



# Artificial Intelligence

**Systematic (Uninformed) Search**

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# Agenda

- § Basic Terminology and Concepts
- § Modeling a Search Problem
- § Systematic (Uninformed, Blind) Search Strategies
  - Tree Search vs. Graph Search
  - Breadth-First search
  - Depth-First search
  - Depth-Limited search
  - Iterative Deepening search
  - Uniform Cost search

## Recommended Reading

### § AIMA Chapter 3: Solving Problems by Searching

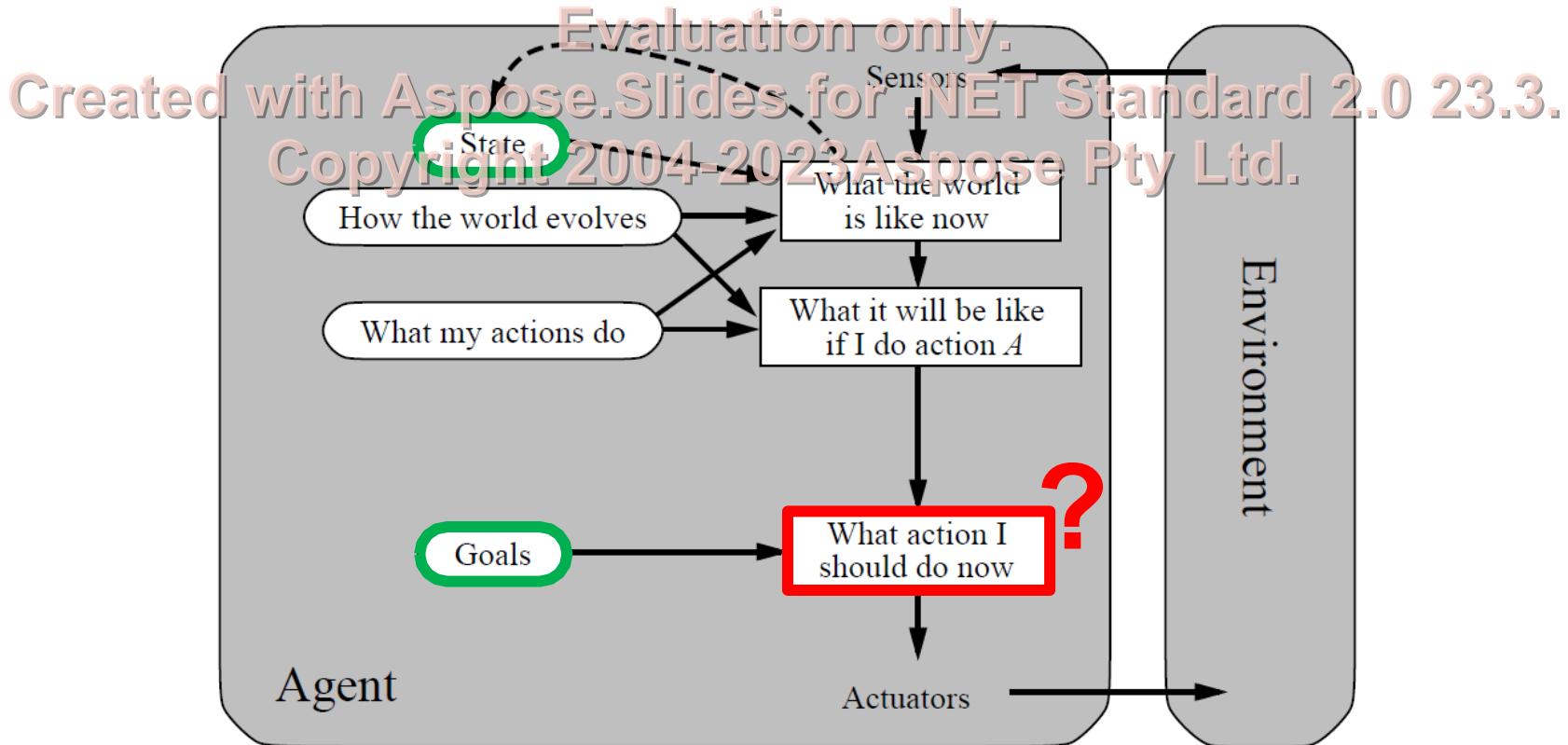
- 3.1 Problem Solving Agents
- 3.2 Example Problems
- 3.3 Searching for Solutions *Evaluation only.*
- 3.4 Uninformed Search Strategies, the following subchapters:
  - 3.4.1 Breadth-first search
  - 3.4.2 Uniform-cost search
  - 3.4.3 Depth-first search
  - 3.4.4 Depth-limited search
  - 3.4.5 Iterative deepening depth-first search
  - 3.4.7 Comparing uninformed search strategies

# Basic Terminology and Concepts

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# How can a Goal-based Agent reach a Goal?

- § Agent perceives the world being in different states
- Initial state: the current state of the world
  - Goal(s): a future state of the world (desirable for the agent)



# Discrete State-Based Search Problems

## § Discrete

- Finite number of states & actions

## § Single agent

- Do not consider action-based changes by other agents

## § Static

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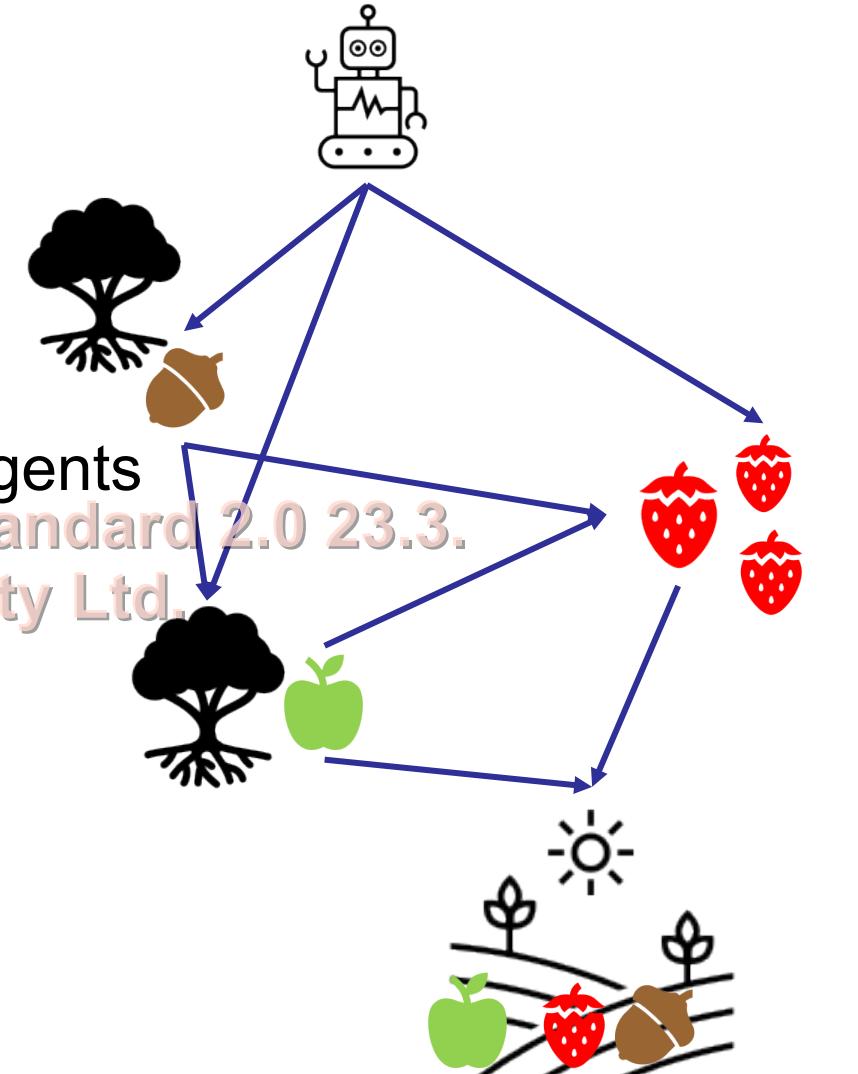
- World does not change while agent is deliberating

## § Observable

- Agent has access to relevant knowledge

## § Deterministic

- Each action has exactly one successor state



## Definition: State Space

A state space is a 4-tuple  $\Theta = (S, A, T, I)$  where:

- $S$  is a finite set of states
- $A$  is a finite set of actions
- $T \subseteq S \times A \times S$  is the transition relation
- $I \in S$  is the initial state

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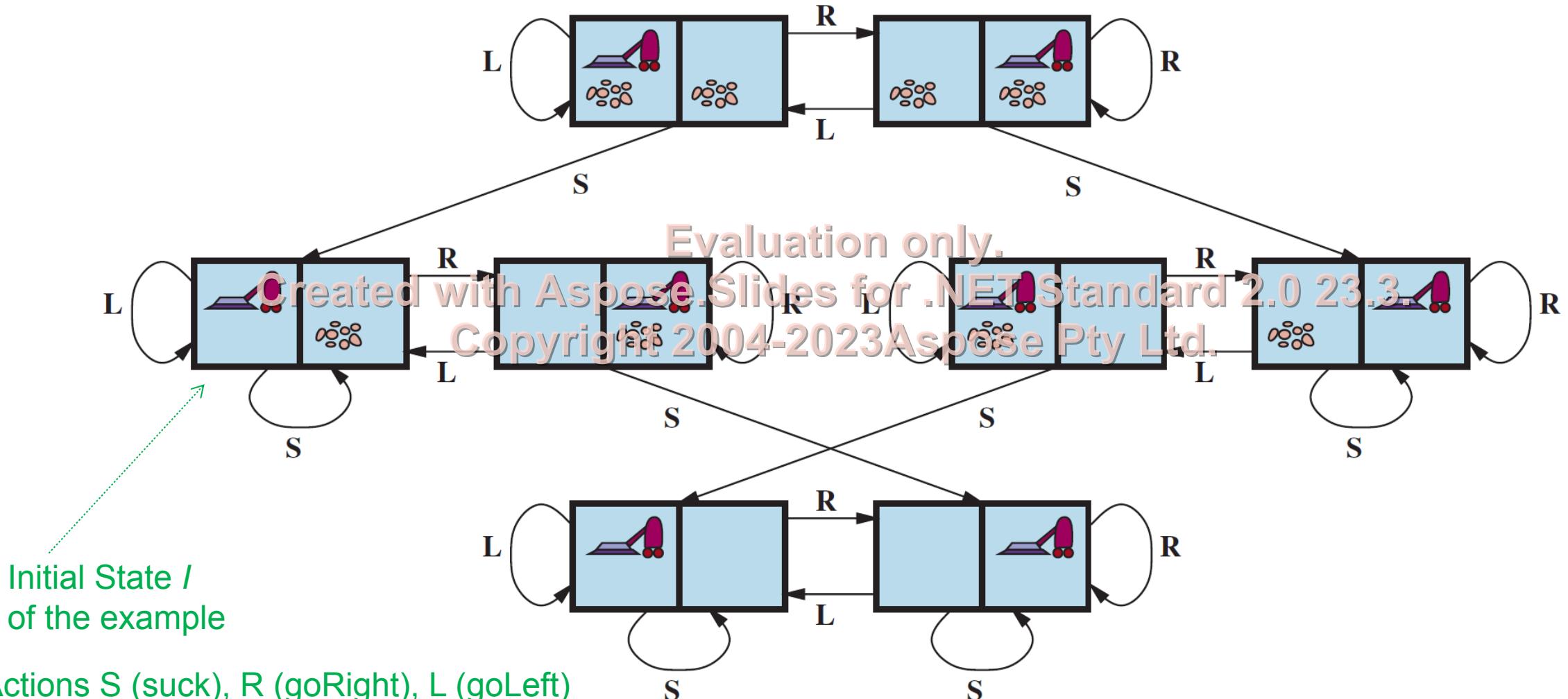
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Note:

- We require that  $T$  is deterministic, i.e., for all  $s \in S$  and  $a \in A$ , there is at most one state  $s'$  such that  $(s, a, s') \in T$ . If such  $(s, a, s')$  exists, then  $a$  is applicable to  $s$
- We say that  $\Theta$  has the transition  $(s, a, s')$  if  $(s, a, s') \in T$ . We also write  $s \xrightarrow{a} s'$ , or  $s \rightarrow s'$  when not interested in  $a$
- The state space forms a directed graph in which the nodes are states and the links between nodes are actions



## Vacuum Cleaner World: Example of a State Space Graph



Initial State /  
of the example

Actions S (suck), R (goRight), L (goLeft)

8 possible states

Arrows annotated with actions show possible state transitions in the state space graph

## Definition: Search Problem

A *search problem* is a 6-tuple  $\Pi = (S, A, c, T, I, S^G)$  where:

