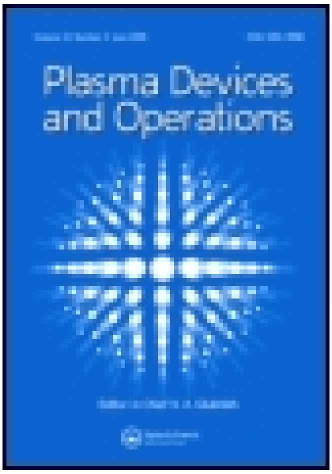

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NUMERICAL ALGORITHM FOR FIELD LINE RECONSTRUCTION FROM VECTOR FIELD DISTRIBUTION

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The algorithm of the vector field force line tracing is studied in this article. The program based on this algorithm allows the build-up of force lines starting from any prescribed points. The results of testing and the examples of force lines calculated by the program are presented.

Keywords: Magnetic field; Vector field; Force line; Numerical simulations; Tokamaks

INTRODUCTION

The term *magnetic field line*, that is a line of magnetic flux or magnetic force, means a curve with a tangent co-directional to magnetic flux density $\vec{B}$ or magnetic field strength $\vec{H}$ at any point of the curve [1].

Field $\vec{B}(\vec{r}) = \vec{B}(x, y, z)$ is a vector field with vector field lines governed by differential equations [2]

$$\mathrm{d}\vec{r} \times \vec{B}(\vec{r}) = 0,$$

where $\vec{r} = (x, y, z)$ is the radius-vector of a point, or, in terms of Cartesian coordinates x, y, z,

$$\frac{\mathrm{d}x}{B_x(x,y,z)} = \frac{\mathrm{d}y}{B_y(x,y,z)} = \frac{\mathrm{d}z}{B_z(x,y,z)}.$$

There is a variety of vector fields, including magnetic, electric and current density fields, etc., which can be geometrically described by a set of vector lines with the relative density proportional to the field at any point.

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If magnetic field in a curl-free region (*i.e.* without currents) is presented in terms of a scalar potential or, generally, a vector potential, the field lines can be determined as a system of lines orthogonal to equipotential surfaces [1].

In practice, a 3D distribution of $\vec{B}(\vec{r})$ is available, which is obtained by calculation or magnetic measurement. For the field analysis in a working area it is often necessary to reconstruct a set of field lines.

In particular, a knowledge of the field lines configuration and value of induction enables estimation of field characteristics controlling the plasma equilibrium and stability conditions [3].

In tokamak devices a crucial problem is to limit the heat load on divertor targets. To reduce the heat load, divertor plates should be oriented practically parallel to the field lines along which plasma diffuses outward [4]. Then the plasma is distributed over a larger area on the targets that can result in a lower heat load.

Design and optimization of various electrostatic optic devices for production, focusing, bending, and separation of a charged beam involve analysis of optical behavior of the beam. Typically this analysis is reduced to geometrical optics and can be performed relying on the analysis of particle trajectory along the electric field lines [5].

Here is described a computation technique to reconstruct field lines using a set of close located discrete points on the lines. The technique is implemented in the common program. In the inputs an arbitrary 3D vector field is given analytically or tabulated. Data interpolation is possible at any point of the calculation domain. This approach has been implemented in the software package KLONDIKE [6] intended for 3D simulation and analysis of magnetostatic fields.

ALGORITHM OF 3D FIELD LINE RECONSTRUCTION

Mathematically, the problem of field line reconstruction is formulated as a first-order differential equation

$$\frac{d\vec{r}}{dl} = \frac{\vec{f}}{|\vec{f}|} \tag{1}$$

for the initial condition

$$\vec{r}(0) = \vec{r}_0 \tag{2}$$

Here, $\vec{f} = \vec{f}(\vec{r})$ is the vector field, $\vec{r} = \vec{r}(l)$ is the radius-vector for a point at a field line, l is the path along the field line.

Variation of numerical value of $\vec{r}(0)$ gives different particular solutions forming integral curves.

For convenience, a unit vector $\vec{u}$, which is co-directed with the field, is introduced

$$\vec{u} = \frac{\vec{f}}{|\vec{f}|} \tag{3}$$

Then Eq. (1) can be re-written as

$$\frac{d\vec{r}}{dl} = \vec{u}(\vec{r}) \tag{4}$$

The easiest way to solve this equation by the explicit method [6] is to replace a derivative $d\vec{r}/dl$ with the ratio of differences $\Delta\vec{r}/\Delta l$:

$$\frac{\Delta\vec{r}_n}{\Delta l_n} = \frac{\vec{r}_{n+1} - \vec{r}_n}{\Delta l_n} = \vec{u}(\vec{r}_n) \tag{5}$$

where n is the step number. The next calculation point $\vec{r}_{n+1}$ can be found by solving Eq. (5) as:

$$\vec{r}_{n+1} = \vec{r}_n + \vec{u}(\vec{r}_n) \cdot \Delta l_n \tag{6}$$

The sequence $\{\vec{r}_n\}$ can be treated as a function given at mesh points.

