Lecture 14:

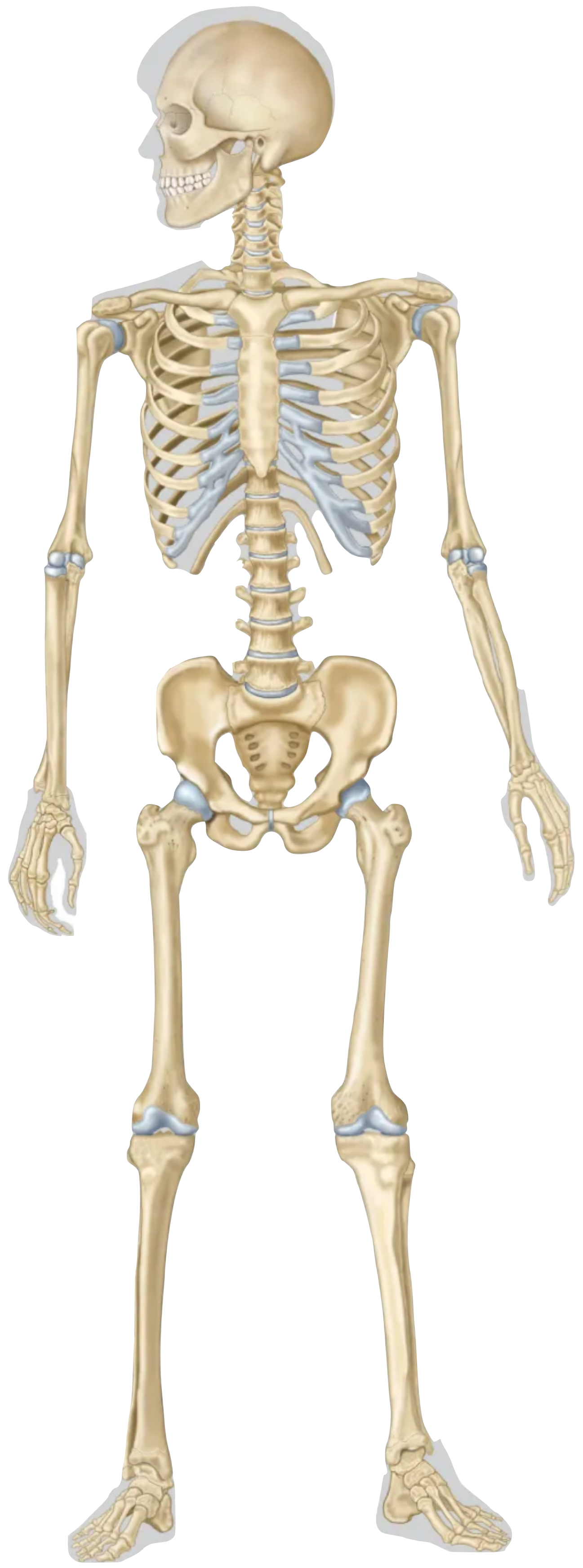
Character Modeling

FUNDAMENTALS OF COMPUTER GRAPHICS

Animation & Simulation

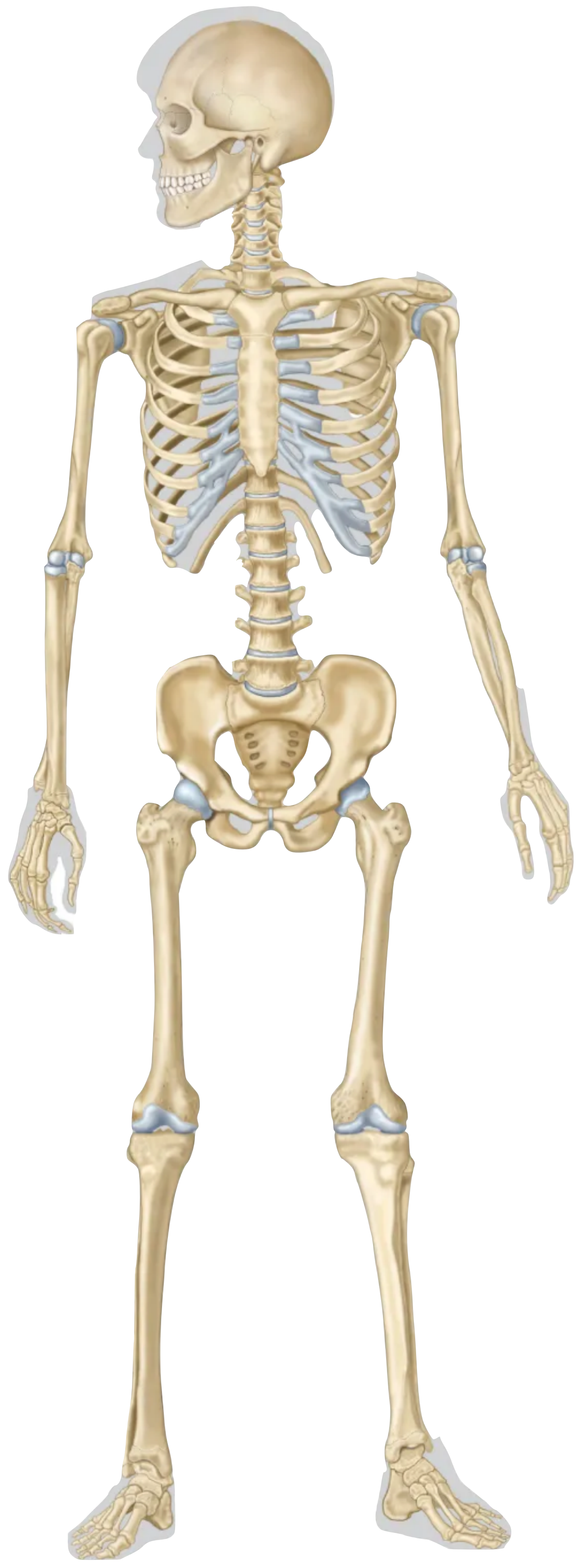
Stanford CS248B, Fall 2022

What is a pose?





How do we model a pose?



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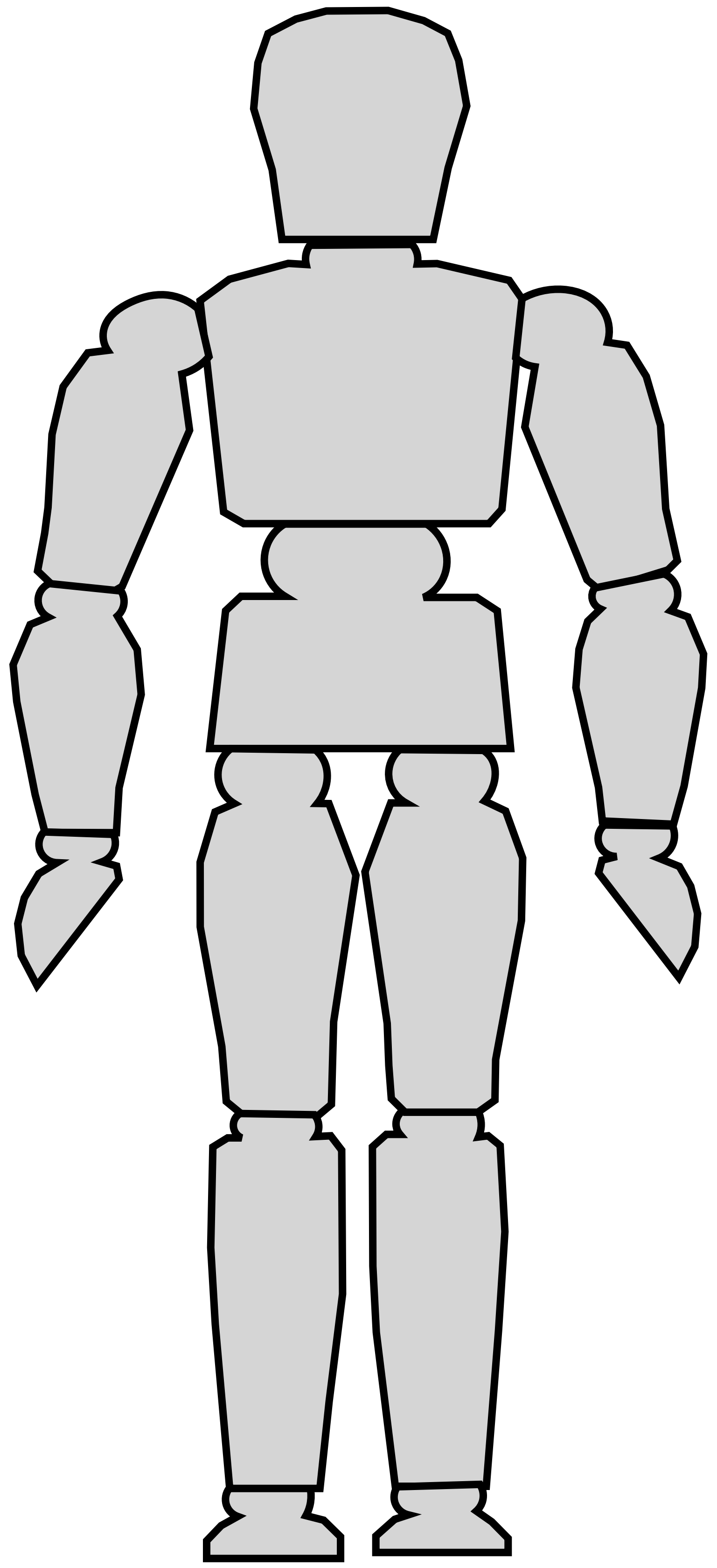

