

4.2 已知正弦电流的波形如图所示，试求此正弦电流的幅值、周期、频率、角频率和初相，并写出该电流表达式。 W4-3

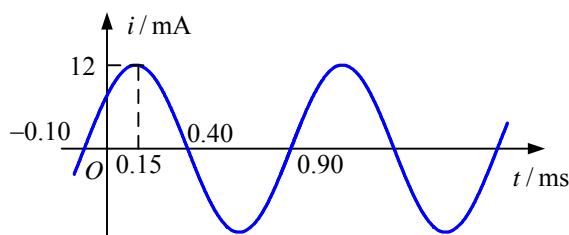


图 题 4.2

【解】

- (1) 电流的幅值为 12mA；
- (2) 周期为 $T = 0.9 - (-0.1) = 1\text{ms}$ ；
- (3) 频率 $f = \frac{1}{T} = \frac{1}{1\text{ms}} = 1000\text{Hz}$ ；
- (4) 角频率 $\omega = 2\pi f = 2000\pi \text{ rad/s}$ ；
- (5) 初相 $2000\pi + \varphi = 0$, $t = 0.15\text{ms}$, 初相为 $\varphi = -\frac{3}{10}\pi$ ；
- (6) 电流表达式 $i(t) = 12\cos(6280t - \frac{3}{10}\pi)\text{mA}$ ；

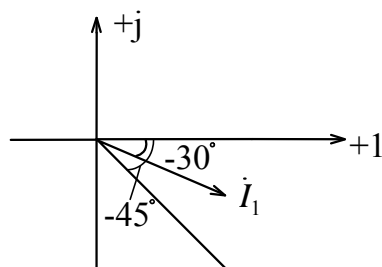
4.3 将下列各正弦量表示为有效值相量，并绘出相量图。 W4-4

- (1) $i_1(t) = 2\cos(\omega t - 30^\circ)\text{A}$ ； $i_2(t) = 3\sin(\omega t + \pi/4)\text{A}$ 。
- (2) $u_1(t) = 100\cos(314t + 2\pi/3)\text{V}$ ； $u_2(t) = -250\cos(314t)\text{V}$ 。

【解】(1) $i_1(t) = 2\cos(\omega t - 30^\circ)\text{A}$ ，相量 $\dot{I}_1 = \sqrt{2}\angle -30^\circ\text{A}$ ；

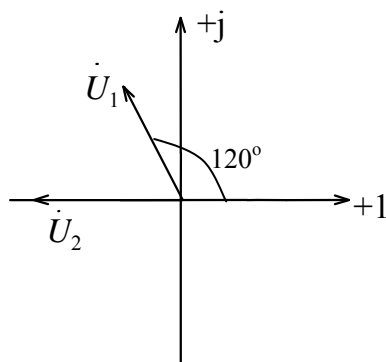
$$i_2(t) = 3\sin(\omega t + 45^\circ)\text{A} = 3\cos(\omega t + 45^\circ - 90^\circ)\text{A} = 3\cos(\omega t - 45^\circ)\text{A},$$

$$\text{相量 } \dot{I}_2 = \frac{3\sqrt{2}}{2}\angle -45^\circ\text{A}$$



(2) $u_1(t) = 100\cos(314t + 120^\circ)\text{V}$ ，相量 $\dot{U}_1 = \frac{100\sqrt{2}}{2}\angle 120^\circ\text{V}$ ；

$u_2(t) = -250\cos(314t)\text{V} = 250\cos(314t - 180^\circ)\text{V}$ ，相量 $\dot{U}_2 = \frac{250\sqrt{2}}{2}\angle -180^\circ$



4.4 设角频率为 ω ，写出下列电压、电流相量所代表的正弦电压和电流：6.2

(a) $\dot{U}_m = 10 \angle -10^\circ \text{V}$ ； (b) $\dot{U} = (-6 - j8) \text{V}$ ； (c) $\dot{I}_m = (0.2 - j20.8) \text{A}$ ； (d) $\dot{I} = -30 \text{A}$ 。

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【解】(a) $u_m = 10 \cos(\omega t - 10^\circ) \text{V}$

(b) $\dot{U} = \sqrt{(-6)^2 + (-8)^2} \angle \arctan \frac{-8}{-6} = 10 \angle 233.1^\circ \text{V}$, $u = 10\sqrt{2} \cos(\omega t + 233.1^\circ) \text{V}$

