- 13-6根据热力学第二定律()。
- (A) 自然界中的一切自发过程都是不可逆的;
- (B) 不可逆过程是不能向相反方向进行的过程;
- (C) 热量可以从高温物体传到低温物体,但不能从低温物体传到高温物体;
- (D) 任何过程总是沿着熵增加的方向进行.

(A)

13-37 有v mol定体热容3R/2的理想气体,从状态A(p_A 、 V_A 、 T_A)分别经如图所示的ADB过程和ACB过程,到达状态B(p_B 、 V_B 、 T_B)。问在这两个过程中气体的熵变各为多少?图中AD是等温线。

解:
$$\Delta S_{ADB} = \int_{A}^{D} \frac{dQ}{T} + \int_{D}^{B} \frac{dQ}{T}$$

$$= vR \ln \frac{V_{D}}{V_{A}} + vC_{p,m} \ln \frac{T_{B}}{T_{D}}$$

$$= vR \ln \frac{V_{B}}{V_{A}} + vC_{V,m} \ln \frac{T_{B}}{T_{A}}$$

$$= vR \ln \frac{V_{B}}{V_{A}} + vC_{V,m} \ln \frac{T_{B}}{T_{A}}$$

$$\Delta S_{ACB} = \int_{A}^{C} \frac{dQ}{T} + \int_{C}^{B} \frac{dQ}{T}$$

$$\begin{aligned} c_B &= \int_A^C \frac{\mathrm{d}\mathcal{Q}}{T} + \int_C^B \frac{\mathrm{d}\mathcal{Q}}{T} \\ &= \nu C_{p,m} \ln \frac{T_C}{T_A} + \nu C_{V,m} \ln \frac{T_B}{T_C} = \nu C_{V,m} \ln \frac{T_B}{T_A} + \nu R \ln \frac{T_C}{T_A} \\ &= \nu R \ln \frac{V_B}{V_A} + \nu C_{V,m} \ln \frac{T_B}{T_A} \end{aligned}$$

13-38 气缸内有0.1mol的氧气,(视为刚性分子的理想气体),作如图所示的循环过程,其中ab为等温过程,bc为等体过程,ca为绝热过程. 已知 $V_b = 3V_a$,

求: (1) 该循环的效率?

(2) 从状态b到状态c,氧气的熵变AS.

解: (1) ab, 吸热
$$Q_{ab} = \nu RT \ln \frac{V_b}{V_a} = \nu RT \ln 3$$
 p_a bc, 放热 $|Q_{bc}| = \nu C_{V,m} (T_b - T_c)$ ca, 绝热 $V_c^{\gamma-1} T_c = V_a^{\gamma-1} T_a$

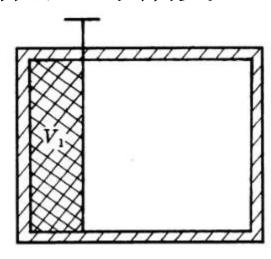
$$\frac{T_c}{T_a} = \left(\frac{V_a}{V_c}\right)^{\gamma - 1} = \left(\frac{1}{3}\right)^{2/5} \quad \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{vC_{V,m}\left(1 - (1/3)^{2/5}\right)T}{vRT\ln 3} = 19.1\%$$

(2)
$$\Delta S = \int_{b}^{c} \frac{dQ}{T} = \nu C_{V,m} \ln \frac{T_{c}}{T_{b}}$$
 $\Delta S = \int_{b}^{a} \frac{dQ}{T} + \int_{c}^{a} \frac{dQ}{T} = \nu R \ln \frac{V_{a}}{V_{b}} = -0.91 \text{J/K}$

13-40 有一体积为 2.0×10^{-2} m³的绝热容器,用一隔板将其分为两部分,如图所示。开始时在左边(体积 V_1 = 5.0×10^{-3} m³)一侧充有1mol理想气体,右边一侧为真空。现打开隔板让气体自由膨胀而充满整个容器,求熵变。

解: 理想气体绝热自由膨胀过程,气体内能不变,温度不变。

假设一<mark>可逆等温膨胀过程连接</mark>初、 末态



$$\Delta S = \int_{1}^{2} \frac{dQ}{T} = \int_{V_{1}}^{V_{2}} \nu R \frac{dV}{V} = \nu R \ln \frac{V_{2}}{V_{1}} = 11.52 \text{ J/K}$$