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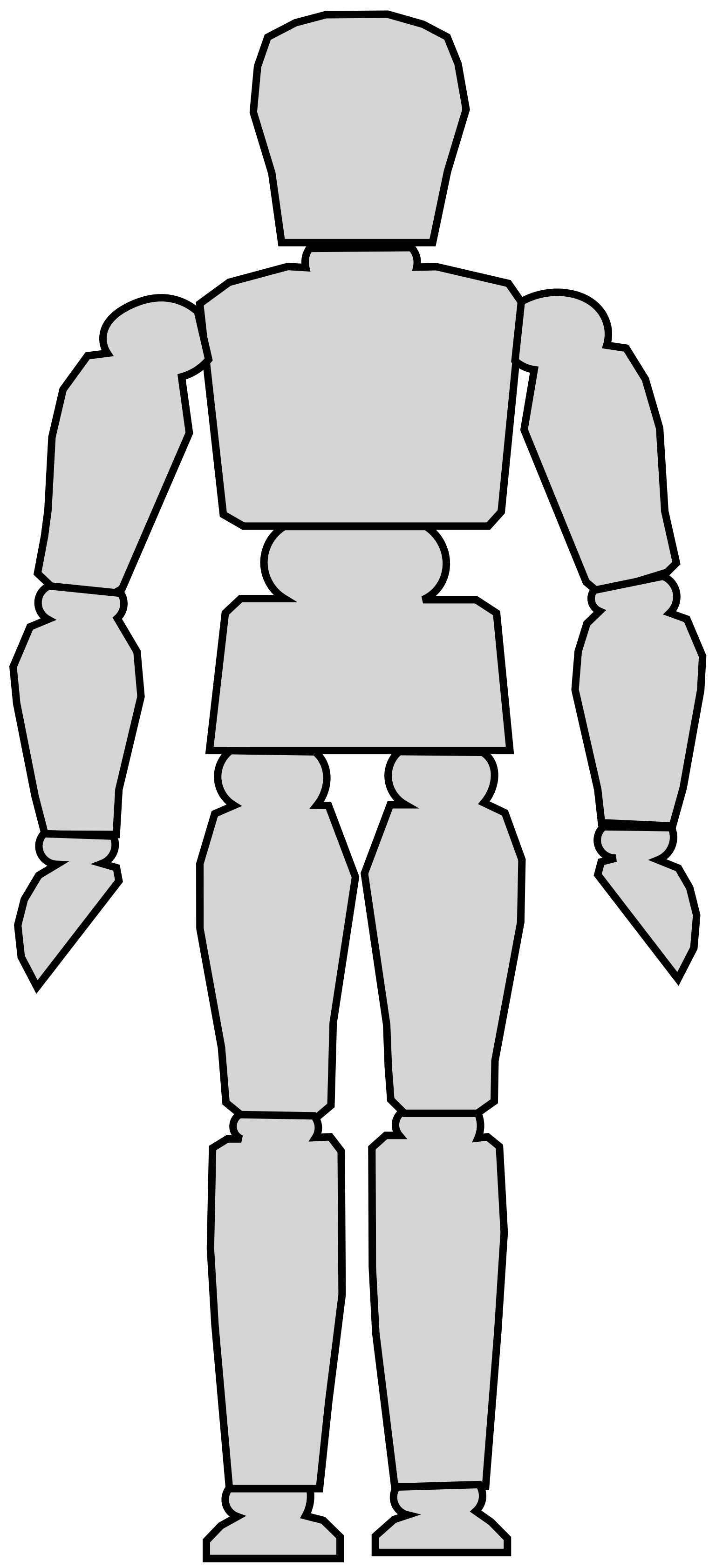

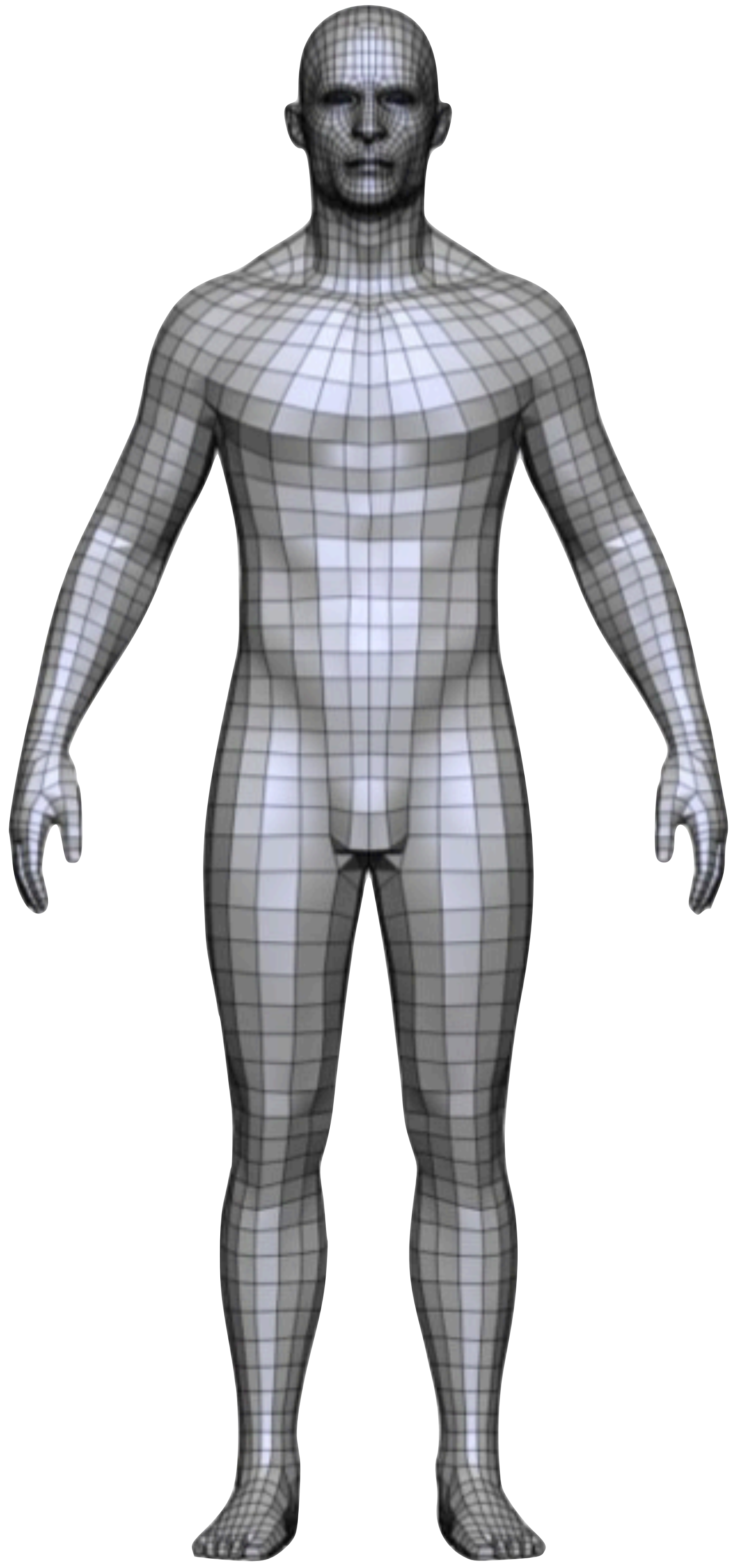

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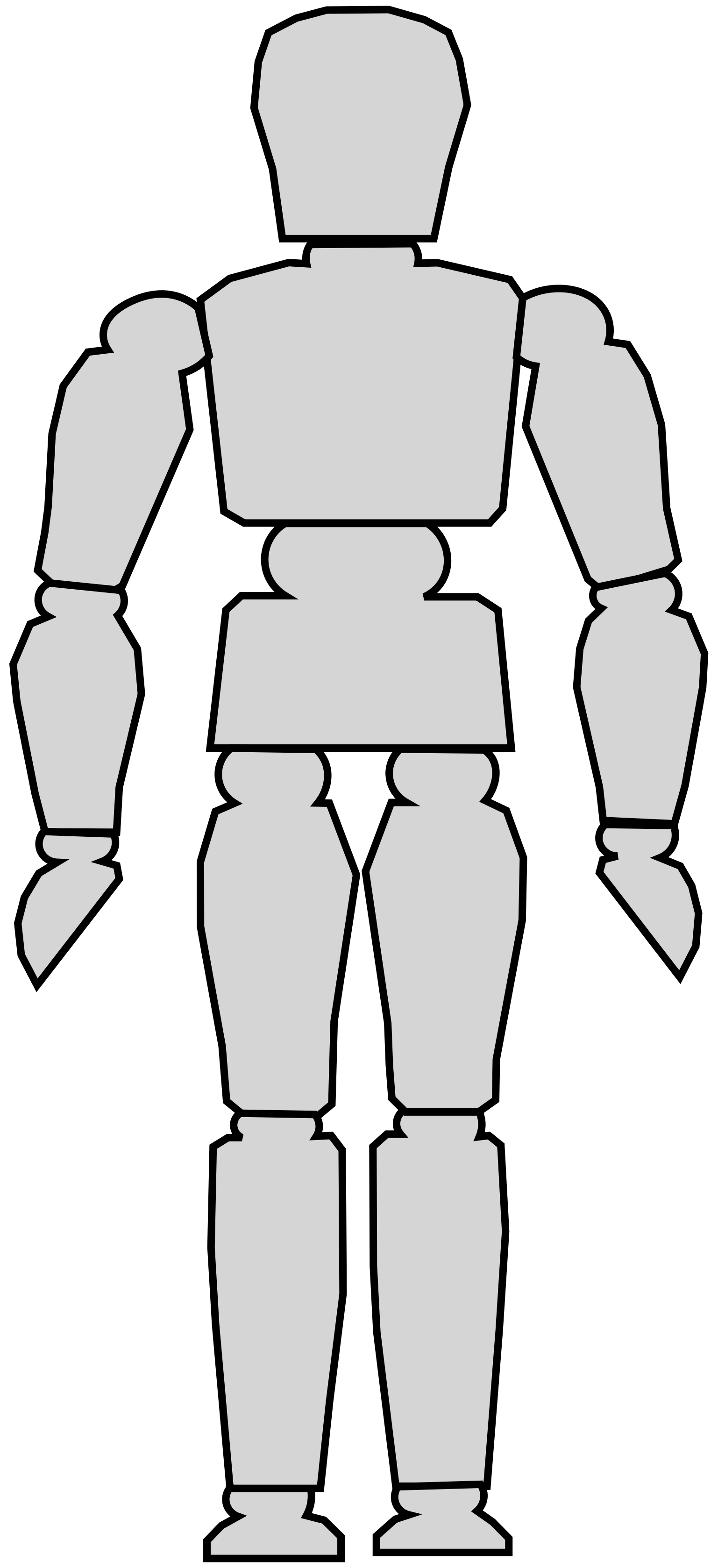

Modeling with transformations

- From computer graphics, we learn to create models that consist of multiple 3D parts using affine transformations.
- A point in homogeneous coordinates, $\mathbf{p} = \{x, y, z, 1\}$ can be transformed by:

Modeling with transformations

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Translate

$$\begin{bmatrix} x + t_x \\ y + t_y \\ z + t_z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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Scale

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Rotate (about x axis)

