

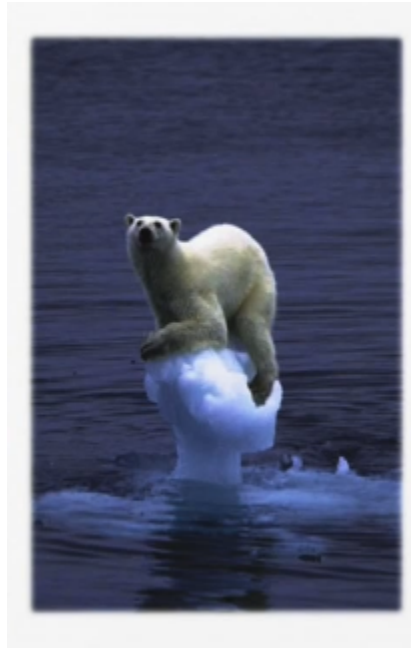
7 Week 7. Thermonuclear fusion: an overview, with Ambrogio Fasoli

7.1 The basics

7.1.1 Why fusion?

HDI correlated with electricity use per capita (though Canada is using almost three times as much as the Netherlands so there's room for improvement there). So to develop humankind we need better power sources.

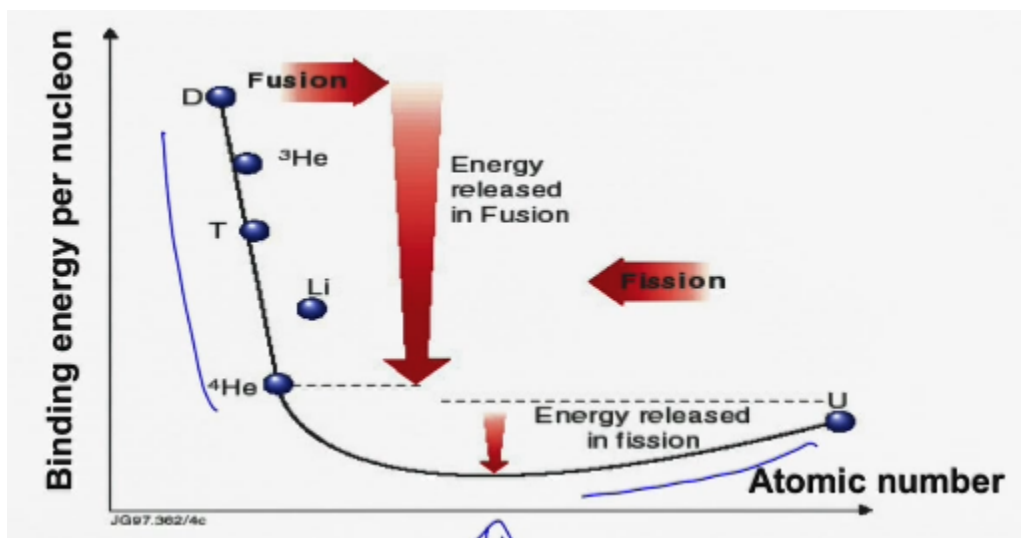
We have to fight global warming, stop using fossil fuels and save this polar bear.



We also have to make sure the power sources are safe for the population. Energy sources that endanger people are **bad**.

And fusion fulfils all of those criteria!

7.1.2 How does it work



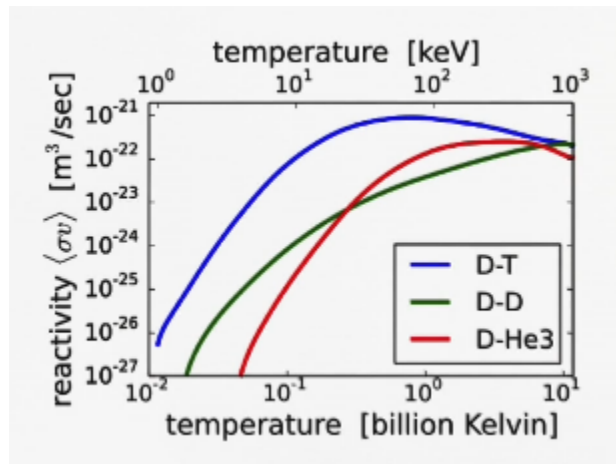
Two possible approaches to nuclear energy generation - split really heavy nuclei, like uranium, or merge small nuclei. Note that as the slope is sharp on the left, so you get a lot of energy per each fusion.