补充1. 如图所示,一定量(刚性)双原子分子理想气体经历的循环过程由直线过程AB、等体过程BC和等压过程CA构成的。求: (1)理想气体在AB过程中内能的该变量ΔE、对外界做的功W和从外界吸收的热量Q; (2)在一个循环中理想气体对外界所做的功; (3)循环的效率;

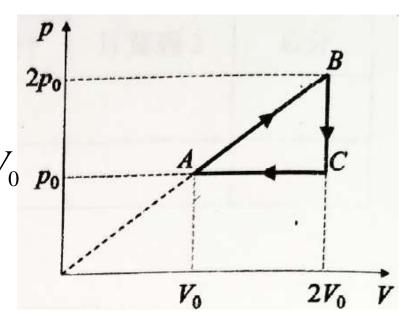
(4)AB过程的摩尔热容

$$(1)i=5,$$

$$\Delta E_{AB} = v \frac{i}{2} R \Delta T = \frac{i}{2} \Delta (pV) = \frac{15}{2} p_0 V_0$$

$$W_{AB} = \int_A^B p dV = \frac{3p_0 V_0}{2}$$

$$Q_{AB} = \Delta E + W = 9 p_0 V_0$$



$$(2) W = \frac{1}{2} p_0 V_0$$

(3)
$$\eta = \frac{W}{Q_1} = \frac{W}{Q_{AB}} = 5.6\%$$

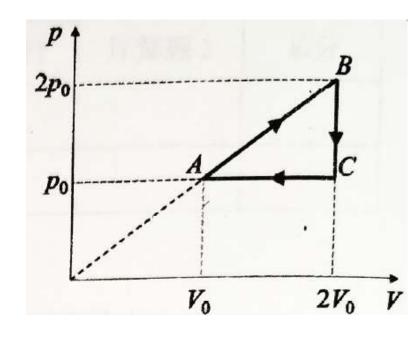
$$(4) AB 过程方程 $p = \frac{p_0}{V_0} V,$$$

$$p dV = V dp \Rightarrow p dV = \frac{1}{2} \nu R dT$$

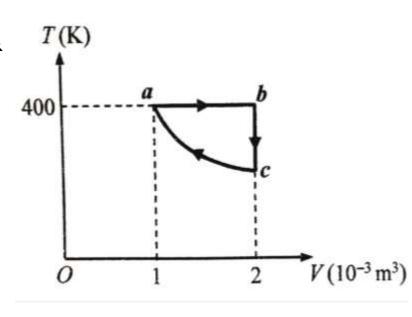
$$dQ_{AB} = dE + pdV = v\frac{i}{2}RdT + \frac{1}{2}vRdT = v\frac{i+1}{2}RdT$$

$$C_m = \frac{i+1}{2}R = 3R = 25 \text{ J/mol/K}$$

或
$$C_m = \frac{Q}{v\Delta T} = \frac{QR}{\Delta(pV)} = 3R$$



补充2. 1mol刚性双原子分子理想气体,其循环过程的T-V曲线如图。已知ca为绝热过程,求: (1)c点的热力学温度; (2)经过一个循环,系统对外界所作的净功; (3)工作在该循环下热机的效率。

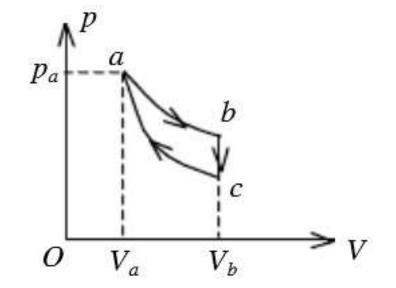


解:pV图如图

(1)
$$i = 5, \gamma = 7/5$$

$$\frac{T_c}{T_a} = \left(\frac{V_a}{V_c}\right)^{\gamma - 1} = \left(\frac{1}{2}\right)^{2/5}$$

$$T_c = (1/2)^{2/5} T_a = 303.14 \text{K}$$



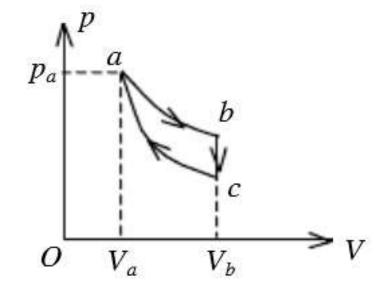
(2)
$$W_{ab} = \nu RT \ln \frac{V_b}{V_a} = \nu RT \ln 2 = 2304 J$$