It is known [7] that in this approach the error is proportional to the first order of the step size $O(\Delta l)$. To ensure required solution accuracy and adjust the step size, the common "step-halfstep" procedure can be applied. Each $(n-1)$th calculation step yields a step size $\Delta l_n^{(k)}$, $k=1$, for the next iteration (the initial step is user-defined). A radius-vector $\vec{r}_{n+1}^{(k)}$ for the estimated step size is initially approximated in accordance with Eq. (6)

$$\vec{r}_{n+1}^{(k)} = \vec{r}_n + \vec{u}(\vec{r}_n) \cdot \Delta l_n^{(k)} \tag{7}$$

After that, the same radius-vector is approximated using a halved step:

$$\vec{r}_{n+1/2}^{(k)} = \vec{r}_n + \vec{u}(\vec{r}_n) \cdot \frac{\Delta l_n^{(k)}}{2} \tag{8}$$

$$\vec{r}_{n+1}^{*(k)} = \vec{r}_{n+1/2}^{(k)} + \vec{u}(\vec{r}_{n+1/2}^{(k)}) \cdot \frac{\Delta l_n^{(k)}}{2} \tag{9}$$

If a relative difference $\delta_{n+1}^{(k)}$ of approximations (8) and (9)

$$\delta_{n+1}^{(k)} = \frac{|\vec{r}_{n+1}^{(k)} - \vec{r}_{n+1}^{*(k)}|}{\Delta l_n^{(k)}} \tag{10}$$

is higher than a given acceptable relative errors ε, *i.e.* $\delta_{n+1}^{(k)} > \varepsilon$, then a halved step is used for the next iteration $(k+1)$

$$\Delta l_n^{(k+1)} = \frac{\Delta l_n^{(k)}}{2} \tag{11}$$

At the $(k+1)$th iteration it is not necessary to calculate $\vec{r}_{n+1}^{(k+1)}$, the value estimated at the previous stage is applicable:

$$\vec{r}_{n+1}^{(k+1)} = \vec{r}_{n+1/2}^{(k)} \tag{12}$$

If $\delta_{n+1}^{(k)} \le \varepsilon$, then $\vec{r}_{n+1}^{*(k)}$ is assumed to be the final estimate (since Eq. (9) gives a more accurate approximation). The step $\Delta l_{n+1}^{(l)}$ of the next iteration is taken equal to $\Delta l_n^{(k)}$. This means, however, that Δl is expected to diminish. That might be unwanted if the field line curvature tends to be decreasing. A possible improvement is achieved assuming that a double Δl is applied at the next iteration if $\delta_{n+1}^{(k)} < \alpha \cdot \varepsilon$, where $\alpha < 1$ is a known or user-defined constant coefficient. In the original implemented software $\alpha = 1/4$ is used. A decision algorithm is formulated as:

$$\Delta l_{n+1}^{(1)} = c \cdot \Delta l_n^{(k)} \tag{13}$$

where

$$c = \begin{cases} 1, & \text{if } \delta_{n+1}^{(k)} \geq \alpha \cdot \varepsilon \\ 2, & \text{if } \delta_{n+1}^{(k)} < \alpha \cdot \varepsilon \end{cases} \tag{14}$$

The total length of a field line is obtained by summing all steps Δl_n:

$$L = \sum_n \Delta l_n \tag{15}$$

A more fast, yet equally accurate procedure, can be proposed as an alternative. The procedure implies a two-stage calculation of $\vec{r}_{n+1}^{(k)}$ (7), $\vec{r}_{n+1/2}^{(k)}$ (8) and $\vec{r}_{n+1}^{*(k)}$ (9) with a single adjustment of point location. Then the calculation algorithm can be re-written, being still an explicit method, in the form

$$\begin{aligned} \vec{r}_{n+1}^{(k,l)} &= \vec{r}_n + \vec{u}(\vec{r}_n) \cdot \Delta l_n^{(k)} \\ \vec{r}_{n+1}^{(k)} &= \vec{r}_n + \frac{\vec{u}(\vec{r}_n) + \vec{u}(\vec{r}_{n+1}^{(k,1)})}{2} \cdot \Delta l_n^{(k)} \end{aligned} \tag{16}$$

