Proving $\frac{n}{0}$ is not ∞

The inverse of division is multiplication, so:

$$\frac{6}{2} = 3$$

$$3 * 2 = 6$$

So if we pretend for a moment that $\frac{n}{0} = \infty$, then the following is true (where n is any number):

$$\frac{n}{0} = \infty$$

$$\infty * 0 = n$$

However, a fundemental rule in multiplication is that anything times 0 is 0, due to the following:

$$n * 0 = \sum_{k=0}^{n} 0 = 0$$

So $\infty * 0 \neq n$, thus proving that anything divided by 0 is not infinity.

Proving $\frac{0}{0}$ is undefined

Let $\alpha = 1 * 0$ and $\beta = 2 * 0$. As anything times 0 is 0, $\alpha = \beta$. Now let's set up the following equation:

$$\alpha = \beta$$

Expanded, this would be:

$$1 * 0 = 2 * 0$$

So far so good. Now let's try dividing 0 on both sides:

$$\frac{1 * 0}{0} = \frac{2 * 0}{0}$$

If we pretend for a moment that $\frac{0}{0}$ is defined, then the following occurs:

$$\frac{1 * \cancel{0}}{\cancel{0}} = \frac{2 * \cancel{0}}{\cancel{0}}$$

Leaving:

$$1 = 2$$

For obvious reasons this is false, but just to prove it is false, if $1 = 2$, then the following can be concluded:

$$1 + 1 = 1$$

$$1 + \cancel{1 + (-1)} = \cancel{1 + (-1)}$$

$$1 + 0 = 0$$

$$1 = 0$$