision will help eradicate stocks with consistent negative performance, and retain the stock rewarding us with consistent returns. It helps capture a short-term momentum of the stock and serves as an effective, and intuitive way to guide our hold or sell decision.

The volatility/risk associated with the stock

While a major driving force behind our decision to hold or sell is the recent returns of the stock, we also need to consider the magnitude of fluctuations of the stock prices while making our decision in order to ensure a degree of certainty. In our model, we use Mean Absolute Deviation to measure the risk associated with the stock.

$$\text{MAD} = \frac{1}{N}\sum_{i=1}^{N} |\text{Return on day i} - \text{Mean return over the past 60 days}|$$

$$\text{Return on day } i = \frac{\text{Closing price on day } i - \text{Opening price on day } i}{\text{Opening price on day } i}$$

Mean Absolute Deviation (MAD) presents a simple way to capture the volatility of the stock without being influenced by outliers. Incorporating MAD into our model can help assess uncertainty associated with the returns of a stock, and ensure that the decision to hold or sell a stock is based not only on the return but also the price swings.

The overall trend of the stock

In addition to average returns and volatility of the stocks, we utilize the Exponential Moving Average (EMA) on the 60-day window to asses the directional momentum of the stock. EMA is a method of averaging over windows in a way that puts higher weight on the most recent data points or events; this makes it more responsive and receptive to any significant short-term price movement. It is recursively defined as:

$$\text{EMA}_t = \alpha \cdot P_t + (1-\alpha) \cdot \text{EMA}_{t-1}$$

where, $\alpha = \frac{2}{n+1}$, P_t is the price at time t, and EMA_{t-1} is the previous EMA.

In order to asses the momentum, we compute EMA over two different window sizes:

- EMA(12): A short-term EMA calculated using a window of 12 data points, which is sensitive to sudden fluctuations.
- EMA(26): A longer-term EMA calculated using a window of 26 data points, which reflects the broader trend.

The difference between these two EMAs serves as a momentum indicator by breaking down into two cases ($E_i = E_{i_{12}} - E_{i_{26}}$ for stock i):

- Bullish Takeover ($\text{EMA}_{diff} > 0$): When the shorter term EMA overtakes the longer term EMA, this indicates a positive momentum, which suggest that the recent price changes are stronger than the long term trend and favors a hold decision.
- Bearish Takeover ($\text{EMA}_{diff} < 0$): When the shorter term EMA falls below the longer term EMA, this indicates a negative momentum; this signals that the selling pressure is increasing and favors a sell decision.

Now that we have some the basic components to make our decision in place, we can use them to come up with our objective function. Some of the main ideas that we use to do this are:

- Each asset i has a binary decision variable x_i, where:

$$x_i = \begin{cases} 1, & \text{if the asset is held for another period} \\ 0, & \text{if the asset is sold} \end{cases}$$

- The average return (R_i) of the stock i over n days. ($R_i = \sum_j r_j = \sum_j \frac{c_j - o_j}{o_j}$) where $j \in \{1, 2, ..., n\}$
- The Mean Absolute Deviation (α_i) of the returns over n days.
  We calculate it using: $\alpha_i = \frac{1}{n}\sum_j |$ r_j - M(r)$|$ where $j \in \{1, 2, ..., n\}$
- We calculate the 12-day exponential moving average ($E_{i_{12}}$) and the 26-day exponential moving average ($E_{i_{26}}$). If $E_{i_{12}} \geq E_{i_{26}}$, then the asset shows positive momentum, and we assign the difference $E_i = E_{i_{12}} - E_{i_{26}}$ as its momentum score.

We use these to come up with a final score to assign to each stock: $D_i = 0.4R_i - 0.3\alpha_i + 0.3E_i$. The weightage associated with each component of the score is subjective and purely based on preference. If an individual prefers ensuring that the asset that are held have consistent positive returns, then they can increases the weightage for R_i. Similarly, if an individual prefers selling an asset that is highly volatile, they can increase the value associated with α_i.

Objective Function:

$$\max \sum_{i=1}^{5} x_i D_i$$

where x_i is the binary decision variable and D_i the final score (weight) assigned to each stock.