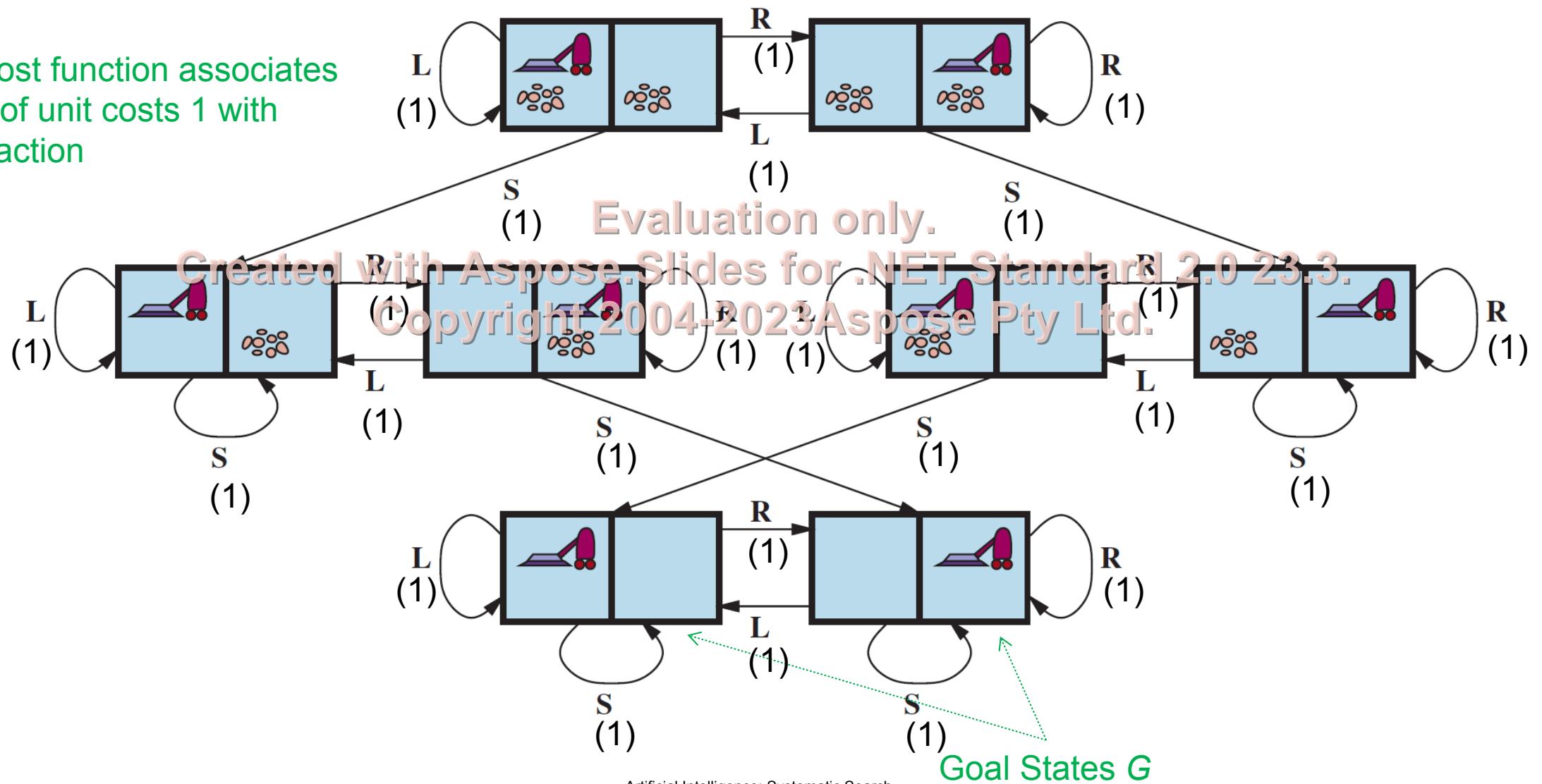
- $S, A, T, I$  are given through the state space
  - $c : S \times A \mapsto \mathbb{R}_0^+$  is the cost function
  - $S^G \subseteq S$  is the set of goal states
- Evaluation only.  
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We say that the search *problem* has *unit costs* if, for all  $a \in A$  and all  $s \in S$ ,  $c(s, a) = 1$ .



# Vacuum Cleaner World: Example of Cost Function and Goal State(s)

The cost function associates costs of unit costs 1 with each action



## Definition: Reachability of States

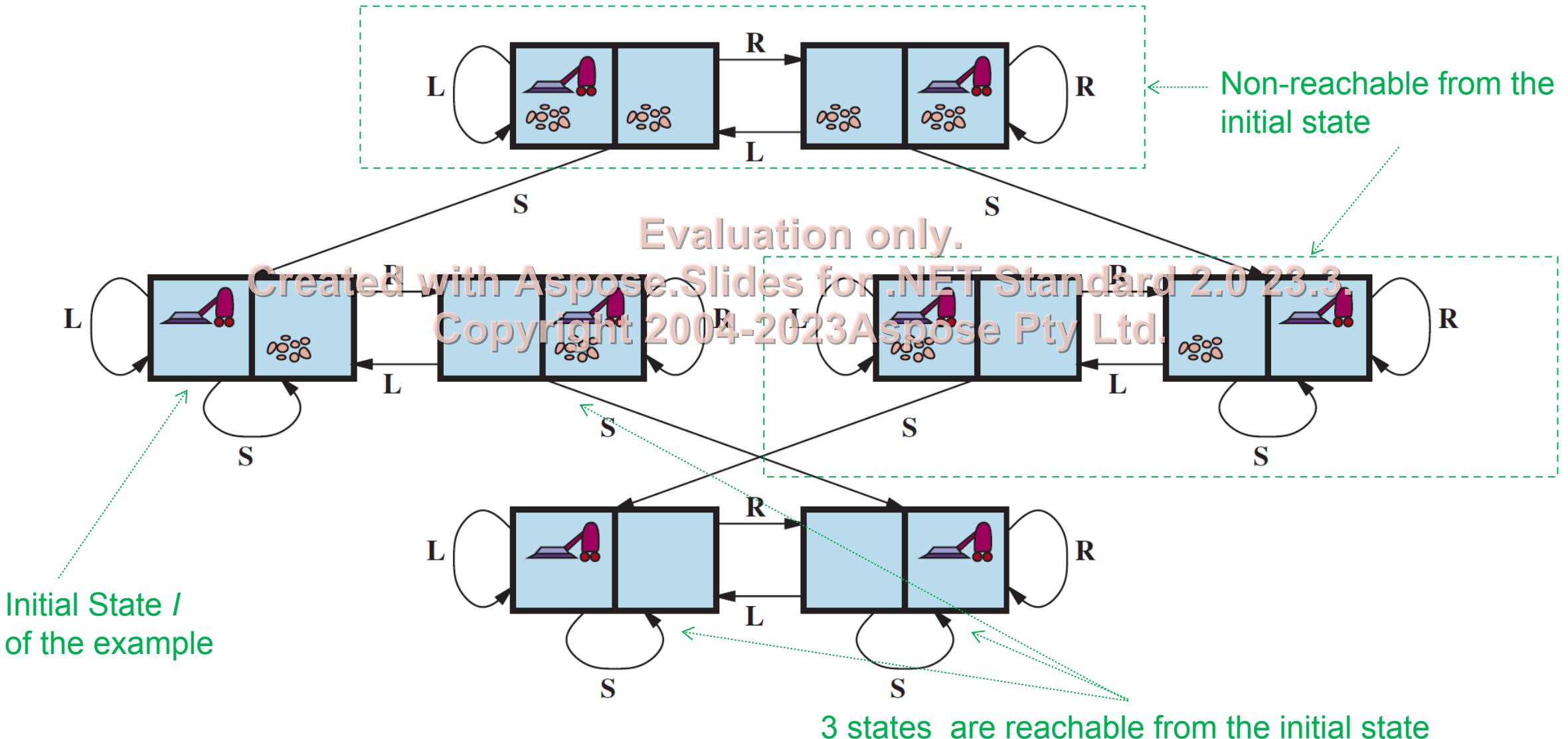
- $s'$  successor of  $s$  if  $s \rightarrow s'$ ;  $s$  predecessor of  $s'$  if  $s \rightarrow s'$
- $s'$  reachable from  $s$  if there exists a sequence of transitions:

$$s = s_0 \xrightarrow{a_1} s_1 \xrightarrow{a_2} \dots \xrightarrow{a_{n-1}} s_{n-1} \xrightarrow{a_n} s_n = s'$$

- $n = 0$  possible; then  $s = s'$  Evaluation only.
- $(a_1, \dots, a_n)$  is called (action) path from  $s$  to  $s'$
- $(s_0, \dots, s_n)$  is called (state) path from  $s$  to  $s'$
- The cost of this path is  $\sum_{i=1}^n c(s_{i-1}, a_i)$
- $s'$  is reachable (without reference state) means reachable from  $I$
- $s$  is solvable if some  $s' \in S^G$  is reachable from  $s$ ; else  $s$  is a dead end (trap state)



# Vacuum Cleaner World: Example of Reachable and Non-Reachable States



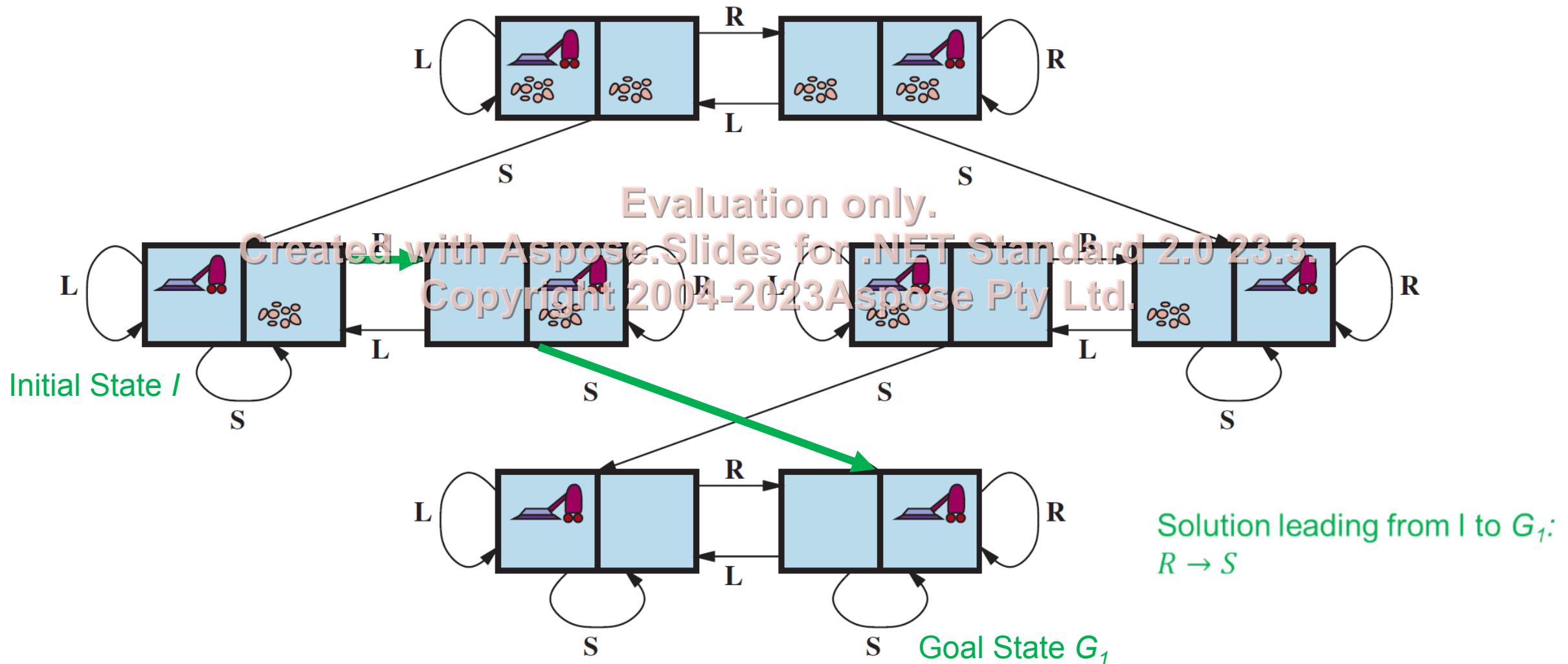
## Definition: (Optimal) Problem Solution

Let  $\Pi = (S, A, c, T, I, S^G)$  be a search problem, and let  $s \in S$

- A **solution** for  $s$  is an action path  $(a_1, a_2, \dots, a_n)$  from  $I$  to  $s$  **Evaluation only.**
- A solution for  $s$  is called a **solution for  $\Theta$**  and denoted by  $\theta$ .  
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- The **set of all solutions** for  $\Theta$  is denoted by  $S^\Theta$   
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- If a solution exists, then  $\Theta$  is **solvable**, otherwise  $\Theta$  is **unsolvable**
- A solution is **optimal** if its cost is minimal among all solutions for  $s$ . For a state  $s$ , the optimal cost  $c^*(s)$  is the cost of a cheapest path reaching  $s$



## Vacuum Cleaner World: Example of a Solution



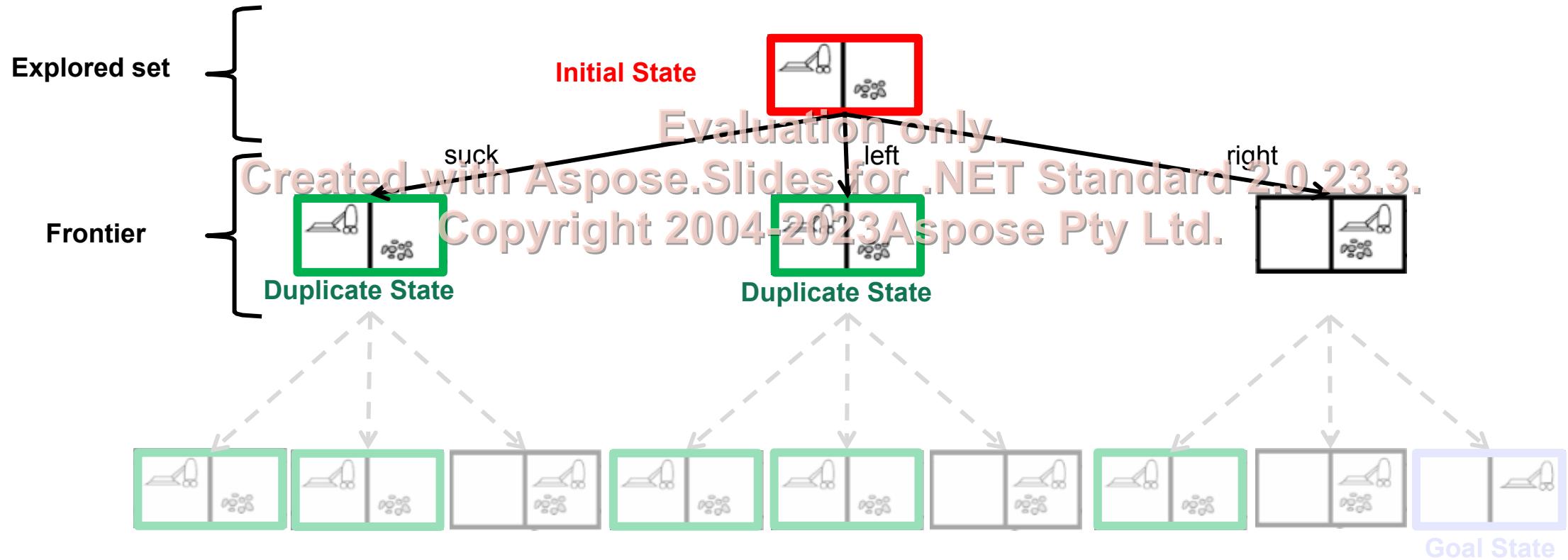
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**Modeling a Search Problem**  
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## Search Tree

- The **search node**  $n$  contains a state reached by the search, plus information about how it was reached.
- Generating all successors of a node, by applying all actions applicable to the node's state  $s$  is called **node expansion**. We also say the state  $s$  is **expanded**.
- The **search strategy** is the method for deciding which node is expanded next.
- The **frontier** is a set of nodes that are candidates for expansion, also called open list.
- The **explored set** is the set of all states which were already expanded (visited), also called closed list.
- A **duplicate (repeated) state** is a state, which is generated during expansion and which is already contained in the explored set. Duplicate states never contribute to an optimal solution.

