

Tema 5.

Fractali.

Termenul *fractal* a fost generalizat pentru a include obiecte din afara definitiei originale a lui Mandelbrot. Prin *obiect fractal* vom intelege orice

obiect care are proprietatea de autoasemanare (self-similarity in lb. engleza).

Obiectele obtinute in cele ce urmeaza sunt aproximatii ale unui obiect fractal

ideal, fiind obtinute intr-un numar finit de iteratii.

1. Multimea Julia-Fatou : se obtine utilizand un proces iterativ.

Plecand de la $z_0 \in \mathbb{C}$ se obtin numerele complexe $(z_n)_{n \geq 0}$ astfel :

$z_{n+1} = z_n^2 + c$, unde $c \in \mathbb{C}$. Un numar complex $x \in \mathbb{C}$ apartine multimii

Julia-Fatou J_c daca, plecand cu $z_0 = x$, urmatoarele conditii nu sunt

indeplinite : $(\exists z \in \mathbb{C})(\lim_{n \rightarrow \infty} z_n = z)$ sau $\lim_{n \rightarrow \infty} |z_n| = \infty$.

In programul [urmator](#) s-au generat 2 (aproximari ale) multimi Julia-Fatou

corespunzatoare valorilor c_1 si c_2 pentru $c \in \mathbb{C}$ indicate in figurile de mai jos.

Cele 2 conditii de mai sus au fost utilizate in program sub forma

$(\exists n_0 > 0)(z_{n_0} = z_{n_0+1})$ si $(\exists n_0 \geq 0)(\exists M > 0)(|z_{n_0}| > M)$ i.e., intr-un numar finit de

iteratii n_0 , se testeaza daca sirul (z_n) devine constant sau $|z_n|$

depaseste

M (ales suficient de mare).

Daca dupa terminarea celor n_0 iteratii nici o conditie nu a fost

adevarata atunci

punctul respectiv apartine multimii Julia-Fatou si a fost colorat cu rosu in figura.

2. [Multimea Mandelbrot](#) : se obtine tot printr-un proces iterativ.

Un numar $c \in \mathbb{C}$ apartine multimii Mandelbrot M daca $\lim_{n \rightarrow \infty} |z_n| \neq \infty$, unde

sirul $(z_n)_{n \geq 0}$ este obtinut astfel : $z_0 = 0 + 0i$ iar $z_{n+1} = z_n^2 + c$, $\forall n \geq 0$.

- a. Construiti multimea Mandelbrot bazandu-va pe urmatoarea proprietate a acesteia (mai bine zis pe negatia ei): daca numarul complex c apartine multimii

Mandelbrot atunci $|z_n| \leq 2$, $\forall n \geq 0$. In concluzie procesul iterativ se opreste daca $|z_n|$ depaseste 2.

- b. Realizati o clasificare a punctelor care nu apartin multimii Mandelbrot, colorandu-le cu culori diferite in functie de numarul de iteratii care a fost necesar pentru a detecta neapartenenta.

3. In programul [urmator](#) sunt generati, recursiv, fractali construiti prin geometria „turtle” (in acest stil de grafica imaginile sunt obtinute prin deplasarea unui cursor pe ecran, acesta deplasandu-se conform unor comenzi : desenare, rotatie catre stanga sau dreapta cu un anumit unghi):

- [curba lui Koch \(fulg de zapada\)](#),
- [arbori binari](#),
- [arborele lui Perron](#),
- [curba lui Hilbert](#).

4. Desenati imaginile urmatoare (utilizand geometria „turtle”): [1](#), [2](#), [3](#).

$$c = -0.12375 + 0.056805i$$

(0.8, 0.4)

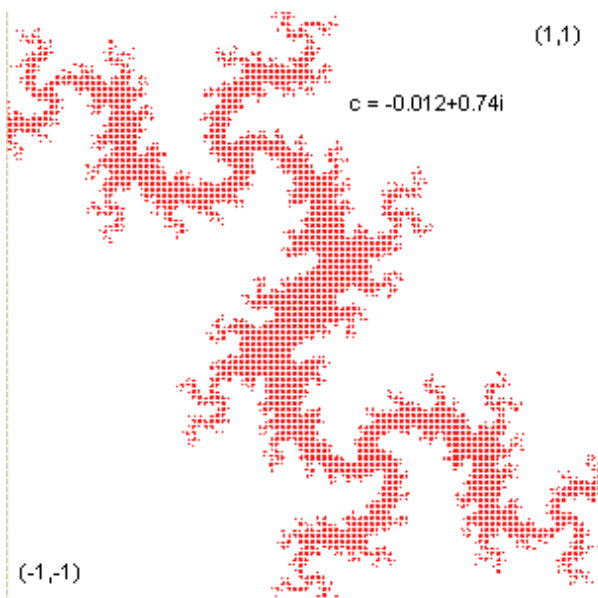


(-0.8, -0.4)

