Identidades matemáticas

$\vec{u}\cdot(\vec{u}\times\vec{v})=0 \qquad \vec{u}\times(\vec{v}\times\vec{w})=(\vec{u}\cdot\vec{w})\vec{v}-(\vec{u}\cdot\vec{v})\vec{w}$

$(\vec{u}\cdot\vec{v})\times(\vec{w}\cdot\vec{z})=(\vec{u}\cdot\vec{w})(\vec{v}\cdot\vec{z})-(\vec{u}\cdot\vec{z})(\vec{v}\cdot\vec{w})$

$\vec{u}\cdot(\vec{v}\times\vec{w})=\vec{v}\cdot(\vec{w}\times\vec{u})=\vec{w}\cdot(\vec{u}\times\vec{v})$

$(\vec{u}\times\vec{v})^2=(uv)^2-(\vec{u}\cdot\vec{v})^2$

$\vec{\nabla}\cdot(\vec{u}\times\vec{v})=\vec{v}\cdot(\vec{\nabla}\times\vec{u})-\vec{u}\cdot(\vec{\nabla}\times\vec{v})$

$\vec{\nabla}(\vec{u}\cdot\vec{v})=(\vec{u}\cdot\vec{\nabla})\vec{v}+(\vec{v}\cdot\vec{\nabla})\vec{u}+\vec{u}\times(\vec{\nabla}\times\vec{v})+\vec{v}\times(\vec{\nabla}\times\vec{u})$

$\vec{\nabla}\times(\vec{u}\times\vec{v})=\vec{u}(\vec{\nabla}\cdot\vec{v})-\vec{v}(\vec{\nabla}\cdot\vec{u})+(\vec{v}\cdot\vec{\nabla})\vec{u}-(\vec{u}\cdot\vec{\nabla})\vec{v}$

$\vec{\nabla}\cdot(f\vec{u})=\vec{\nabla}f\cdot\vec{u}+f\vec{\nabla}\cdot\vec{u}$

$\vec{\nabla}\times(f\vec{u})=\vec{\nabla}f\times\vec{u}+f\vec{\nabla}\times\vec{u}$

$\vec{\nabla}\times(\vec{\nabla}\times\vec{u})=\vec{\nabla}(\vec{\nabla}\cdot\vec{u})-\vec{\nabla}^2\vec{u}$

$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{u})=0 \qquad \vec{\nabla}\times(\vec{\nabla}f)=0$

$\vec{\nabla}\cdot(f\vec{\nabla}g)=\vec{\nabla}f\cdot\vec{\nabla}g+f\vec{\nabla}^2g$

$\vec{\nabla}\times\vec{r}=0 \qquad \vec{\nabla}\cdot\vec{r}=3$

$\vec{\nabla}\left(1/r^k\right)=-\dfrac{k}{r^{k+2}}\vec{r}=-\dfrac{k}{r^{k+1}}\hat{r} \;,\quad \forall k\in\mathbb{Z}$

$\vec{\nabla}\cdot(\vec{r}/r^3)=-\vec{\nabla}^2(1/r)=4\pi\delta^3(\vec{r})$

$$\begin{cases}x=r\sin\theta\cos\varphi\\ y=r\sin\theta\sin\varphi\\ z=r\cos\theta\end{cases}\begin{cases}r=\sqrt{x^2+y^2+z^2}\\ \theta=\arccos(z/r)\\ \varphi=\mathrm{sgn}Si\arccos(x/\sqrt{x^2+y^2})\end{cases}$$

$$\begin{cases}\hat{e}_x=\sin\theta\cos\varphi\,\hat{e}_r+\cos\theta\cos\varphi\,\hat{e}_\theta-\sin\varphi\,\hat{e}_\varphi\\ \hat{e}_y=\sin\theta\sin\varphi\,\hat{e}_r+\cos\theta\sin\varphi\,\hat{e}_\theta+\cos\varphi\,\hat{e}_\varphi\\ \hat{e}_z=\cos\theta\,\hat{e}_r-\sin\theta\,\hat{e}_\theta\end{cases}$$

$$\begin{cases}\hat{e}_r=\sin\theta\cos\varphi\,\hat{e}_x+\sin\theta\sin\varphi\,\hat{e}_y+\cos\theta\,\hat{e}_z\\ \hat{e}_\theta=\cos\theta\cos\varphi\,\hat{e}_x+\cos\theta\sin\varphi\,\hat{e}_y-\sin\theta\,\hat{e}_z\\ \hat{e}_\varphi=-\sin\varphi\,\hat{e}_x+\cos\varphi\,\hat{e}_y\end{cases}$$

$\vec{\nabla}f=\partial_r(f)\hat{r}+\partial_\theta(f)\hat{\theta}+\frac{1}{r\sin\theta}\partial_\varphi(f)\hat{\varphi}$

$\vec{\nabla}\cdot\vec{u}=\frac{1}{r^2}\partial_r(r^2u_r)+\frac{1}{r\sin\theta}\partial_\theta(\sin\theta u_\theta)+\frac{1}{r\sin\theta}\partial_\varphi u_\varphi$