(c) $\dot{I}_m = \sqrt{0.2^2 + (-20.8)^2} \angle \arctan \frac{-20.8}{0.2} = 20.8 \angle -89.4^\circ \text{A}$, $i = 20.8 \cos(\omega t - 89.4^\circ) \text{A}$

(d) $\dot{I} = 30 \angle 180^\circ \text{A}$, $i = 30\sqrt{2} \cos(\omega t + 180^\circ) \text{A}$

4.7 图(a)电路，已知 $i_{S1} = I_m \cos(\omega t + \pi/6) \text{A}$ ， $i_{S2} = I_m \cos(\omega t - \pi/3) \text{A}$ ，求电压 u_C 和 u_L ；
图(b)电路，已知 $u_{S1} = U_m \cos \omega t \text{V}$ ， $u_{S2} = U_m \cos(\omega t + \pi/2) \text{V}$ ，求电流 i_C 和 i_L 。新编

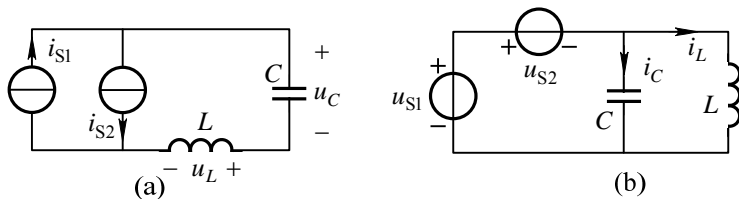


图 题 4.7

【解】(a) 依题意得，电流的相量形式为 $\dot{I}_{S1} = \frac{I_m}{\sqrt{2}} \angle 30^\circ \text{A}$ ， $\dot{I}_{S2} = \frac{I_m}{\sqrt{2}} \angle -60^\circ \text{A}$ ，

这样

$$\begin{aligned} \dot{U}_C &= \frac{1}{j\omega C} (\dot{I}_{S1} - \dot{I}_{S2}) = \frac{1}{j\omega C} \times \frac{I_m}{\sqrt{2}} \times \left[\left(\frac{\sqrt{3}}{2} + j0.5 \right) - \left(0.5 - j\frac{\sqrt{3}}{2} \right) \right] \\ &= \frac{1}{j\omega C} \times \frac{I_m}{\sqrt{2}} \times \left[\left(\frac{\sqrt{3}}{2} - 0.5 \right) + j \left(\frac{\sqrt{3}}{2} + 0.5 \right) \right] = \frac{1}{j\omega C} \times \frac{I_m}{\sqrt{2}} \times \sqrt{2} \angle \arctan \frac{\frac{\sqrt{3}}{2} + 0.5}{\frac{\sqrt{3}}{2} - 0.5} \\ &= \frac{I_m}{j\omega C} \angle 75^\circ = \frac{I_m}{\omega C} \angle (75^\circ - 90^\circ) = \frac{I_m}{\omega C} \angle -15^\circ \text{V} \end{aligned}$$

$$\begin{aligned} \dot{U}_L &= j\omega L (\dot{I}_{S1} - \dot{I}_{S2}) = j\omega L \times \frac{I_m}{\sqrt{2}} \times \left[\left(\frac{\sqrt{3}}{2} + j0.5 \right) - \left(\frac{\sqrt{3}}{2} - j0.5 \right) \right] \\ &= j\omega L \times \frac{I_m}{\sqrt{2}} \times \sqrt{2} \angle 75^\circ = \omega L I_m \angle 75^\circ + 90^\circ = \omega L I_m \angle 165^\circ \text{V} \end{aligned}$$

所以 $u_C = \frac{\sqrt{2} I_m}{\omega C} \cos(\omega t - 15^\circ) \text{V}$ ， $u_L = \sqrt{2} \omega L I_m \cos(\omega t + 165^\circ) \text{V}$ 。