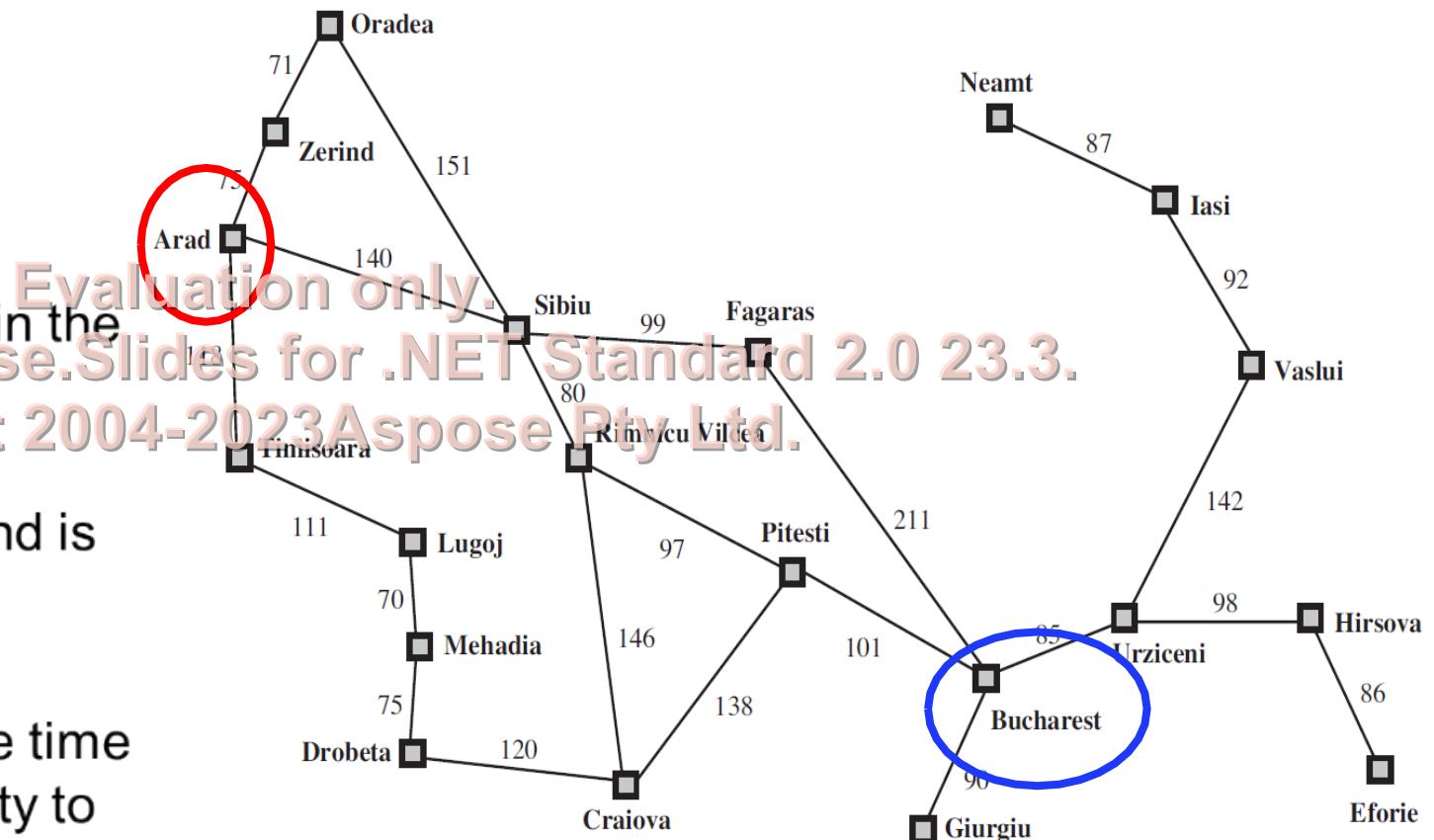


# Vacuum Cleaner World: Example Search Tree



## Romania Travel Example: State Space

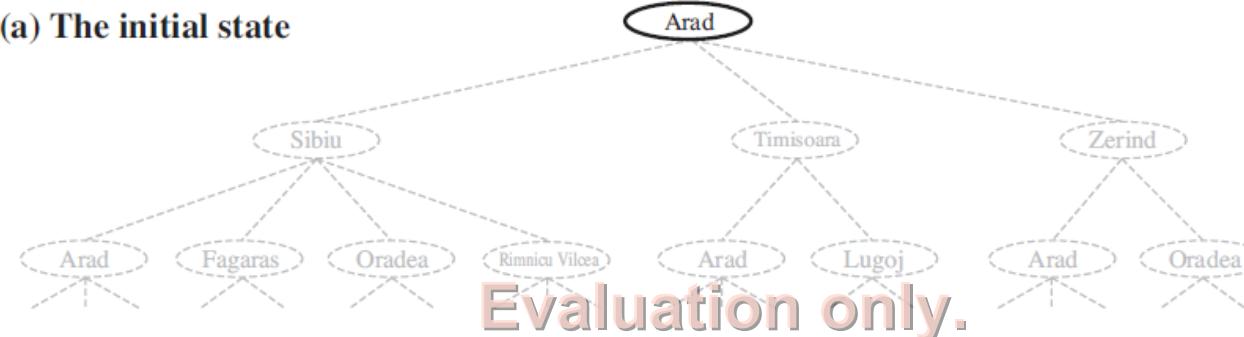
- States:  $\{In(Arad), In(Oradea), In(Zerind), In(Sibiu), \dots\}$
- Initial state:  $In(Arad)$
- Goal state:  $\{In(Bucharest)\}$
- Example actions: applicable actions in the state  $In(Arad)$  are  $\{Go(Sibiu), Go(Timisoara), Go(Zerind)\}$
- Example transition from Arad to Zerind is given by  $(In(Arad), Go(Zerind), In(Zerind))$
- The cost of each action describes the time which is needed to travel from one city to another



Russell, S., & Norvig, P. (2010). Figure 3.2

# Romania Travel Example: Search builds a Search Tree when expanding the State Space Graph

(a) The initial state



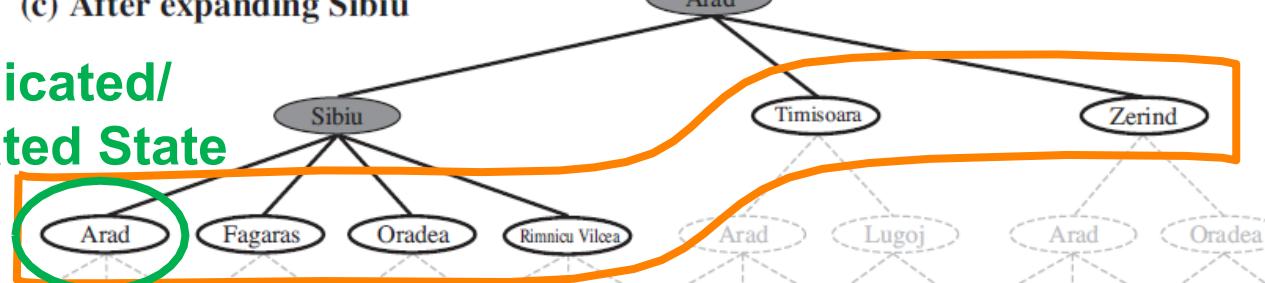
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(b) After expanding Arad



(c) After expanding Sibiu

Duplicated/  
Repeated State

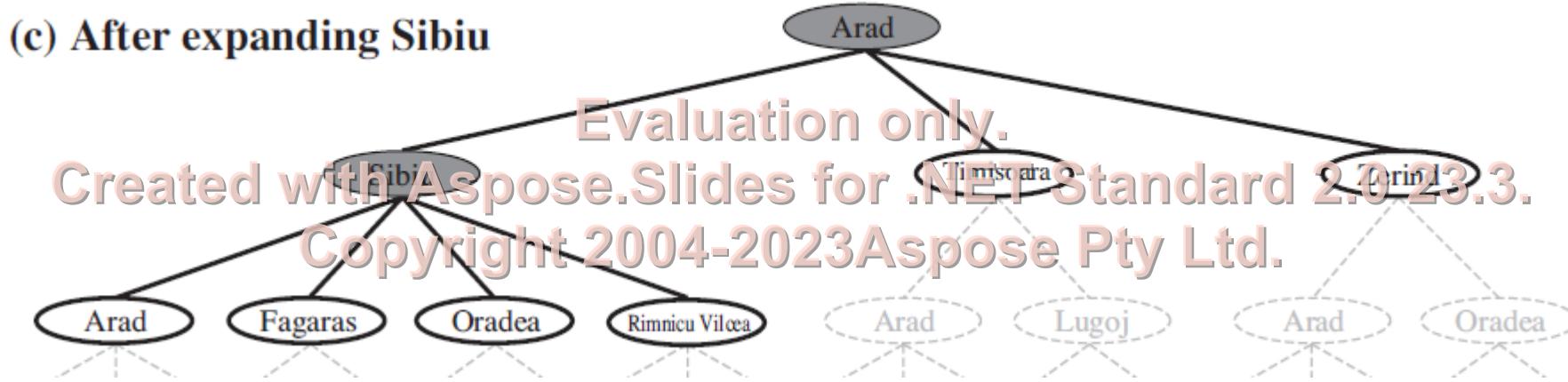


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Search tree = sequential description of the visiting order

## Romania Travel Example: Duplicate / Repeated States

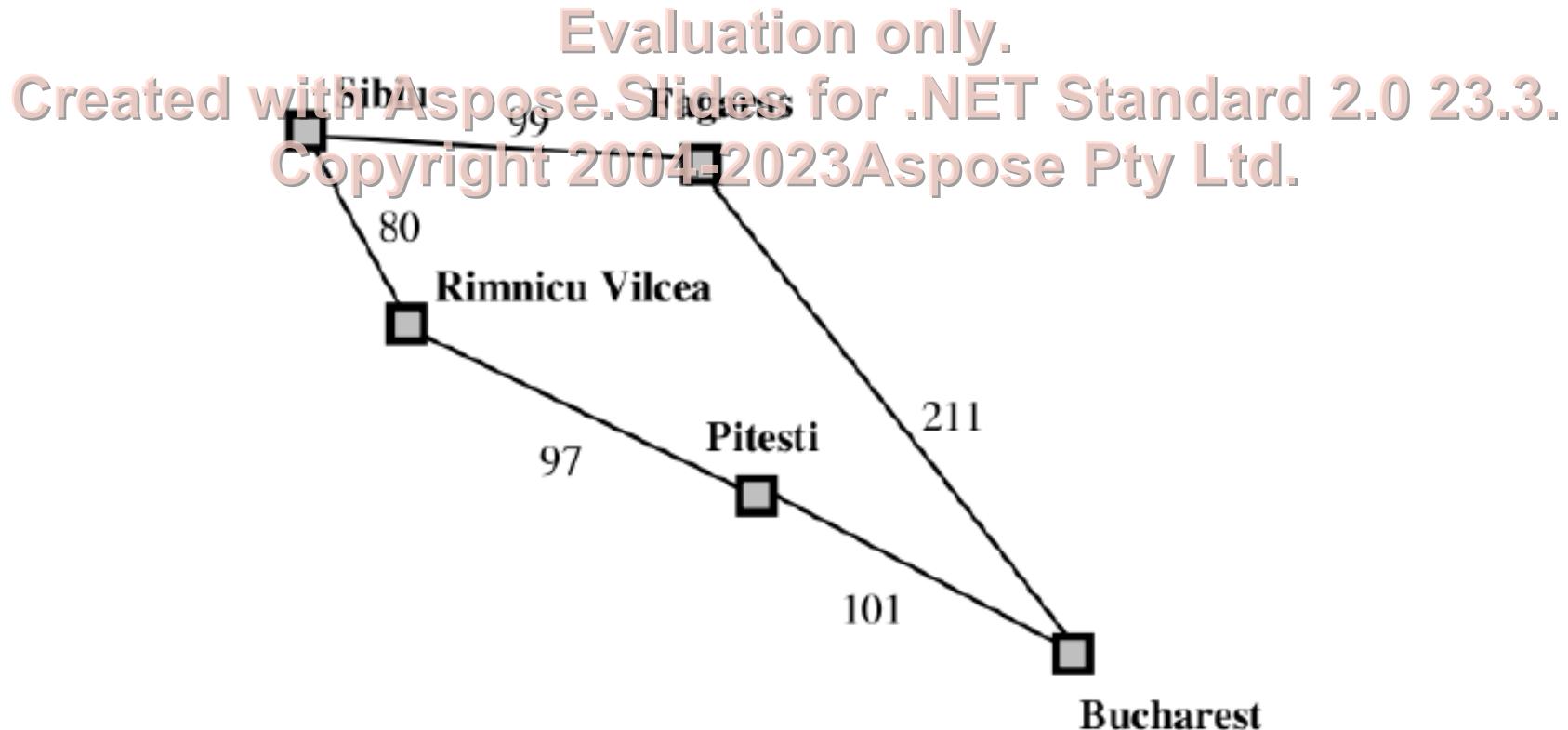
§ ... created by loopy paths in the state space graph