$\vec{\nabla}^2f=\frac{1}{r^2}\partial_r(r^2\partial_rf)+\frac{1}{r^2\sin\theta}\partial_\theta(\sin\theta\partial_\theta f)+\frac{1}{r^2\sin^2\theta}\partial_\varphi^2f$

$\vec{\nabla}^2f=\frac{1}{r}\partial_r(r\partial_rf)+\frac{1}{r^2}\partial_\varphi^2f$

$\int\cos^3x\,dx=\sin x-\frac{1}{3}\sin^3x$

$\int\sin^2x\,dx=\frac{x}{2}-\frac{1}{4}\sin(2x)$

$\int_0^\infty x^ne^{-ax}\,dx=n!/a^{n+1}$

$\int_{-1}^{1}\dfrac{1-x^2}{(1+ax)^5}\,dx=\frac{4}{3}\frac{1}{(1-a^2)^3}$

$\int\dfrac{1}{1-(x/a)^2}\,dx=a\cdot arctanh(x/a)+C$

$\int\dfrac{1}{\sqrt{1-(x/a)^2}}\,dx=a\cdot arcsin(x/a)+C \qquad (a>0)$

$sin(\alpha\pm\beta)=sin(\alpha)\cdot cos(\beta)\pm cos(\alpha)\cdot sin(\beta)$

$cos(\alpha\pm\beta)=cos(\alpha)\cdot cos\beta\mp sin(\alpha)\cdot sin(\beta)$

$tan(\alpha\pm\beta)=\frac{tan(\alpha)\pm tan(\beta)}{1\mp tan(\alpha)\cdot tan(\beta))}$

Constantes

$\mu_0=4\pi\cdot10^{-7}\;\mathbf{H/m}$

$\epsilon_0=1/\mu_0c^2=8{,}854\cdot10^{-12}\;\mathbf{F/m}$

$\eta_0=\sqrt{\frac{\mu_0}{\epsilon_0}}=\mu_0c=377\Omega \qquad m_ec^2=511\text{keV}$

$\vec{H}=\frac{\vec{B}}{\mu_0}-\vec{M} \qquad \vec{D}=\epsilon_0\vec{E}+\vec{P}$

$\vec{\beta}=\vec{v}/c\simeq1-1/(2\gamma^2) \qquad \gamma=1/\sqrt{1-\beta^2}$

Conceptos básicos de Electromagnetismo

$\vec{\nabla}\cdot\vec{E}=\frac{\rho}{\epsilon_0} \qquad \vec{\nabla}\times\vec{B}-\partial_t\vec{E}/c^2=\mu_0\vec{j}$

$\vec{\nabla}\times\vec{E}+\partial_t\vec{B}=0 \qquad \vec{\nabla}\cdot\vec{B}=0$

$\vec{F}=q(\vec{E}+\vec{v}\times\vec{B}) \qquad \vec{j}=\rho\vec{v} \qquad \partial_t\rho+\vec{\nabla}\cdot\vec{j}=0$

$\vec{S}=\frac{1}{\mu_0}(\vec{E}\times\vec{B})$

$u=\frac{1}{2}(\epsilon_0\vec{E}^2+\frac{\vec{B}^2}{\mu_0})=\frac{1}{2}\epsilon_0c^2\left(\frac{E^2}{c^2}+B^2\right)$

$\partial_tu+\vec{\nabla}\cdot\vec{S}=-\vec{E}\cdot\vec{j}$

Tensor de estrés de Maxwell:

$$\mathbf{T}=\frac{\epsilon_0\vec{E}^2+\frac{1}{\mu_0}\vec{B}^2}{2}\mathbb{1}-(\epsilon_0\vec{E}\circ\vec{E}+\tfrac{1}{\mu_0}\vec{B}\circ\vec{B})$$

$\mathrm{T}^M_{ij}=\varepsilon_0c^2\left[\frac{E_iE_j}{c^2}+B_iB_j-\frac{1}{2}\delta_{ij}\left(\frac{E^2}{c^2}+B^2\right)\right]$

$(T^{ij}=T^{ji}) \qquad Tr[\mathbf{T}]=u$

Conservación del momento lin. EM ($\vec{p}\equiv\vec{S}/c^2$):

$\vec{\nabla}\mathbf{T}-\partial_t\vec{p}=\vec{f} \qquad \partial_kT^{ik}-\frac{1}{c^2}\partial_tS^i=f^i$

donde $\vec{f}=\rho\vec{E}+\vec{j}\times\vec{B}$

$\vec{E}=-\vec{\nabla}\phi-\partial_t\vec{A} \qquad \vec{B}=\vec{\nabla}\times\vec{A}$

$\phi'=\phi-\partial_tf \qquad \vec{A}'=\vec{A}+\vec{\nabla}f$

Conceptos básicos de Electromagnetismo

- Temp.: $\phi=0$ - Coulomb: $\vec{\nabla}\cdot\vec{A}=0$
- Axial: $A^k=0\;(k=1,2,3)$
- Lorenz: $\vec{\nabla}\cdot\vec{A}+\partial_t\phi/c^2=0\;\leftrightarrow\;\partial_iA^i=0$

Ecs de ondas ($\rho=\vec{j}=0$):

$\vec{\nabla}^2E=\partial_t\vec{E}/c^2 \qquad \vec{\nabla}^2B=\partial_t\vec{B}/c^2$