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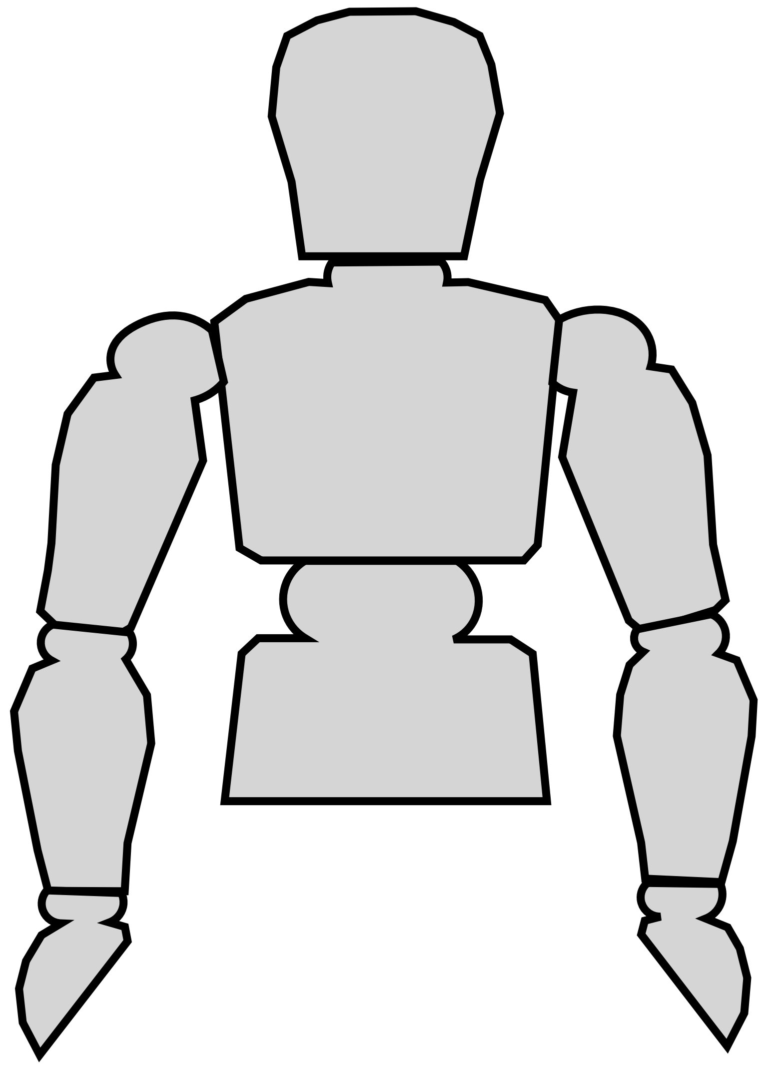
Make a human figure by transforming parts

for each part
- **scale** to the right size
- **rotate** to the right orientation
- **translate** to the right position

Make a human figure by transforming parts



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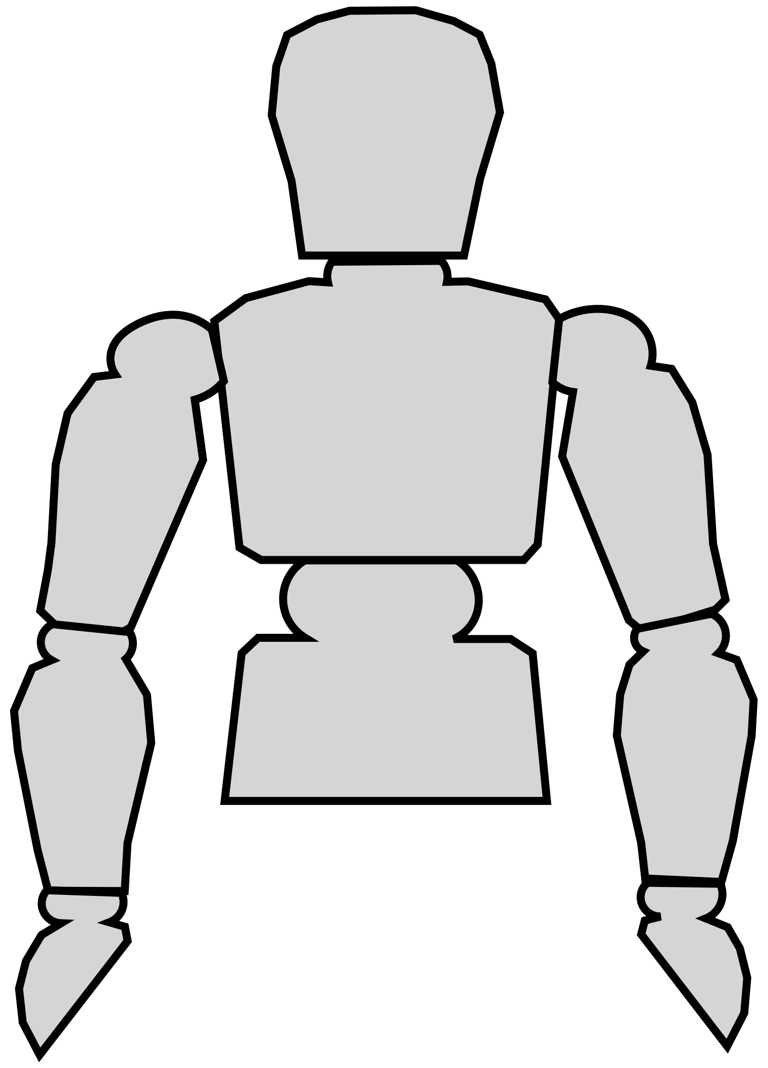

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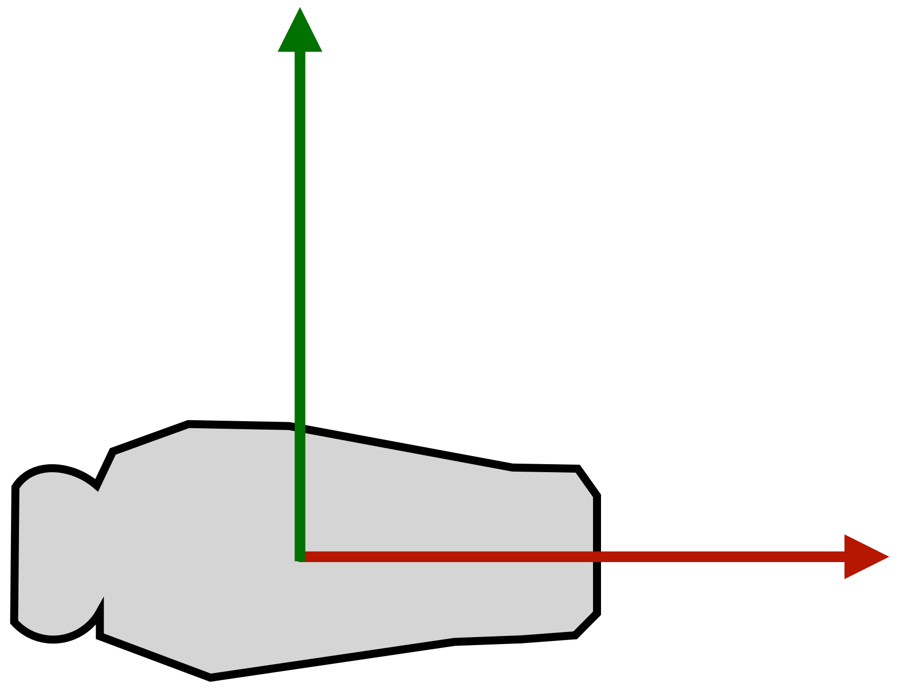

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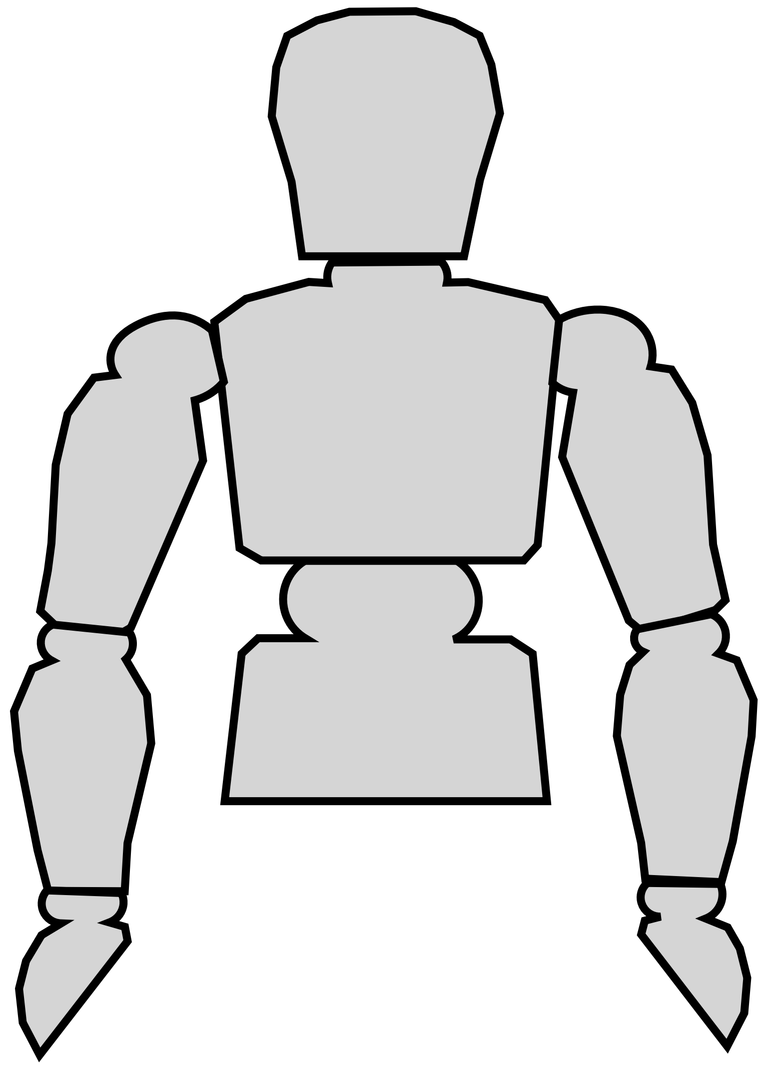