Constraints:

- The minimum maximization should be set greater than 0, ensuring that if all the stocks are performing poorly, none are selected. This prevents the model from being forced to hold the stock with the least negative performance, effectively leading to selling all assets when their performance is below a certain threshold.

$$\sum_{i=1}^{5} x_i D_i \geq \epsilon$$

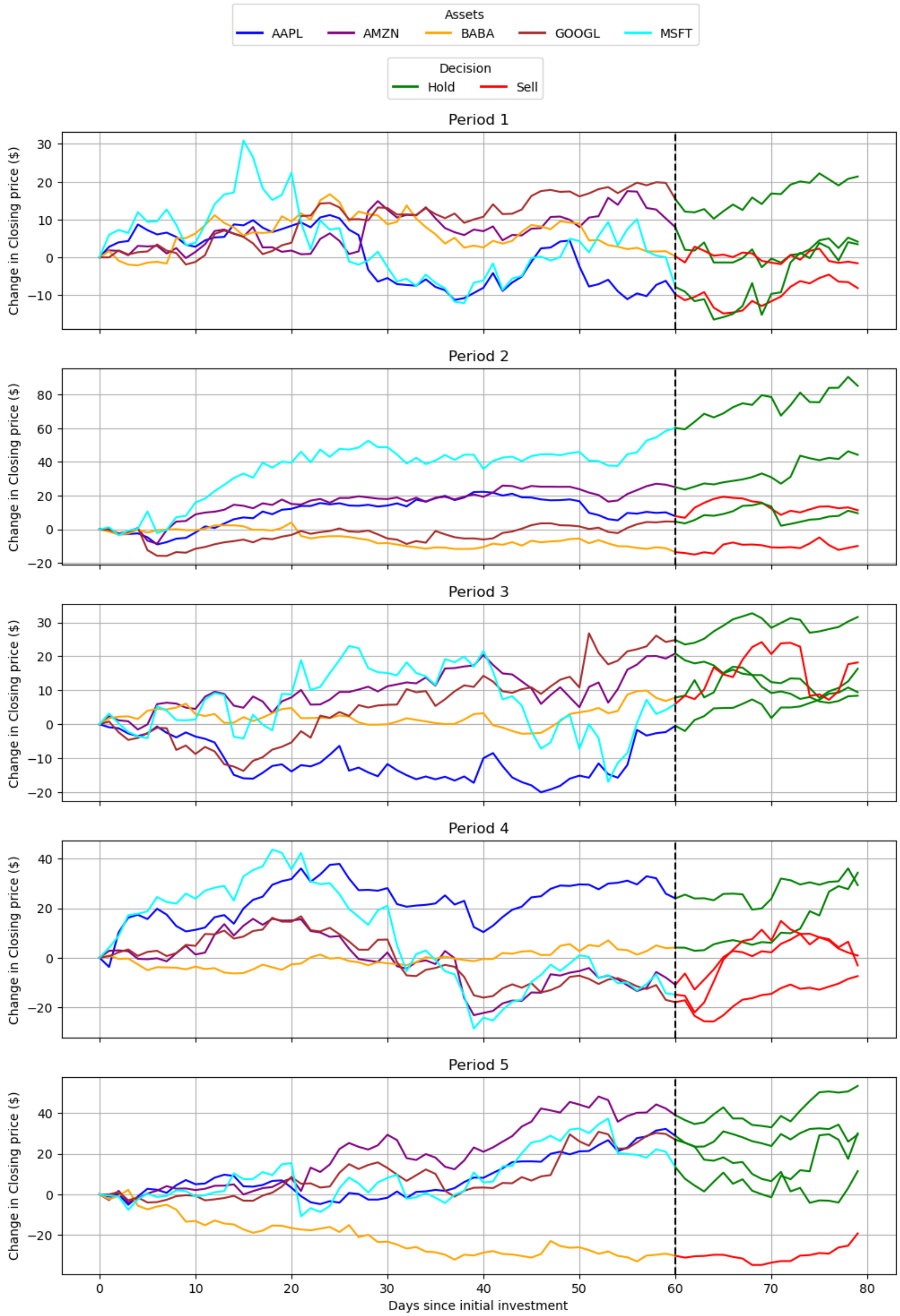
- The sum of risk terms cannot exceed the gamma value

$$\sum_{i=1}^{5} \alpha_i \leq \gamma$$

Solving the model:

The problem formulated above can be considered as a knapsack problem, where we choose a combination of stocks to hold based on the constraints mentioned above. The first constraint forms the basis for the qualification of a combination of stock to be selected; it gives the minimum threshold that the sum of these stocks should cross in order to be selected. The second constraint can be interpreted as the total allowable budget, i.e. the total risk allowable across all the selected stocks. For this report, we choose $\epsilon = 0.5$ and $\gamma = 0.025$. A detailed analysis and explanation of the choice of these hyper-parameter is presented in the analysis section. The knapsack problem is solved in python using dynamic programming, and the Pandas and Numpy libraries.