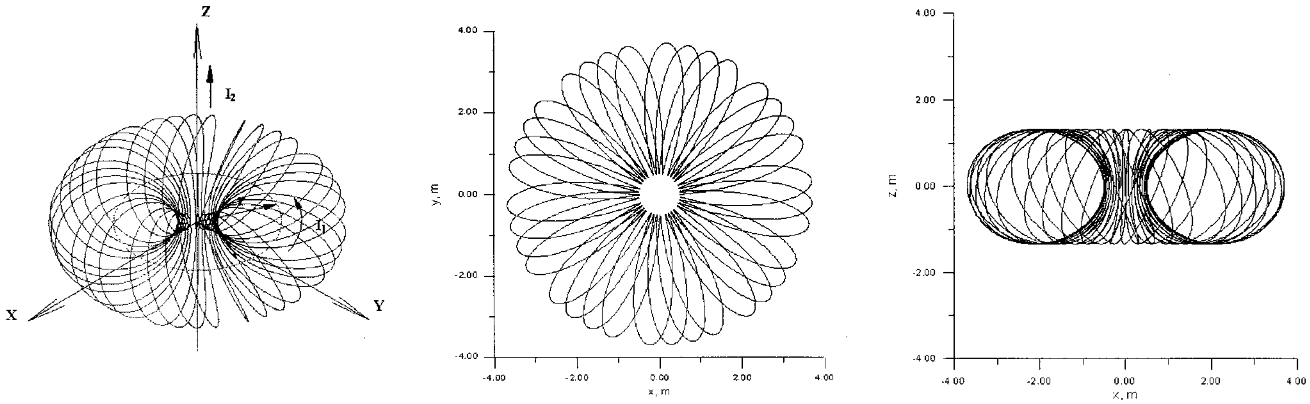
$$\begin{aligned} \vec{r}_{n+1/2}^{(k,1)} &= \vec{r}_n + \vec{u}(\vec{r}_n) \cdot \frac{\Delta l_n^{(k)}}{2} \\ \vec{r}_{n+1/2}^{(k)} &= \vec{r}_n + \frac{\vec{u}(\vec{r}_n) + \vec{u}(\vec{r}_{n+1/2}^{(k,1)})}{2} \cdot \frac{\Delta l_n^{(k)}}{2} \end{aligned} \tag{17}$$

$$\begin{aligned} \vec{r}_{n+1}^{*(k,l)} &= \vec{r}_{n+1/2}^{(k)} + \vec{u}(\vec{r}_{n+1/2}^{(k)}) \cdot \frac{\Delta l_n^{(k)}}{2} \\ \vec{r}_{n+1}^{*(k)} &= \vec{r}_{n+1/2}^{(k)} + \frac{\vec{u}(\vec{r}_{n+1/2}^{(k)}) + \vec{u}(\vec{r}_{n+1}^{*(k,1)})}{2} \cdot \frac{\Delta l_n^{(k)}}{2} \end{aligned} \tag{18}$$

The use of the adjusted iterative process gives a required accuracy with a sufficient reduction of the number of iterations and a runtime. It can be shown that the error of this method is proportional to $O(\Delta l^2)$.

TABLE I Comparison of Calculated Results.

Eps	*Method*	*N*	N_{calc}	*S*	*X*
1Å-2	1	206	417	6.4375	1.0491
	2	25	133	6.2500	1.0005
1Å-3	1	1613	3234	6.3008	1.0061
	2	50	262	6.2500	0.9995
1Å-4	1	25,741	51,494	6.2844	1.0004
	2	201	1025	6.2813	1.000001



a) b) c)

FIGURE 1 Magnetic surface of a helical toroidal field generated by a combination of an infinite straight conductor and a ring conductor: (a) axonometric view, (b) top view, (c) lateral view.

VERIFICATION AND EXPERIMENTAL DATA

To assess the capability of the proposed algorithm to accurately reconstruct field lines, a test run was performed for a vector field $\vec{f}(\vec{r}) = \vec{e}_z \times \vec{r}$, where $\vec{e}_z$ was a unit vector directed along the Z axis. The field lines form concentric circles around the Z axis. The initial point was taken as $\vec{r}_0 = (1, 0, 0)$. The calculation was made over a full circle taking the end point nearest to the initial point. Four parameters were compared: (i) number of steps N, (ii) number of vector field calculations N_{calc}, (iii) divergence between a calculated field line length and the unit radius circle length $S = 2\pi \approx 6.2832$, (iv) deviation of the end point coordinate x_N from the initial coordinate $x_0 = 1.0$. Comparison results are presented in Table I. The following notations are used:

Eps is the value of ε,

Method 1 – algorithm based on Eqs. (7)–(9),

Method 2 – algorithm based on Eqs. (16)–(18) with correction of a point location.

Method 2 uses, on the average, 5 calculations of the function $\vec{f}(\vec{r})$ for every iteration compared with 2 calculations for method 1. It is evident, however, that the total number of field calculations is smaller in method 2. This discrepancy increases with improving the approximation accuracy. Also, method 2 gives a smaller actual error under similar accuracy requirements.

Figure 1 illustrates a reconstruction of a magnetic surface formed by an infinite helical field line going round a closed toroidal surface. The field is generated by a combination of a ring conductor with the radius $R_{\text{ring}} = 1\,\text{m}$ and a straight conductor. Each conductor is assumed to be a 1D current filament.

The straight conductor passes through the center of the ring along its vertical axis and may be treated practically infinite. The reconstructed field is a helical toroidal field modelled as a nest of magnetic surfaces with the magnetic axis coincident with the ring conductor. The pitch of the helical line is dictated by the ratio of the ring and straight currents $I_1/I_2 = 20$.

CONCLUSIONS

The computational procedure developed at the Efremov Institute offers an efficient and accurate 3D reconstruction of a field line for any vector field. The procedure has successfully been applied to visualize calculated and measured data and for 3D analysis of field configuration in a working zone and stray fields of electromagnetic systems in various electrophysical devices.

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