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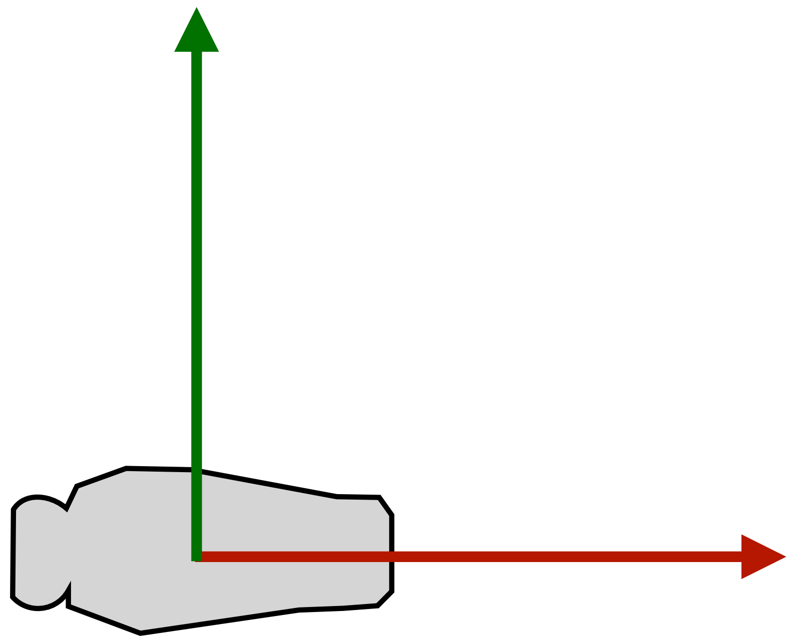

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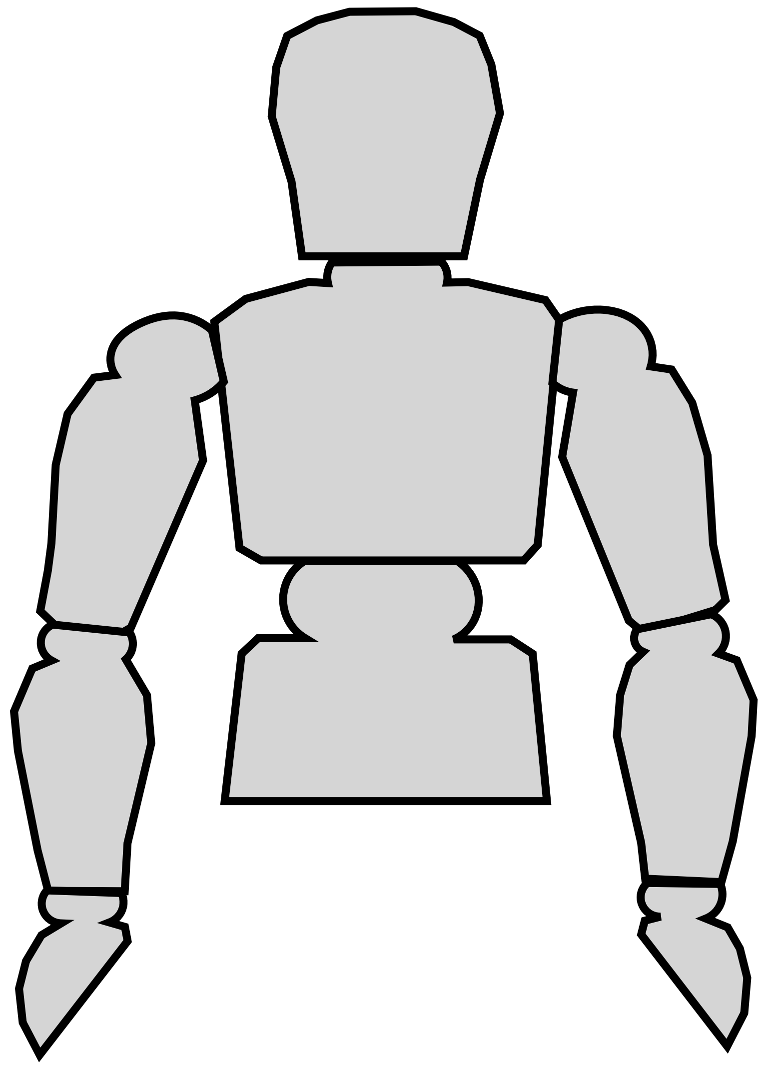

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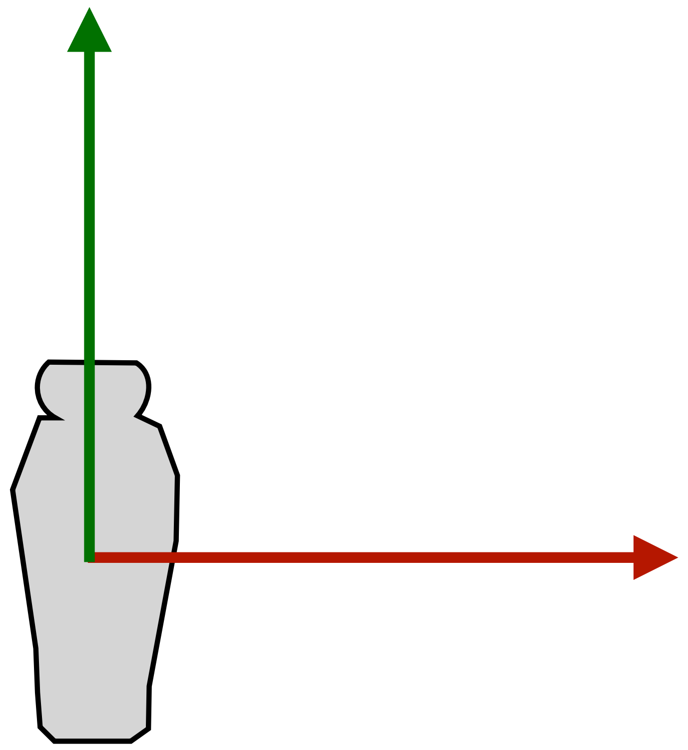

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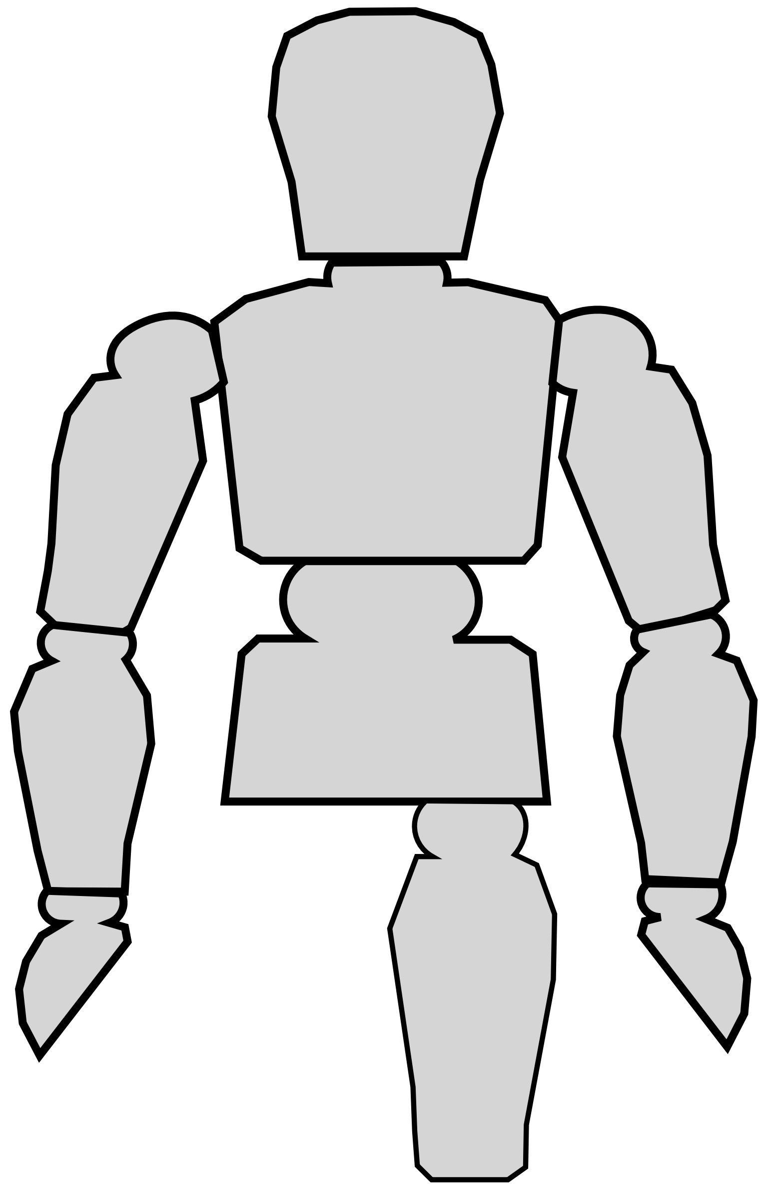

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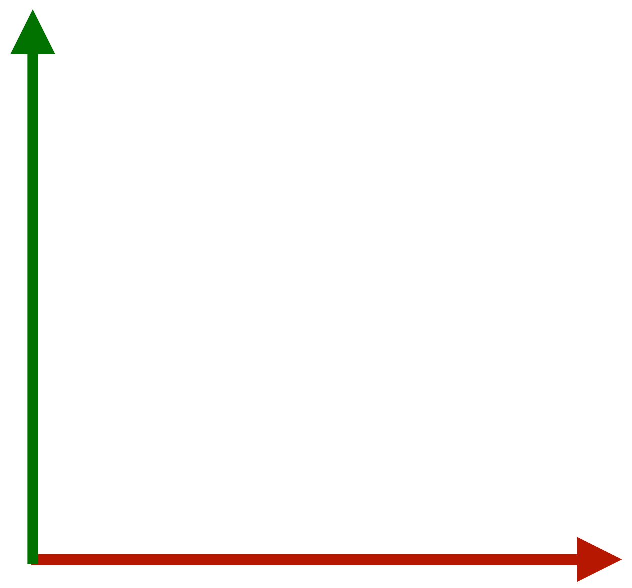