7.1.3 Fusion reactions for a terrestrial reactor

Three possibilities: DT, DHe3, DD fuel cycles.

At low kinetic energies coulomb repulsion deflects particles. This prevents fusion from occurring (essentially interception by atomic forces). High kinetic energies are equivalent to saying we need high temperature.

You don't actually need 380keV to get into the potential well on the left - you can do with 10keV due to tunneling (a quantum effect).

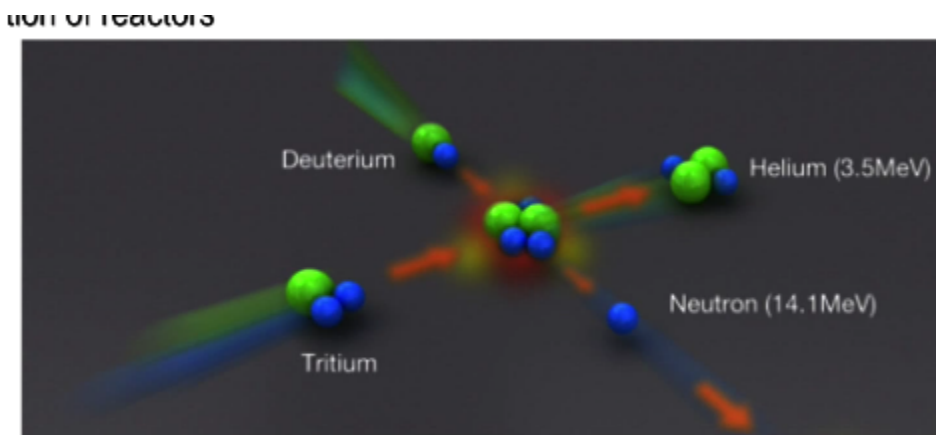


Cross section times average velocity over distribution functions (AKA **reactivity**) - this is a parameter that describes how easy it is for particles to fuse. The DT reaction has the largest one at that for the smallest temperatures, so we're focusing on that for now.

At these temperatures (10keV or so), pretty much all matter is in the plasma state! So we're gonna need to go through this to understand that.

Thermonuclear fusion - as Coulomb collisions have a humongous cross section, any monoenergetic beam would be scattered in a plasma before the particles have time to fuse. Thus, an important condition is that our plasma is in a **thermalized** Maxwellian distribution (this would mean that the plasma is kind of in a stationary state).

7.1.4 The DT cycle



This is the current approach to terrestrial fusion. Deuterium is ubiquitous in the oceans. Tritium has a half-life of 12.5 years and has to be **bred** from lithium. That can also be found in the ocean.

We look into the $Li - 6$ reaction due to its large cross-section. Smack a neutron into the $Li - 6$ atom and one T and one $He - 4$ pops out. Lithium can be supplied by using a **blanket** mounted on the inside wall of the vacuum vessel.

7.1.5 Schematic of a fusion reactor

We inject deuterium into the reactor and lithium into the blanket. The tritium stays in a closed system. Helium can be bottled and made into balloons or sold at high prices. The energy is removed from the system by a conventional fluid based system.

7.1.6 Why fusion is neat

Fusion energy density: $3.5e8 MJ/kg$. Compare to $30 MJ/kg$ for coal, $50 MJ/kg$ for oil, $8.5e7 MJ/kg$ for uranium fission. $9e10 MJ/kg$ for $E = mc^2$ total conversion.

The fuels are virtually inexhaustible. Lithium specifically can be mined (one example - Nevada), both lithium and deuterium can be obtained from sea water. This is so uniformly spread over the Earth that you could almost call it open source.