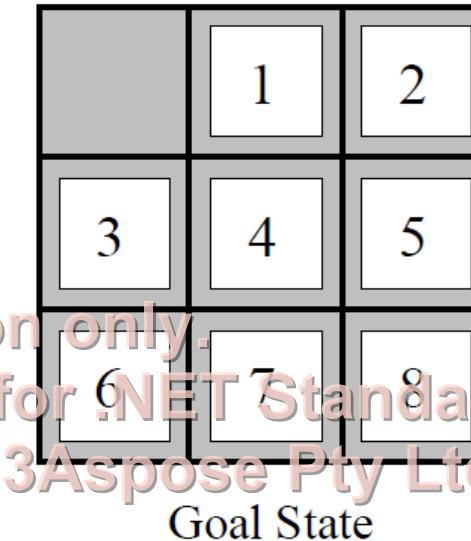
§ Loopy paths can never contribute to the optimal solution

## Romania Travel Example: Redundant Paths

- § Two possible paths from Sibiu to Bucharest
- § The route via Fagaras is a more costly way to get to Bucharest  
 $99 + 211 = 310$  vs.  $80 + 97 + 101 = 278$

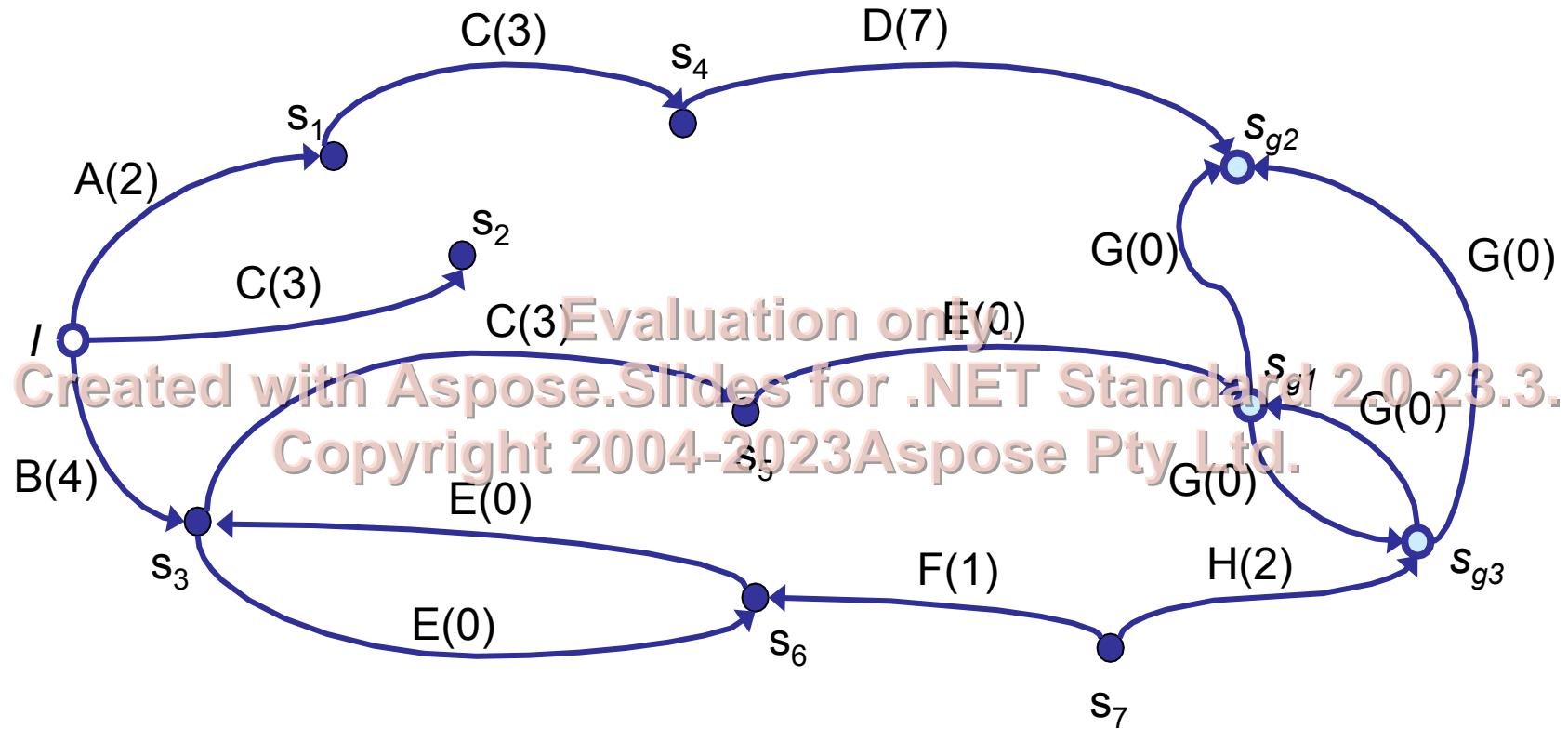


## The 8-Puzzle Example



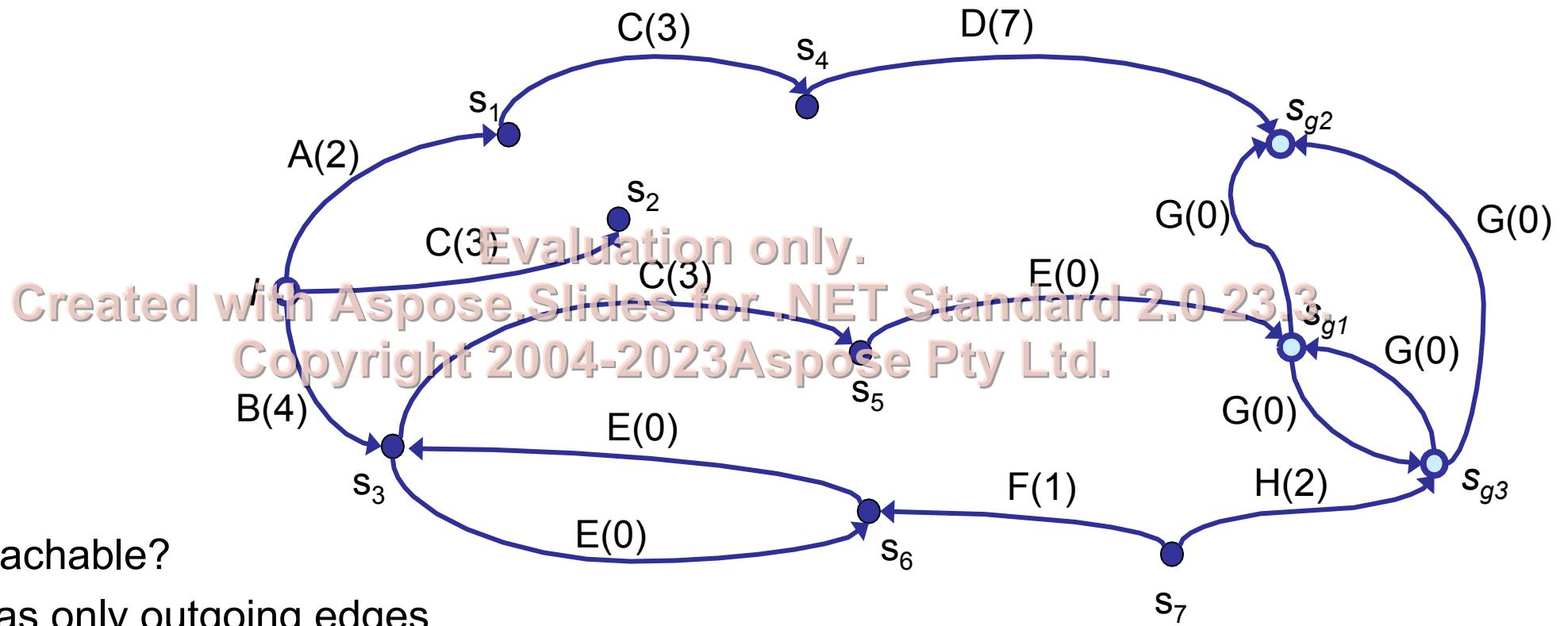
- § States: \_\_\_\_\_
- § Initial (Start) State: \_\_\_\_\_
- § Actions: \_\_\_\_\_
- § Goal states: \_\_\_\_\_
- § Path Costs: \_\_\_\_\_

## A State Space Graph with Costs



- Unit costs? No (see numbers in brackets)
- Actions applicable in initial state  $I$ :  $A, B, C$
- Deterministic  $T$ ? No (see action  $G$  in state  $s_{g1}$ )

## Reachable and Solvable States



§ All states reachable?

- No:  $s_7$  has only outgoing edges

§ All states solvable?

- No:  $s_2$  has no outgoing edges (dead end)

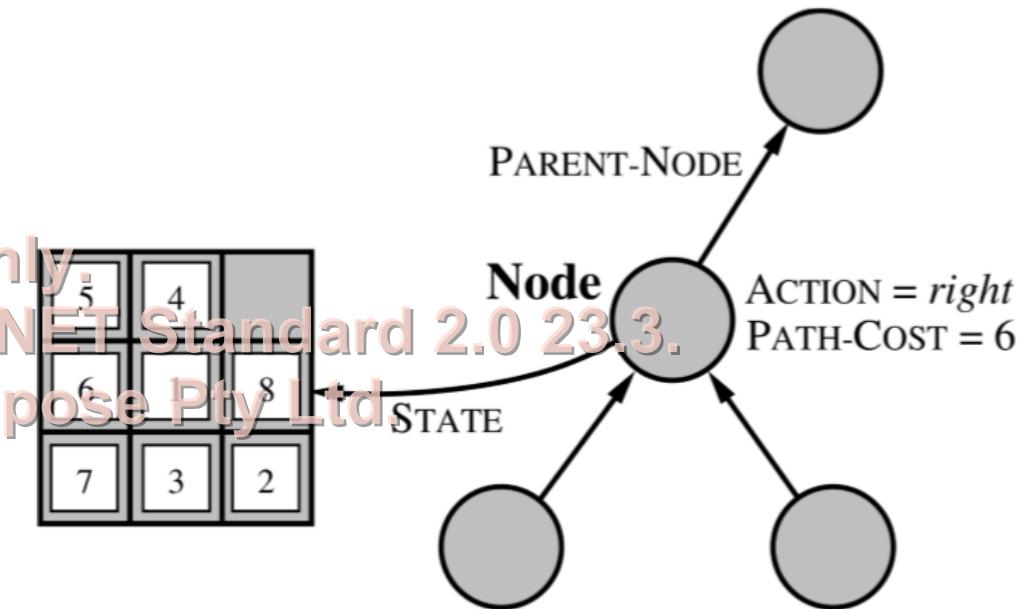
§ Optimal solutions?

$B - E^* - C - E - G^*$  with costs:  $4+0+3+0+0 = 7$

## Implementation – Data Structure of a Search Node

Let *Node* be a search node

- § *Node.STATE* is a state from the state space which the node *Node* contains
- § *Node.PARENT-NODE* is the node in the search tree whose node expansion generates the node *Node*
- § *Node.ACTION* is the action which was applied to the parent node to generate the node *Node*
- § *Node.PATH-COST* is the cost of the path from the initial state to the node *Node* (as indicated by the parent pointers)



## Implementation – Operations on the State Space

- § `problem.InitialState` returns the initial state of a problem
- § `GoalTest(Node.STATE)` returns a Boolean, “true” iff a state is a goal state, otherwise “false” *Evaluation only.*
- § `Cost(Node.ACTION)` returns the cost function value of the action
- § `Actions(Node.STATE)` returns the set of actions that are applicable to the state
- § `ChildState(Node.STATE,Node.ACTION)` returns the outcome state *childState* if the action is applicable to the state *state*

## Implementation – Operations on Search Nodes

Let *Node* be a search node

- § `Solution(Node)` returns the path to node *Node*. By back chaining over *Node.PARENT* and collecting *Node.ACTION* in each step
- § `ChildNode(Node.STATE,Node.ACTION)` generates the outcome *childNode* corresponding to the application of the action in the state

## Implementation – Operations on the Frontier

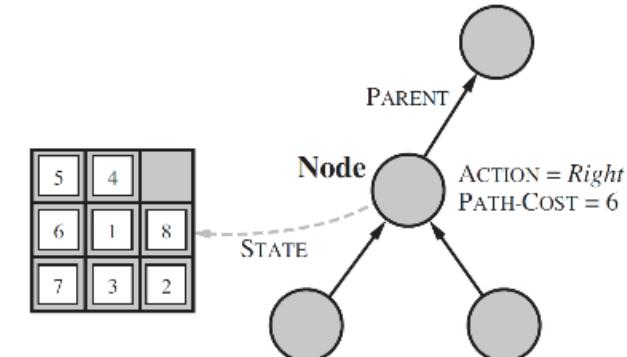
- §  $\text{Empty}(\text{frontier})$  returns true iff there are no more elements in the frontier
- §  $\text{Pop}(\text{frontier})$  returns the first node in the frontier and removes it from the list
- §  $\text{Insert}(\text{Node}, \text{frontier})$  adds a node to the frontier

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# Formulating Search Problems

## 1) Blackbox description

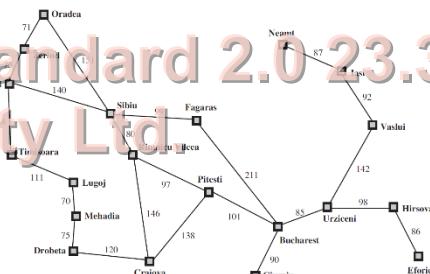
- Application programming interface (API) to construct the state space



## 2) Whitebox description

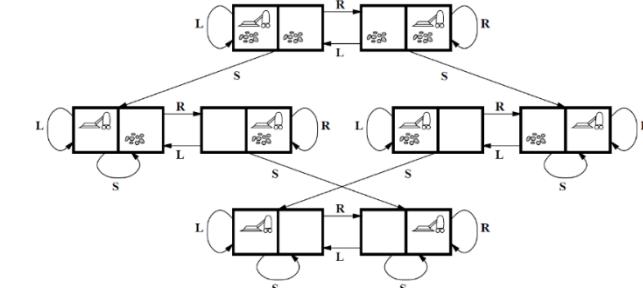
- Accessible, but compact representation of states, actions, goal test, ...