(b) 电压的相量形式 $\dot{U}_{S1} = \frac{U_m}{\sqrt{2}} \angle 0^\circ \text{V}$ ， $\dot{U}_{S2} = \frac{U_m}{\sqrt{2}} \angle 90^\circ \text{V}$

$$\dot{U}_{S1} - \dot{U}_{S2} = \frac{U_m}{\sqrt{2}} \angle 0^\circ \text{V} - \frac{U_m}{\sqrt{2}} \angle 90^\circ \text{V} = \frac{U_m}{\sqrt{2}} (1 - j) \text{V} = U_m \angle -45^\circ \text{V}$$

这样就有

$$\dot{I}_C = j\omega C \times (\dot{U}_{S1} - \dot{U}_{S2}) = \omega C U_m \angle 45^\circ \text{A},$$

$$\dot{I}_L = \frac{1}{j\omega L} \times (\dot{U}_{S1} - \dot{U}_{S2}) = \frac{1}{j\omega L} \times U_m \angle -45^\circ \text{V} = \frac{U_m}{\omega L} \angle -135^\circ \text{A}$$

$$\text{所以 } i_C = \sqrt{2}\omega C U_m \cos(\omega t + 45^\circ) \text{A}, \quad i_L = \frac{\sqrt{2}U_m}{\omega L} \cos(\omega t - 135^\circ) \text{A}。$$

4.8 图示电路中，若电流 $i = \cos(314t) \text{A}$ ，试求电压 u_R 、 u_L 、 u_C 和 u ，并画出相量图。

W4-5

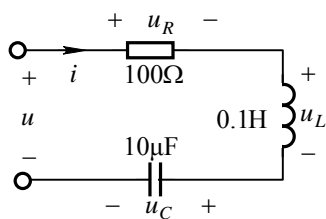


图 题 4.8

$$\text{【解】 } j\omega L = j314 \text{rad/s} \times 0.1 \text{H} = j31.4\Omega, \quad -j\frac{1}{\omega C} = -j\frac{1}{314 \text{rad/s} \times 10\mu\text{F}} = -j318.5\Omega$$

$$\text{总阻抗为 } Z = 100 + j(-318.5 + 31.4) = 100 - j287.1\Omega$$

$$\text{这样 } \dot{U}_R = R \times \dot{I} = 100\Omega \times \frac{\sqrt{2}}{2} \angle 0^\circ \text{A} = 70.7 \angle 0^\circ \text{V},$$

$$\dot{U}_L = j\omega L \times \dot{I} = 31.4 \angle 90^\circ \Omega \times \frac{\sqrt{2}}{2} \angle 0^\circ \text{A} = 22.2 \angle 90^\circ \text{V},$$

$$\dot{U}_C = -j\frac{1}{\omega C} \times \dot{I} = 318.5 \angle -90^\circ \Omega \times \frac{\sqrt{2}}{2} \angle 0^\circ \text{A} = 225.2 \angle -90^\circ \text{V}$$

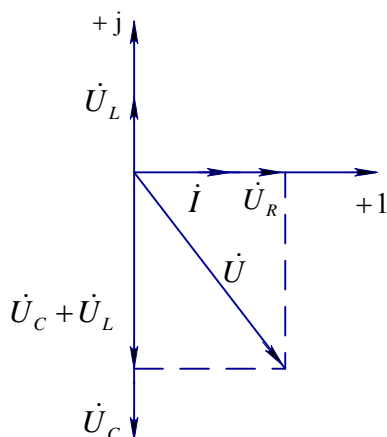
$$\dot{U} = Z \times \dot{I} = (100 - j287.1)\Omega \times \frac{\sqrt{2}}{2} \angle 0^\circ \text{A} = 214.9 \angle -70.8^\circ \text{V}$$

最后，

$$u_R = 100 \cos(314t) \text{V}, \quad u_L = 31.4 \cos(314t + 90^\circ) \text{V},$$

$$u_C = 318.5 \cos(314t - 90^\circ) \text{V}, \quad u = 304 \cos(314t - 70.8^\circ) \text{V}。$$

相量图如下，



4.9 图示电路，已知 $i_R = \sqrt{2} \cos(\omega t) \text{ A}$ ， $\omega = 2 \times 10^3 \text{ rad/s}$ ，求各元件电压、电流及总电压 u ，并作各电压、电流的相量图。6.7

