Summary of Rules for Error Propagation

Suppose you measure some quantities $a, b, c, ...$ with uncertainties $\delta a, \delta b, \delta c, ...$. Now you want to calculate some other quantity Q which depends on a and b and so forth. What is the uncertainty in Q? The answer can get a little complicated, but it should be no surprise that the uncertainties $\delta a, \delta b$, etc. "propagate" to the uncertainty of Q. Here are some rules which you will occasionally need; all of them assume that the quantities a, b, etc. have errors which are *uncorrelated* and *random*. [1]

1. Addition or subtraction: If

$$Q = a + b + ... + c - (x + y + ... + z)$$

then

$$\delta Q = \sqrt{(\delta a)^2 + (\delta b)^2 + ... + (\delta c)^2 + (\delta x)^2 + (\delta y)^2 + ... + (\delta z)^2}.$$

2. Multiplication or division: If

$$Q = \frac{ab...c}{xy...z}$$

then

$$\frac{\delta Q}{|Q|} = \sqrt{(\frac{\delta a}{a})^2 + (\frac{\delta b}{b})^2 + ... + (\frac{\delta c}{c})^2 + (\frac{\delta x}{x})^2 + (\frac{\delta y}{y})^2 + ... + (\frac{\delta z}{z})^2}.$$

3. Measured quantity times exact number: If A is known exactly (e.g. $A = 2$ or $A = \pi$) and

$$Q = Ax$$

then

$$\delta Q = |A|\, \delta x$$

or equivalently

$$\frac{\delta Q}{|Q|} = \frac{\delta x}{|x|}$$

4. Uncertainty in a power: If n is an exact number and

$$Q = x^n$$

then

$$\frac{\delta Q}{|Q|} = |n|\, \frac{\delta x}{|x|}.$$

5. General formula for error propagation: If $Q = Q(x)$ is any function of x, then

$$\delta Q = |\frac{dq}{dx}|\, \delta x.$$

[1] These rules can all be derived from the Gaussian equation for normally-distributed errors, but you are not expected to be able to derive them, merely to be able to use them.