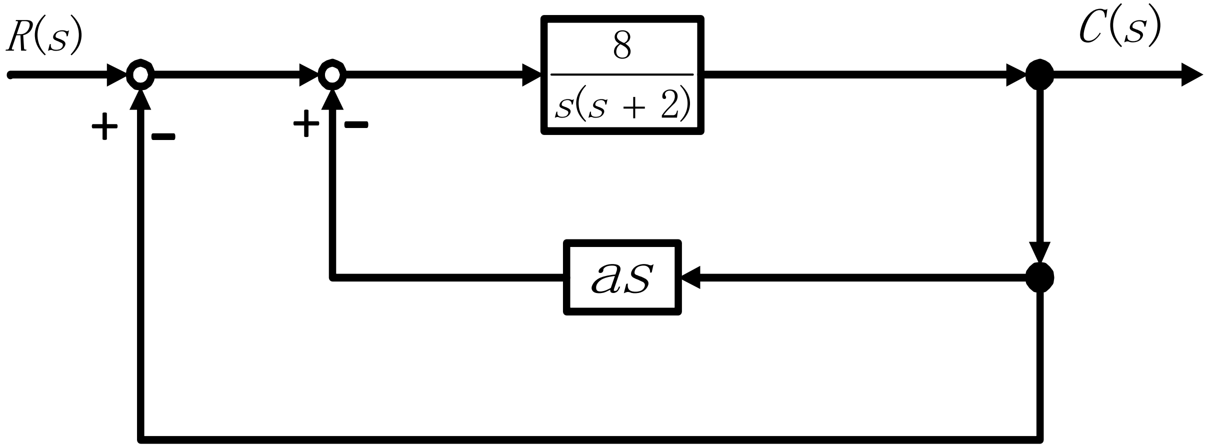
习题课

习题一



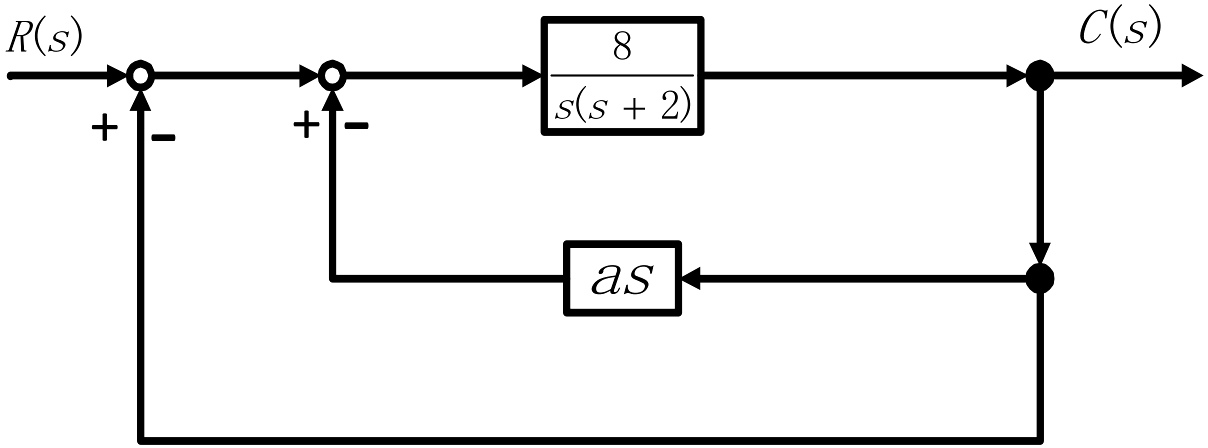
1、当 a = 0时，确定系统的阻尼比ξ，自然频率ω_n和单位斜坡输入时的稳态误差

2、当 ξ= 0.7时，确定参数a的值以及单位斜坡输入时的稳态误差

3、在保证 ξ= 0.7和稳态误差ess(∞) = 0.25时，确定参数a及前向通道增益K

习题课

习题一



1、当 a = 0时

$$G(s) = \frac{8}{s(s+2)} = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$$

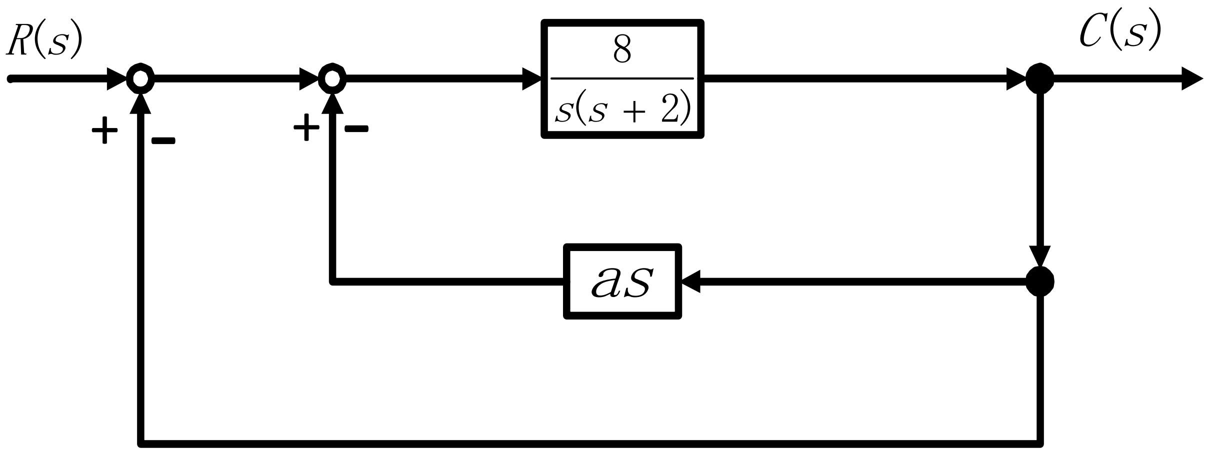
$$\omega_n = 2\sqrt{2} = 2.83, \zeta = \sqrt{2}/4 = 0.35$$

Ⅰ型系统 $$K_v = 4$$

$$e_{ss}(\infty) = 1/K_v = 0.25$$

习题课

习题一



2、当 $a \neq 0, \zeta = 0.7$

$$G(s) = \frac{8}{s(s+2+8a)} = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$$

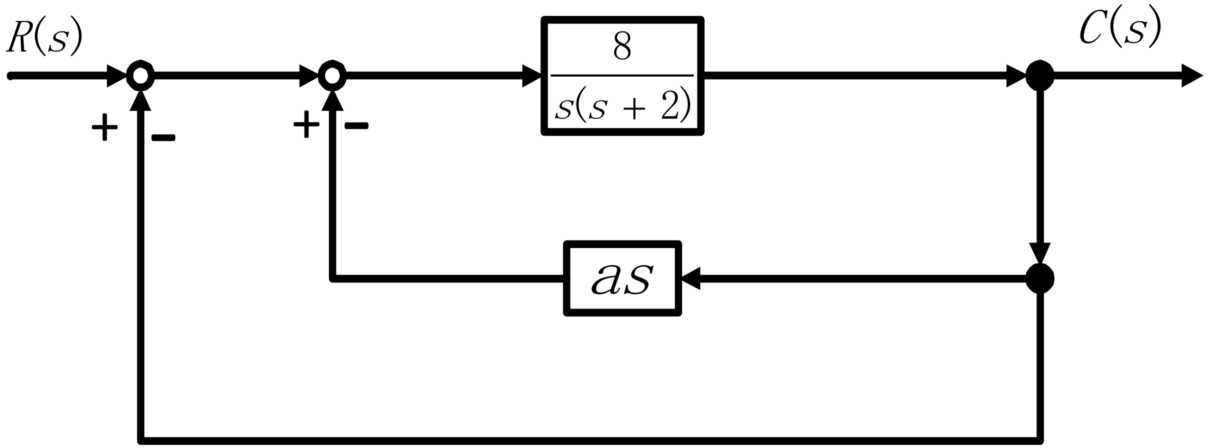
$$\omega_n = 2\sqrt{2} = 2.83, a = \frac{\zeta\omega_n - 1}{4} = 0.245$$

$$K_v = 8/(2+8a) = 2.02$$

$$e_{ss}(\infty) = 1/K_v = 0.495$$

习题课

习题一



3、当 $\zeta = 0.7, e_{ss}(\infty) = 0.25$

$$G(s) = \frac{K}{s(s+2+Ka)} = \frac{\omega_n^2}{s(s+2\zeta\omega_n)} \qquad K_v = K/(2+Ka)$$