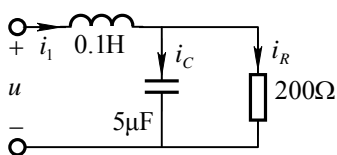
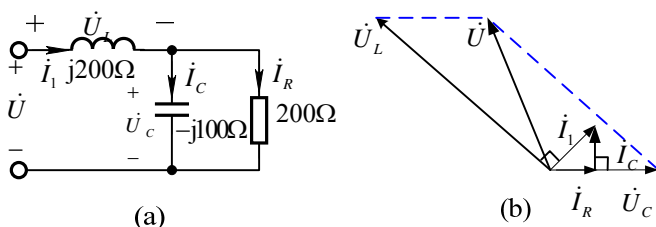


图 题 4.9

【解】感抗 $X_L = \omega L = (2 \times 10^3) \text{ rad/s} \times 0.1 \text{ H} = 200 \Omega$

$$\text{容抗 } X_C = -\frac{1}{\omega C} = -\frac{1}{(2 \times 10^3) \text{ rad/s} \times (5 \times 10^{-6}) \text{ F}} = -100 \Omega$$

原电路的相量模型如图(a)所示。



由已知得 $\dot{I}_R = 1 \angle 0^\circ \text{ A}$ ，按从右至左递推的方法求得各元件电压、电流相量如下：

$$\dot{U}_C = \dot{I}_R R = 200 \angle 0^\circ \text{ V}$$

$$\dot{I}_C = \frac{\dot{U}_C}{jX_C} = \frac{200 \angle 0^\circ \text{ V}}{-j100 \Omega} = 2 \angle 90^\circ \text{ A}$$

$$\dot{I}_1 = \dot{I}_C + \dot{I}_R = (1 \angle 0^\circ + 2 \angle 90^\circ) \text{ A} = (1 + 2j) \text{ A} = \sqrt{5} \angle 63.43^\circ \text{ A}$$

$$\dot{U}_L = jX_L \dot{I}_1 = j200 \times \sqrt{5} \angle 63.43^\circ \text{ V} = 200\sqrt{5} \angle 153.43^\circ \text{ V}$$

$$\dot{U} = \dot{U}_L + \dot{U}_C = (200\sqrt{5} \angle 153.43^\circ + 200 \angle 0^\circ) \text{ V} = 200\sqrt{2} \angle 135^\circ \text{ V}$$

由以上各式画出电压、电流相量图如图(b)所示。由各相量值求得各元件电压、电流瞬时值分别为

$$i_C = 2\sqrt{2} \cos(\omega t + 90^\circ) \text{A}, i_1 = \sqrt{10} \cos(\omega t + 63.43^\circ) \text{A}$$

$$u_R = u_C = 200\sqrt{2} \cos(\omega t) \text{V}, u_L = 200\sqrt{10} \cos(\omega t + 153.43^\circ) \text{V}$$