Sols. (parte real):

$\vec{E}(\vec{x},t)=\vec{E}(\vec{k},\omega)\,e^{i(\vec{k}\cdot\vec{x}-\omega t)}$

$\vec{B}(\vec{x},t)=\vec{B}(\vec{k},\omega)\,e^{i(\vec{k}\cdot\vec{x}-\omega t)}$

(caso general $\rho\neq0$ y $\vec{j}\neq0$):

$\partial_t^2\phi/c^2-\vec{\nabla}^2\phi=\frac{\rho}{\epsilon_0}+\partial_t(\vec{\nabla}\cdot\vec{A}+\partial_t\phi/c^2)$

$\partial_t^2\vec{A}/c^2-\vec{\nabla}^2\vec{A}=\mu_0\vec{j}-\vec{\nabla}(\vec{\nabla}\cdot\vec{A}+\partial_t\phi/c^2)$

$\partial_t^2A^\mu/c^2-\vec{\nabla}^2A^\mu=\mu_0j^\mu$

Conceptos básicos de Electromagnetismo

- Punto reposo: $\phi(\vec{r})=\frac{q}{4\pi\epsilon_0r}$, y $\vec{A}=0$
- Dipolo el: $\phi(\vec{r})=\frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}=\frac{1}{4\pi\epsilon_0}\frac{p_ix^i}{(x_jx^j)^{3/2}}$
- Cuadrupolo el: $\phi(\vec{r})=\frac{1}{4\pi\epsilon_0}\frac{\vec{r}\,\mathbf{Q}\,\vec{r}}{r^5}=\frac{1}{4\pi\epsilon_0}\frac{x_iQ^{ij}x_j}{(x_kx^k)^{5/2}}$
- Dipolo mag: $\phi(\vec{r})=0$, y $\vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}\Longrightarrow$

$A^\mu=\frac{\mu_0}{4\pi}\dfrac{\varepsilon_{\mu\nu\sigma}m_\nu x_\sigma}{(x_\alpha x^\alpha)^{3/2}}$

- Cuadrupolo mag: $\vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\frac{\vec{r}\times\mathbf{Q}\vec{r}}{r^5}\Longrightarrow$

$\Rightarrow A^\mu=\frac{\mu_0}{4\pi}\dfrac{\varepsilon_{\mu\nu\sigma}x^\nu Q^\lambda_\sigma x_\lambda}{(x_\alpha x^\alpha)^{5/2}}$

Ctes. opticas: ($\tilde{\gamma}$ cte. de prop., $\tilde{n}$ el índ. de refracción, $\tilde{\epsilon}$ la cte. dieléctrica y $\tilde{\eta}$ la impedancia):

$\tilde{\gamma}=i\frac{\omega}{x}\sqrt{\frac{\tilde{\epsilon}}{\epsilon_0}} \qquad \tilde{\eta}=\sqrt{\frac{\mu_0}{\tilde{\epsilon}}}$

$\tilde{\epsilon}=\epsilon_0\left(1-i\dfrac{\sigma}{\omega\epsilon_0}\right) \qquad \tilde{n}=c\mu_0/\tilde{\eta}$

Coefs de reflexión y transmisión ($\tilde{n}=n+i\alpha$):

$R=\frac{(n_1-n_2)^2+(\alpha_1-\alpha_2)^2}{(n_1+n_2)^2+(\alpha_1+\alpha_2)^2} \qquad T=\frac{4n_1n_2+4\alpha_1\alpha_2}{(n_1+n_2)^2+(\alpha_1+\alpha_2)^2}$

Relatividad

Boost $\vec{\beta}=(\beta,0,0)=(v/c,0,0)$:

$ct'=\gamma(ct-\beta x) \quad \vec{x}'=\vec{x}+\frac{\gamma-1}{\beta^2}(\vec{\beta}\cdot\vec{x})\vec{\beta}-\gamma x^0\vec{x}$

$$\begin{pmatrix}ct'\\x'\\y'\\z'\end{pmatrix}=\begin{pmatrix}\gamma&-\gamma\beta&0&0\\-\gamma\beta&\gamma&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}ct\\x\\y\\z\end{pmatrix}$$

$\vec{x}'=\Lambda\vec{x} \quad\Longrightarrow\quad x'^i=\Lambda^i_jx^j$

$$\Lambda=\left(\begin{array}{c|c}\gamma&-\gamma\vec{\beta}\\\hline-\gamma\vec{\beta}&\mathbb{1}+\dfrac{\gamma-1}{\beta^2}\vec{\beta}\circ\vec{\beta}\end{array}\right)=$$

$$\begin{pmatrix}\gamma&-\gamma\beta_x&-\gamma\beta_y&-\gamma\beta_z\\-\gamma\beta_x&1+\frac{\gamma-1}{\beta^2}\beta_x^2&\frac{\gamma-1}{\beta^2}\beta_x\beta_y&\frac{\gamma-1}{\beta^2}\beta_x\beta_z\\-\gamma\beta_y&\frac{\gamma-1}{\beta^2}\beta_x\beta_y&1+\frac{\gamma-1}{\beta^2}\beta_y^2&\frac{\gamma-1}{\beta^2}\beta_y\beta_z\\-\gamma\beta_z&\frac{\gamma-1}{\beta^2}\beta_x\beta_z&\frac{\gamma-1}{\beta^2}\beta_y\beta_z&1+\frac{\gamma-1}{\beta^2}\beta_z^2\end{pmatrix}$$

