A Basic Concept First

Multiplication is defined as follows:

$$a * b = \sum_{c=1}^{a} b$$

Such that $a * b$ is equivilent to the following:

$$\underbrace{b + b + \ldots + b}_{a \text{ times}}$$

Explaining Why $\frac{n}{0}$ is not ∞

The inverse of division is multiplication, such as the following:

$$\frac{6}{2} = 3$$

$$3 * 2 = 6$$

So if we pretend for a moment that $\frac{n}{0} = \infty$, then the following is true (where n is any number):

$$\frac{n}{0} = \infty$$

$$\infty * 0 = n$$

However, a fundemental rule in multiplication is that anything times 0 is 0, due to the following:

$$a * 0 = \sum_{c=1}^{a} 0 = 0$$

So $\infty * 0 \neq n$, thus proving that anything divided by 0 is not infinity.

Explaining Why $\frac{0}{0}$ is undefined

Let n and k be numbers such that $n \neq k$, and let $\alpha = n * 0$ and $\beta = k * 0$. As anything times 0 is 0, $\alpha = \beta$. Now let's set up the following equation:

$$\alpha = \beta$$

Expanded, this would be:

$$n * 0 = k * 0$$

So far so good. Now let's try dividing 0 on both sides:

$$\frac{n * 0}{0} = \frac{k * 0}{0}$$

If we pretend for a moment that $\frac{0}{0}$ is defined, then the following occurs:

$$\frac{n * \cancel{0}}{\cancel{0}} = \frac{k * \cancel{0}}{\cancel{0}}$$

Leaving:

$$n = k$$

However, n and k are defined such that $n \neq k$, so division by 0 is undefined.