$$u = 400 \cos(\omega t + 135^\circ) \text{V}$$

4.10 图示各电路，已标明电压表和电流表的读数，试求电压 u 和电流 i 的有效值。6.6

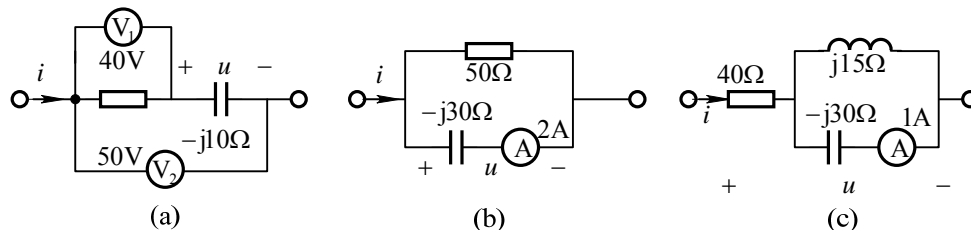


图 题 4.10

【解】(a) RC 串联电路中电阻电压与电容电压相位正交，各电压有效值关系为

$$U = \sqrt{U_2^2 - U_1^2} = \sqrt{50^2 - 40^2} \text{V} = 30 \text{V}$$

电流 i 的有效值为

$$I = I_C = \frac{U}{|X_C|} = \frac{30 \text{V}}{10 \Omega} = 3 \text{A}$$

$$(b) \quad U = |X_C| I_C = 30 \Omega \times 2 \text{A} = 60 \text{V}$$

$$I_R = \frac{U}{R} = \frac{60 \text{V}}{50 \Omega} = 1.2 \text{A}$$

RC 并联电路中电阻电流与电容电流相位正交，总电流有效值为

$$I = \sqrt{I_C^2 + I_R^2} = \sqrt{2^2 + 1.2^2} \text{A} = 2.33 \text{A}$$

$$(c) \quad U_C = |X_C| I_C = 30 \Omega \times 1 \text{A} = 30 \text{V}$$

$$\text{由} \quad U_L = U_C = X_L I \Rightarrow I_L = \frac{U_C}{X_L} = \frac{30 \text{V}}{15 \Omega} = 2 \text{A}$$

并联电容、电感上电流相位相反，总电流为

$$I = |I_L - I_C| = 1 \text{A}$$

电阻电压与电容电压相位正交，总电压为：

$$U = \sqrt{U_C^2 + U_R^2} = \sqrt{30^2 + 40^2} = 50 \text{V}$$

4.11 在图示电路中各元件电压、电流取一致的参考方向。设有效值 $I_1 = 1 \text{A}$ ，取 \dot{I}_1 为参考相量，画出各电流、电压相量图，再根据相量图写出各元件电压、电流有效值相量。6.8

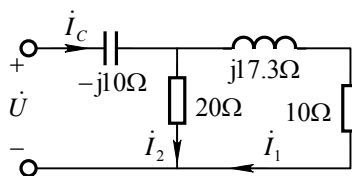
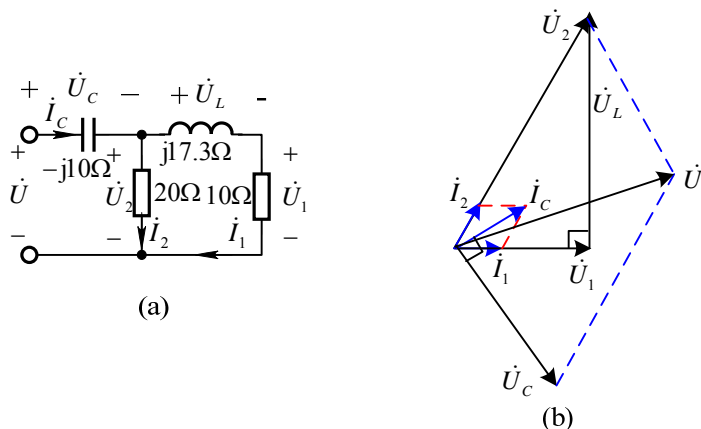


图 题 4.11

【解】对原电路再进行一些标注，如下电路图(a)，



从右至左递推求得各元件电压、电流相量分别为：

$$R: \dot{I}_1 = 1\angle 0^\circ \text{ A}, \dot{U}_1 = 10\text{ V}$$

$$L: \dot{I}_L = \dot{I}_1 = 1\angle 0^\circ \text{ A}, \dot{U}_L = 17.3\angle 90^\circ \text{ V}$$

$$\dot{U}_2 = (10 + j17.3)\text{ V} = 20\angle 60^\circ \text{ V}, \dot{I}_2 = \dot{U}_2 / 20\Omega = 1\angle 60^\circ \text{ A}$$

$$C: \dot{I}_C = \dot{I}_1 + \dot{I}_2 = 1.732\angle 30^\circ \text{ A}, \dot{U}_C = -j10\dot{I}_C = 17.32\angle -60^\circ \text{ V}$$

各元件电压、电流相量图如图(b)所示。

4.13 求图示电路 a、b 两点之间的等效阻抗 Z_{ab} ，设角频率为 ω 。

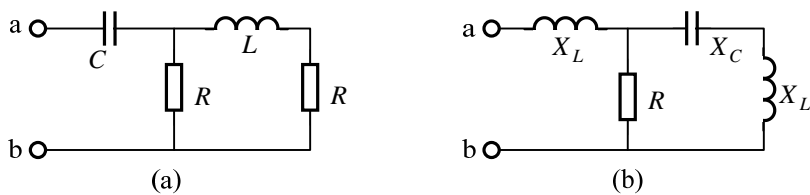
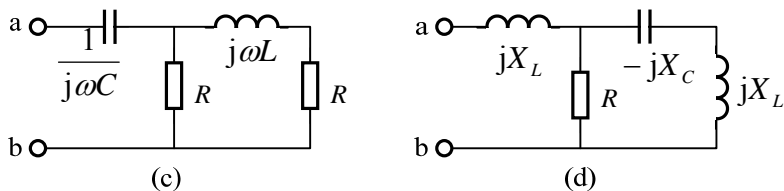