Vels:

$u'_{\|}=\frac{u_{\|}-v}{1-\frac{\vec{v}\cdot\vec{u}}{c^2}}$, y $\vec{u}'_\perp=\frac{\vec{u}_\perp}{\gamma(1-\frac{\vec{v}\cdot\vec{u}}{c^2})}$

$\vec{E}'=\gamma(\vec{E}+\vec{v}\times\vec{B})-\frac{\gamma^2}{\gamma+1}(\vec{v}\cdot\vec{E})\frac{\vec{v}}{c^2}$

$\vec{B}'=\gamma(\vec{B}-\vec{v}\times\vec{E}/c^2)-\frac{\gamma^2}{\gamma-1}(\vec{v}\cdot\vec{B})\frac{\vec{v}}{c^2}$

$E'_{\|}=E_{\|} \qquad \vec{E}'_\perp=\gamma(\vec{E}_\perp+\vec{v}\times\vec{B}_\perp)$

$B'_{\|}=B_{\|} \qquad \vec{B}'_\perp=\gamma(\vec{B}_\perp-\vec{v}\times\vec{E}_\perp/c^2)$

Rapidez: $\cosh\xi=\gamma \quad \sinh\xi=\gamma\beta \quad \tanh\xi=\beta$

$\xi=acosh(\gamma)\simeq\ln(2\gamma) \quad (\simeq\;ultrarrel.)$

$x^i=\begin{pmatrix}ct\\\vec{x}\end{pmatrix} \qquad v^i=\frac{dx^i}{d\tau}=\begin{pmatrix}\gamma_vc\\\gamma_v\vec{v}\end{pmatrix}$

($d\tau=dt/\gamma$ y $ds=cd\tau$)

$a^i=\frac{dv^i}{d\tau}=\begin{pmatrix}\gamma_v\dot{\gamma_v}c\\\gamma_v^2\vec{a}+\gamma_v\dot{\gamma_v}\vec{v}\end{pmatrix}=\begin{pmatrix}\gamma_v^4\,\frac{\vec{a}\cdot\vec{v}}{c}\\\gamma_v^4\left(\vec{a}+\frac{\vec{a}\cdot\vec{v}}{c^2}\vec{v}\right)\end{pmatrix}$

$p^i=\begin{pmatrix}E/c\\\vec{p}\end{pmatrix} \quad f^i=\frac{dp^i}{d\tau}=\begin{pmatrix}\gamma_v\,\frac{\vec{f}\cdot\vec{v}}{c}\\\gamma_v\vec{f}\end{pmatrix} \quad J^i=\begin{pmatrix}c\rho\\\vec{j}\end{pmatrix}$

$F_c=\gamma ma_c \qquad F_t=\gamma^3ma_t \qquad p=\gamma mv$

$E_T=\gamma mc^2 \qquad E^2=c^2|\vec{p}|^2+(mc^2)^2$

Relatividad

$\omega'=\omega\gamma(1-\beta\cos\theta) \qquad I'(t)=\frac{1-\beta}{1+\beta}I$

$\tan\theta'=\dfrac{\sin\theta}{\gamma(\cos\theta-\beta)}$

a'=cte:

$v(t)=\dfrac{a't}{\sqrt{1+(\frac{a't}{c})^2}} \quad x(t)=\frac{c^2}{a'}\left[\sqrt{1+(\frac{a't}{c})^2}-1\right]$

$\tau(t)=ln\left[\frac{a't}{c}+\sqrt{1+(\frac{a't}{c})^2}\right]$

Adición de vels.: $v=\dfrac{v_1+v_2}{1+\frac{v_1v_2}{c^2}}$

Cálculo tensorial

Producto exterior ($\circ$):

$$\vec{E}\circ\vec{E}=\begin{pmatrix}E_x^2&E_xE_y&E_xE_z\\E_yE_x&E_y^2&E_yE_z\\E_zE_x&E_zE_y&E_z^2\end{pmatrix}$$

$x^i=\begin{pmatrix}ct\\\vec{x}\end{pmatrix} \qquad x_i=g_{ij}x^j=\begin{pmatrix}ct\\-\vec{x}\end{pmatrix}$

$F_{\alpha\beta}=g_{\alpha\mu}g_{\nu\beta}F^{\mu\nu}$

$\delta^i_j\delta^k_i=\delta^k_j \qquad \delta^i_i=\delta^i_j\delta^j_i=3$

$\delta^i_j=g^{ik}g_{kj}=\mathbb{1}_{ij}\neq\delta_{ij}=g_{ik}\delta^k_j$

$\delta_{ij}=g_{ij}=g^{ij}=\delta^{ij} \qquad \frac{\partial x^\alpha}{\partial x^\beta}=\delta^\alpha_\beta$

$$\varepsilon_{ijk}=\begin{cases}1\Leftrightarrow(i,j,k)\;\text{permutación par de }(1,2,3)\\-1\Leftrightarrow(i,j,k)\;\text{permutación impar de }(1,2,3)\\0\Leftrightarrow\;\text{hay valores repetidos en los índices}\end{cases}$$

$$\varepsilon^{ijk}\varepsilon_{lmn}=\begin{vmatrix}\delta^i_l&\delta^i_m&\delta^i_n\\\delta^j_l&\delta^j_m&\delta^j_n\\\delta^k_l&\delta^k_m&\delta^k_n\end{vmatrix}\Rightarrow\varepsilon^{ijk}\varepsilon_{imn}=\delta^j_m\delta^k_n-\delta^j_n\delta^k_m$$