I型系统 $e_{ss}(\infty) = 1/K_v = 2/K + a$

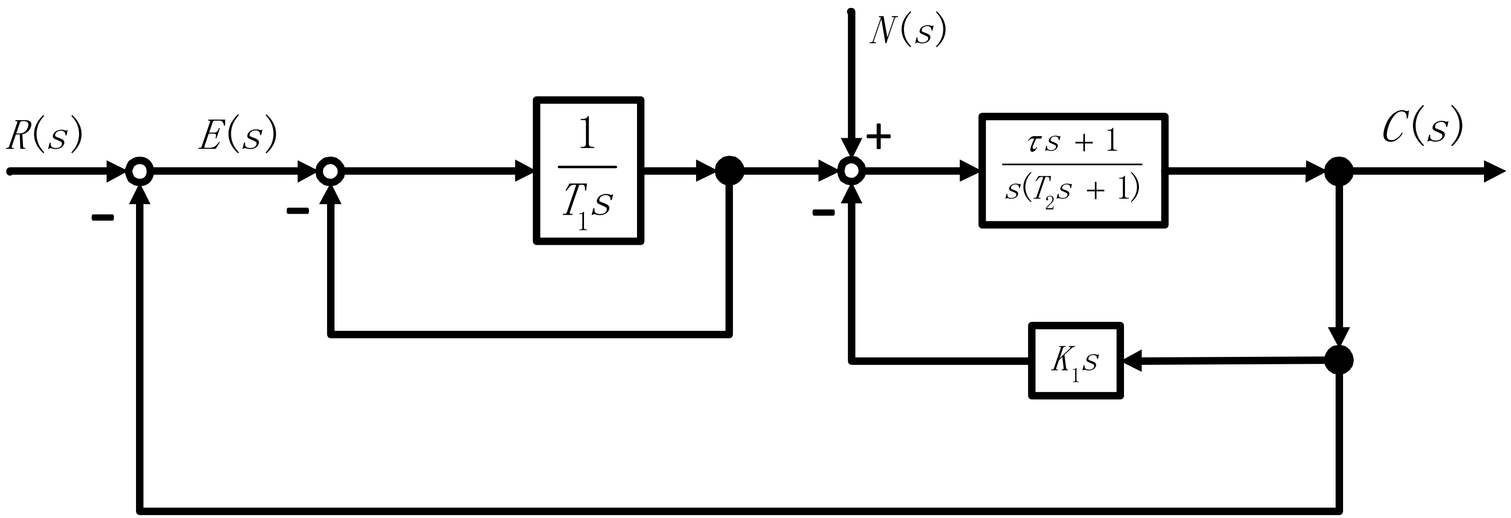
代入

$\zeta = 0.7, e_{ss}(\infty) = 0.25$

$$\begin{cases} \omega_n^2 = K \\ 1.4 \bullet \omega_n = 2 + Ka \\ 2/K + a = 0.25 \end{cases} \Rightarrow \begin{cases} \omega_n = 5.6 \\ K = 31.36 \\ a = 0.186 \end{cases}$$

习题课

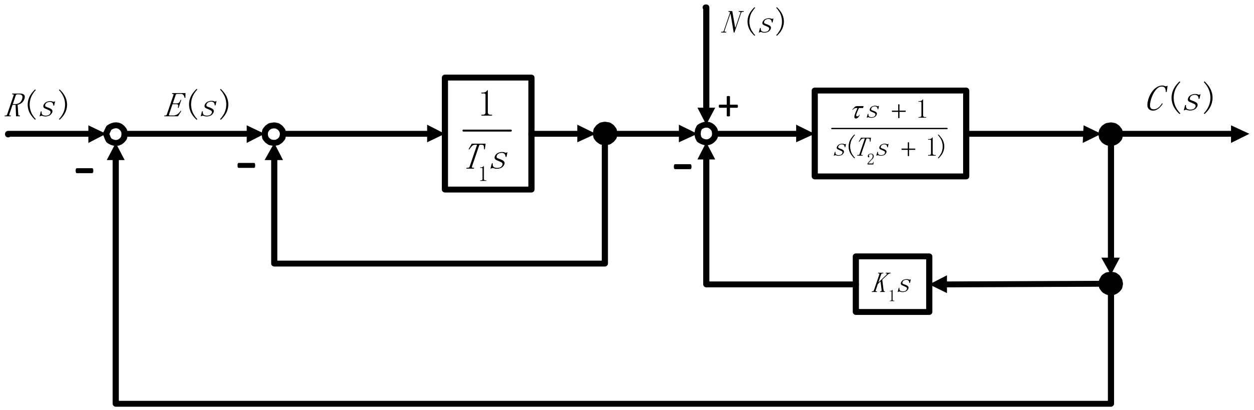
习题二



试鉴别系统对输入***r(t)***和扰动***n(t)***的型别

习题课

习题二



当n(t) = 0 时，开环传递函数

$$G_r(s) = \frac{\frac{1}{T_1 s}}{1+\frac{1}{T_1 s}} \bullet \frac{\frac{\tau s+1}{s(T_2 s+1)}}{1+\frac{K_1 s(\tau s+1)}{s(T_2 s+1)}} = \frac{\tau s+1}{s(T_1 s+1)\left[(T_2 s+1)+K_1(\tau s+1)\right]}$$

I型系统

当r(t) = 0 时，开环传递函数

前向通道 $G_n(s) = \frac{\tau s+1}{s(T_2 s+1)}$ 反馈通道 $H_n(s) = \frac{1}{T_1 s+1} + K_1 s = \frac{K_1 T_1 s^2 + K_1 s + 1}{Ts+1}$

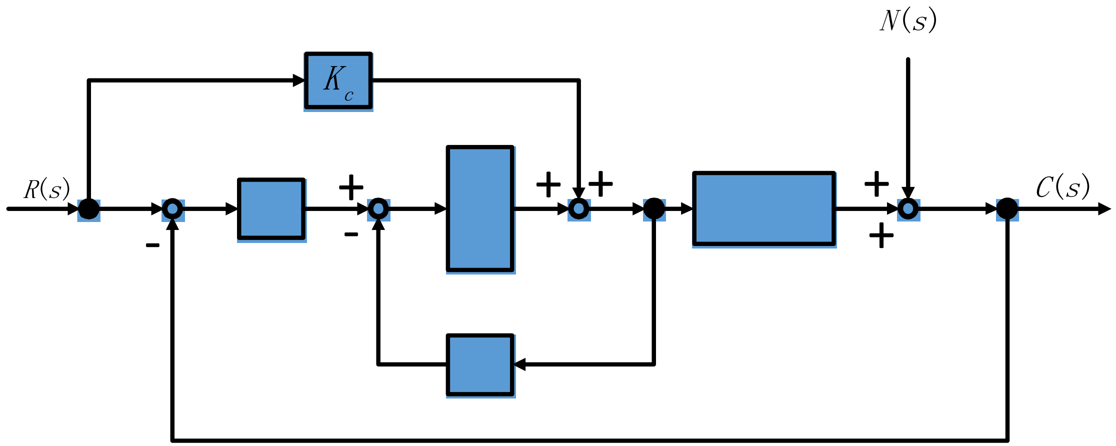
$$E_n(s) = R(s) - C_n(s) = -C_n(s) = -\frac{G_n(s)}{1+G_n(s)H_n(s)}N(s)$$

$$= -\frac{(\tau s+1)(T_1 s+1)}{s(T_1 s+1)(T_2 s+1)+(\tau s+1)(K_1 T_1 s^2+K_1 s+1)}N(s)$$