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## 3) Explicit description

- Explicit representation of all states in the state-space graph

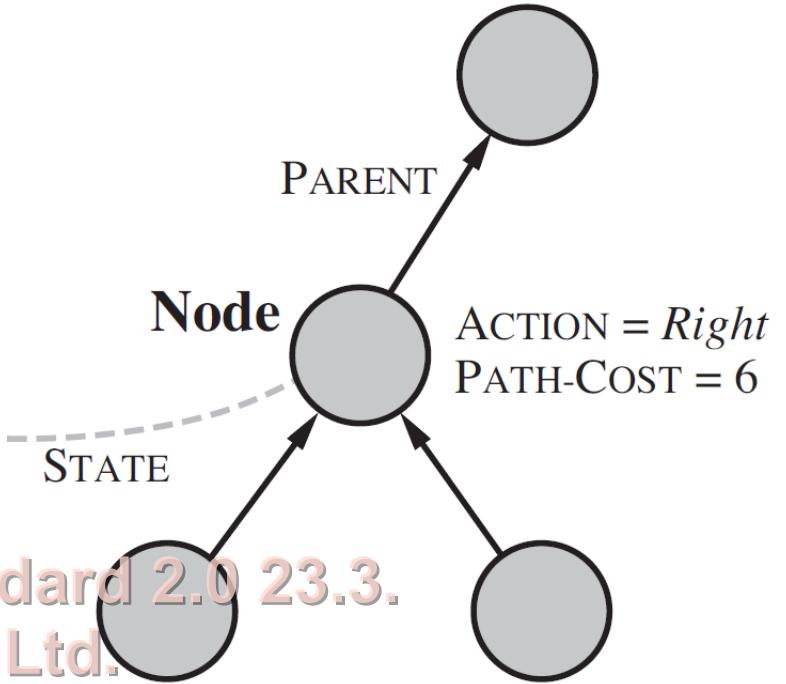
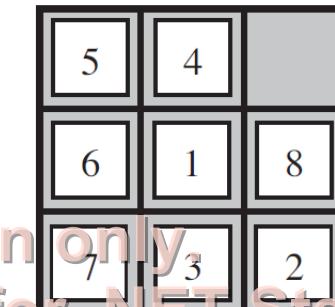


## Blackbox Description

Application programming interface (API) to construct the state space

- § *problem.InitialState* is given
- § *GoalTest(Node.STATE)* is given for all *states*
- § *Cost(Node.Action)* is given for all *actions*
- § *Actions(Node.STATE)* is given for all *states*
- § *ChildState(Node.STATE,Node.ACTION)* is given for all *states* and *actions*

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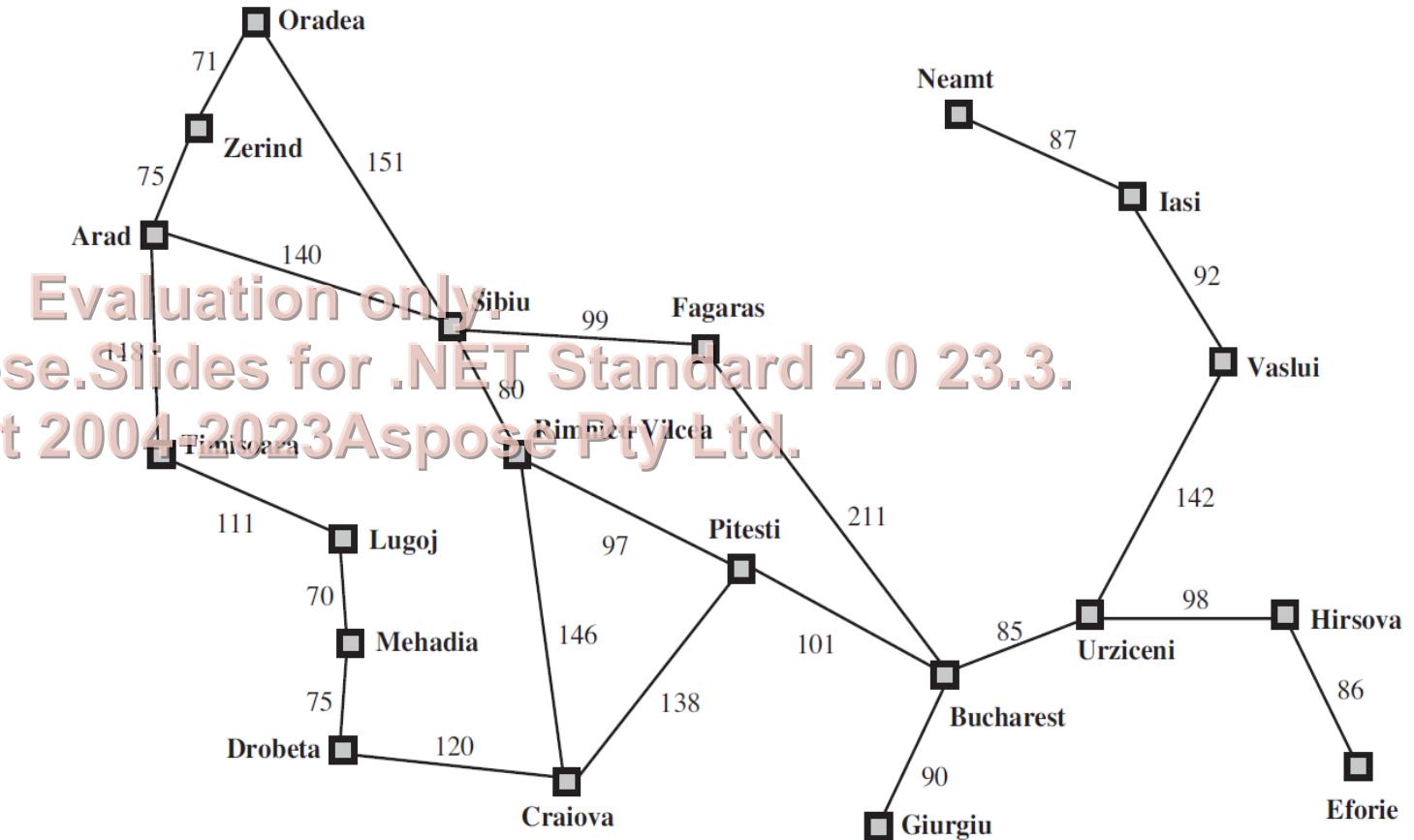


## Whitebox Description: Romania Travel Example

§ Given:

- State space (in some compact encoding, not as a state space graph)
- Goal test
- Cost function

§ The only difference to the blackbox description is that all states in the state space are directly accessible and not only via the *childstate* function



# Whitebox Description: Vacuum Cleaner Agent

## § State space:

- States: 2 positions, dirt or no dirt  
→ 8 states
- Actions: Left (L),  
Right (R),  
Suck (S)

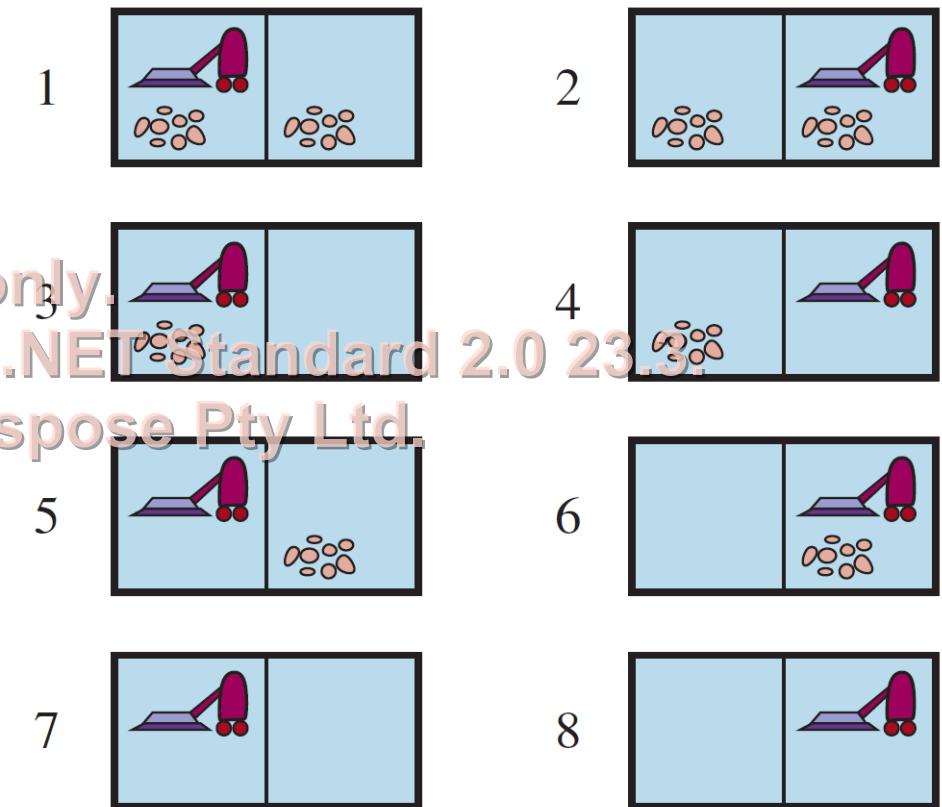
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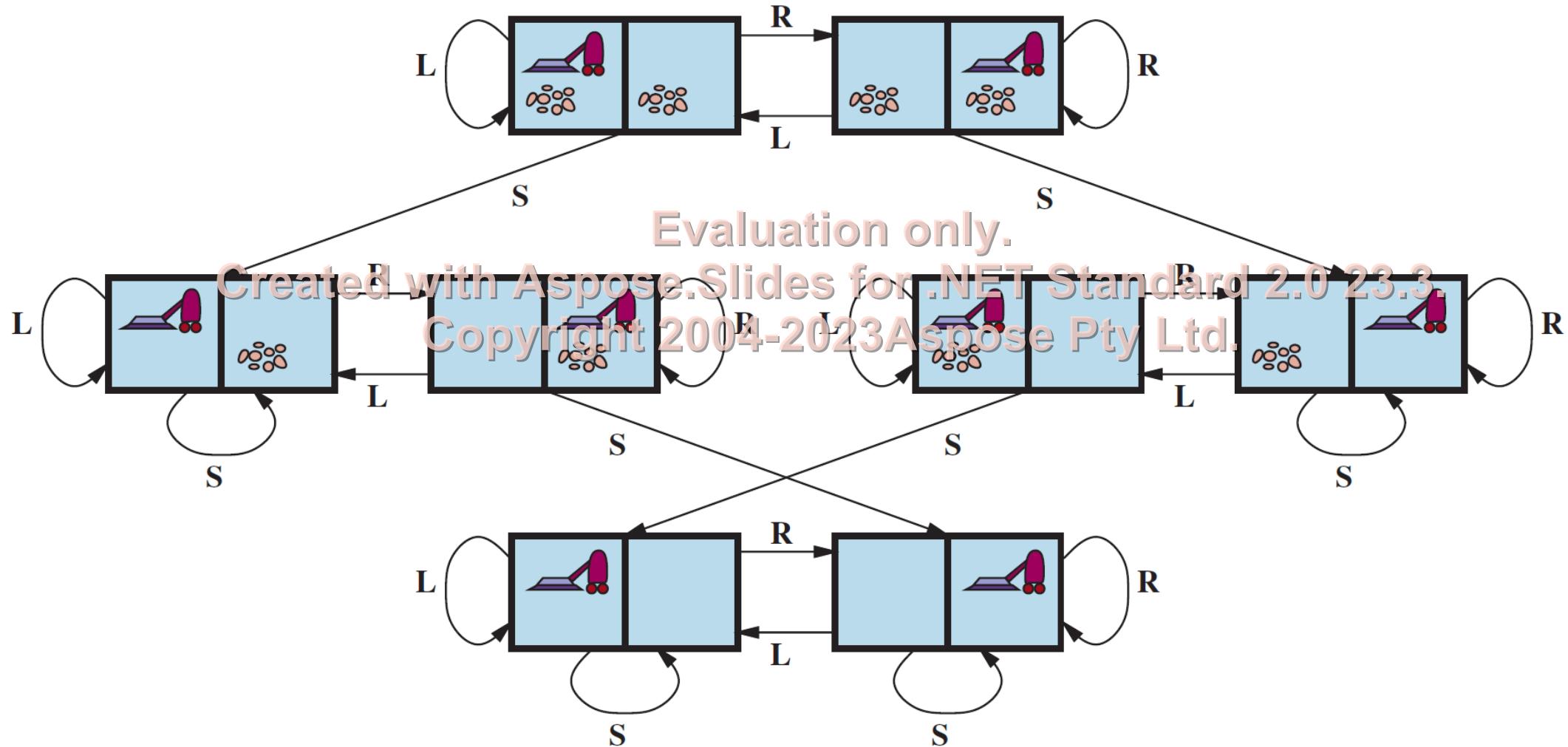
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## § Cost function: Unit costs

