Ray Sphere intersection

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Let $c = |\mathbf{o} - \mathbf{c}|^2 - r^2$ and $b = \mathbf{d} \cdot (\mathbf{o} - \mathbf{c})$. The condition for interesection is $b^2 > c$. We are asked to interpret this inequality. First, let $\mathbf{b} = (\mathbf{o} - \mathbf{c})$.We know that $\mathbf{d}$ is the normalized ray direction. Thus

$$
\begin{gathered}
b^2 > c \\
\leftrightarrow (\mathbf{d} \cdot \mathbf{b})^2 > |\mathbf{b}|^2 - r^2 \\
\leftrightarrow \left(\frac{\mathbf{d} \cdot \mathbf{b}}{|\mathbf{b}|} \right)^2 |\mathbf{b}|^2 > |\mathbf{b}|^2 - r^2
\end{gathered}
$$

However we know that

$$\left(\frac{\mathbf{d} \cdot \mathbf{b}}{|\mathbf{b}|} \right) = p, \tag{1}$$

is the scalar projection of $\mathbf{d}$ onto $\mathbf{b}$.

Therefore we have

$$(p|\mathbf{b}|)^2 > |\mathbf{b}|^2 - r^2$$

We can write $|\mathbf{b}|^2 - r^2 = w$. The quantity w can be thought to represent the distance from the ray origin to the surface of the sphere. So the inequality above is

$$(p|\mathbf{b}|)^2 > w. \tag{2}$$

Essentially, the scaled scalar projection of the ray onto the origin-center vector has to be larger than the distance to the surface.

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I get the physical significance of vector addition & subtraction. But I don't understand what do dot & cross products mean?

Perhaps you would find the geometric interpretations of the dot and cross products more intuitive:

The dot product of A and B is the length of the *projection* of A onto B multiplied by the length of B (or the other way around--it's commutative).

The magnitude of the cross product is the *area of the parallelogram with two sides A and B*. The orientation of the cross product is orthogonal to the plane containing this parallelogram.

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The product may be understood geometrically as the *projection* of one vector onto another, multiplied by the length of the vector that it is projected onto. If one takes the dot product of two vectors $\vec{a}$ and $\vec{b}$, we can apply this procedure to find the correct formula for the dot product:

Let's call the angle between the vectors φ. Then, the projection of $\vec{a}$ onto $\vec{b}$ is $|\vec{a}|\cos\varphi$. Multiplying by the length of $\vec{b}$ gives us

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\varphi$$

This is the correct expression for the dot product. Of course, the dot product is symmetric so we might as well picture it as projecting $\vec{b}$ along $\vec{a}$.

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answered May 8 '14 at 20:39
Danu
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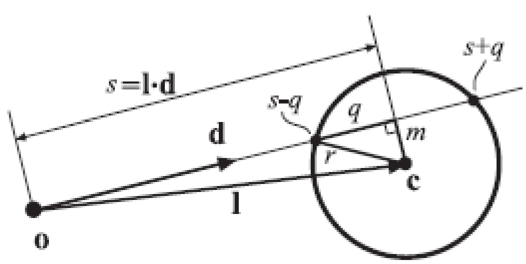

Figure 1: Ray sphere intersection