0型系统

习题课

习题三

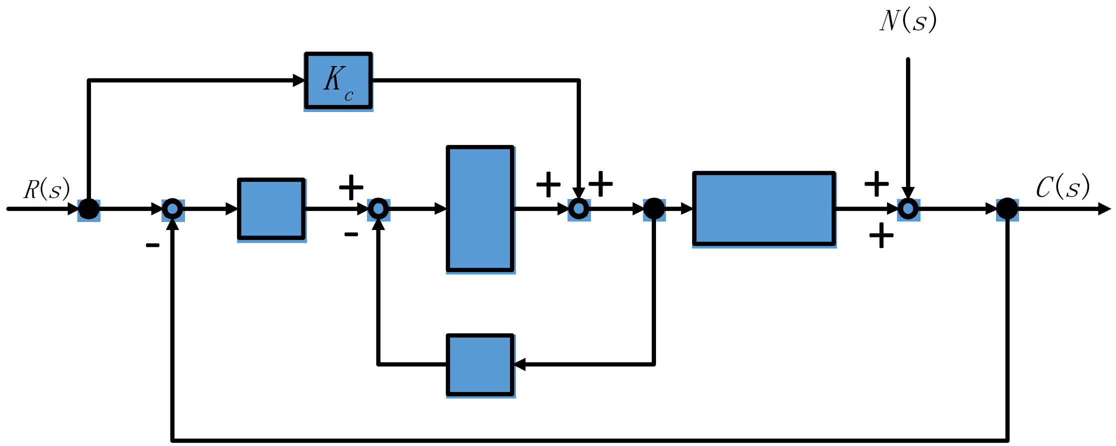


1、计算扰动 **n(t) = t** 引起的稳态误差

2、设计K_c，是系统在 **r(t) = t** 作用下无稳态误差

习题课

习题三



1、n(t) = t

$$N(s)=\frac{1}{s^2}\qquad \frac{C(s)}{N(s)}=\frac{s(s+K_2K_3)(Ts+1)}{s(s+K_2K_3)(Ts+1)+K_1K_2K_4}$$

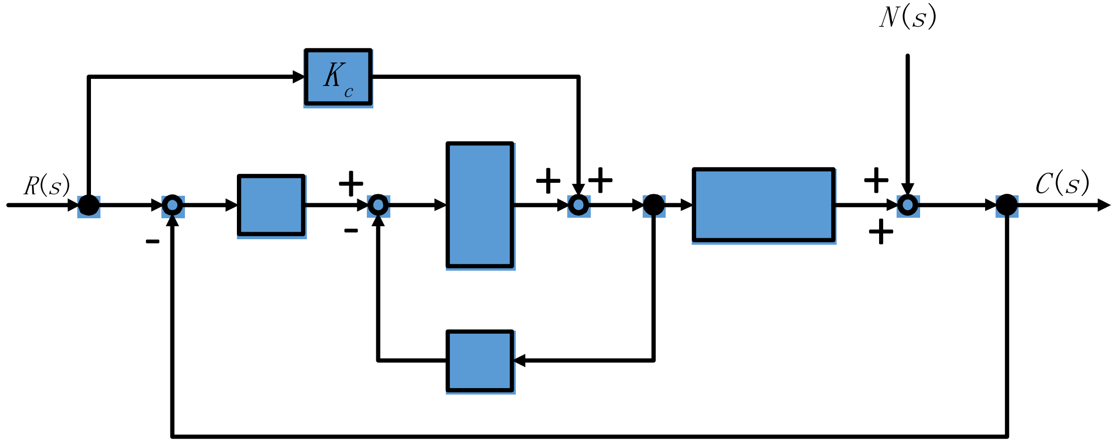
$$E_n(s)=-C(s)=-\frac{s(s+K_2K_3)(Ts+1)}{s(s+K_2K_3)(Ts+1)+K_1K_2K_4}N(s)$$

$$e_{ssn}(\infty)=\lim_{s\to0}sE_n(s)=-\lim_{s\to0}s\frac{s(s+K_2K_3)(Ts+1)}{s(s+K_2K_3)(Ts+1)+K_1K_2K_4}\bullet\frac{1}{s^2}$$

$$e_{ssn}(\infty)=-\frac{K_3}{K_1K_4}$$

习题课

习题三



2、r(t) = t

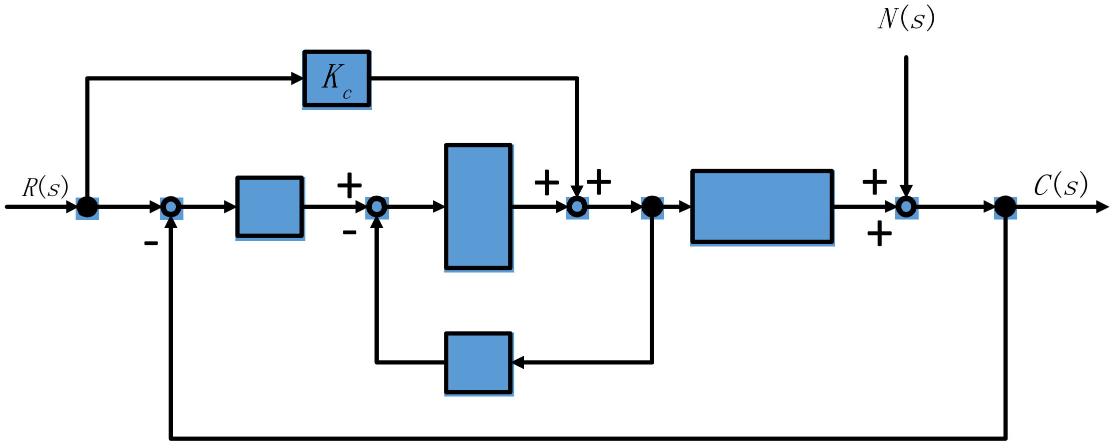
两个单独回路 $L_1 = -\frac{K_2K_3}{s}, L_2 = -\frac{K_1K_2K_4}{s^2(Ts+1)}, \Delta = 1 - L_1 - L_2$

两条前向通道 $p_1 = \frac{K_1K_2K_4}{s^2(Ts+1)}, \Delta_1 = 1 \quad p_2 = \frac{K_cK_4}{s(Ts+1)}, \Delta_2 = 1$

$$\Phi(s) = \frac{C(s)}{R(s)} = \frac{\sum p_i\Delta_i}{\Delta} = \frac{K_cK_4s + K_1K_2K_4}{s^2(Ts+1) + K_2K_3s(Ts+1) + K_1K_2K_4}$$

习题课

习题三



$$\Phi(s)=\frac{C(s)}{R(s)}=\frac{\sum p_i\Delta_i}{\Delta}=\frac{K_cK_4s+K_1K_2K_4}{s^2(Ts+1)+K_2K_3s(Ts+1)+K_1K_2K_4}$$

$$E_r(s)=R(s)-C(s)=[1-\Phi(s)]\bullet R(s)$$

$$=\frac{s^2(Ts+1)+K_2K_3s(Ts+1)-K_cK_4s}{s^2(Ts+1)+K_2K_3s(Ts+1)+K_1K_2K_4}\bullet R(s)$$

$$e_{ssr}(\infty)=\lim_{s\to 0}sE_r(s)=\lim_{s\to 0}s\cdot\frac{s^2(Ts+1)+K_2K_3s(Ts+1)-K_cK_4s}{s^2(Ts+1)+K_2K_3s(Ts+1)+K_1K_2K_4}\cdot\frac{1}{s^2}$$

$$=\lim_{s\to 0}\frac{s(Ts+1)+K_2K_3Ts+\left(K_2K_3-K_cK_4\right)}{s^2(Ts+1)+K_2K_3s(Ts+1)+K_1K_2K_4}=0$$