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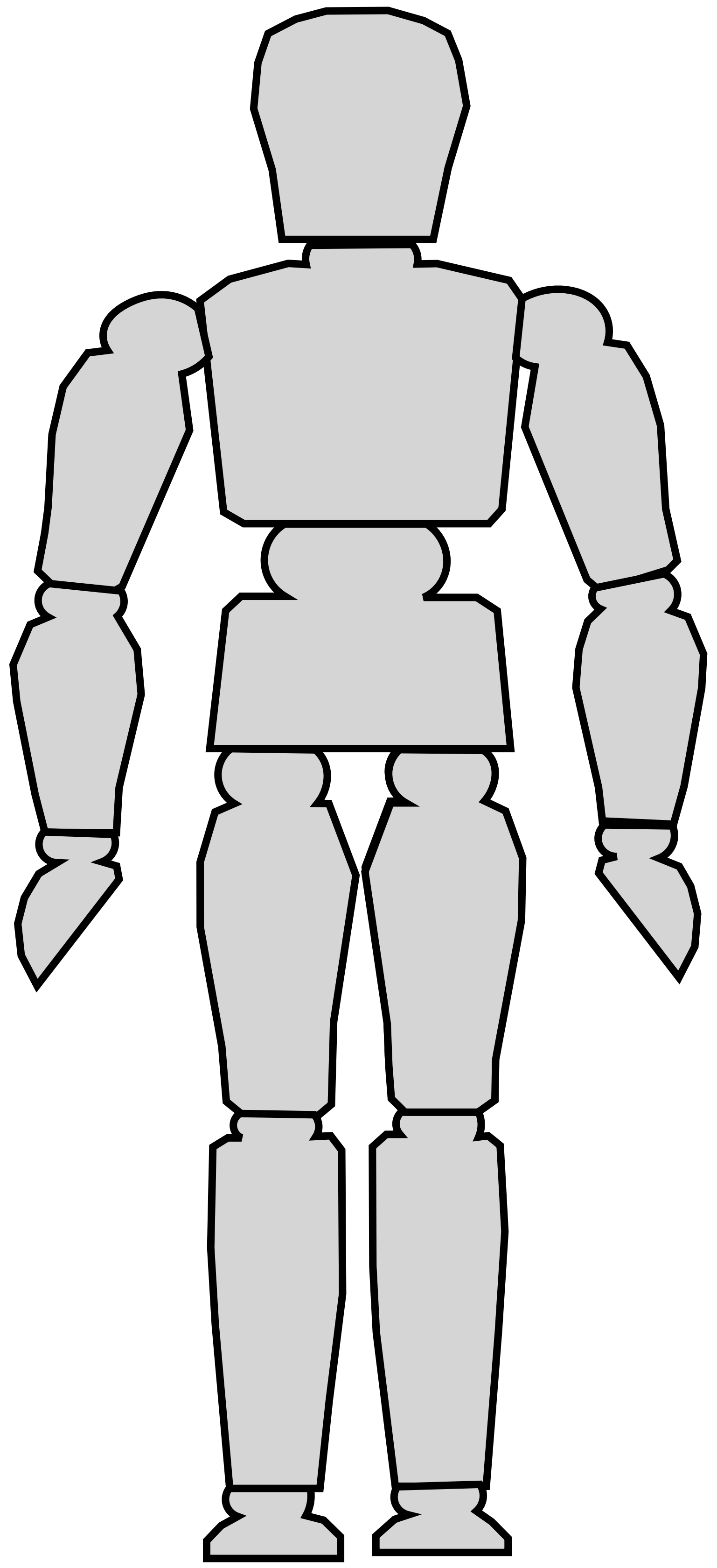

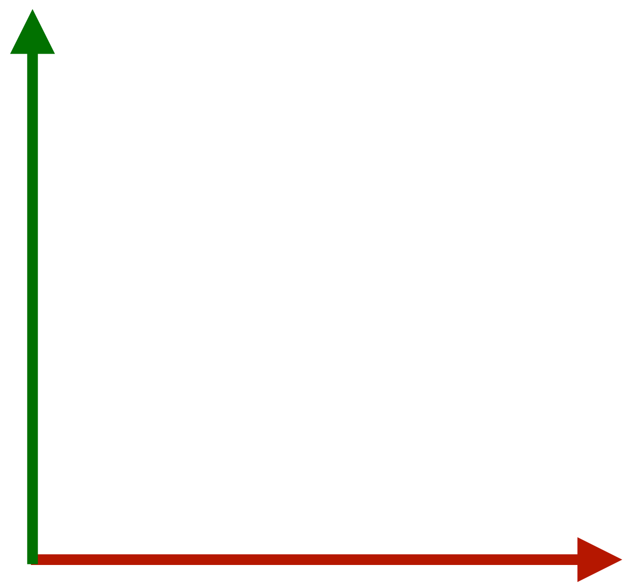

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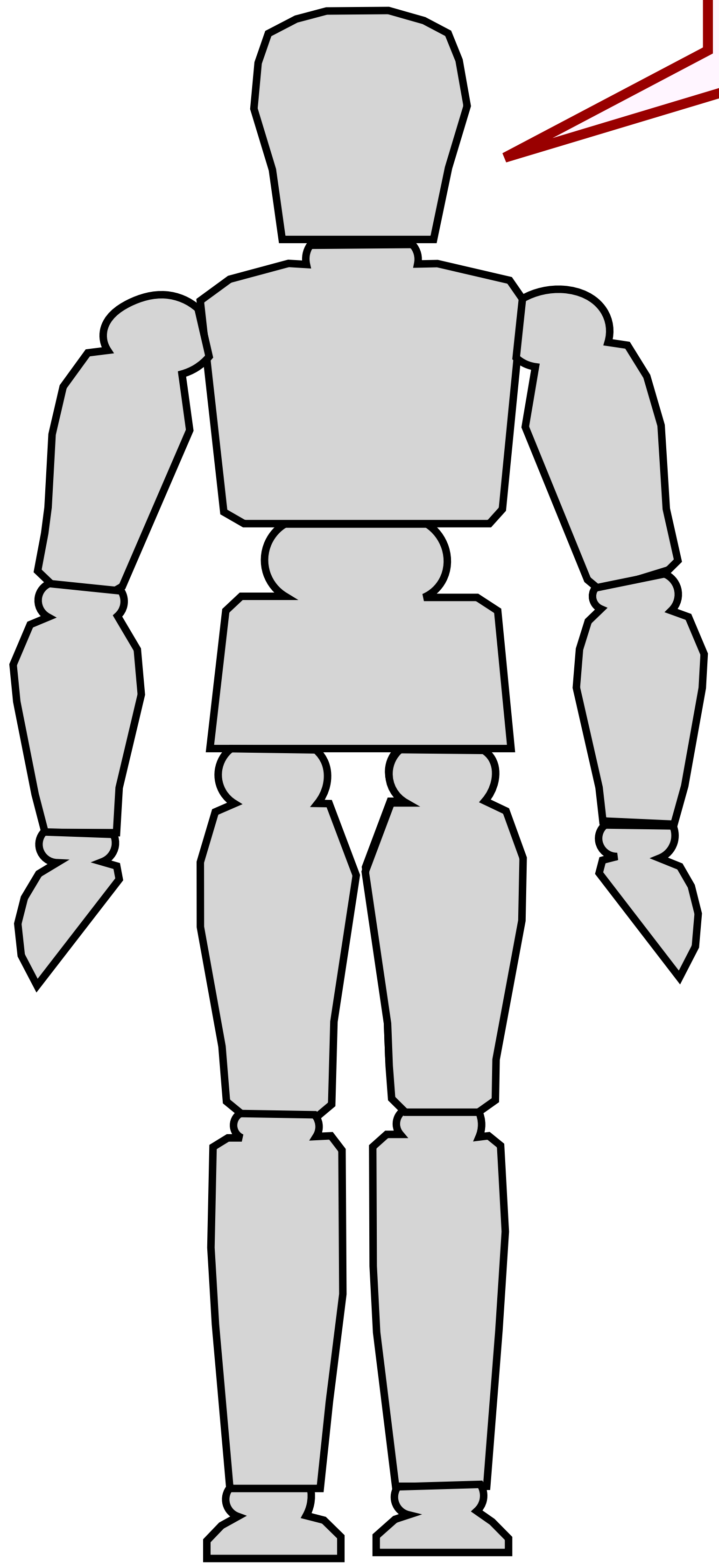

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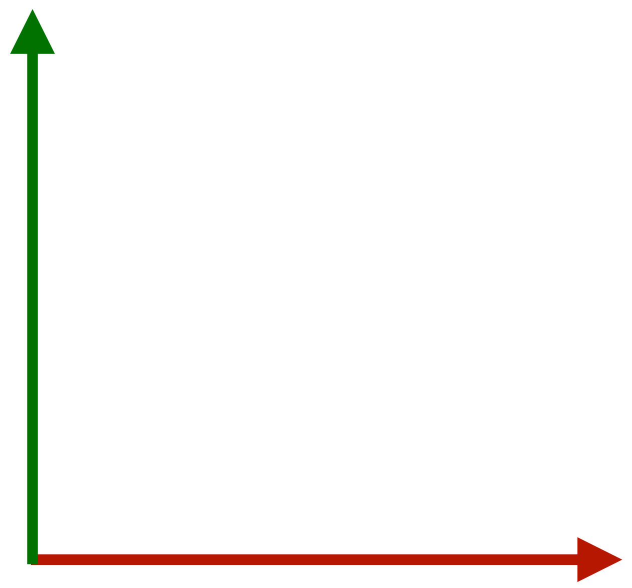
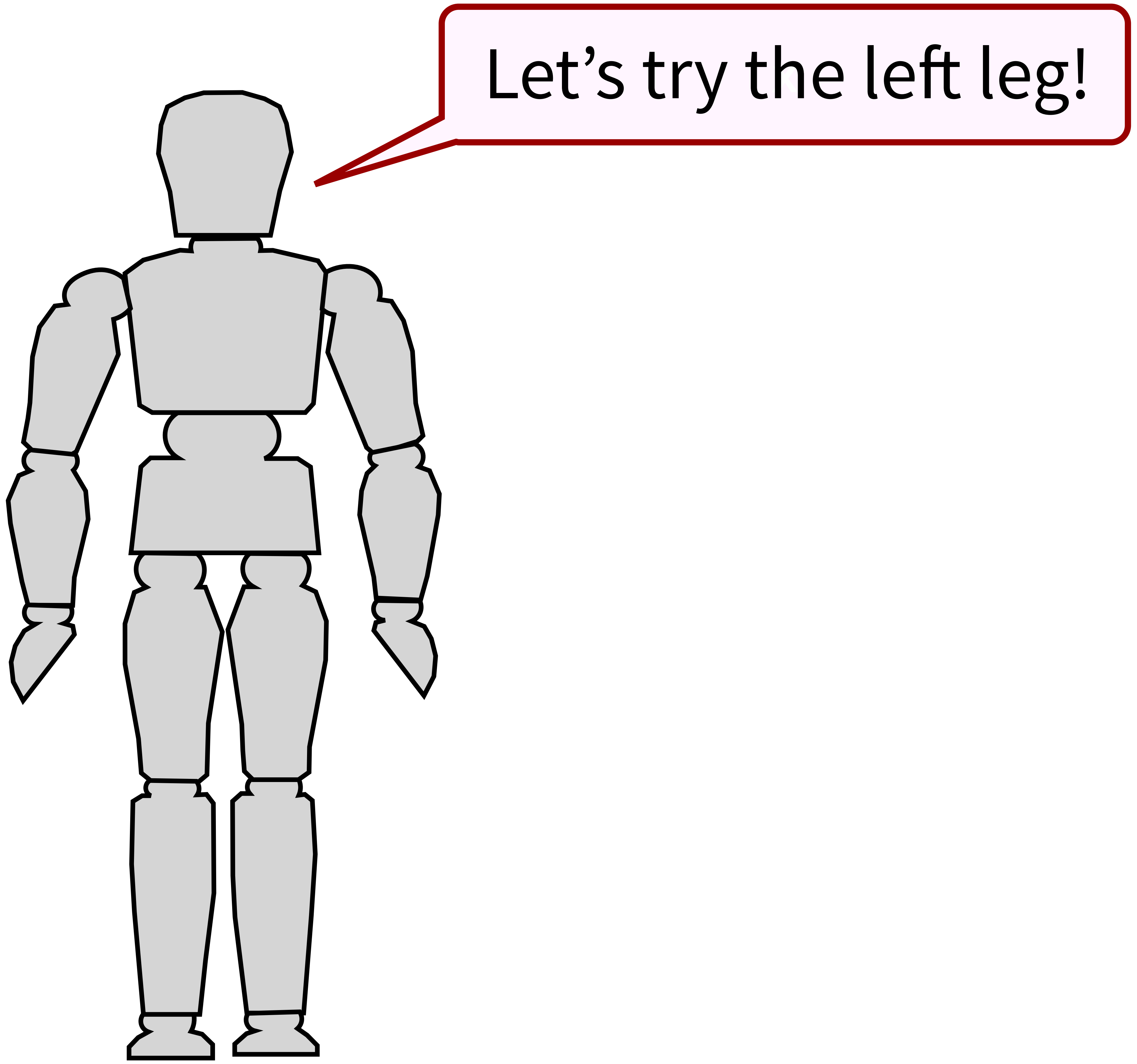


Now make it move!