## § Goal states: no dirt in the rooms



# Explicit Description: State Space Graph of Vacuum Cleaning Agent



# Systematic (Uninformed, ~~Evaluation only~~, Blind) Search Strategies

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# Tree Search vs. Graph Search

## § Tree search

- We assume that the search space has tree structure
- When performing tree search, we do not need to remember visited states, because a state can only be visited via exactly one path from the initial state
- Consequently, when performing tree search on a graph, we will not know whether we generate repeated states

## § Graph search

- Remember visited states (keep an explored set)
- Use *duplicate elimination*: If a generated state is in the explored set, skip it, otherwise explore it



# Comparing Tree Search with Graph Search

```
function TREE-SEARCH(problem) returns a solution or failure
    initialize the frontier using the initial state of problem
    loop do
        if the frontier is empty then return failure
        choose a node from the frontier and remove it from the frontier
        if the node contains a goal state then return the corresponding solution
        expand the chosen node, adding the resulting nodes to the frontier
```

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```
function GRAPH-SEARCH(problem) returns a solution or failure
    initialize the frontier using the initial state of problem
    initialize the explored set to be empty
    loop do
        if the frontier is empty then return failure
        choose a node from the frontier and remove it from the frontier
        if the node contains a goal state then return the corresponding solution
        add the node to the explored set
        expand the chosen node, adding the resulting nodes to the frontier
        only if not in the frontier or explored set
```



# Criteria for Evaluating Search Algorithms

## Guarantees:

- **Soundness:** Is an action path returned a solution to the problem?
- **Completeness:** Is the strategy guaranteed to find a solution when there is one?
- **Optimality:** Are the returned solutions ~~Evaluation only~~ be optimal?

**Complexity:** ~~Created with Aspose.Slides for .NET Standard 2.0 23.3.~~

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- **Time Complexity:** How long does it take to find a solution? (Measured in node expansions)
  - **Space Complexity:** How much memory does the search require? (Measured in nodes stored in memory)

## Typical state space features governing complexity:

- **Branching factor  $b$ :** How many successors does each state have?
- **Goal depth  $d$ :** The number of actions required to reach the shallowest goal state.



# Overview on Discussed Search Strategies

§ **No** information on the length or cost of a path to the solution, only state space and goal states are given

- 1) Breadth-first search
- 2) Depth-first search *Evaluation only.*
- 3) Depth-limited search *Created with Aspose.Slides for .NET Standard 2.0 23.3.  
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- 4) Iterative deepening search

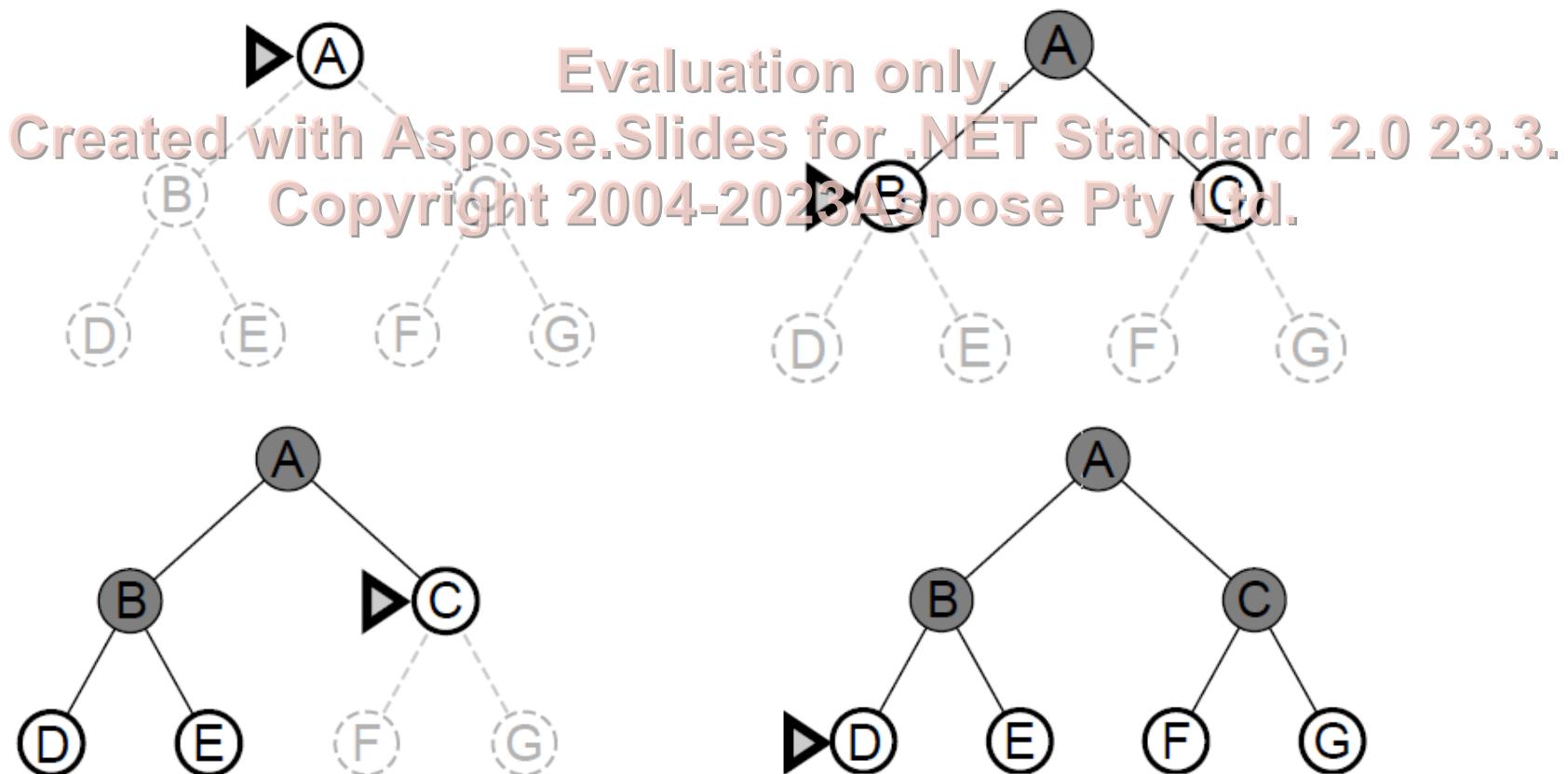
§ **Only current path costs** influence search

- 5) Uniform cost search



## (1) Breadth-First Search (BFS)

- § Nodes are expanded in the order they are produced
  - Frontier is a **FIFO** queue



# Breadth-First Search (BFS)

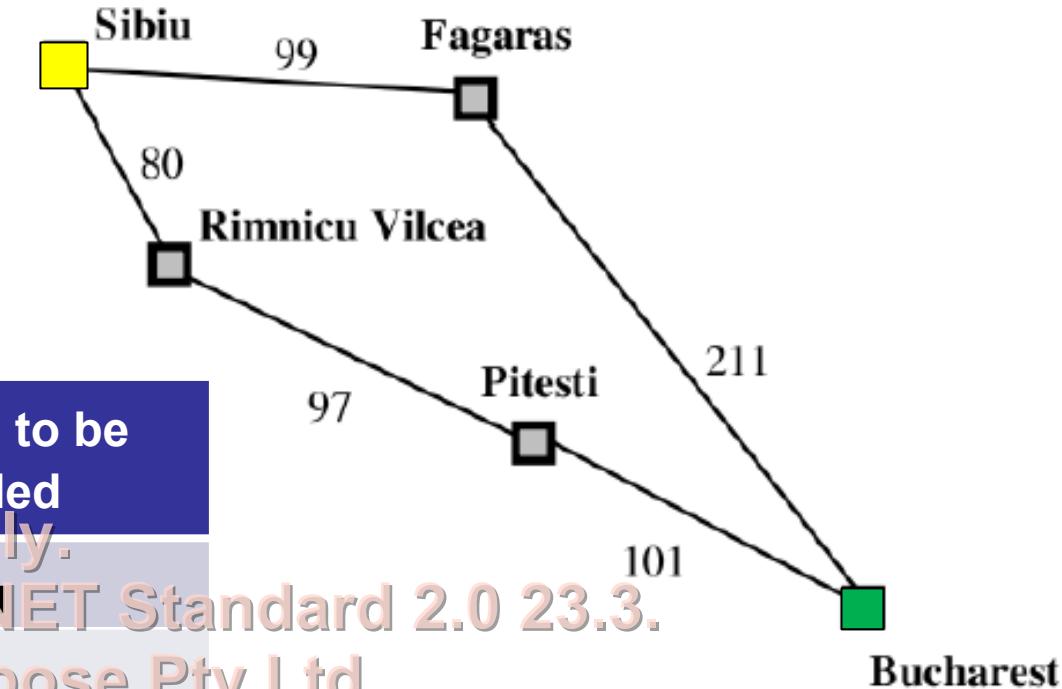
```
function BREADTH-FIRST-SEARCH(problem) returns a solution or failure
    node ← a node with node.State = problem.InitialState
    PathCost = 0
    if GoalTest(node.State) then return Solution(node)
    frontier ← a FIFO queue with node as the only element
    explored ← an empty set
    loop do
        if EMPTY?(frontier) then return failure
        node ← POP(frontier) /* chooses the shallowest node in frontier*/
        add node.State to explored
        for each action in Actions(node.State) do
            child ← ChildNode(problem, node, action)
            if child.State is not in explored or frontier then
                if GoalTest(child.State) then return Solution(child)
                frontier ← Insert(child, frontier)
```

- § Duplicate check against explored set and frontier: No need to re-generate a state already in the (current) last layer
- § Goal test at node-generation time (as opposed to node-expansion time): The search strategy already knows that this is a shortest path so it can just stop



## Breadth-First Search (BFS) in Romania Travel Example

Search Step	Frontier	Explored set	Next node to be expanded
0	Sibiu		Evaluation only.
1	Rimnicu Vilcea, Fagaras	Sibiu	Rimnicu Vilcea
2	Fagaras, Pitesti	Sibiu, Rimnicu Vilcea	Fagaras



When expanding Fagaras, the goal test on Bucharest returns True, so the solution returned is:

**Sibiu → Fagaras → Bucharest**

## Properties of Breadth-First Search (BFS)

- § Always finds the shallowest goal state first
  - § Completeness is obvious
    - Incomplete for search spaces with infinite branching (non-finite action space)
  - § The solution is optimal, provided every action has identical, non-negative (unit) costs
- Evaluation only.  
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- ∅ The Romania travel example has non-unit action costs
  - ∅ The solution found is sub-optimal

## Time and Space Complexity of Breadth-First Search (BFS)

- Exponential time complexity
  - Let  $b$  be the maximum branching factor and  $d$  the depth of a solution path
  - Maximum number of node generations is

Evaluation only

$$1 + b + b^2 + b^3 + \dots + b^d = \sum_{n=0}^d b^n \in O(b^d)$$

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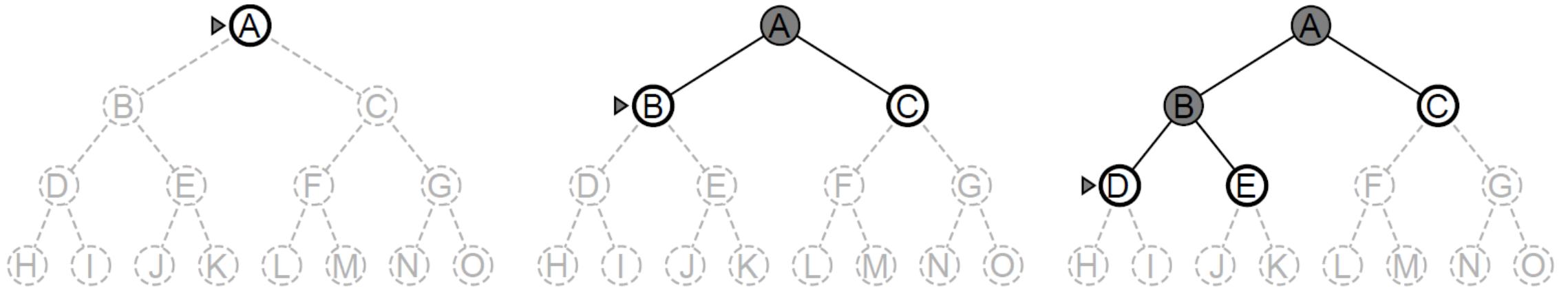
- Exponential space complexity
  - Every node generated is kept in memory:  $\sum_{n=0}^d b^n$
  - Space needed for the frontier is:  $O(b^d)$
  - Space needed for the explored set:  $O(b^{d-1})$

## (2) Depth-First Search (DFS)

- § Always expand the deepest (most recent) node in the frontier
  - Frontier is a **LIFO** queue
  - If a node has no children, search backs up to the next deepest node that has unexplored children

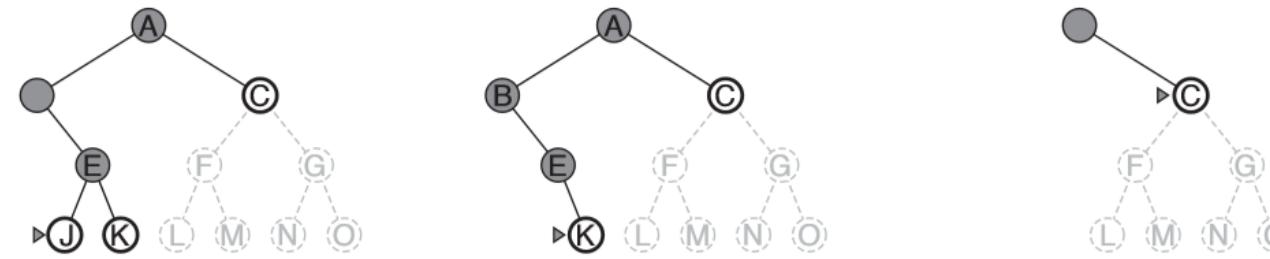
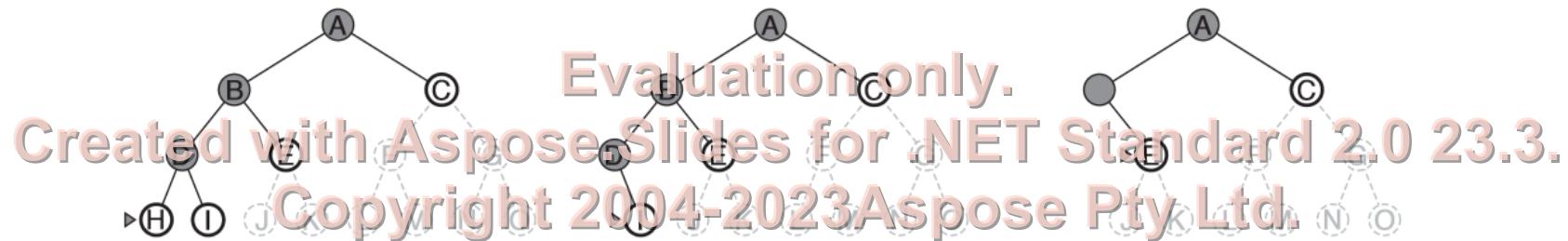
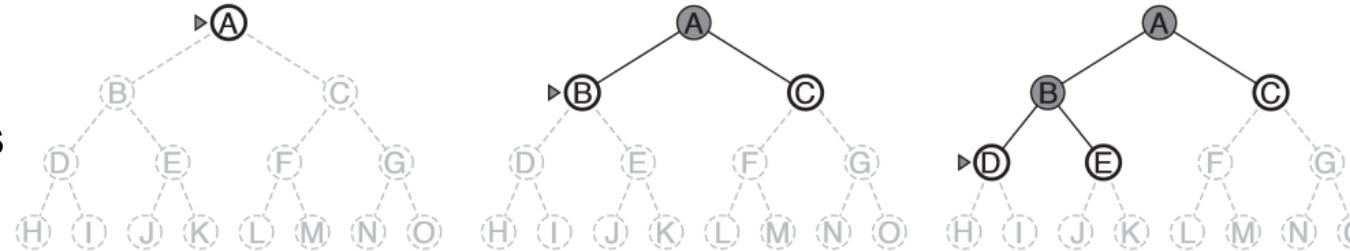
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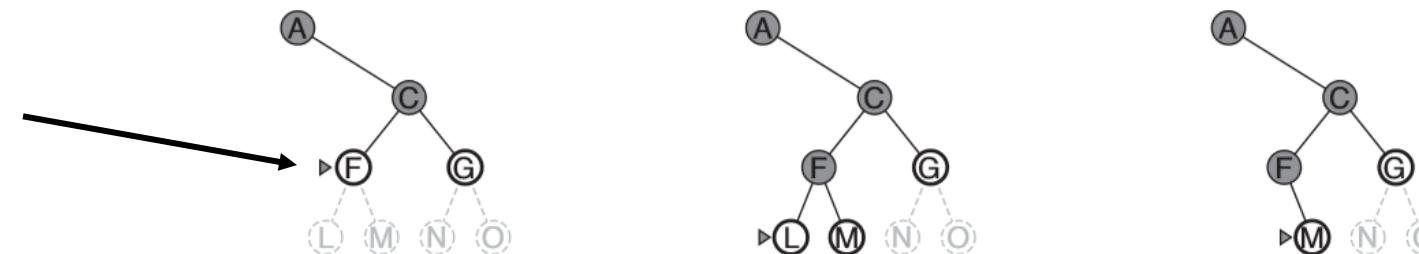


## Example of Depth-First Search (DFS)

Nodes at depth 3 have no successors and M is the only goal node



The pseudo code of the algorithm executes the goal test when expanding F and can thus terminate with a solution 2 steps earlier



Russell & Norvig, Fig. 3.16.

# Depth-First Search (DFS) Algorithm