$$K_2K_3-K_cK_4=0$$

$$\Rightarrow\quad K_c=\frac{K_2K_3}{K_4}$$

习题课

习题四

设单位反馈系统的开环传递函数如下：

$$G(s) = \frac{K^*(s+2)}{(s+6)(s^2+2s+s)}$$

试绘制K在正负反馈时的根轨迹图，并确定系统无超调时K的范围

习题课

习题四

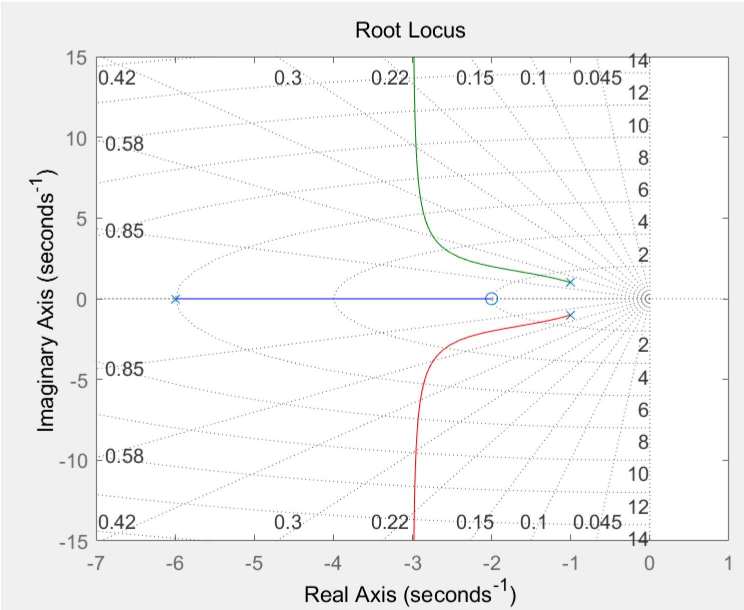
$$G(s)=\frac{K^*(s+2)}{(s+6)(s^2+2s+2)}=\frac{K^*(s+2)}{(s+6)(s+1-j)(s+1+j)}$$

负反馈： $K:0\to+\infty$ 实轴上的分布： $[-6,-2]$

根轨迹的渐近线： $\sigma_\alpha=\dfrac{-1+j-1-j-6+2}{3-1}=-3,\ \varphi_\alpha=\pm\dfrac{\pi}{2}$

根轨迹的起始角： $\theta_{p1}=180°+\varphi_{z1p1}-\theta_{p2p1}-\theta_{p3p1}=180°+45°-90°-\arctan\dfrac{1}{5}=123.69°$

$\theta_{p2}=-123.69°$



习题课

习题四

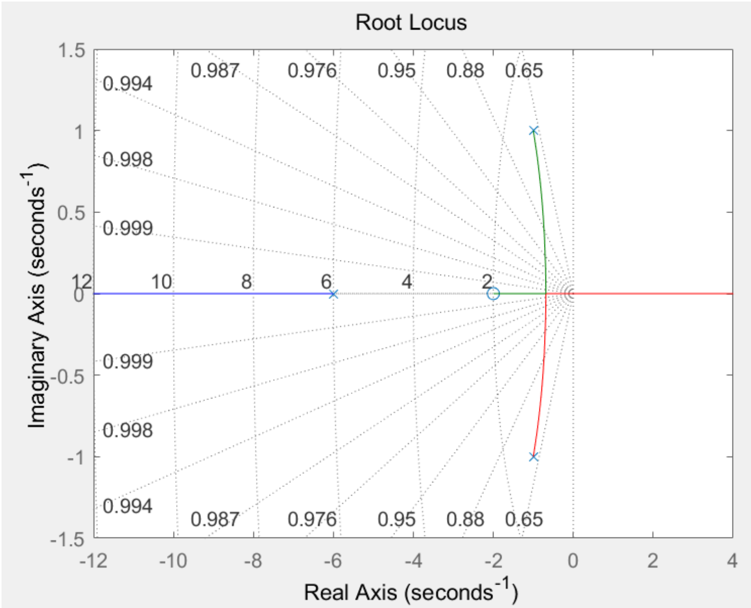
$$G(s)=\frac{K^*(s+2)}{(s+6)(s^2+2s+2)}=\frac{K^*(s+2)}{(s+6)(s+1-j)(s+1+j)}$$

正反馈： $K:-\infty\to 0$ 　　实轴上的分布： $[-6,-\infty),[-2,+\infty)$

根轨迹的分离点： $\frac{1}{d+6}+\frac{1}{d+1-j}+\frac{1}{d+1+j}=\frac{1}{d+2}$ 　　$d=-0.685$

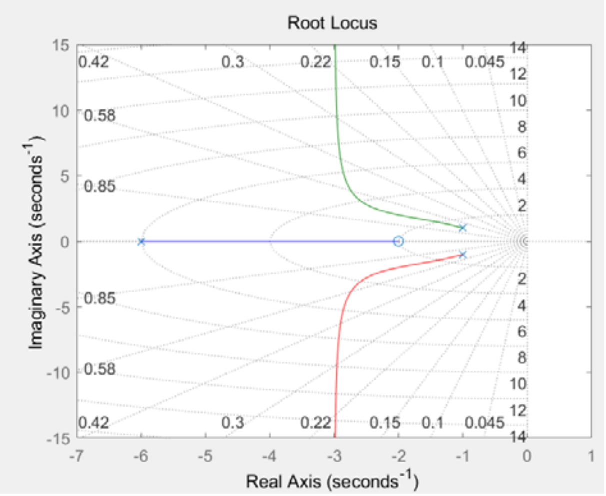
根轨迹的起始角： $\theta_{p1}=\varphi_{z1p1}-\theta_{p2p1}-\theta_{p3p1}=45°-90°-\arctan\frac{1}{5}=-56.31°$

$\theta_{p2}=56.31°$



习题课

习题四 $$G(s)=\frac{K^*(s+2)}{(s+6)(s^2+2s+2)}=\frac{K^*(s+2)}{(s+6)(s+1-j)(s+1+j)}$$



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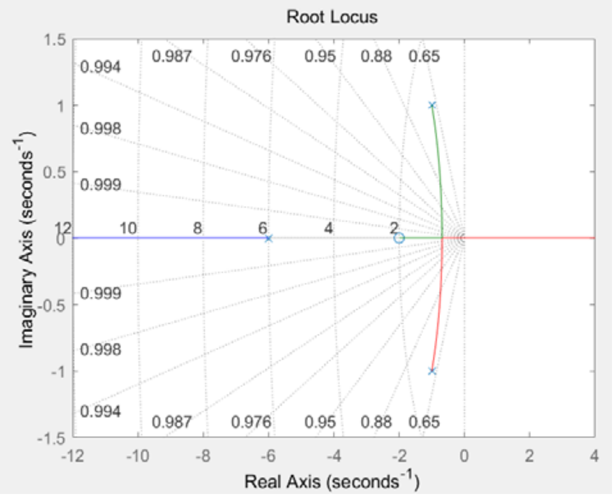


√

使系统无超调，闭环极点须位于实轴之上

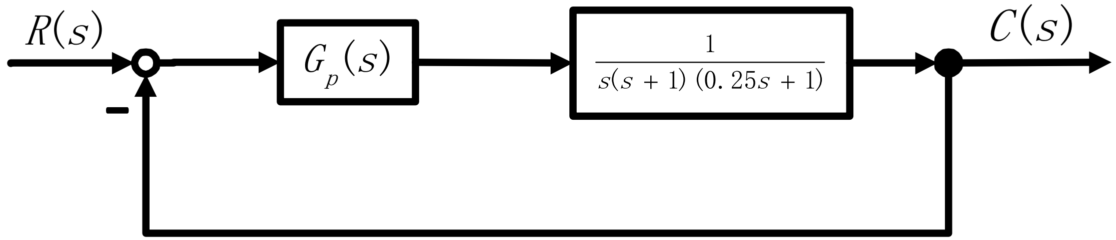
系统闭环特征方程 $D(s)=(s+6)(s^2+2s+2)+K^*(s+2)=s^3+8s^2+14s+12+K^*(s+2)=0$