The environmental impact? No greenhouse gases whatsoever, helium is neat. The radioactive elements, depending on the materials used, take a short time to become safe.

It is impossible to weaponize a fusion reactor. You can't lose control of the reaction (no criticality) and there's very little fuel in the reactor at any given time (about $1g$) .



Let's go do that.

7.2 Power balance in reactors

Each DT reaction gives $3.5 MeV$ to the α and $14.1 MeV$ to the neutron, for a total of $17.6 MeV$.

Fusion power density for a particular relative velocity (or energy). n_D, n_T are densities of deuterium and tritium. σ is the reaction cross-section.

$$R_{DT}(v)\Delta E_f = (n_D n_T \sigma_{DT}(v) v) \Delta E_f \quad (1)$$

We can integrate that over velocity distributions for both D and T.

Assuming neutrality ($n_D = n_T = n_e/2 = n/2$), neglecting impurities and density of alpha particles:

$$\text{Power density} = n_D n_T \langle \sigma v \rangle_{DT} \Delta E_f = \frac{1}{4} n^2 \langle \sigma v \rangle_{DT} \Delta E_f \quad (2)$$

To produce $1GW$ of fusion power at $T = 20keV$, $n = 5e20 m^{-3}$, use tabularized data for the average $\langle \sigma v \rangle$. Divide desired fusion power by fusion power density. The result: $14m^3$ - apparently a reasonable volume.

7.3 Generalities

For steady state operation - conservation of energy. The losses include radiation, convection and conduction ('direct losses'). Our power input is whatever we do to heat the plasma and the fusion power produced inside.

The radiative losses include bremsstrahlung - electrons are accelerated in an electric field, producing x-ray radiation (which can pass through the plasma and the containment vessel - thus escaping). Power density of that $\sim n^2 T_e^{1/2}$.

The direct losses are described by the energy confinement time τ_E . Power density of that $= 3nT/\tau_E$.

7.4 Calculation of power balance - breakeven

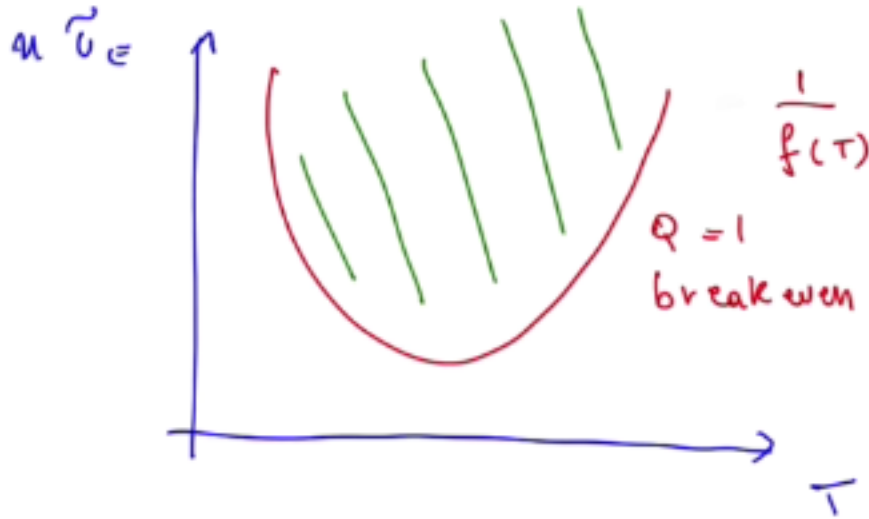
Def. physics fusion gain $Q = \text{Fusion power}/\text{Input power} = P_f/P_{in}$. A reactor needs $Q > 1$. At $Q = 1$ we have a breakeven situation. $P_i n + P_\alpha = P_{losses} = P_{directlosses} + P_{bremsstrahlung}$, so we get $Q = \frac{P_f}{P_{directlosses} + P_{bremsstrahlung} - P_\alpha}$.