```
function DEPTH-FIRST-SEARCH(problem) returns a solution or failure
  node ← a node with node.State = problem.InitialState
  PathCost = 0
  if GoalTest(node.State) then return Solution(node)
  frontier ← a LIFO queue with node as the only element
  explored ← an empty set
  loop do
    if EMPTY?(frontier) then return failure
    node ← POP(frontier) /* chooses the deepest node in frontier */
    add node.State to explored
    for each action in Actions(node.State) do
      child ← ChildNode(problem, node, action)
      if child.State is not in explored or frontier then
        if GoalTest(child.State) then return Solution(child)
        frontier ← Insert(child, frontier)
```



## Properties of Depth-First Search (DFS)

§ In general, solution found is not optimal

§ Incomplete!

Evaluation only.

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§ Completeness can be guaranteed only for graph search (we need to remember the visited nodes) or acyclic finite state spaces

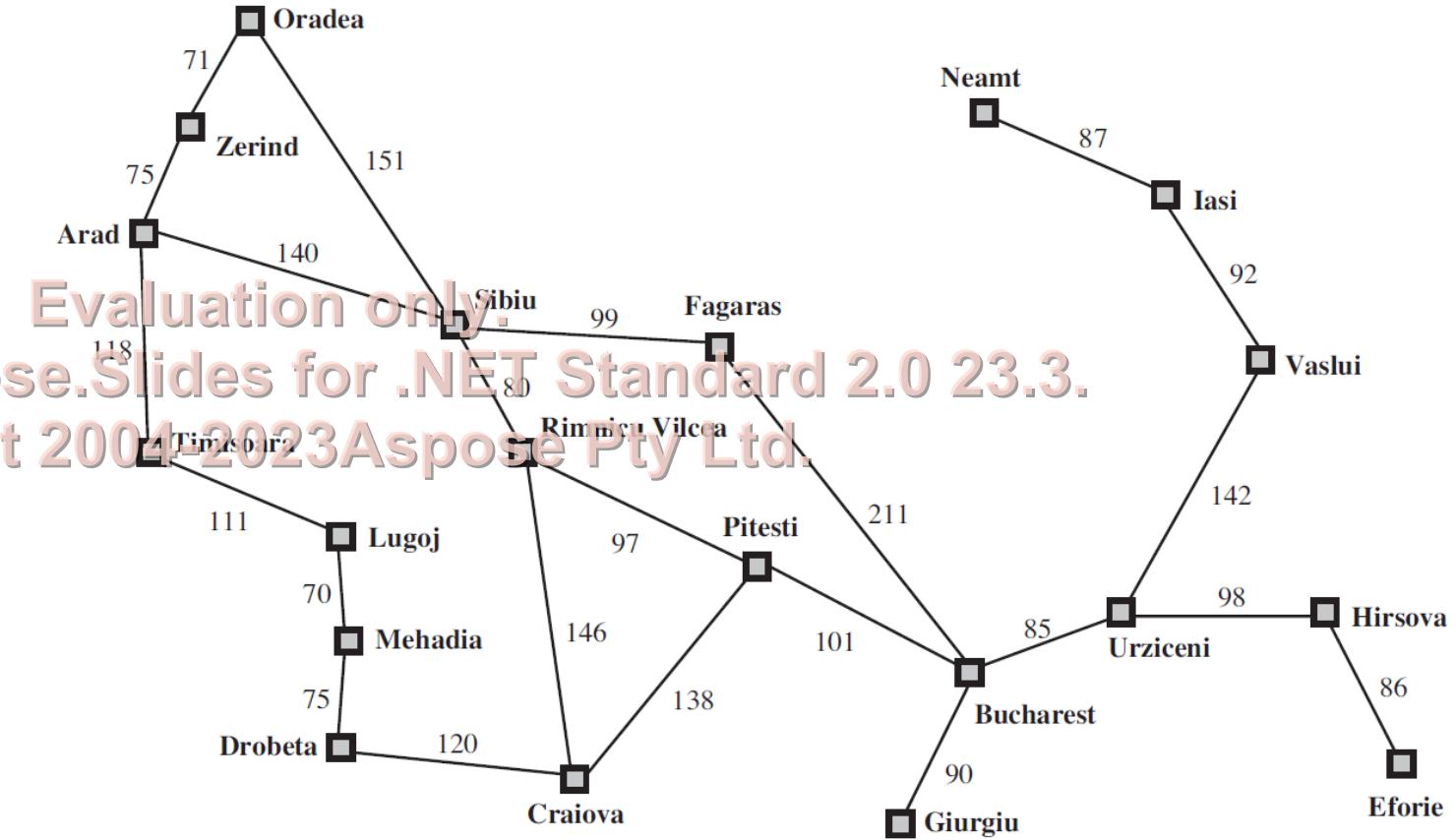
- In infinite state spaces, descends forever on infinite paths
- Tree search may loop forever in repeated states

## Time and Space Complexity of Depth-First Search (DFS)

- **Exponential time complexity:**  $O(b^m)$ 
  - Where  $m$  is the maximum depth of the graph (longest path)
  - In the worst case, all nodes have to be visited until a solution is found
  - This can be even larger than ~~the state space~~  
~~Evaluation only.~~ if we do tree search on graphs,  
i.e., do ~~not remember already visited nodes~~
- **Linear space complexity:**  $O(bm)$  or  $O(m)$ 
  - We need  $O(m)$  to store the nodes along the current path and  $O(b)$  to store all neighbours (frontier at each level)
  - With clever indexing (backtracking search), we can eliminate  $O(b)$  space and compute the neighbors dynamically in an efficient way using only  $O(m)$

## (3) Depth-Limited Search (DLS)

- § Depth-first search with an imposed cutoff on the maximum depth of a path
- § For example, for route planning with  $n$  cities: the maximum depth is  $n-1$  to visit each city once
- § In the Romania travel example, a depth of 9 is sufficient - every city can be reached in at most 9 steps



# Depth-Limited Search (DLS) Algorithm

```
function DEPTH-LIMITED-SEARCH(problem, limit) returns a solution or failure/cutoff
  startNode  $\leftarrow$  a node with currentNode.State = problem.InitialState
  return RECURSIVE-DLS(startNode, problem, limit)  
  