带入分离点 $d=-0.685$

$$K^*=-4.443 \qquad K^*\le -4.443$$

习题课

习题五



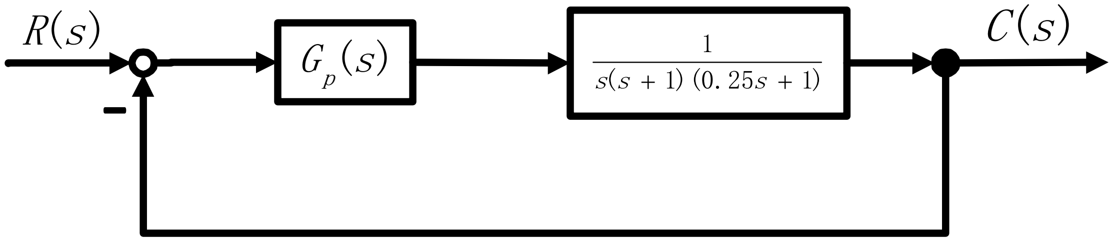
1、当Gp(s) = Kp时，绘制以Kp为参数变量的根轨迹

2、为使系统的阶跃响应呈现衰减振荡形式，试确定Kp范围

3、当Gp(s) = Kp(1+0.5s)时，试绘制以Kp为参数变量的根轨迹

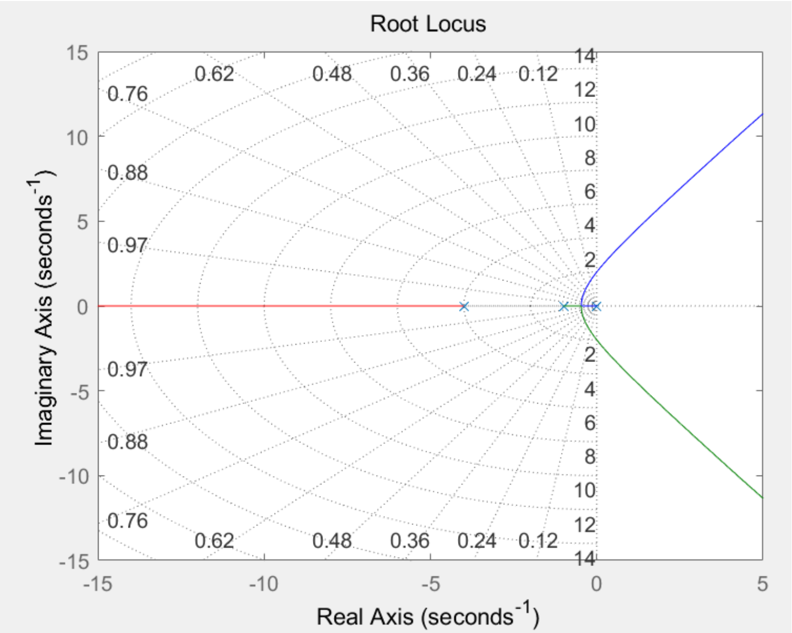
习题课

习题五



1、当Gp(s) = Kp时，绘制以Kp为参数变量的根轨迹

$$G(s)=\frac{K_p}{s(s+1)(0.25s+1)}=\frac{4K_p}{s(s+1)(s+4)}$$



渐近线 $\sigma_\alpha=-\frac{5}{3},\varphi_\alpha=\pm 60°,180°$

分离点 $\frac{1}{d}+\frac{1}{d+1}+\frac{1}{d+4}=0 \quad d=-0.465$

$$4K_p=0.465\times 0.535\times 3.535=0.879$$

$K_p=0.22$ 分离点处根轨迹增益

$$s^3+5s^2+4s+4K_p=0$$

s^3	1	4
s^2	5	$4K_p$
s^1	$\frac{20-4K_p}{5}$	0
s^0	$4K_p$	

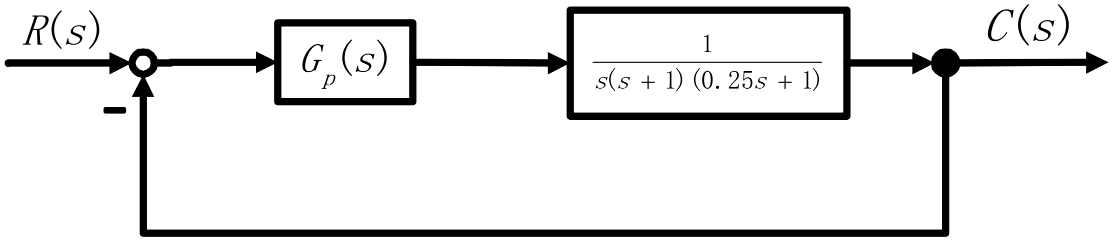
$K_p=5$ 全零行

$$5s^2+4K_p=5s^2+20=0$$

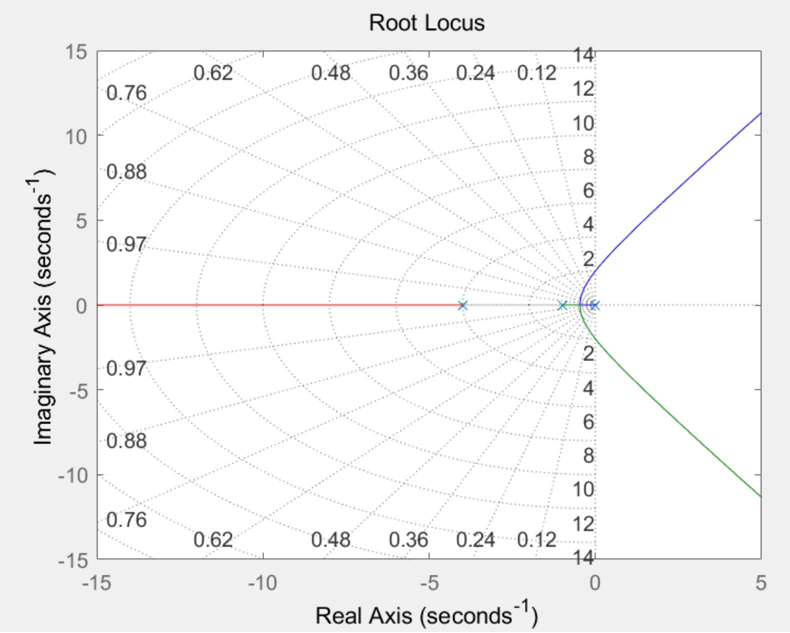
$$\omega=\pm 2$$

习题课

习题五



2、为使系统的阶跃响应呈现衰减振荡形式，试确定Kp范围



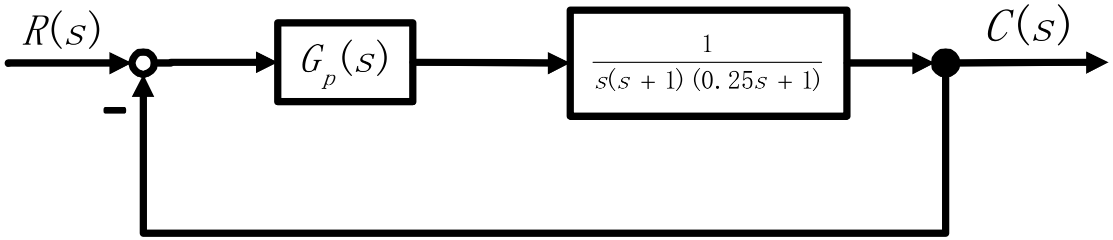
$K_p = 0.22$　分离点处根轨迹增益

$K_p = 5$　全零行

$$0.22 < K_p < 5$$

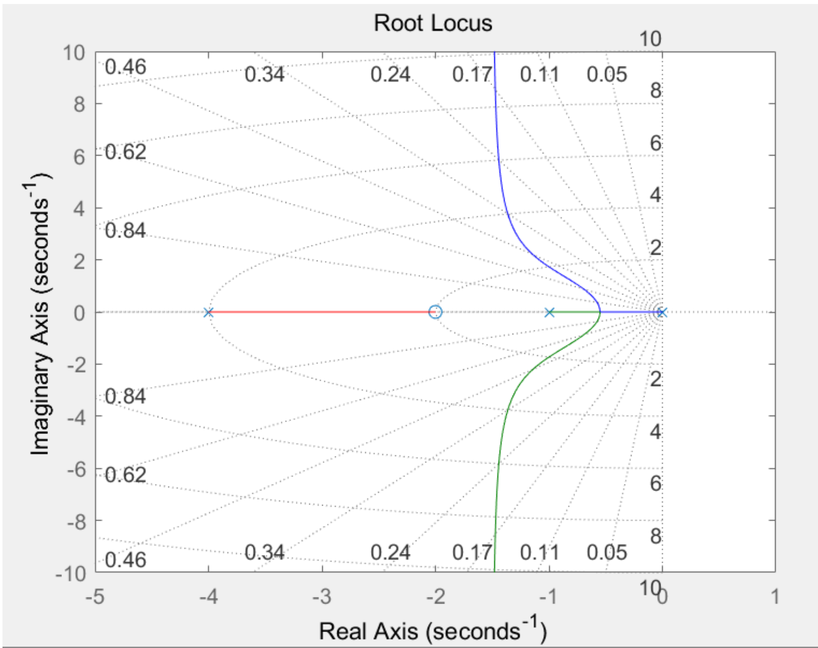
习题课

习题五



3、当Gp(s) = Kp(1+0.5s)时，试绘制以Kp为参数变量的根轨迹

$$G_p(s) = K_p(1 + 0.5s) \qquad G(s) = \frac{K_p(1 + 0.5s)}{s(s + 1)(0.25s + 1)} = \frac{2K_p(s + 2)}{s(s + 1)(s + 4)}$$



渐近线

$$\sigma_\alpha = \frac{-5 + 2}{3 - 1} = -1.5, \varphi_\alpha = \pm 90°$$

分离点 $\frac{1}{d} + \frac{1}{d + 1} + \frac{1}{d + 4} = \frac{1}{d + 2}$