$\Rightarrow\varepsilon^{ijk}\varepsilon_{ijn}=2\delta^k_n\Rightarrow\varepsilon^{ijk}\varepsilon_{ijk}=6$

$\vec{A}\cdot\vec{B}=A_iB_i \qquad (\vec{A}\times\vec{B})_k=\varepsilon_{ijk}A_iB_j$

$\vec{\nabla}\cdot\vec{A}=\partial_iA_i \qquad (\vec{\nabla}\times\vec{A})_k=\varepsilon_{ijk}\partial_iA_j$

Grupo de Poincaré: Λ tq $\Lambda^Tg\Lambda=g$

Grupo Lorentz restringido (forma general de una matriz):

$\Lambda=e^{-i(\vec{\theta}\cdot\vec{J}+\vec{\eta}\cdot\vec{K})}$

Álgebra de Lie:

$[J_i,J_j]=i\epsilon_{ijk}J_k \qquad [K_i,K_j]=-i\epsilon_{ijk}J_k$

$[J_i,K_j]=i\epsilon_{ijk}K_k$

Matrices de rotación:

$R_{\vec{n}}(\theta)=e^{\phi\vec{G}\cdot\vec{n}}=\mathbf{1}+\phi\vec{G}\cdot\vec{n}$

Teoría de Campos

$\partial^i=\begin{pmatrix}\frac{1}{c}\partial_t\\-\vec{\nabla}\end{pmatrix} \qquad A^i=\begin{pmatrix}\phi/c\\\vec{A}\end{pmatrix}$

$$F^{ik}\equiv\partial^iA^k-\partial^kA^i=\left(\begin{array}{c|c}0&-\vec{E}/c\\\hline\vec{E}/c&\vec{B}_\wedge\end{array}\right)=$$

$$\begin{pmatrix}0&-E_x/c&-E_y/c&-E_z/c\\E_x/c&0&-B_z&B_y\\E_y/c&B_z&0&-B_x\\E_z/c&-B_y&B_x&0\end{pmatrix}$$

$$F^{ik^*}\equiv G^{ik}\equiv\tfrac{1}{2}\varepsilon^{iklm}F_{lm}=\left(\begin{array}{c|c}0&-\vec{B}\\\hline\vec{B}&-\vec{E}/c_\wedge\end{array}\right)=$$

$$\begin{pmatrix}0&-B_x&-B_y&-B_z\\B_x&0&E_z/c&-E_y/c\\B_y&-E_z/c&0&E_x/c\\B_z&E_y/c&-E_x/c&0\end{pmatrix}$$

$F'^{\alpha\beta}=\Lambda^\alpha_\mu\Lambda^\beta_\nu F^{\mu\nu}\;;\;\vec{E}'=\gamma(\vec{E}+c\vec{\beta}\times\vec{B})$

$\vec{B}'=\gamma(\vec{B}-\dfrac{\vec{\beta}\times\vec{E}}{c})-\dfrac{\gamma^2}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\dot{\vec{\beta}})$

Invariantes bajo transformaciones de Lorentz:

$F_{\mu\nu}F^{\mu\nu}=-2\left(\frac{\vec{E}^2}{c^2}-\vec{B}^2\right)\;;\;F^*_{\mu\nu}F^{\mu\nu}=-\frac{4}{c}\vec{E}\cdot\vec{B}$

Identidad de Bianchi: $\partial_\mu F^{*\mu\nu}=0$

Ecuación de Klein-Gordon: $\partial^i\partial_i\phi+\mu^2\phi=0$

($\mu\equiv masa\;de\;Proca$)

Teoría de Campos

$S = S_{libre} + S_{int} + S_{EM} = -mc^2 \int dt \sqrt{1-v^2/c^2} + \frac{1}{c}\int d^4x j^\mu A_\mu - \frac{1}{c\mu_0}\int d^4x F_{\mu\nu}F^{\mu\nu}$

Ecs. Maxwell: $\partial_\nu F^{\mu\nu} = \mu_0 j^\mu, \quad \partial_\nu F^{\mu\nu^*} = 0$

Tma. continuidad: $\partial^\alpha A_\alpha = 0$

$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)}\right) - \frac{\partial \mathcal{L}}{\partial \phi_i} = 0$

Dens. lag. libre: $\mathcal{L} = -\frac{\epsilon_0 c^2}{4} F^{\mu\nu}F_{\mu\nu}$

Dens. lag. Maxwell: $\mathcal{L} = -\frac{\epsilon_0 c^2}{4} F^{\mu\nu}F_{\mu\nu} - J_\mu A^\mu$