function RECURSIVE-DLS(node, problem, limit) returns a solution or failure/cutoff
  if GoalTest(node.State) then return Solution(node)
  else if limit = 0 then return cutoff
  else
    cutoffOccurred  $\leftarrow$  false
    for each action in Actions(node.State) do
      child  $\leftarrow$  ChildNode(problem, node, action)
      result  $\leftarrow$  RECURSIVE-DLS(child, problem, limit-1)
      if result = cutoff then cutoffOccurred  $\leftarrow$  true
      else if result  $\neq$  failure then return result
    if cutoffOccurred then return cutoff
    else return failure
```



Limit must not be smaller than the depth of the shallowest goal state,  
otherwise DLS is incomplete



## Properties and Complexity of Depth-Limited Search (DLS)

- Complete if the depth limit is not smaller than the depth of the shallowest goal state
- First solution found may not be optimal

Evaluation only.

- Time and space complexity as with DFS, but  $m = l$  (the depth-limit)  
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- **Exponential time complexity:**  $O(b^l)$
- **Linear space complexiy:**  $O(bl)$  or  $O(l)$  with backtracking search

## (4) Iterative Deepening Search (IDS)

§ Use depth-limited search and in every iteration increase search depth by 1

**Evaluation only.**

function ITERATIVE-DEEPENING-SEARCH(*problem*) returns a solution or failure.  
for *depth* = 0 to  $\infty$  do  
  *result*  $\leftarrow$  DEPTH-LIMITED-SEARCH(*problem*, *depth*)  
  if *result*  $\neq$  cutoff then return *result*

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# Illustration of Iterative Deepening Search (IDS)

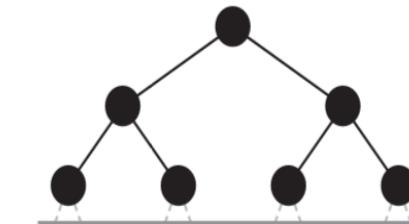
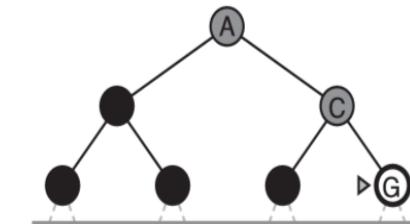
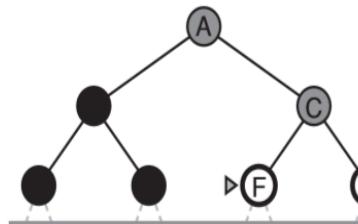
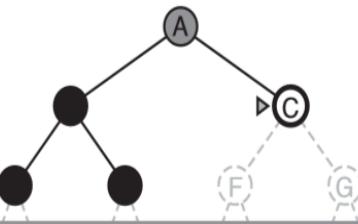
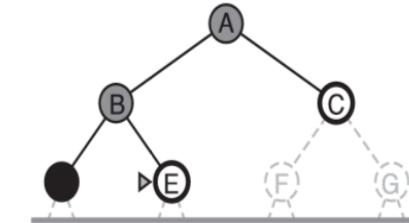
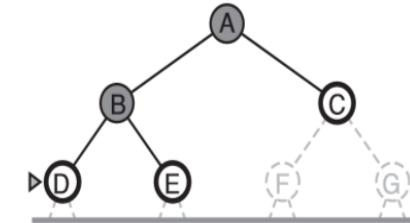
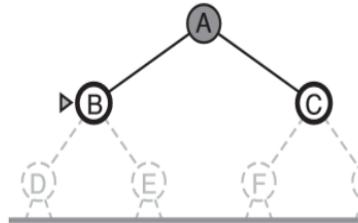
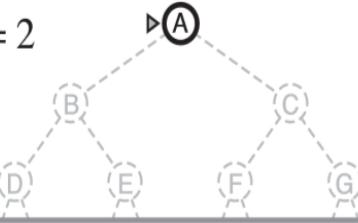
Limit = 0



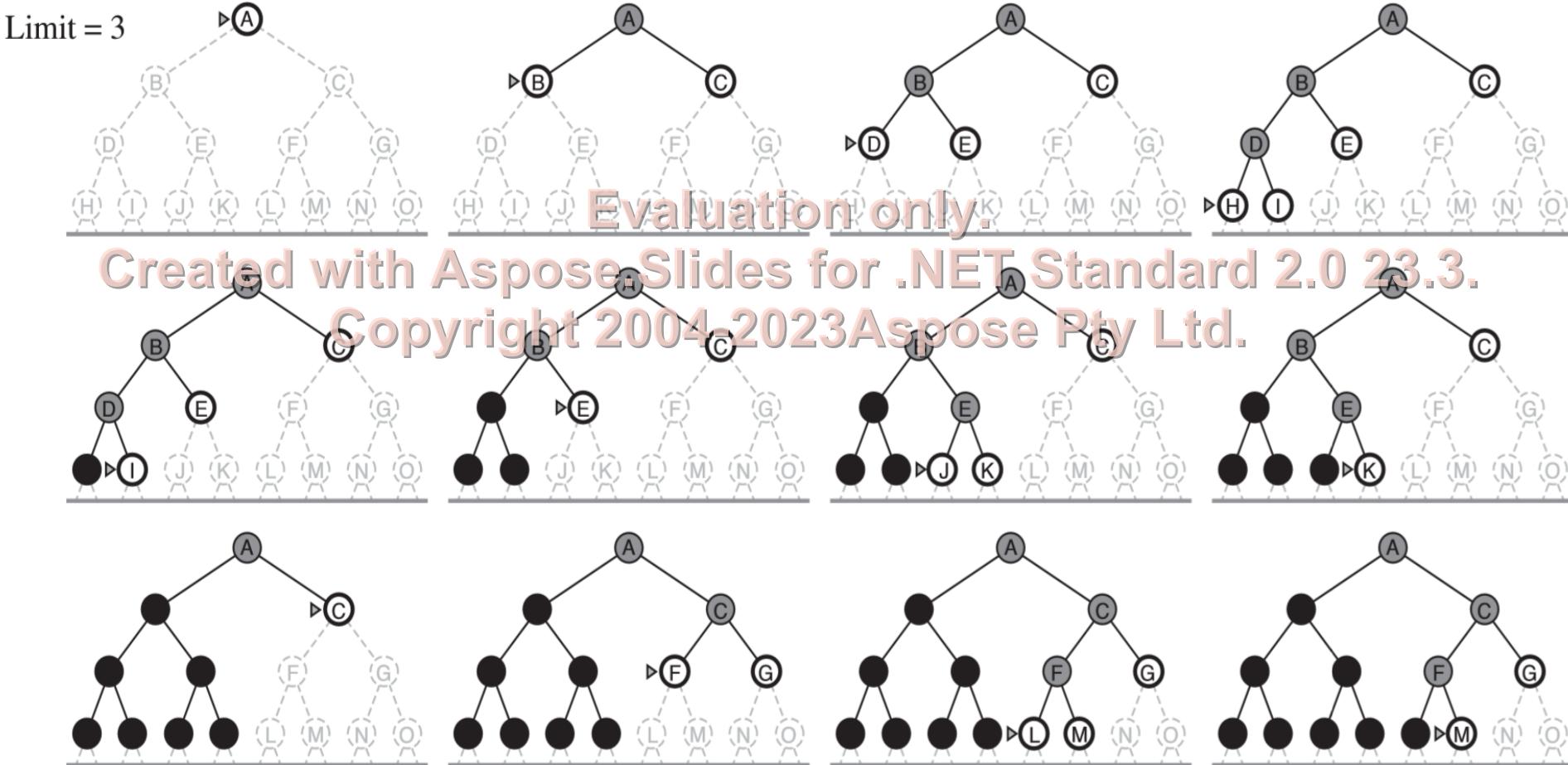
Limit = 1



Limit = 2



## Illustration Iterative Deepening Search (IDS) Continued



# Properties and Complexity of Iterative Deepening Search (IDS)

- Combines advantages of BFS and DFS
  - Optimal for unit action costs: extension to general action costs possible
  - Complete (for finite branching)
  - Complexity as for DLS: exponential time  $O(b^d)$  with depth  $d$  set to limit  $l$  and linear space  $O(bl)$  or  $O(l)$  with backtracking search
- Evaluation only.  
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## Time complexity:

Breadth-First-Search	$b + b^2 + \dots + b^{d-1} + b^d \in O(b^d)$
Iterative Deepening Search	$(d)b + (d - 1)b^2 + \dots + 3b^{d-2} + 2b^{d-1} + 1b^d \in O(b^d)$

**Example:**  $b = 10, d = 5$

Breadth-First Search	$10 + 100 + 1,000 + 10,000 + 100,000 = 111,110$
Iterative Deepening Search	$50 + 400 + 3,000 + 20,000 + 100,000 = 123,450$ $= 5 \times 10^1 + 4 \times 10^2 + 3 \times 10^3 + 2 \times 10^4 + 1 \times 10^5$

## (5) Uniform-Cost Search (UCS)

- Consider the **path costs** for each node  $g(n)$
- Organize the frontier as a priority queue and expand the node with the lowest path costs first **Evaluation only.**

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- Finds an optimal solution if all actions have non-negative costs. In this case:

$$g(\text{successor}(n)) \geq g(n)$$

must hold for all  $n$

# Uniform-Cost Search (UCS) Algorithm

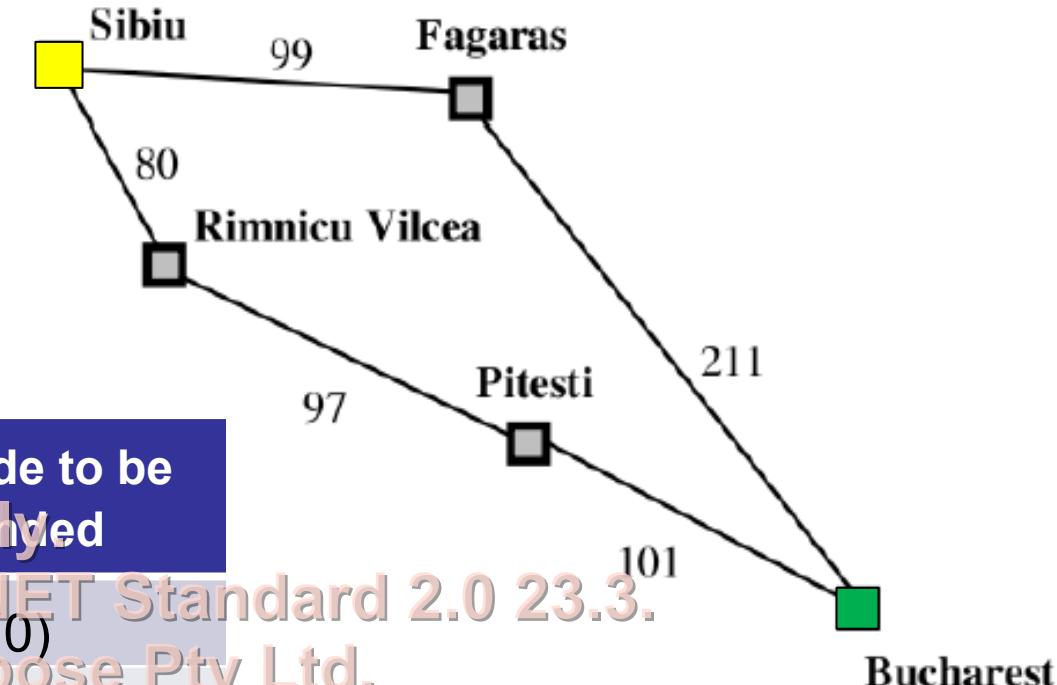
```
function UNIFORM-COST SEARCH(problem) returns a solution or failure
    node ← a node with node.State = problem.InitialState
    frontier ← a priority queue ordered by ascending PathCost, with node as the only element
    explored ← empty set of states
    loop
        if Empty?(frontier) then return failure
        node ← Top(frontier)
        if GoalTest(node.State) then return Solution(node)
        explored ← explored ∪ node.State
        for action in Actions(node.State) do
            child ← ChildNode(problem, node, action)
            if child.State ∉ [explored ∪ States(frontier)] then Insert(child, child.PathCost, frontier)
            else if there exists otherNode ∈ frontier s.t. otherNode.State = child.State and
                child.PathCost < otherNode.PathCost then replace otherNode in frontier with child
```

§ Goal test at node-expansion time

§ Duplicates in frontier replaced in case of cheaper path



## Example of Uniform-Cost Search (UCS)



Search Step	Frontier	Explored set	Next node to be expanded
0	S (0)		S (0)
1	RV (80), F (99)	S	RV (80)
2	F (99), P (177)	S, RV	F (99)
3	P (177), B (310)	S, RV, F	P (177)
4	B (278)	S, RV, F, P	<b>B (278)</b>

The solution is returned with cost 278:

**Sibiu → Rimnicu Vilcea → Pitesti → Bucharest**

## Properties and Complexity of Uniform-Cost Search (UCS)

- Optimal for non-negative action costs
  - Whenever a node is selected for expansion, the optimal path to this node has been found
  - Does not care about the number of actions on a path, but only about the total path costs Created with Aspose.Slides for .NET Standard 2023.3.
- Complete, if all action costs  $> 0$  Copyright 2004-2023 Aspose Pty Ltd.
- Exponential time and space complexity depend on length of optimal solution:  
$$O(b^{1+\lfloor C^*/\varepsilon \rfloor})$$
  - $C^*$  path cost of optimal solution, action costs  $\geq \varepsilon$
  - If all action costs are equal and optimal solution has length  $d$ , then  $b^{d+1}$

## UCS and Dijkstra's Algorithm

**Lemma.** Uniform-cost search is equivalent to Dijkstra's algorithm on the state space graph. (Obvious from the definition of the two algorithms)

The only differences are:

- (a) We generate only a part of the graph incrementally, whereas Dijkstra inputs and processes the whole graph
- (b) We stop when we reach any goal state (rather than a fixed target state given the input)

### Dijkstra's algorithm:

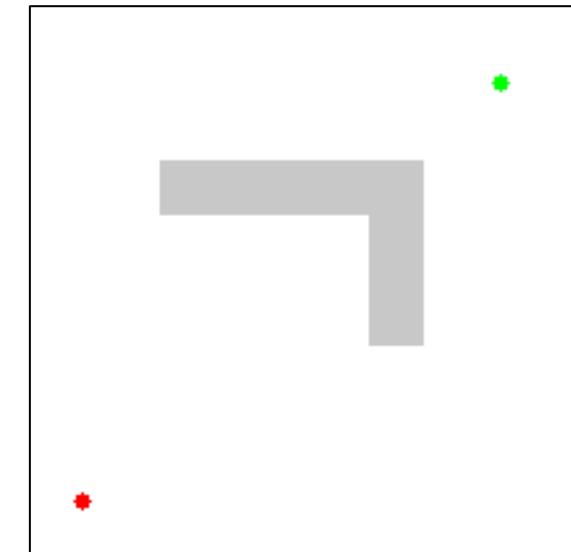
*Initialise the cost of each node to  $\infty$  and the cost of the source to 0*

*While there are unknown nodes left in the graph*

*Select an unknown node with the lowest cost  
and mark as known*

*For each node  $b$  adjacent to  $a$*

*If  $cost(a) + cost(a, b) < cost(b)$  do*  
*$cost(b) = cost(a) + cost(a, b)$*   
*$parent(b) = a$*



# Summary of Algorithm Properties and Complexity

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes <sup>a</sup>	Yes <sup>a,b</sup>	No <sup>d</sup>	No	Yes <sup>a</sup>
Time	$O(b^d)$	$O(b^{1+ C^* /\varepsilon})$ <i>Evaluation only.</i>	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^d)$	$O(b^{1+ C^* /\varepsilon})$	$O(b^m)$	$O(lb)$	$O(bd)^e$
Optimal?	Yes <sup>c</sup>	Yes <sup>c</sup>	No	No	Yes <sup>c</sup>

## Where:

- $b$  branching factor
- $d$  depth of solution
- $m$  maximum depth of the search tree
- $l$  depth limit
- $C^*$  cost of the optimal solution
- $\varepsilon$  minimal cost of an action

## Superscripts:

- <sup>a</sup>  $b$  is finite
- <sup>b</sup> if action costs not less than  $\varepsilon$
- <sup>c</sup> if action costs are all identical
- <sup>d</sup> Yes for finite search spaces
- <sup>e</sup>  $O(b)$  can be eliminated by backtracking search



## Summary

- § DFS is often used because of its minimal memory requirements
  - Compact encodings of exponential-size explored node set exists
- § BFS is rarely found in practice due to its exponential space complexity
  - This does not mean that there are no applications for which this would be the search method of choice!
- § DLS prevents infinite descends on infinite paths
- § IDS is the preferred uninformed search method when there is a large search space and the depth (length) of the solution is not known
- § UCS finds optimal solutions when all actions have non-negative action costs (and thus is optimal for problems with non-unit action costs)

## Working Questions

1. Which concepts are used to describe search problems?
2. Which concepts are used to describe search algorithms and search spaces?
3. What is the difference between tree and graph search?
4. What is the set of explored nodes used for?  
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5. Why don't we need a set of explored nodes when the search space is a tree?
6. Can you explain how BFS, DFS, DLS, IDS, UCS work?
7. What properties are used to characterize search algorithms?
8. Compare uninformed search methods based on time complexity, space complexity, optimality, completeness.