$$d = -0.55$$

习题课

习题六

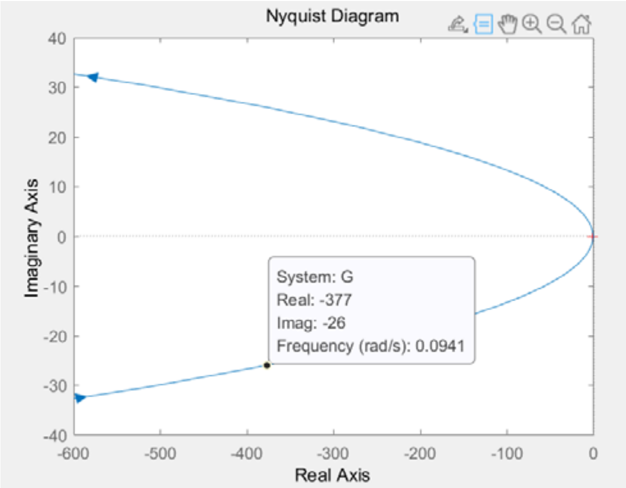
设单位反馈系统开环传递函数如下，试用奈奎斯特判据判断系统稳定性

（1） $G(s) = \dfrac{250(s+1)}{s^2(s+5)(s+15)}$ （2） $G(s) = \dfrac{(s+1)^2}{s^2(3s+1)(0.1s+1)^2}$

习题课

习题六

（1） $G(s)=\dfrac{250(s+1)}{s^2(s+5)(s+15)}$



$$G(\mathrm{j}\omega)=\frac{250(\mathrm{j}\omega+1)}{-\omega^2(\mathrm{j}\omega+5)(\mathrm{j}\omega+15)}$$

$$=-\frac{250\left(75+19\omega^2\right)}{\omega^2\left(25+\omega^2\right)\left(225+\omega^2\right)}-\mathrm{j}\frac{250\left(55-\omega^2\right)}{\omega\left(25+\omega^2\right)\left(225+\omega^2\right)}$$

$G(\mathrm{j}0+)=-\infty-\mathrm{j}\infty \qquad G(\mathrm{j}\infty)=0$

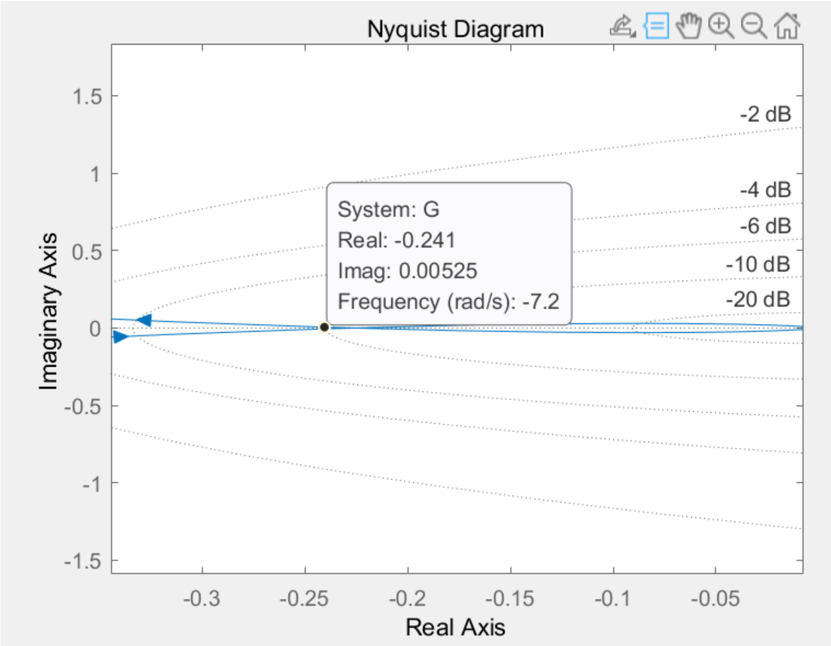
与实轴交点 $\mathrm{Im}[G(j\omega)]=0$

$\omega_x=7.42, G(j\omega_x)=\mathrm{Re}[G(j\omega_x)]=-0.23$

$\nu=2 \qquad N_-=0, N_+=0$

$N=N_+-N_-=0 \qquad Z=P-2N=0$

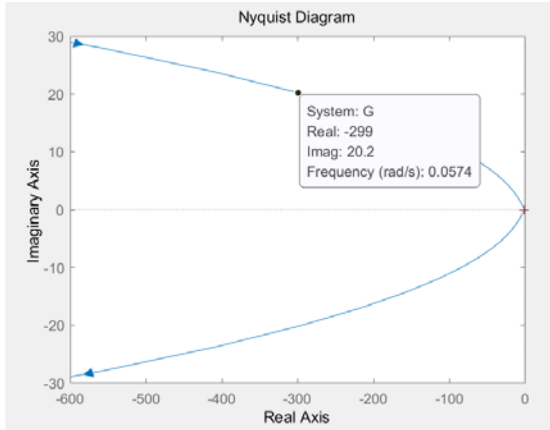
系统稳定



习题课

习题六

（2） $$G(s)=\frac{(s+1)^2}{s^2(3s+1)(0.1s+1)^2}$$



$$G(j\omega)=\frac{(j\omega+1)^2}{-\omega^2(j3\omega+1)(j0.1\omega+1)^2}$$

$$=-\frac{1+4.79\omega^2+0.55\omega^4}{\omega^2\left(1+9\omega^2\right)\left(1+0.01\omega^2\right)^2}+j\frac{0.03\omega^4-2.01\omega^2+1.2}{\omega\left(1+9\omega^2\right)\left(1+0.01\omega^2\right)^2}$$