Dens. lag. de radiación:

$$\mathcal{L} = -\frac{z_0 c^2}{4}\partial^\mu A^\nu \partial_\nu A_\mu - J^\mu A_\mu$$

Dens. lag. de Proca: $\mathcal{L} = -\frac{\epsilon_0 c^2}{4} F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\mu^2 A^\alpha A_\alpha$

Masa de Proca: $\mu = \frac{mc}{\hbar}$

Corriente/Carga de Noether:

$$J_k^\mu = -\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\partial_\nu \phi - \mathcal{L}\delta_\nu^\mu\right)\left(\frac{\delta x^\mu}{\delta \omega^k}\right) + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\left(\frac{\delta \phi}{\delta \omega^k}\right), \quad Q_k = \frac{1}{c}\int J_k^0 dV$$

Tensor energía-momento:

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\partial^\nu \phi - \mathcal{L}\eta^{\mu\nu}$$

Para una trsf. Lor. (externa) de parámetros $\omega^{\rho\sigma}$ (0i boosts, ij rotaciones) y $\frac{\delta x^\mu}{\delta \omega^k} = \frac{1}{2}(\delta_\rho^\mu \delta_\sigma^\nu - \delta_\sigma^\mu \delta_\rho^\nu)x_\nu$:

$$J_{\rho\sigma}^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\frac{1}{2}(\delta_\rho^\nu x_\sigma - \delta_\sigma^\nu x_\rho)\partial_\nu \phi - \mathcal{L}\frac{1}{2}(\delta_\rho^\mu x_\sigma - \delta_\sigma^\mu x_\rho)$$

Tensor energía momento (GLR):

$T^{\mu\sigma\rho} = -[T^{\mu\rho}x^\sigma - T^{\mu\sigma}x^\rho]$ con $\partial_\mu T^{\mu\sigma\rho} = 0$

Carga conservada ($\mu = 0$) $M^{\rho\sigma} = \int d^3x T^{0\rho\sigma}$

Tma conservación:

$\partial_0 J_k^0 \to Q_k(t) = \int_v dx^3 J_k^0 \to \frac{d}{dt}Q_k = 0$

(Conservación de la carga de noether)

Tensor energía momento para EM:

$$\Theta^{ik} = \left(\begin{array}{c|c} u & c\vec{g} \\ \hline c\vec{\eta} & -T^{ij} \end{array}\right)$$

Con $\vec{g} = \epsilon_0(\vec{E}\times\vec{B}) = \frac{\vec{S}}{c^2}$ y $T^{ij} = \frac{\partial \mathcal{L}}{\partial(\partial_i \phi)}\left(\frac{\partial \phi}{\partial x_j}\right) - g^{ij}\mathcal{L}$

y $u = \frac{1}{2}(\vec{E}\cdot\vec{D} + \vec{B}\cdot\vec{H})$

$$T_{ij}^M = \varepsilon c^2\left[\frac{E_iE_j}{c^2} + B_iB_j - \frac{1}{2}\delta_{ij}\left(\frac{E^2}{c^2} + B^2\right)\right]$$

Cons. energía en campo EM: $\frac{\partial u}{\partial t} + \vec{\nabla}\cdot\vec{S} + \vec{E}\cdot\vec{J} = 0$

Tma conservación: $\partial_i T_j^i = 0$

$\Theta^{ik} = \epsilon_0 c^2(F^{ij}F_j^k + \frac{1}{4}F^2)$

$\partial_i \Theta^{ik} = -F^{jl}J_l; \quad F_i = \int T_{ij}dS_j$

Presión de radiación: $\mathcal{P}_{rad}^{ab} = \frac{dF_i}{dS_i}$

D'Alembertiano: $\partial_i \partial^i = \frac{1}{c^2}\partial_t^2 - \vec{\nabla}^2$

Dens. hamiltoniana: $\mathcal{H} = \sum_j \Pi_j(\partial_0 \phi_k) - \mathcal{L}$, y el hamiltoniano $H = \int d^3x \mathcal{H} = \int d^3x T_0^0$ donde $\Pi_j = \frac{\partial \mathcal{L}}{\partial(\partial_0 \phi_k)}$ la dens. momento canónico.

Radiación

Lenard Wiehart:

$\phi(t,\vec{r}) = \frac{q}{4\pi\epsilon_0}\left.\frac{1}{s}\right|_{ret}$; $\vec{A}(t,\vec{r}) = \frac{q}{4\pi\epsilon_0}\left.\frac{\vec{v}}{c^2 s}\right|_{ret}$

con $s = R - \vec{R}\cdot\vec{\beta}$

$$\vec{E}(t,\vec{r}) = \frac{q}{4\pi\epsilon_0}\left[\frac{\vec{n}-\vec{\beta}}{\gamma^2R^2(1-\vec{n}\vec{\beta})^3} + \frac{\vec{n}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{cR(1-\vec{n}\vec{\beta})^3}\right]$$