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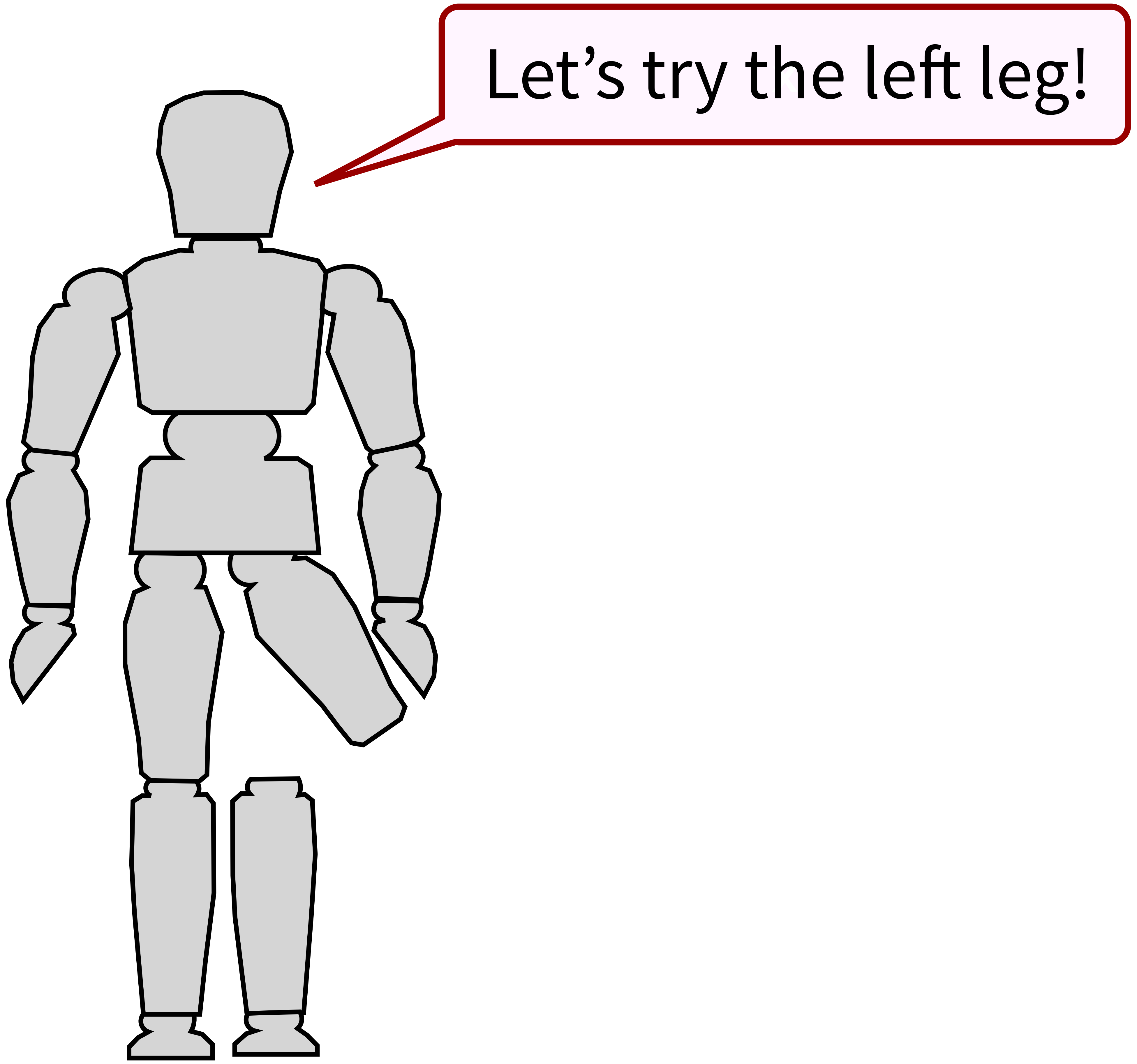

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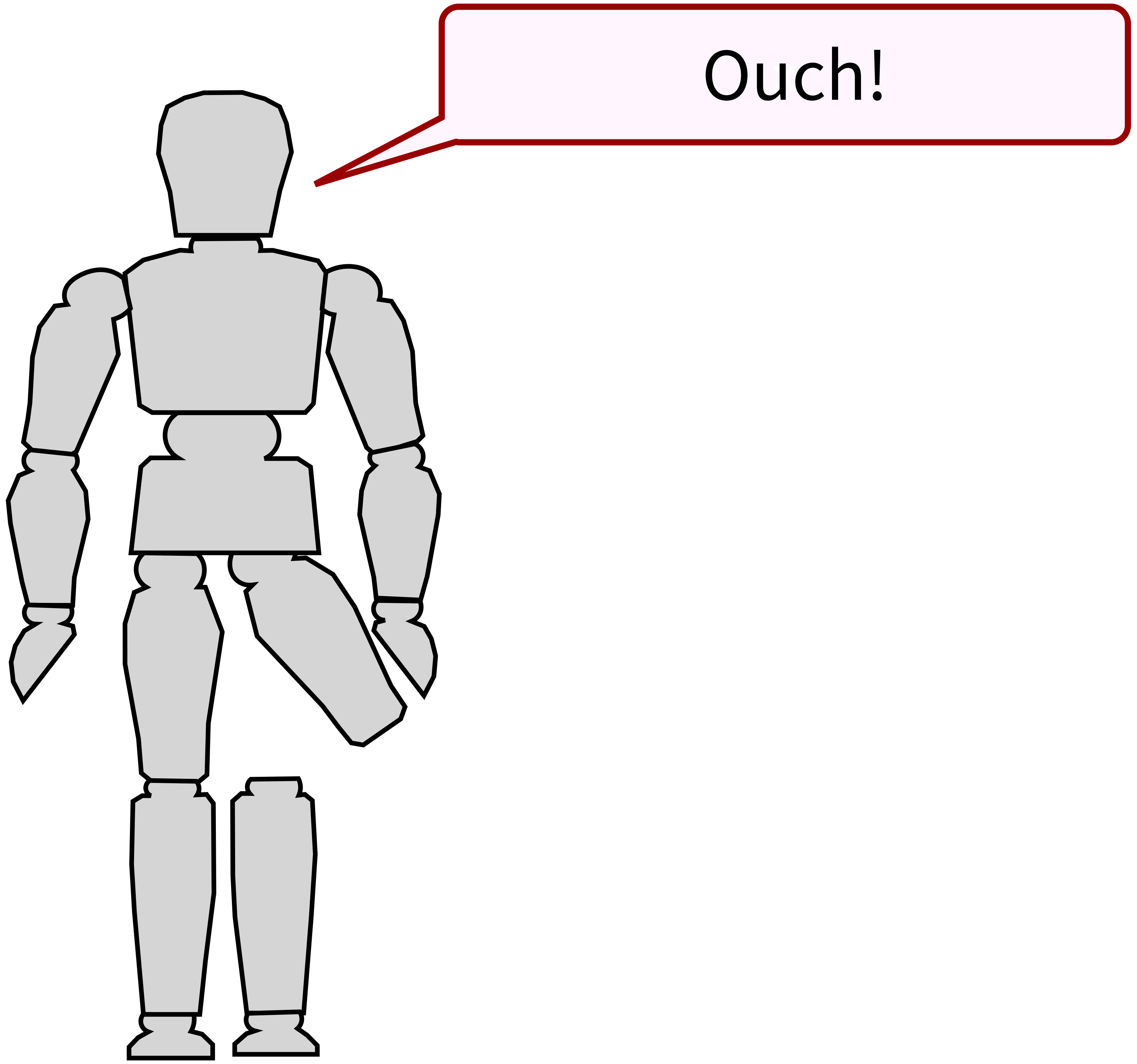