图 题 4.13

【解】(1) 图(a)的电路相量模型为图(c)所示，



等效复阻抗

$$Z_{ab} = \frac{1}{j\omega C} + R // (j\omega L + R) = \frac{1}{j\omega C} + \frac{R \times (j\omega L + R)}{R + j\omega L + R} = \frac{2R^2 - \omega^2 RLC + j(\omega L + \omega R^2 C)}{-\omega^2 LC + j2\omega RC}$$

$$= \frac{\sqrt{(2R^2 - \omega^2 RLC)^2 + (\omega L + \omega R^2 C)^2}}{\sqrt{(\omega^2 LC)^2 + (2\omega RC)^2}} \angle \varphi \Omega, \quad \varphi = \arctan \frac{(\omega L + \omega R^2 C)}{2R^2 - \omega^2 RLC} + \arctan \frac{2R}{\omega L}$$

(b) 图(b)的电路相量模型为图(d)所示,
等效复阻抗

$$Z_{ab} = jX_L + R // (jX_L - jX_C) = jX_L + \frac{R \times (jX_L - jX_C)}{R + jX_L - jX_C} = \frac{(X_L X_C - X_L^2) + j(2RX_L - RX_C)}{R + jX_L - jX_C}$$

$$= \frac{\sqrt{(X_L X_C - X_L^2)^2 + (2RX_L - RX_C)^2}}{\sqrt{(R)^2 + (X_L - X_C)^2}} \angle \varphi \Omega, \quad \varphi = \arctan \frac{2RX_L - RX_C}{X_L X_C - X_L^2} - \arctan \frac{X_L - X_C}{R}$$

4.16 设图示电路中正弦电源角频率分别为 500、1000 和 2000rad/s, 试求此电路在这三种频率下的等效阻抗以及串联等效电路参数。6.11

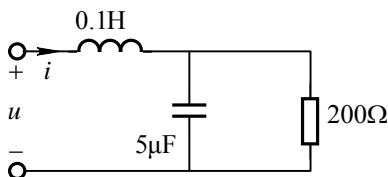


图 题 4.16

【解】利用阻抗的并联及串联等效, 图题 4.16 电路阻抗可表示为

$$Z(\omega) = \frac{jX_C \times 200}{jX_C + 200} + jX_L = \frac{\frac{1}{j\omega C} \times 200}{\frac{1}{j\omega C} + 200} + j\omega L$$

$$Z(\omega) = \frac{jX_C \times 200}{jX_C + 200} + jX_L = \frac{\frac{1}{j\omega C} \times 200}{\frac{1}{j\omega C} + 200} + j\omega L$$

$$= \frac{200}{1 + j200\omega C} + j\omega L = \frac{(200 - 200\omega^2 LC) + j\omega L}{1 + j200\omega C}$$

将 $\omega = 500, 1000, 2000 \text{ rad/s}$ 分别代入上式, 得

$$Z(500) = (160 - j30)\Omega$$

虚部为负值, 故此时等效电路为 RC 串联:

$$R = \text{Re}[Z(500)] = 160\Omega$$

$$X_C = -\frac{1}{\omega C} \text{Im}[Z(500)] = -30\Omega$$

$$C = -\frac{1}{\omega X_C} = 66.6\mu\text{F}$$

$Z(1000) = 100\Omega$, 虚部为零, 故此时等效电路为电阻 R , $R = 100\Omega$ 。

$Z(2000) = (40 + j120)\Omega$, 虚部为正值, 故此时等效电路为 RL 串联:

$$R = \text{Re}[Z(2000)] = 40\Omega$$

$$X_L = \frac{1}{\omega L} = \operatorname{Im}[Z(2000)] = 120\Omega$$

$$L = \frac{1}{\omega X_L} = 0.06\text{H}$$

注释：因为感抗和容抗是频率的函数，因此正弦电流电路的等效参数一般与频率有关。

4.17 图示电路，已知正弦电源角频率 $\omega = 3\text{rad/s}$ ， $L = 1\text{H}$ ， $R = 4\Omega$ ，问电流 i_2 超前于 i_1 多少度？（2007 秋大工试题）