Multime Julia-Fatou

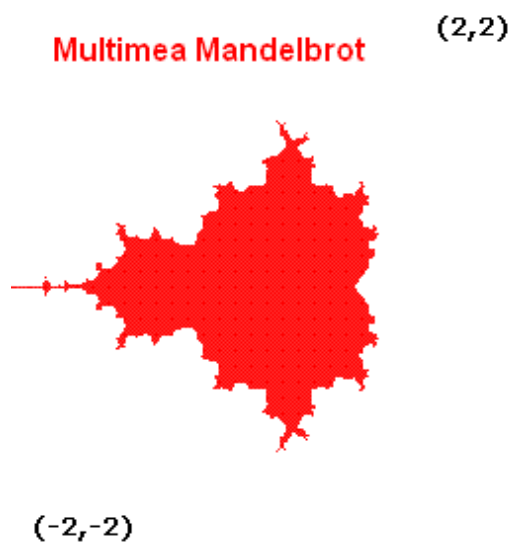
(1,1)

$$c = -0.012 + 0.74i$$

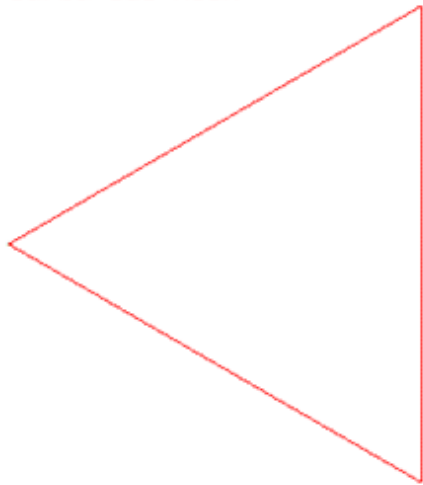


(-1,-1)