Inserting the previously derived expressions:

$$n\tau_E \geq \frac{12T}{\frac{6}{5} \langle \sigma v \rangle_{DT} \Delta E_F + 4AZ_{eff} T^{1/2}} \quad (3)$$

At large temperatures (over 1keV), we can neglect the second term to get

$$\frac{1}{n\tau_E} = f(T) = \frac{\langle \sigma v \rangle_{DT} \Delta E_F}{10T} \quad (4)$$



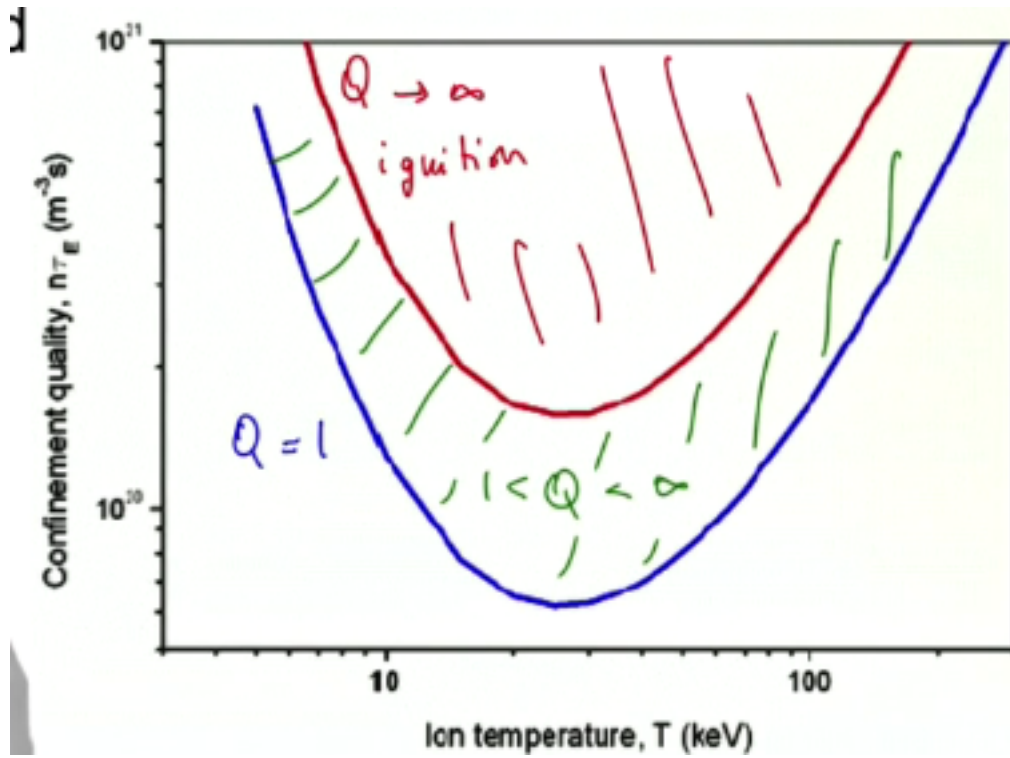
Reactors can work in the green area above the breakeven curve.

7.4.1 Ignition

This is the limit where all the heating the plasma needs comes from the fusion reactors. This would be the Sun on Earth scenario. $P_F/P_{in} = +\infty$ as $P_{in} \rightarrow 0$

$P_i n = 0$ means that $P_\alpha \geq P_{losses} \sim P_{dl}$ (we neglect bremsstrahlung).

$n\tau_E$ at ignition $\geq \frac{12T}{\langle \sigma v \rangle_{DT} \Delta E_\alpha} = 6/f(t)$. It's just a higher parabola!



The burning plasma regime is when alpha particle heating is greater than the input heating, $Q \geq 5$.

7.4.2 Engineering fusion gain

Def. Q_E - ratio of electric power output to input, η_e - efficiency of plasma heating electrical power supply, η_t - efficiency of thermal fusion power conversion into energy. $\eta = \eta_e \eta_t$, $Q_E = \frac{\eta(P_f + P_{in}) - P_{in}}{P_{in}} = \eta Q - (1 - \eta)$. $Q \gg Q_E$, sadly.