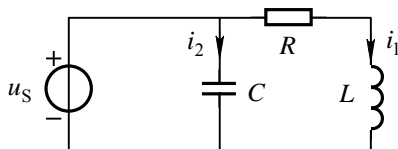


图 题4.17

【解】令 $\dot{U}_S = U_S \angle 0^\circ \text{V}$ ， $\omega L = 3\text{rad/s} \times 1\text{H} = 3\Omega$ ，

$$\dot{I}_2 = \frac{\dot{U}_S}{\frac{1}{j\omega C}} = \omega C U_S \angle 90^\circ \text{A}, \quad \dot{I}_1 = \frac{\dot{U}_S}{j\omega L + R} = \frac{\dot{U}_S}{(4 + j3)\Omega} = \frac{U_S}{5} \angle -\arctan \frac{3}{4} \text{A}$$

所以，电流 i_2 超前于 i_1 角度为： $90^\circ - (-\arctan \frac{3}{4}) = 126.87^\circ$

4.18 图示电路，已知 $R_1 = 10\Omega$ ， $X_C = 17.32\Omega$ ， $I_1 = 5\text{A}$ ， $U = 120\text{V}$ ， $U_L = 50\text{V}$ ，电压 \dot{U} 与电流 i 同相。求 R 、 R_2 和 X_L 。W4-7

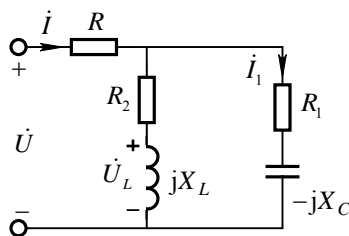
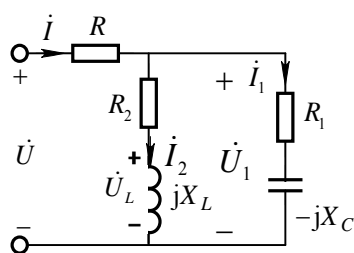


图 题 4.18

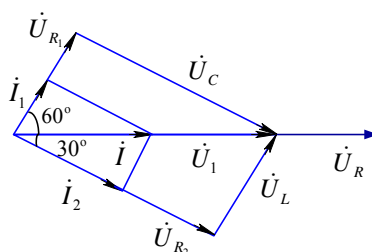
【解】

原电路一些变量进行标定，如下图(a)所示。

设 $\dot{U}_1 = U_1 \angle 0^\circ \text{V}$ ，相量如图(b)所示，



(a)



(b)

通过已知条件, 可知 R_1 与电容 C 串联支路的复阻抗的阻抗角 $\varphi = \arctan \frac{17.32\Omega}{10\Omega} = 60^\circ$,

$$U_1 = \frac{U_{R_1}}{\cos 60^\circ} = 100 \text{ V}, \text{ 根据相量图得到 } \dot{U}_{R_1} = 50 \angle 60^\circ \text{ V}$$

另外根据相量图 (b), $\dot{I}_1 = 5 \angle 60^\circ \text{ A}$, $\dot{I} = 10 \angle 0^\circ \text{ A}$, $\dot{I}_2 = 5\sqrt{3} \angle -30^\circ \text{ A}$,

$$\dot{U}_{R_2} = 50\sqrt{3} \angle -30^\circ \text{ V},$$

$$\text{因此, } R_2 = \frac{\dot{U}_{R_2}}{\dot{I}_2} = \frac{50\sqrt{3} \angle -30^\circ \text{ V}}{5\sqrt{3} \angle -30^\circ \text{ A}} = 10\Omega,$$

$$R = \frac{\dot{U}_R}{\dot{I}} = \frac{\dot{U} - \dot{U}_1}{\dot{I}} = \frac{120 \angle 0^\circ \text{ V} - 100 \angle 0^\circ \text{ V}}{10 \angle 0^\circ \text{ A}} = 2\Omega, \quad X_L = \frac{U_L}{I_2} = \frac{50 \text{ V}}{5\sqrt{3}} = 5.773\Omega.$$