Multime Julia-Fatou

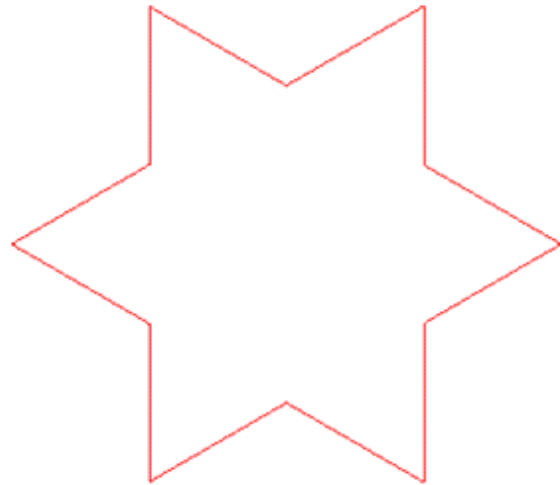


curba lui Koch



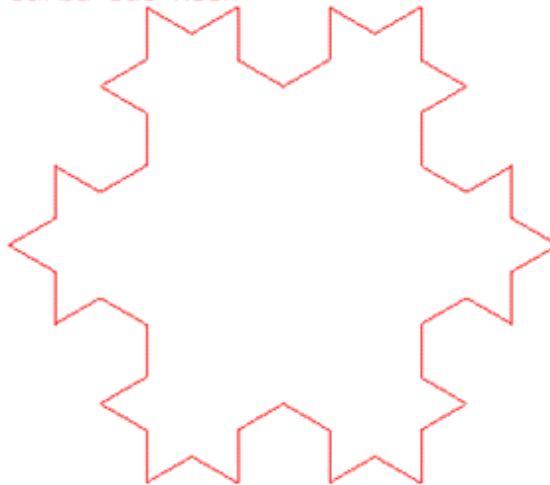
Nivel= 0

curba lui Koch



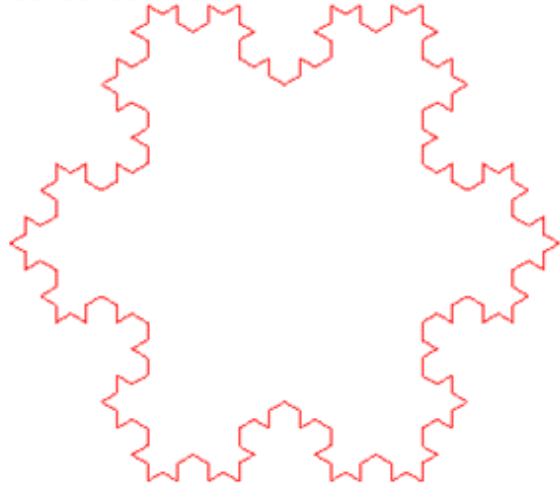
Nivel= 1

curba lui Koch



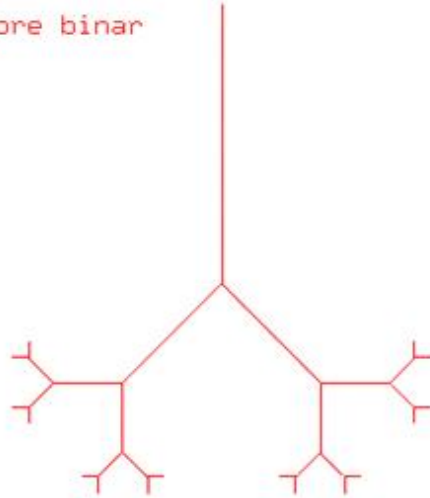
Nivel= 2

curba lui Koch

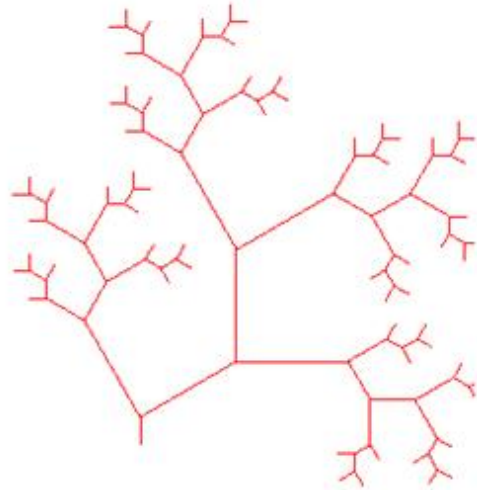
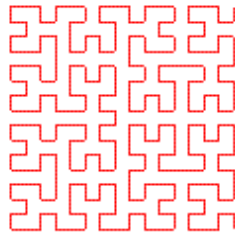


Nivel= 3

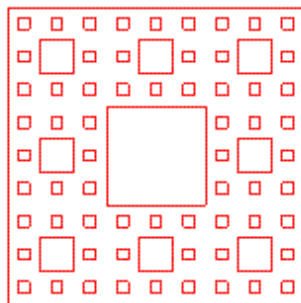
arbore binar



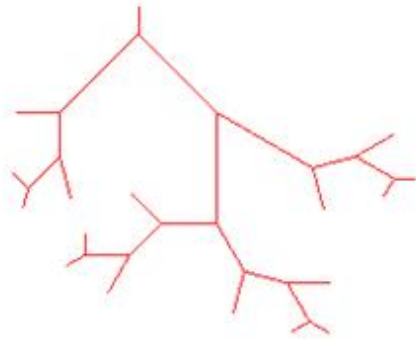
Nivel= 4

arbore Perron
Nivel= 3curba Hilbert
Nivel= 4

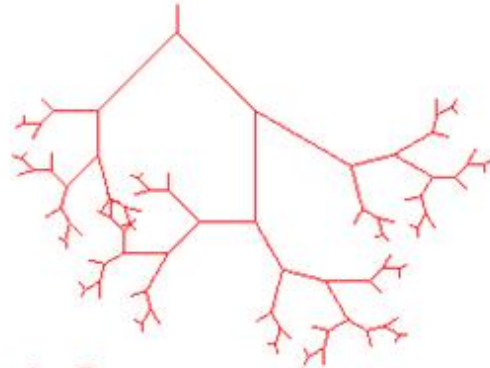
Imaginea 1



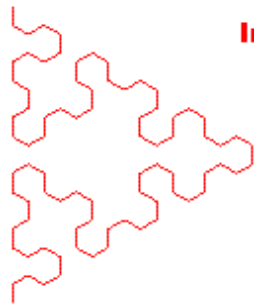
Nivel= 2

Imaginea 2

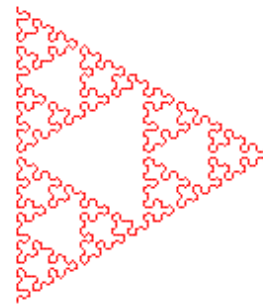
Nivel= 2



Nivel= 3



Nivel= 4

Imaginea 3

Nivel= 6

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