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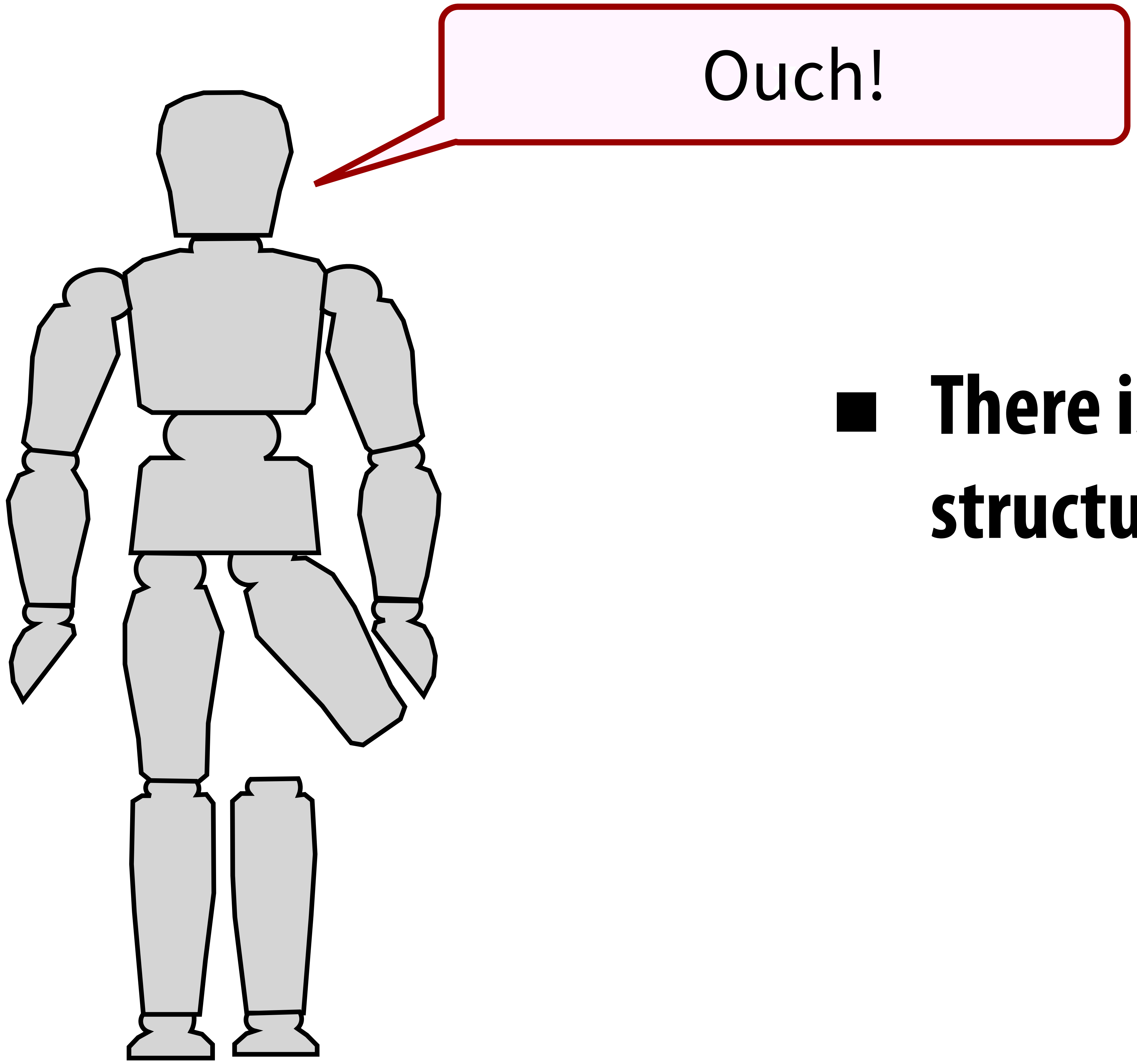

- There is no constraint that enforces the articulated structure to move in the “legal” configuration space.

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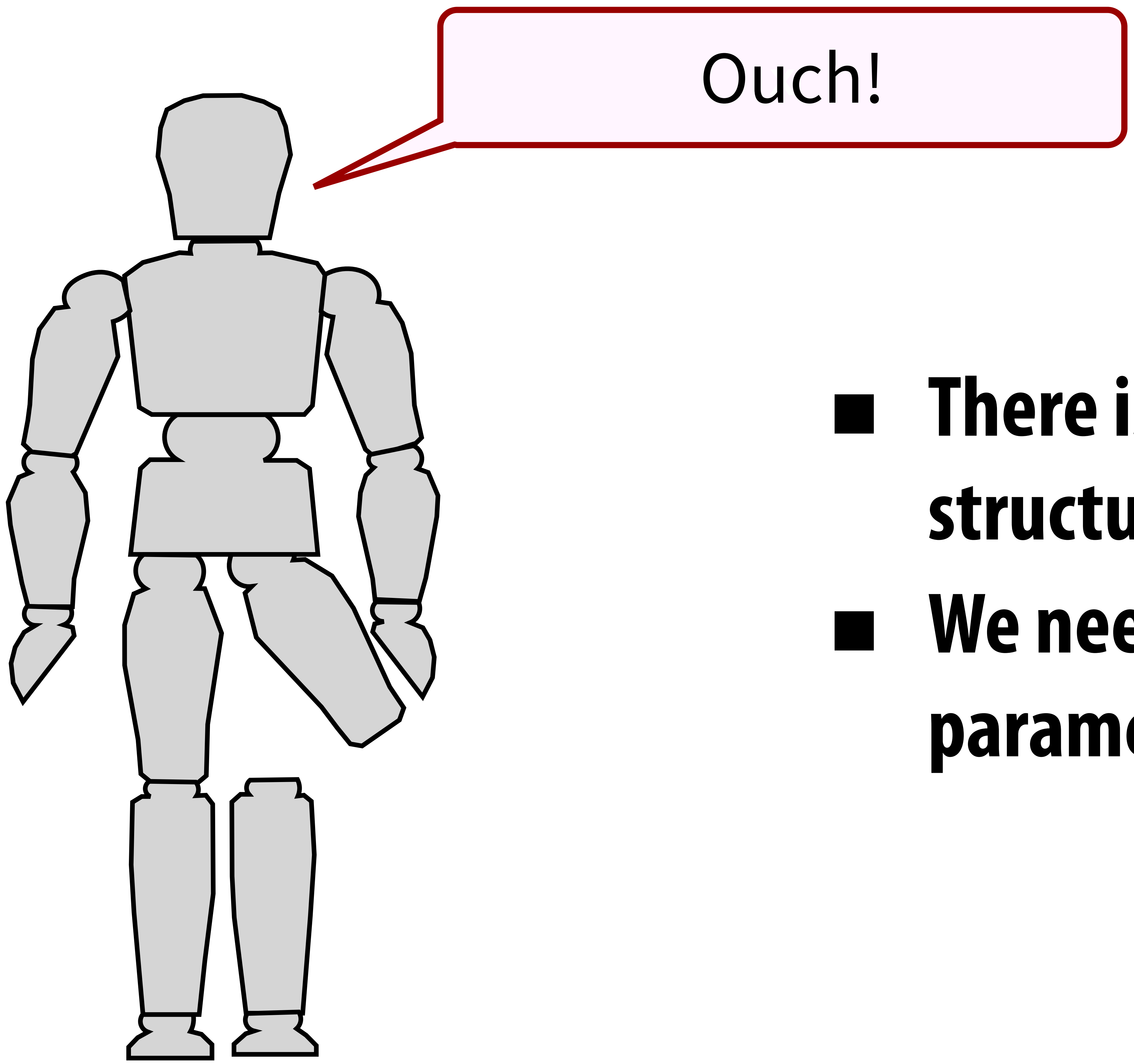
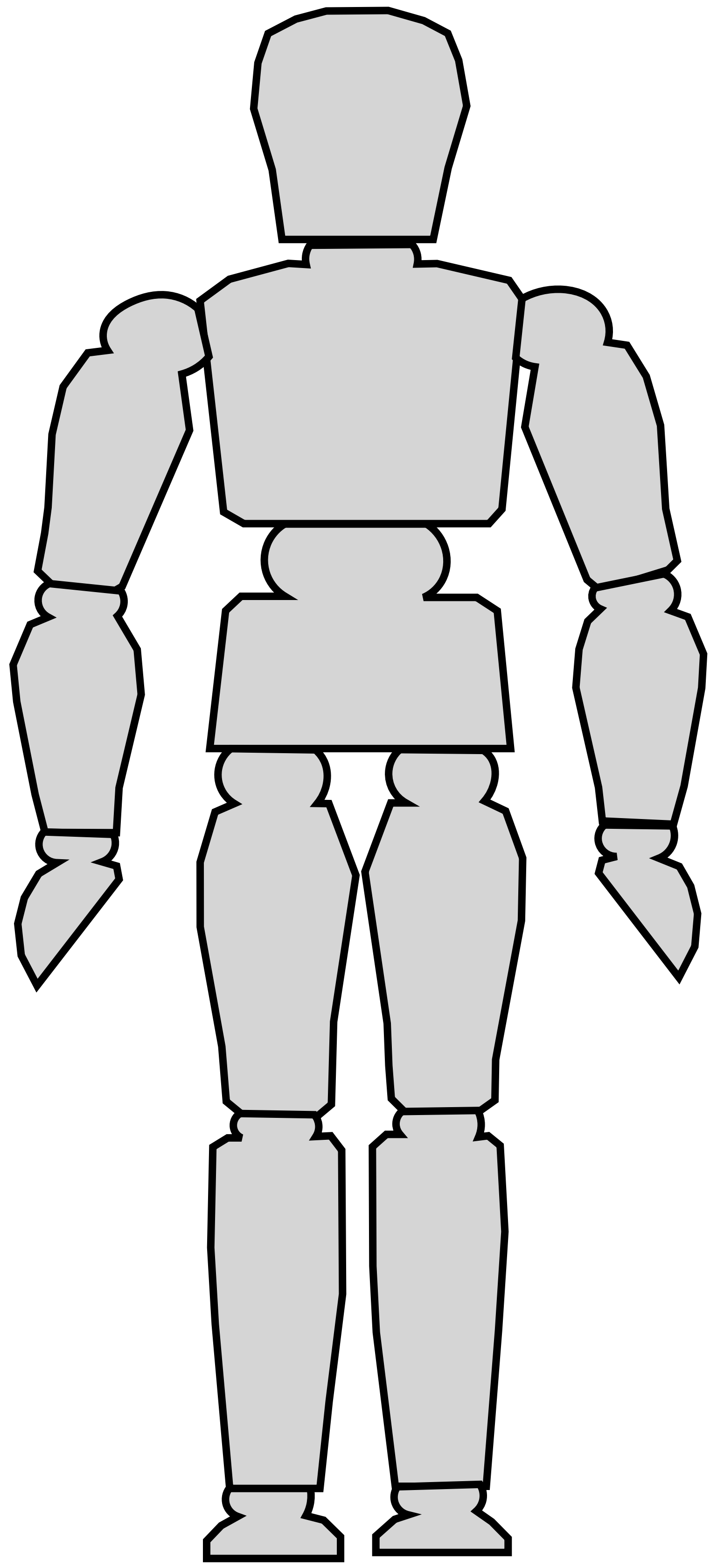


- There is no constraint that enforces the articulated structure to move in the "legal" configuration space.
- We need a hierarchical structure and a compact parameterized space to represent poses.

How to represent a pose?