$$G(j0+)=-\infty+j\infty \qquad G(j\infty)=0$$

与实轴交点 $\mathrm{Im}[G(j\omega)]=0$

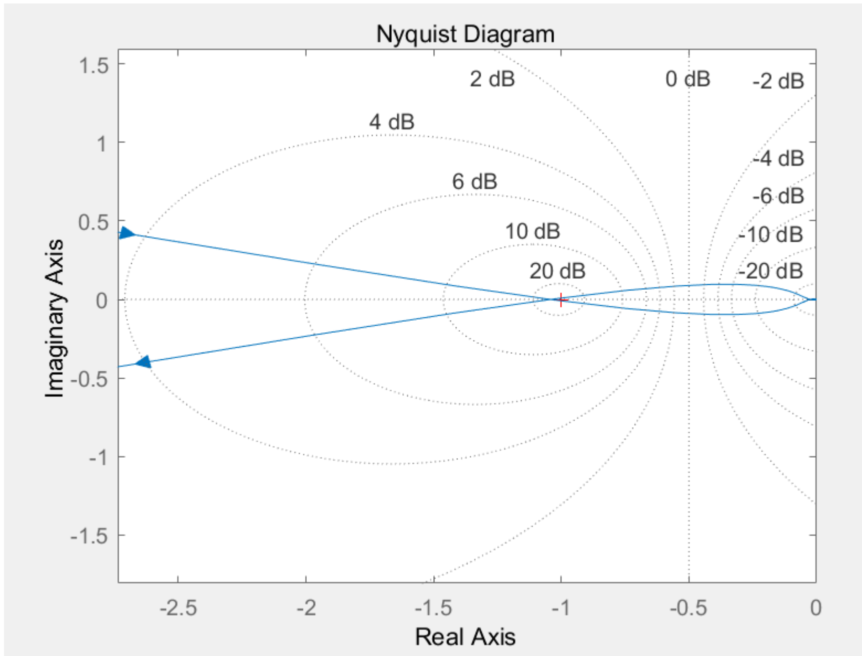
$$\omega_{x2}=8.149, G(j\omega_{x2})=\mathrm{Re}[G(j\omega_{x2})]=-0.025$$

$$\omega_{x1}=0.776, G(j\omega_{x1})=\mathrm{Re}[G(j\omega_{x1})]=-1.04$$

曲线在II、III象限变化

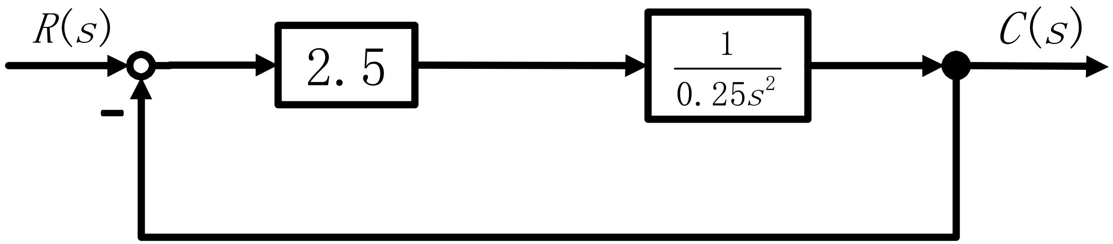
$$N_-=1, N_+=1 \qquad N=N_+-N_-=0$$

$$Z=P-2N=0$$ 系统稳定



习题课

习题七

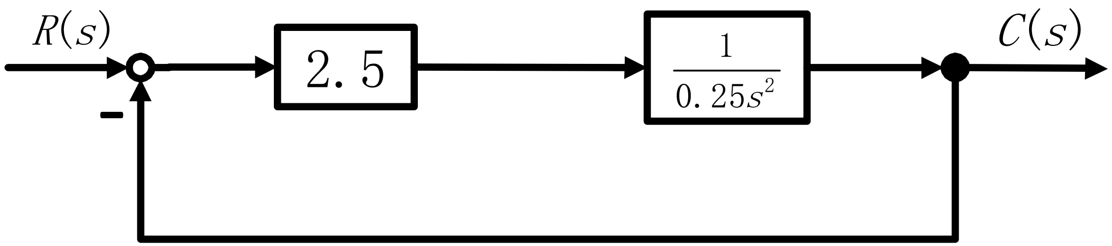


1、试设计串联校正网络，使系统相角裕度为50度、截止频率为6rad/s

2、若不采用串联校正，采用速度反馈校正，确定使系统单位阶跃响应的超调量不超过15%的反馈系数

习题课

习题七



1、试设计串联校正网络，使系统相角裕度为50度、截止频率为6rad/s

$$G_0(s) = \frac{10}{s^2} \qquad \omega_c' = \sqrt{10} = 3.16,\ \gamma' = 0°$$

试用超前校正网络

$$G_c(s) = \frac{1+T_2 s}{1+T_1 s},\ T_2 > T_1$$

要求 $\omega_c = \omega_c'' = 6$

$$L_0(\omega_m) = 20\lg 10 - 20\lg \omega_m^2 = -11.126dB$$

$$L_c(\omega_m) = -L_0(\omega_m) = 11.126dB$$

$$\frac{0 - 11.126}{\lg\frac{1}{T_2} - \lg \omega_m} = 20$$

$$\lg T_2 = -0.22,\ T_2 = 0.6$$

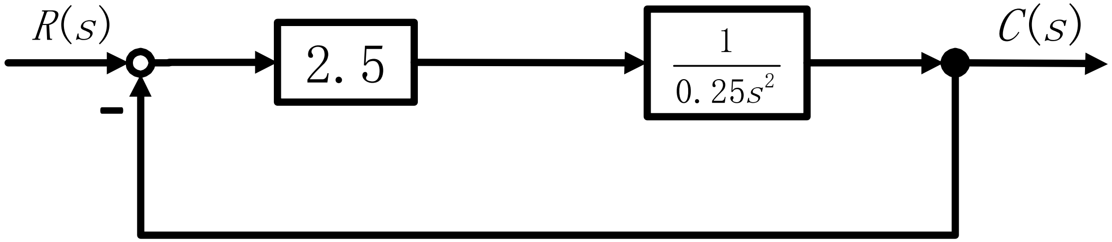
$$\gamma = 180° + \angle G(j6) = \arctan 6T_2 - \arctan 6T_1$$