4 Reporting Results



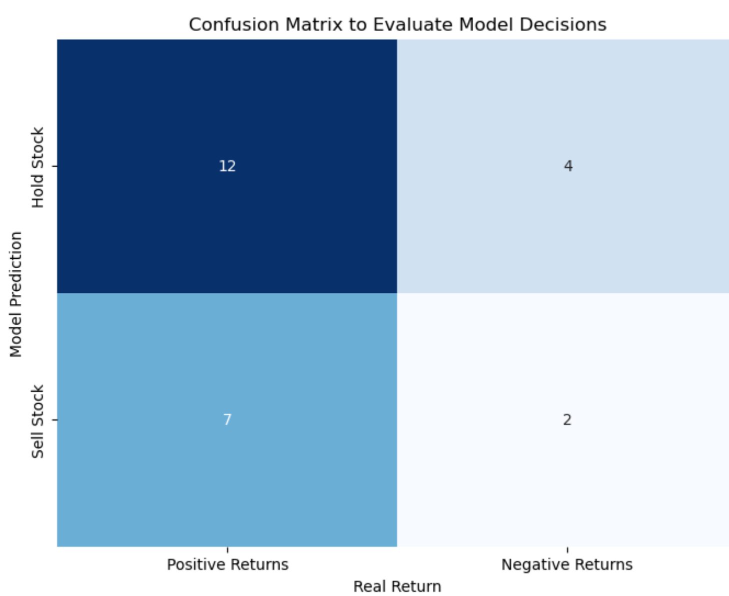
Each plot above shows the performance of the five stocks over a period of approximately 3 months. The variable that is being tracked in each of the sub-plot is the difference between the closing price of the stock on a particular day and the closing price of the stock on the day when it was bought. The dashed-line indicated the time-point when a decision to hold or sell was made for each of the stocks. The data from the period before the dashed line was used to derive our decision to buy or sell a stock. The data after the dashed line demonstrates the performance of the stock for 20 days after it was held of sold. Each stock is color coded based on the legend at the top of the figures. After the dashed line, the stocks which were held are shown in green, and the stocks which were sold are showing in red.

Our model's decision-making process is shown by green ("Hold") and red ("Sell") lines, followed expected patterns:

- Stocks with a steady upward trend (like MSFT and AMZN) were more likely to be classified as "Hold."
- Stocks that showed flat or negative momentum (like BABA and some instances of GOOGL) were often classified as "Sell."

Looking at the results across all five periods, we noticed some clear trends in how stock prices moved and how our model classified them as "Hold" or "Sell." MSFT showed the strongest upward trend, especially in Periods 2 and 4, where it gained over 40 dollars. The model consistently classified MSFT as "Hold," which makes sense given its strong momentum. BABA struggled the most, staying flat or declining across multiple periods, with some drops exceeding 20 dollars. The model marked it as "Sell," which aligned well with its negative trend. AAPL and GOOGL had mixed performance, fluctuating without a clear long-term direction. We saw that sometimes they were classified as "Hold" and other times "Sell," which makes sense given their inconsistency. AMZN had moderate gains, doing better than BABA but lacking the strong momentum of MSFT. The model's classification of AMZN varied depending on its short-term trend.

To evaluate how well these hold/sell decisions aligned with actual stock performance, we constructed a confusion matrix summarizing the model's predictions across all five periods.



Out of all decisions made, 12 were true positives, where the model correctly held stocks that increased in value. However, there were also 7 false negatives, meaning the model sold stocks that ultimately performed well — a missed opportunity for gains. False positives (4 cases) occurred when the model held onto declining stocks, while true negatives (2 cases) reflected correct sell decisions.

When compared to a perfect model — which would have all predictions along the diagonal (i.e., all correct) — our model shows room for improvement, particularly in avoiding false negatives. The performance metrics reinforce this:

- **Accuracy:** 56%
- **Precision:** 75%
- **Recall:** 63.16%
- **F1 Score:** 68.57%

An accuracy of 56% suggests that just over half of the model's predictions correctly aligned with actual outcomes, which indicates moderate effectiveness. The precision value of 75% shows that when the model chose to hold a stock, it was often a sound decision, reflecting a conservative approach that avoids unnecessary risk. However, the recall of 63.16% reveals that the model did miss a number of profitable opportunities by incorrectly selling stocks that later increased in value. The F1 score, sitting at 68.57%, reflects a reasonable balance between precision and recall, but also highlights that there is room for improvement in optimizing the model to both capture more winners and reduce false positives. Compared to a perfect model—where all decisions would be correct—these metrics show that while the model makes informed decisions, enhancements are needed to improve its consistency and profitability.

A pattern we consistently noticed was that "Sell" decisions followed periods of stagnation or decline in the prior 60-day window. This confirms that the model prioritizes recent trends when making decisions. When we look at each period more closely, we can see:

- **Period 1:** Stocks moved up and down moderately. AAPL and AMZN had positive trends, and sell decisions were fairly balanced.
- **Period 2:** MSFT had one of its strongest runs, jumping over 40 dollars. The model held onto MSFT while selling weaker stocks.
- **Period 3:** AAPL's price took a noticeable dip, leading to a mix of "Hold" and "Sell" decisions.
- **Period 4:** This was a more volatile period, especially for MSFT and GOOGL. The model correctly held onto MSFT but decided to sell some weaker stocks.
- **Period 5:** The model made consistent choices based on past observations. BABA kept declining, reinforcing sell decisions.