$\vec{B}(t,\vec{r}) = \vec{n}\times\left.\frac{\vec{E}}{c}\right|_{ret}$

Caso general:

$$\frac{dP(t')}{d\Omega} = |\vec{S}|^{\text{rad}}R^2(1-\vec{n}\vec{\beta}) = \epsilon_0 c|\vec{E}|^2R^2(1-\vec{n}\vec{\beta}) = \frac{q^2}{16\pi^2\epsilon_0 c}\frac{(\vec{n}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}])^2}{(1-\vec{n}\vec{\beta})^5} \; ; \; \frac{dt}{dt'} = 1 - \vec{n}\vec{\beta}$$

$\frac{dP(t')}{d\Omega} = (1-\vec{n}\vec{\beta})\frac{dP(t)}{d\Omega}$ $\quad P(t') = \frac{1}{4\pi\epsilon_0}\frac{2q^2}{3c}\gamma^6[\dot{\vec{\beta}}^2 - (\vec{\beta}\times\dot{\vec{\beta}})^2]$

$P(t') = \frac{q^2\gamma^2}{6\pi\varepsilon_0 c^3 m^2}(F^2 - (\vec{\beta}\cdot\vec{F})^2)$

Radiación Larmor: (no relativista→sí en sistema propio.)

$\frac{dP}{d\Omega} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{4\pi c^3}[\vec{n}\times(\vec{n}\times\dot{\vec{v}})]^2 \qquad P = \frac{1}{4\pi\epsilon_0}\frac{2}{3}\frac{q^2\dot{\vec{v}}^2}{c^3}$

lineal $\frac{dP}{d\Omega} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{4\pi c^3}\frac{a^2\operatorname{sen}^2\theta}{(1-\beta\cos\theta)^5} \qquad P = \frac{1}{4\pi\epsilon_0}\frac{2}{3}\frac{q^2}{c^3}\gamma^6 a^2$

$\frac{dP}{d\Omega} = \frac{1}{4\pi\epsilon_0}\frac{8}{\pi}\frac{q^2}{c^3}\gamma^8 a^2\frac{(\gamma\theta)^2}{(1+(\gamma\theta)^2)^5}$ (lim.ultrrel. θ peq.)

Radiación

circular

$\frac{dP}{d\Omega} = \frac{q^2}{16\pi^2\varepsilon_0 c^3}\frac{1}{(1-\beta\cos\theta)^3}\left(1 - \frac{(1-\beta^2)\sin^2\theta}{(1-\beta\cos\theta)^2}\right) \qquad \frac{dP}{d\Omega} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{4\pi c^3}\frac{a^2}{(1-\beta\cos\theta)^3}\left[1 - \frac{\sin^2\theta\cos^2\phi}{\gamma^2(1-\beta\cos\theta)^2}\right]$

$P = \frac{1}{4\pi\epsilon_0}\frac{2}{3}\frac{q^2}{c^3}\gamma^4 a^2$

circ. ultrar

$$\frac{dP}{d\Omega} = \frac{1}{4\pi\epsilon_0}\frac{2q^2}{\pi c^3}\frac{\gamma^6 a^2}{(1+(\gamma\theta)^2)^3}\left[1 - \frac{4(\gamma\theta)^2\cos^2\phi}{(1+(\gamma\theta)^2)^2}\right] \qquad P = \frac{1}{4\pi\epsilon_0}\frac{2}{3}\frac{q^2c}{R^2}\gamma^4\beta^4 \quad \text{con} \quad a = \frac{u^2}{R}$$

$\longrightarrow \delta\varepsilon[MeV] = 8{,}85\cdot 10^{-2}\frac{\varepsilon^4[GeV]}{R[m]}\left(\frac{q}{e}\right)^2\left(\frac{m_ec^2}{mc^2}\right)^4$

con radio $R[m] = \frac{\varepsilon[GeV]}{0{,}3\cdot B[T]}\frac{m}{m_e}\frac{e}{q}\begin{cases} m_e,\ e = \text{del electrón} \\ m,\ q = \text{de la partícula}\end{cases}$

Sincrotrón radia /vuelta: $P[W] = 10^6\cdot\delta\varepsilon[MeV]\cdot I[A]$

Cambio radio: $\frac{2}{3}\frac{q^2/(4\pi\varepsilon_0)}{mc^2}\left(\frac{qcB}{mc^2}\right)^3 c\Delta t = \Delta(1/R)$

$\dot{\vec{\beta}} = \frac{1}{\gamma mc}\left[\vec{F} - (\vec{F}\cdot\vec{\beta})\vec{\beta}\right] \qquad \frac{d\gamma}{dt'} = \gamma^3(\vec{\beta}\cdot\dot{\vec{\beta}})$

$\vec{F} = \frac{d\vec{p}}{dt'} = \gamma mc\left[\dot{\vec{\beta}} - \gamma^2(\vec{\beta}\cdot\dot{\vec{\beta}})\vec{\beta}\right]$

Reacción de radiación (despreciable si $T \ll \tau$, ó $E^{\text{rad}} \ll E_0^c$):

$\vec{F}_{\text{rad}} = m\tau\ddot{\vec{v}} \qquad \tau = \frac{q^2}{6\pi\epsilon_0 mc^3}$

Tma conservación energía: $\frac{dE^{\text{cin}}}{dt'} + \frac{dE^{\text{rad}}}{dt'} = 0$