4.28 求图示电路的戴维南等效电路。6.20

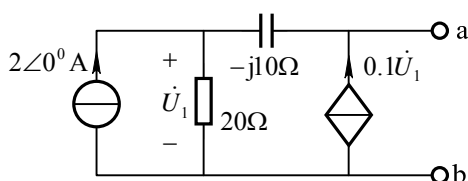
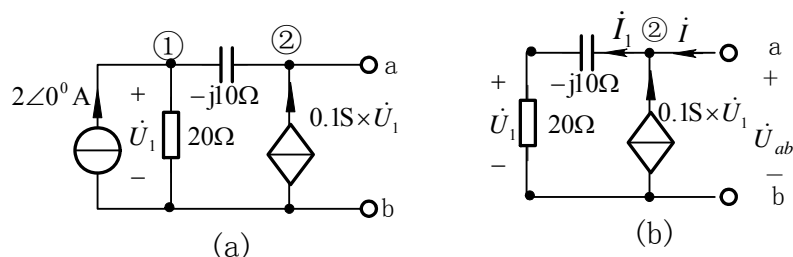


图 题 4.28



【解】对原电路一些变量进行标定, 如图(a)。

(1) 求开路电压 \dot{U}_{oc}

对图(a)电路列节点电压方程

$$\begin{cases} (\frac{1}{20} + \frac{1}{-j10})S \times \dot{U}_{n1} - \frac{1}{-j10} \times \dot{U}_{n2} = 2 \angle 0^\circ \text{ A} \\ -\frac{1}{-j10} S \times \dot{U}_{n1} + \frac{1}{-j10} S \times \dot{U}_{n2} = 0.1S \times \dot{U}_1 \end{cases}$$

受控源控制量 \dot{U}_1 即为节点电压 \dot{U}_{n1} , 即

$$\dot{U}_1 = \dot{U}_{n1} \quad (1)$$

将式(1)代入上述方程组, 解得

$$\dot{U}_{n1} = -40 \text{ V}, \quad \dot{U}_{n2} = \dot{U}_{oc} = 40\sqrt{2} \angle 135^\circ \text{ V}$$

(2) 求等效阻抗 Z_i

在 a、b 端外施电压源 \dot{U}_{ab} ，求输入电流 \dot{I} ， \dot{U}_{ab} 与 \dot{I} 的比值即为等效阻抗 Z_i ，电路图(b)。

$$\text{由节点②得 } \dot{I} = \dot{I}_1 - 0.1S \times \dot{U}_1 = \frac{\dot{U}_1}{20\Omega} - \frac{\dot{U}_1}{10\Omega}$$

$$\text{又 } \dot{U}_{ab} = (20 - j10)\Omega \dot{I}_1 = (20 - j10) \times \frac{\dot{U}_1}{20}$$

$$\text{得 } Z_i = \frac{\dot{U}_{ab}}{\dot{I}} = \frac{(20 - j10) \times \frac{\dot{U}_1}{20}}{(\frac{1}{20} - \frac{1}{10})\dot{U}_1} = 22.36 \angle 153.43^\circ \Omega$$

4.29 设图示一端口网络中 $u_S = 200\sqrt{2} \cos \omega t \text{ V}$ ， $\omega = 10^3 \text{ rad/s}$ ，求其戴维南等效电路。6.21

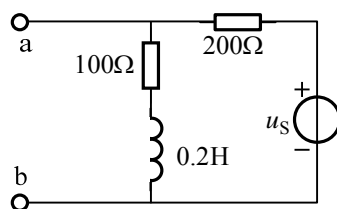


图 题 4.29 (a)

【解】(1) 对原电路(a)，感抗 $X_L = \omega L = 10^3 \text{ rad/s} \times 0.2 \text{ H} = 200\Omega$ ，由分压公式得端口开路电压

$$\dot{U}_{oc} = \frac{(100 + j200)\Omega}{(100 + j200 + 200)\Omega} \times 200 \angle 0^\circ \text{ V} = 124 \angle 29.7^\circ \text{ V}$$

求等效阻抗，将电压源作用置零，

$$Z_i = (100 + j200)\Omega // 200\Omega = \frac{200\Omega \times (100 + j200)\Omega}{(200 + 100 + j200)\Omega} = 124 \angle 29.7^\circ \Omega$$