7.5 Plasma confinement approaches

We need both $n\tau_E = 10^{20} m^{-3}s$ and $T \geq 10 keV$. Magnetic confinement has τ_E about 1s. Inertial confinement goes with $10^{-9}s$ and much higher densities.

7.6 Inertial confinement

We consider the ion confinement time $\tau_i = R/c_s$, R being the pellet radius and c_s the speed of sound, at which the ions tend to move.

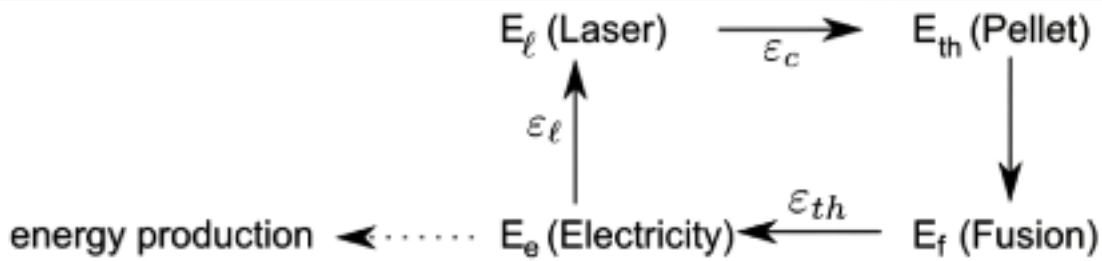
We also define the heating time $\tau_h = \frac{\text{energy in pellet}}{\text{power deposited on surface}} = \frac{3nT4\pi R^3/3}{F4\pi R^2} = \frac{nTR}{F}$, F being the energy flux

We must have heating faster than the confinement time. $\tau_h < \tau_i \rightarrow F > mT^{3/2}/\sqrt{m_i}$. The size of the pellet cancels out. Note that we neglected all compressional effects (shock waves...).

For a standard example, F is about $5e19 W/m^2$. This means lasers. Really big lasers.

7.6.1 ICF energy balance

Laser inputs energy E_L - I took the liberty of changing notation. Thermal energy is scaled by efficiency ϵ_c : $E_{th} = E_L \epsilon_c$. This produces fusion energy E_f and that process also has a finite efficiency in conversion to electricity, ϵ_{th} . We can also recirculate some of the fusion energy to drive the reactor.



It turns out there's a simple condition for the laser energy sufficient for fusion. For typical values (solid deuterium), and relatively low efficiencies (5%, 10%) it turns out to give $10^{15} J$ - 50 times the Hiroshima bomb. We can currently achieve $2MJ$. This requires a density of $3000n_0$, n_0 being solid deuterium density. This means we have to have a pressure of $3000n_0T$, which is $2e17$ Pascal. That's a lot. Can't do that just by firing lasers.

7.6.2 The rocket effect

Generate an implosion by launching a lot of energy into the external surface. Conservation of momentum will cause an inward motion of the external layers and this will compress the plasma.

Pressure $P_{rocket} = \frac{\frac{d(mv)}{dt}}{surface} \sim \frac{V \frac{dm}{dt}}{S}$ at constant v

The energy flux F , at constant v , turns out to be $\frac{v^2 dm}{2S dt}$ and we can derive the $\frac{dm}{dt} = \frac{2SF}{v^2}$.

Cancelling the terms, we get $\frac{P_{rocket}}{P_{laser}} = \frac{2c}{v}$. This is much bigger than 1 and thus a considerable boost

7.6.3 Additional ICF effects

The shockwave, if it goes symmetrically into the center, helps increase pressure.

Alpha production can also help heat the core, but the pellet must be larger than alpha mean free path.

However, instabilities, as always, occur and the laser light can't be too intense. Also, too much density means the laser beam actually gets reflected (plasma frequency cutoff occurs)

7.6.4 Summary of physics

- Radiation phase

Surface heated by laser, gets plasmified

- Blowoff phase

This is where the surface is blown off and the rest is pushed inside.