$E^{\text{rad}} = \int_0^T P(t')dt \qquad \frac{d\vec{p}}{dt'} = \frac{1}{c^2}\frac{d}{dt'}(\vec{E}\cdot\vec{v})$

$\Delta\vec{p} = \vec{F}\cdot\Delta t =$ Impulso $\qquad \omega_c = \frac{qB}{\gamma m}$

BETATRÓN:

1)Condición del betatrón para radio $R = k$ constante: $B_R = \frac{\int_S \vec{B}\cdot\vec{n}dA}{2\pi R^2}$

2) Flujo magnético: $\phi_m = \int_S \vec{B}\cdot\vec{n}dA$

3) Fuerza electromotriz: $\epsilon = -\frac{d\phi_m}{dt}$

4) Trabajo de la fem: $W = q|\epsilon|$ 5) Intensidad: $I = \rho v S = \frac{Nq}{T}$

Radiación multipolar:

Dipolo eléctrico: $\sum_i q_i\vec{r'}_i$

Dipolo magnético: $\frac{1}{2}\sum_i q_i(\vec{r'}_i\times\vec{v'}_i)$

Cuadrupolo eléctrico: $Q_{ij} = \sum_r q_r(3x'_{ij} - r'^2\delta_{ij})$

Dipolo eléctrico/dipolo magnetico:

$\bar{A}(\bar{r}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\dot{\bar{p}} \qquad \bar{A}(\bar{r}) = \frac{-\mu_0}{4\pi c}\frac{e^{ikr}}{r}[\bar{n}\times\dot{\bar{m}})$

$\bar{H} = -\frac{1}{4\pi c}\frac{e^{ikr}}{r}(\bar{n}\times\ddot{\bar{p}}) \qquad \bar{H} = -\frac{1}{4\pi c^2}\frac{e^{ikr}}{r}[(\bar{n}\times\ddot{\bar{m}})\times\bar{n}]$

$\bar{E} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}[\bar{n}\times(\bar{n}\times\ddot{\bar{p}})] \qquad \bar{E} = \frac{\mu_0}{4\pi c}\frac{e^{ikr}}{r}[\bar{n}\times\ddot{\bar{m}}]$

$\bar{P}(t) = p_0 e^{-i\omega t} \qquad \bar{m} = m\cdot e^{-i\omega t}$

Dipolo el. osc. $d \ll \frac{\lambda}{\text{so}} \qquad r \gg \frac{c}{\omega} \qquad P_0 = q\cdot d \qquad \bar{P}(t) = q_0 d\cos(wt)\hat{u}$

$t \to t - \frac{r}{c} \qquad P(t) = \int_\Omega \frac{dP}{d\Omega}d\Omega = \int_\Omega SR^2 d\Omega = \int_\Omega \varepsilon_0 cE^2R^2d\Omega = \frac{\mu_0 P_0^2\omega^4}{12\pi c}$

Campos de radiación de un cuadrupolo oscilante de componentes:

$$Q_{ij}e^{-iwt} = q_r(3x_ix_j - r^2\delta_i^j)e^{-iwt}\left.\begin{matrix} B_{rad} = \frac{\mu_0 k^2 e^{i(kr-wt)}}{4\pi}\frac{1}{r}u_r\times q \\ E_{rad} = c\,(B_{\text{rad}}\times u_r)\end{matrix}\right\} \qquad q = -\frac{i\omega}{2}n_jQ_{ij} \quad \frac{dP}{d\Omega} = \frac{\mu_0 c}{32\pi^2}k^4|u_r\times q|^2$$

Resistencia a la radiación $R = 80\pi^2\left(\frac{d}{\lambda}\right)^2\Omega$; $\qquad I^2R =$ potencia

Directiv. $D = \left(\frac{dP}{d\pi}\right)_{\text{máx}} / \left(\frac{P}{4\pi}\right) \qquad D = 3/2$

Dipolo mag. osc. $I(t) = I_0 cos(\omega t)$

$\bar{m}(t) = I_0\pi a^2 cos(\omega t) \qquad m_0 = I_0\pi a^2 P = \frac{\mu_0 m_0^2\omega^4}{12\pi c^3}$

$R_{rad} = 320\pi^6(a/\lambda)^4\Omega \qquad D = 3/2$

Otros

Th. de Gauss: $\int_V \partial_i K^i d^4x = \int_S n_iK^i dS$

Din. cargas a partir de ec. $\frac{dp^i}{d\tau} = qF^{ij}u_j$:

$\frac{dW}{dt} = q\vec{E}\cdot\vec{v} \qquad \vec{\omega}_c = \frac{e\vec{B}}{\gamma m}$ (frec. sincrotón) $\frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v}\times\vec{B})$

$E = \frac{\sigma 0}{\varepsilon}$ condensador $B = \frac{\mu_0 Ir}{2\pi R}$ hilo $r<R$

$B = \frac{\mu_0 I}{2\pi r} \qquad E = \frac{\lambda}{2\pi\varepsilon_0 R}$ hilo $r>R$

$B = \mu_0 In$ (solenoide); $B = \frac{\mu_0 I}{2R}$ (centro de espira circular); $B = \frac{\mu_0 IR^2}{2(z^2+R^2)^{3/2}}$ (eje de espira)

$\mathcal{E}$