$$T_1 = 0.076$$

$$G(s) = G_c(s)G_0(s) = \frac{10(1+T_2 s)}{s^2(1+T_1 s)}$$

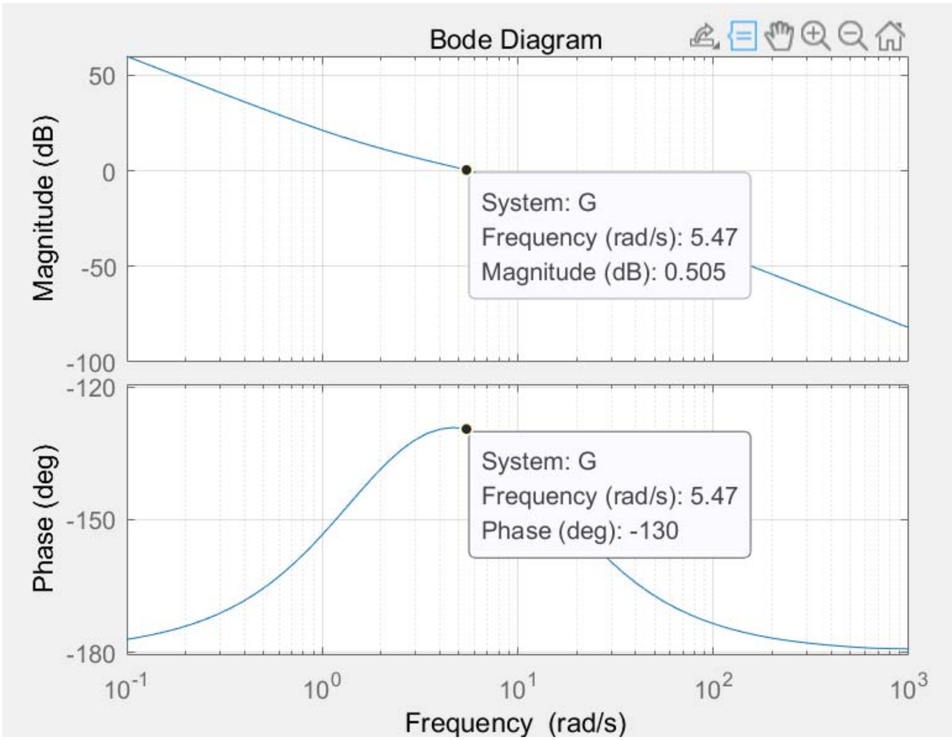
习题课

习题七



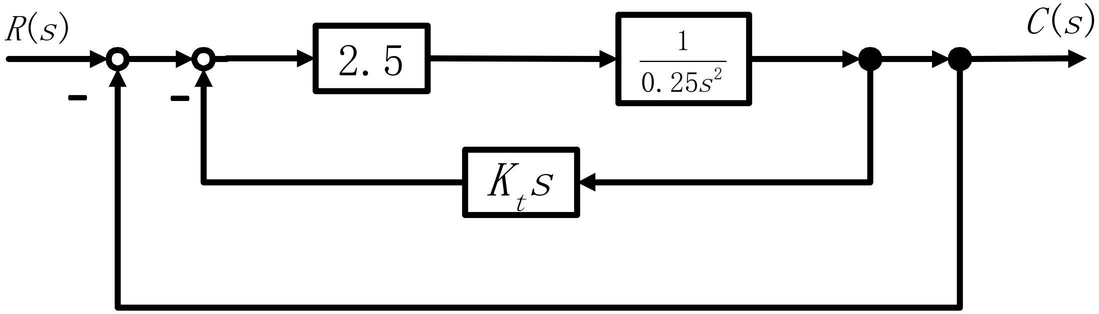
校正前后相角

$$G_c(s) = \frac{1 + 0.6s}{1 + 0.076s} \qquad \varphi_0(\omega) = -180°; \quad \varphi(\omega) = -180° + \arctan 0.6\omega - \arctan 0.076\omega$$



习题课

习题七



2、若不采用串联校正，采用速度反馈校正，确定使系统单位阶跃响应的超调量不超过15%的反馈系数

开环传递函数 $G(s) = \dfrac{10 / s^2}{1 + 10K_t s / s^2} = \dfrac{10}{s(s+10)K_t}$

闭环传递函数 $\Phi(s) = \dfrac{10}{s^2 + 10K_t s + 10} = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$$\omega_n^2 = 10, 2\zeta\omega_n = 10K_t \qquad \sigma\% = 100e^{-\pi\zeta/\sqrt{1-\zeta^2}}\% \le 15\%$$

$$\zeta \ge 0.517$$

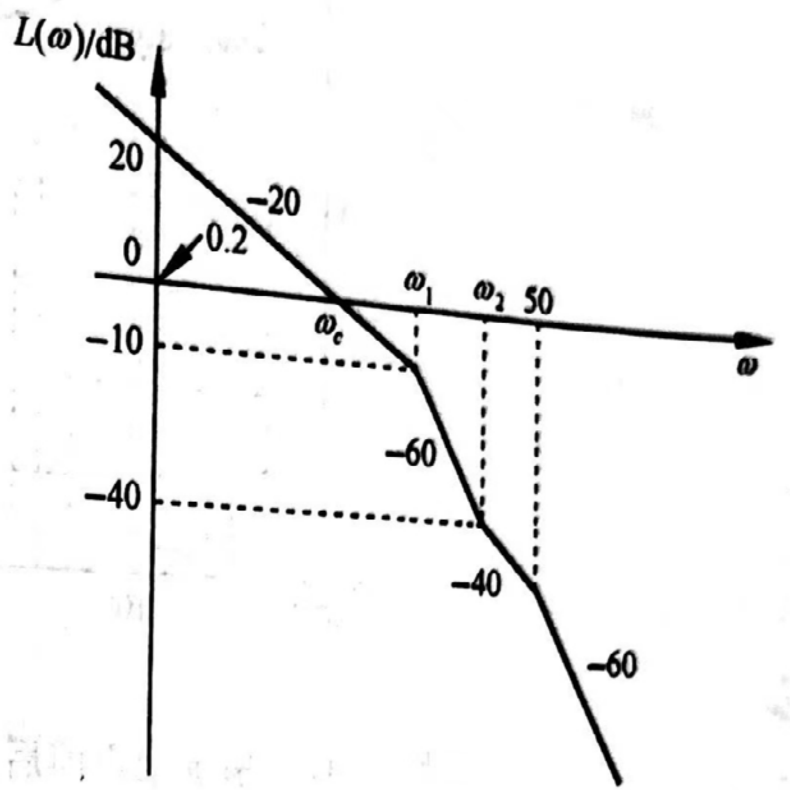
取 $\zeta = 0.55$ $\quad K_t = \dfrac{2\xi\omega_n}{10} = 0.348$

$$\sigma\% = 100e^{-\pi\xi/\sqrt{1-\xi^2}}\% = 12.6\% < 15\%$$

习题课

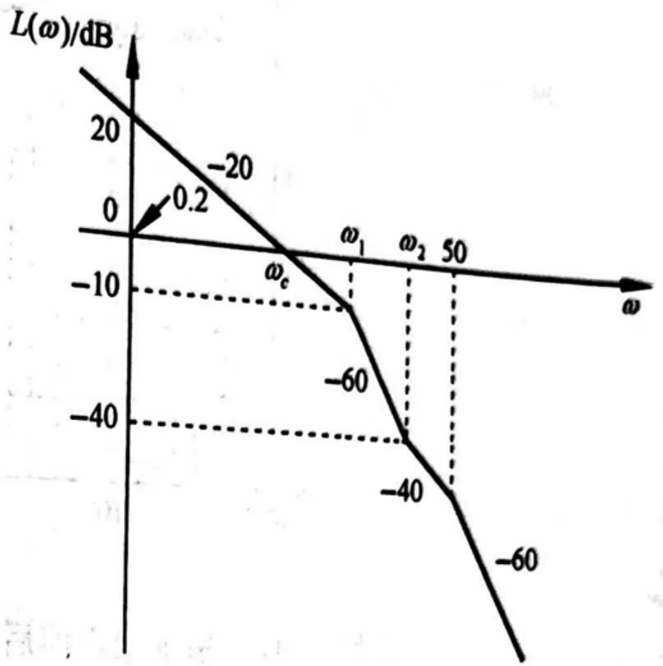
习题八

已知某最小相位系统开环对数特性曲线渐近线如图，试写出开环传递函数G0(s)的一种表达式



习题课

习题八



$$G_0(s) = \frac{K(\frac{1}{\omega_2}s + 1)}{s(\frac{1}{\omega_1}s + 1)^2(\frac{1}{50}s + 1)}$$

$$20\lg\frac{K}{0.2} = 20 \Rightarrow K = 2 \qquad 20\lg\frac{\omega_c}{0.2} = 20 \Rightarrow \omega_c = 2$$

$$20\lg\frac{\omega_1}{\omega_c} = 10 \Rightarrow \omega_1 = 6.32 \qquad 60\lg\frac{\omega_2}{\omega_1} = 30 \Rightarrow \omega_2 = 20$$

$$G_0(s) = \frac{2(\frac{1}{20}s + 1)}{s(\frac{1}{6.32}s + 1)^2(\frac{1}{50}s + 1)}$$

$$\gamma = 180° + \angle G(\omega_c)$$

$$= (90° + \arctan\frac{\omega_c}{20} - \arctan\frac{\omega_c}{6.32} - \arctan\frac{\omega_c}{50})\Big|_{\omega_c=2} = 58.3°$$