$$W_{bc} = 0$$

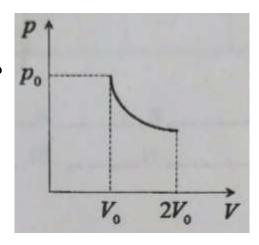
$$W_{ca} = \frac{p_a V_a - p_c V_c}{1 - \gamma} = -\nu \frac{i}{2} R(T_a - T_c) = -2012 J$$

$$W = 292 \, \text{J}$$

(3)
$$\eta = \frac{W}{Q_{ab}} = \frac{W}{W_{ab}} = 12.7\%$$



补充3. 一定量的单原子分子的理想气体经历准静态过程 $pV^2 = 常数$,体积变为原来的两倍。已知 p_0 和 V_0 ,求(1)整个过程中,气体对外界做的功;(2)整个过程中,气体内能的改变量;(3)该过程的摩尔热容;(4)整个过程中,1mol这种气体的熵变。



P: (1)
$$W = \int_{V_0}^{2V_0} p dV = \int_{V_0}^{2V_0} \frac{p_0 V_0^2}{V^2} dV = \frac{p_0 V_0}{2}$$

(2)
$$i = 3, C_{V,m} = \frac{3}{2}R$$

$$p_2 = \frac{p_0 V_0^2}{(2V_0)^2} = \frac{p_0}{4}$$

$$\Delta E = vC_{V,m}\Delta T = \frac{3}{2}\Delta(pV) = \frac{3}{2}\left(\frac{p_0}{4}2V_0 - p_0V_0\right) = -\frac{3}{4}p_0V_0$$

$$(3) pV^2 = C \Longrightarrow VT = C'$$

$$\Rightarrow \frac{dV}{V} = -\frac{dT}{T}$$

$$\Rightarrow pdV = \frac{C}{V^2}dV = -\frac{C}{V}\frac{dT}{T} = -\nu RdT$$

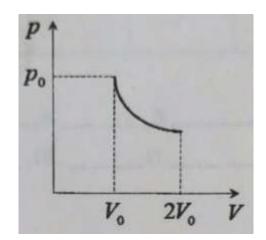
$$dQ = dE + pdV = \frac{3}{2}vRdT - vRdT = \frac{1}{2}vRdT$$

$$C_m = \frac{\mathrm{d}Q}{v\mathrm{d}T} = \frac{1}{2}R$$

或
$$Q = \Delta E + W = -\frac{1}{4}p_0V_0 = vC_m\Delta T = -vC_m\frac{T_0}{2} = -\frac{C_mp_0V_0}{2R}$$

(4)
$$\Delta S = \int \frac{dQ}{T} = \frac{1}{2}R \int \frac{dT}{T} = \frac{1}{2}R \ln \frac{T_2}{T_0} = \frac{1}{2}R \ln \frac{V_0}{2V_0} = -\frac{1}{2}R \ln 2$$

或
$$\Delta S = \int \frac{dE + pdV}{T} = vC_{V,m} \ln \frac{T_2}{T_0} + vR \ln \frac{V_2}{V_0} = -\frac{1}{2}R \ln 2 = -2.88 \text{ J/K}$$



习题13-41 一均匀细杆长L,单位长度的热容为C,开始 时沿细杆方向温度由低到高分布,一端为T₁,一端为T₂ $(>T_1)$ 。在热传导作用下,最后温度均为为 $(T_1+T_2)/2$,求 该过程细杆的熵变。

解:杆上任一小段dx的熵变

过程细杆的熵变。

*** 杆上任一小段dx的熵变**

$$dS_x = \int \frac{dQ}{T} = \int_{T_1 + \frac{T_2 - T_1}{L}x}^{\frac{T_1 + T_2}{2}} \frac{C_l dx dT}{T} \qquad \mathbf{d}x$$

$$= C_l dx \left[\ln \frac{T_1 + T_2}{2} - \ln(T_1 + \frac{T_2 - T_1}{L}x) \right]$$

整个杆的熵变

$$\Delta S = \int dS_x = C_l \int_0^L \left[\ln \frac{T_1 + T_2}{2} - \ln \left(T_1 + \frac{T_2 - T_1}{L} x \right) \right] dx$$

$$= C_l L \left(\ln \frac{T_1 + T_2}{2} - \frac{T_2 \ln T_2 - T_1 \ln T_1}{T_2 - T_1} + 1 \right)$$