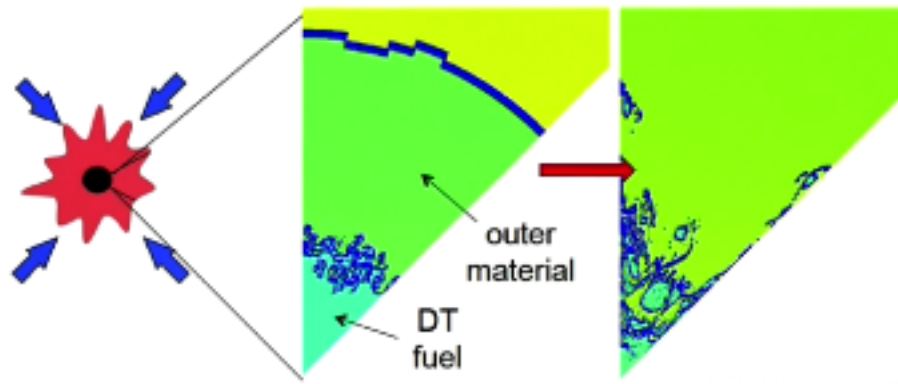
- Implosion phase

Thermal energy is transported to the center, the core is heated

- Thermonuclear burning

The challenges here are as follows: The core cannot be heated before the shock wave hits it (separate it from the rest somehow?). Imposes constraints on laser pulse timing and pellet design.

Also, there's the question of hydrodynamic stability:



The pellet must be very symmetric and the beams have to be really carefully aligned. Shown above is a mixing of the DT fuel and core material due to the Rayleigh Taylor instability - we don't really want that.

7.6.5 Direct and indirect drive

The direct drive is the simpler one - we just smack the pellet with lasers, as symmetrically as possible. There's multiple layers - a low density ablator on the outside, a ball of solid or liquid fuel and a gas layer.

The indirect drive is intended to lower the symmetry requirements. We have a small chamber - the so called hohlraum - around the pellet. It receives laser beams through two holes and acts as a black body source of x-rays. This is inherently more symmetric than injecting the pellet with laser energy.

The Lawrence Livermore Laboratory's National Ignition Facility uses 192 lasers to achieve pulse energy of $2MJ$, with about $500TW$ of power. These fire UV light ($352nm$).

7.6.6 Engineering challenges

Cost, efficiency, reliability. Difficult to find materials for first vacuum chamber wall. Capsules are really complex (3d printing, perhaps?).

A reactor would need to fire a few times a second to function as a power generator.

7.7 Magnetic confinement

Confine particles by making them rotate around magnetic field lines. Macroscopic confinement times allow lower densities.

There's many schemes for this:

- Linear devices - such as magnetic mirrors
- Toroidal configurations
- Stellarators

7.7.1 Simple toroidal device example

A simple toroidal field has curvature and an intensity gradient. This causes drifts - positive charges go to the top, negative charges to the bottom. This causes first an E field, then an ambient magnetic field and this means loss of confinement.

What we do is impose a poloidal field. Through their motions particles stay in regions of different drifts directions.

There's the so called hoop force which tends to move field lines to the outer region of the tokamak. To avoid this, we add a vertical field which counters that.

To sum up:

- Toroidal field coils
- Plasma current generates poloidal field
- Outer poloidal field coils that generate the vertical field, can also help positioning

How to drive the current in the plasma? That's a neat idea - it's basically a transformer. The plasma is the secondary circuit and the primary circuit is a coil inside the core - center - of the tokamak. Drive AC through that and you drive AC through the plasma by Faraday's law.

7.7.2 Progress

Density times confinement time with respect to temperature - we're steadily moving towards the high energy multiplication and sustained burn region. $Q = 5$ is getting really close now. ITER should have Q of about 10 - a reactor needs about 30 or 40. TFTR and JET actually got close to breakeven by significantly increasing the confinement time. ITER aims to push that to several hundreds of second.

ITER is being built in Cadarache in the south of France. A really global project.¹

And now, back to our regularly scheduled programme on magnetic confinement. 5

¹Really makes you wonder why the victory condition in Civilization games is the space race to Alpha Centauri, huh?