One key constraint in our model is that the sum of the final D_i values of all selected assets must be greater than 0. This means the model isn't forced to select stocks just for the sake of it and if all the assets are performing poorly, the model has the option to sell everything rather than hold onto the "least bad" stock. We see this come into play in cases where the model opted to sell multiple assets instead of making risky holds.

Additionally, our model can only classify stocks as "Hold" or "Sell." Since we are using a combinatorial selection approach, it does not allow for rebalancing funds into different stocks. This means that when we sell a stock, the money is effectively removed instead of being reinvested into a stronger asset. For example, in Period 2, the model correctly held onto MSFT, but it didn't reinvest capital from sold assets into better-performing stocks. If we had a rebalancing mechanism, we could have redirected funds from BABA or GOOGL into MSFT or AMZN, increasing returns. Going forward, adding a rebalancing strategy would make sure that capital is always working efficiently, rather than just being removed from the portfolio. We also see room for improving risk weighting, especially for stocks like AAPL and GOOGL, where the model's decisions were less certain.

Overall, the model performed well in identifying strong stocks and filtering out weak ones. It correctly held onto MSFT and AMZN while making smart sell decisions for BABA and some weaker performers. At the same time, we noticed that the lack of re-balancing meant missed opportunities to reallocate funds into better-performing stocks. By incorporating re-balancing and risk-adjusted weighting, we can improve decision accuracy and portfolio efficiency in future versions.

5 Analyzing the Model

The sensitivity analysis heatmap below explores how the model's portfolio allocation responds to changes in two parameters: the Min Score Threshold and the Max Mean Absolute Deviation (MAD) Constraint.

From the plot, we observe that increasing the MAD constraint beyond 0.015 generally leads to higher asset values, particularly when the Min Score Threshold is below 0.95. This suggests that greater flexibility in volatility tolerance (higher MAD) allows the model to hold potentially higher-return assets, provided that their score still exceeds a low threshold. It underscores a key trade-off: more restrictive thresholds improve risk control but may limit performance, while relaxed constraints improve returns but expose the portfolio to greater volatility. It also hints that our current fixed-parameter approach may not be optimal across all market conditions.

To address this issue, we can introduce mechanisms that adjust the decision weights rather than using fixed parameters. We could be optimizing these weights based on recent market conditions to enhance portfolio performance. Additionally, instead of evaluating stocks only at fixed monthly intervals, introducing a stop-loss mechanism would allow the model to react to sudden market shifts more effectively. For example, if a stock drops below a predefined threshold within the month, it could be flagged for immediate selling rather than waiting for the next evaluation period.

Another key limitation of our model is that it treats each stock independently, without considering correlations between assets. In reality, stocks often move together due to broader market trends and industry-specific factors. For example, AMZN and GOOGL are both technology stocks that often move in the same direction due to industry-wide trends. If the model decides to sell GOOGL while holding AMZN, it might overlook the fact that both stocks are affected by similar external factors hence leading to suboptimal diversification. Incorporating a correlation matrix would allow the model to account for these relationships and make more well-rounded investment decisions.

To address these limitations, a correlation matrix could be used to better model inter-dependencies between stocks.

The correlation matrix reveals strong positive relationships among AMZN, GOOGL, MSFT, and even APPL, with correlation coefficients exceeding 0.92. This indicates that these stocks tend to move together due to shared industry trends, making their price fluctuations highly dependent on one another. Consequently, our model often assigns similar hold or sell decisions to these stocks, further reinforcing the idea of momentum-driven selections. However, this also introduces a risk of over-concentration, as investing heavily in highly correlated stocks reduces diversification and exposes the portfolio to industry-specific downturns.

Conversely, BABA exhibits weak or even negative correlation with the other assets, particularly MSFT (-0.2), suggesting that including BABA could serve as a hedge against broader market trends. This highlights a key limitation in our model: BABA is rarely included in our portfolio, suggesting a potential lack of diversification. While BABA's weak or negative correlation with the other stocks makes it a valuable hedge, our model primarily prioritizes recent performance and momentum, often leading to its exclusion. As a result, the portfolio becomes heavily concentrated in highly correlated tech stocks, increasing exposure to sector-specific risks.

While our current model does not directly incorporate correlation into decision-making, doing so could improve risk-adjusted returns by ensuring that assets are selected not just based on individual performance metrics but also on their contribution to overall portfolio diversification. Introducing a diversification constraint or adjusting decision weights to account for correlation could help ensure that the model selects assets not only based on performance but also on their contribution to overall portfolio stability.

Implementing these enhancements would bring our model better for real-world trading hence improving both our risk management and decision-making accuracy.

References

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