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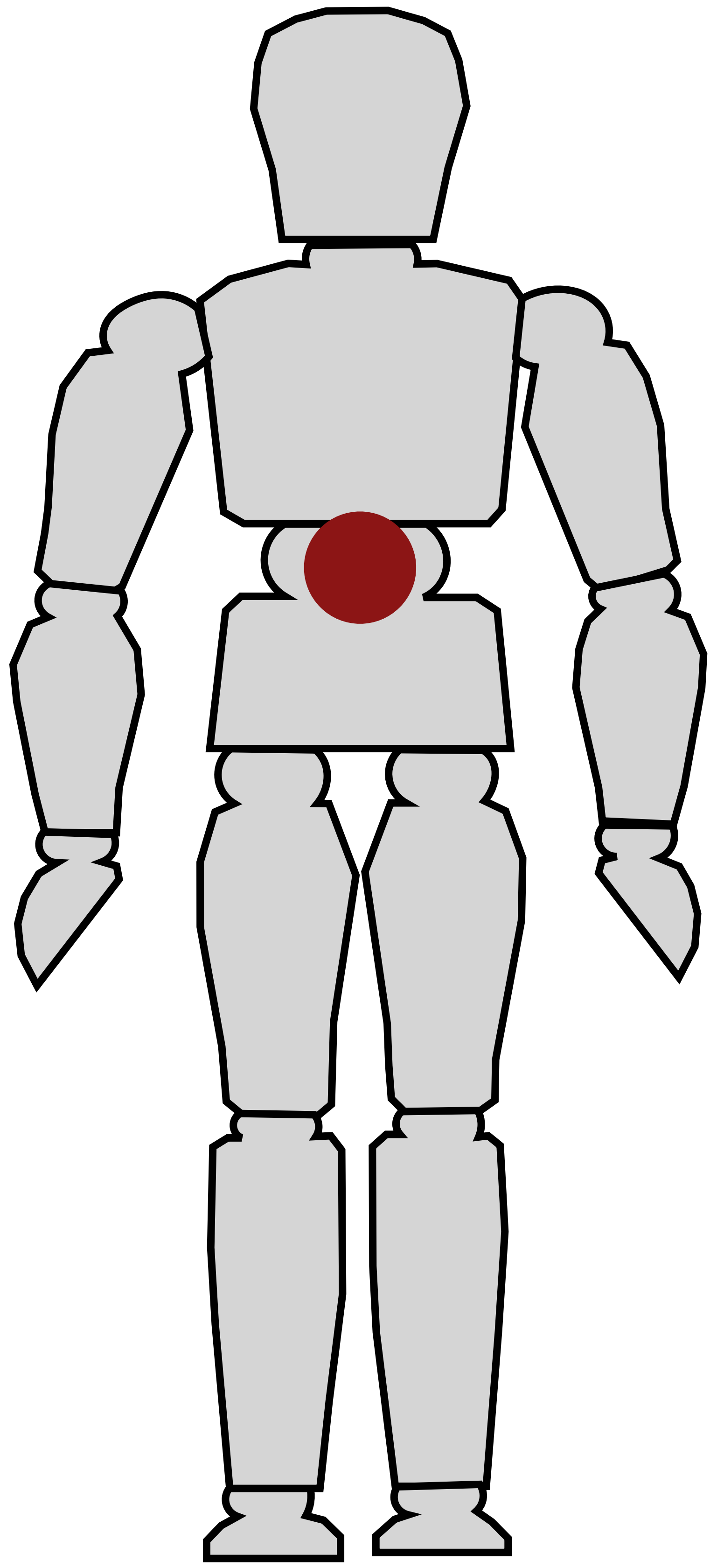

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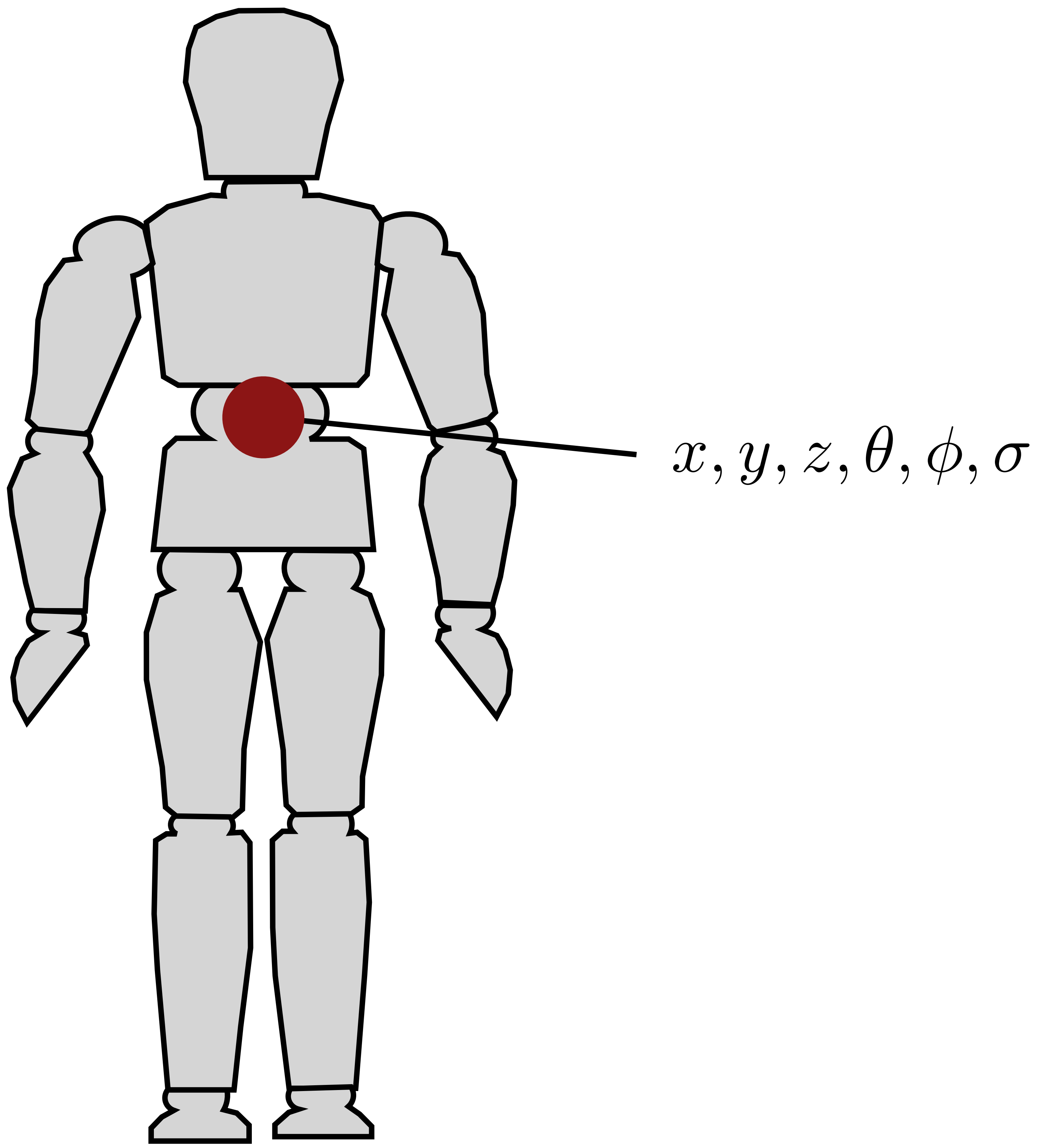

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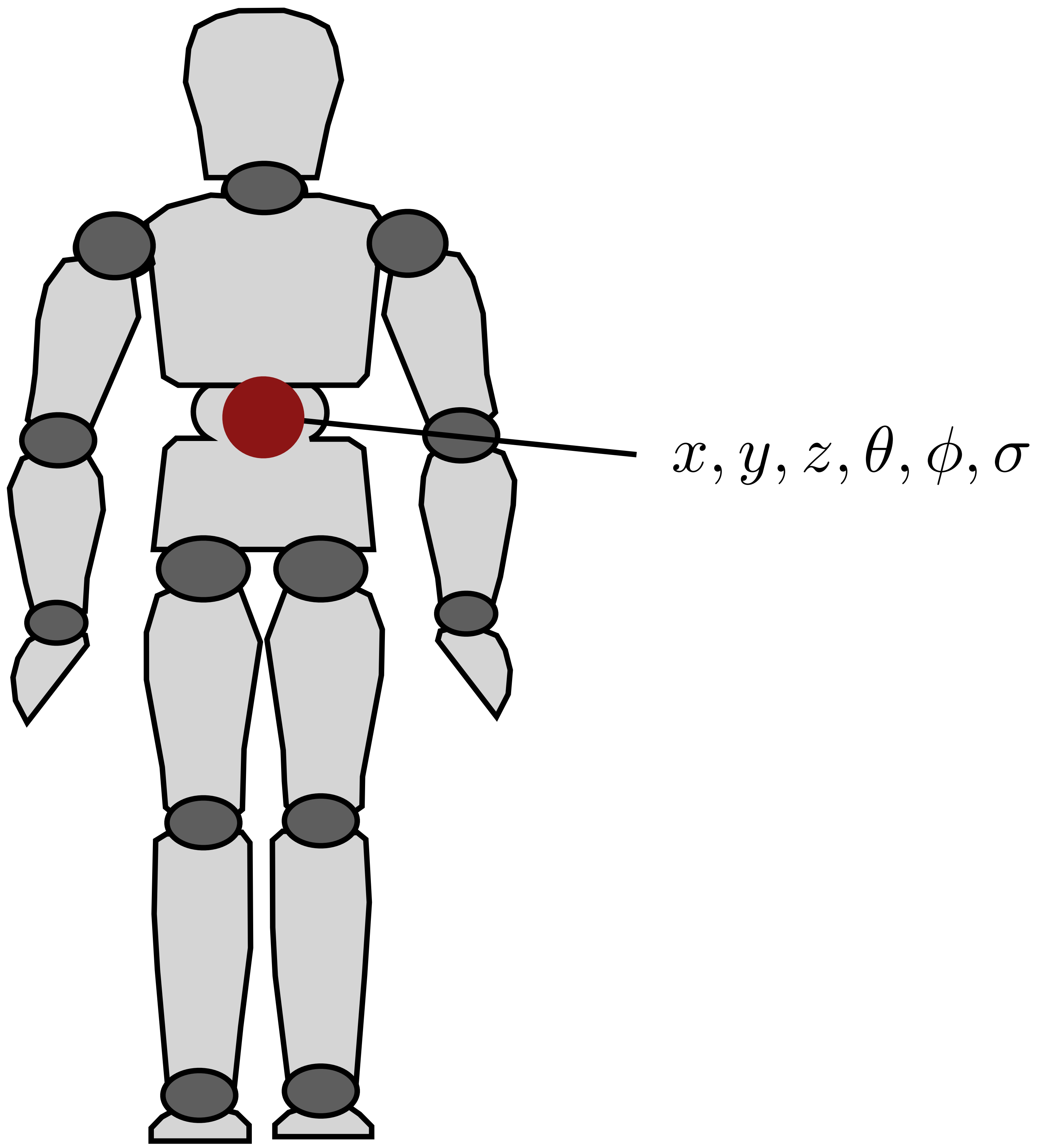

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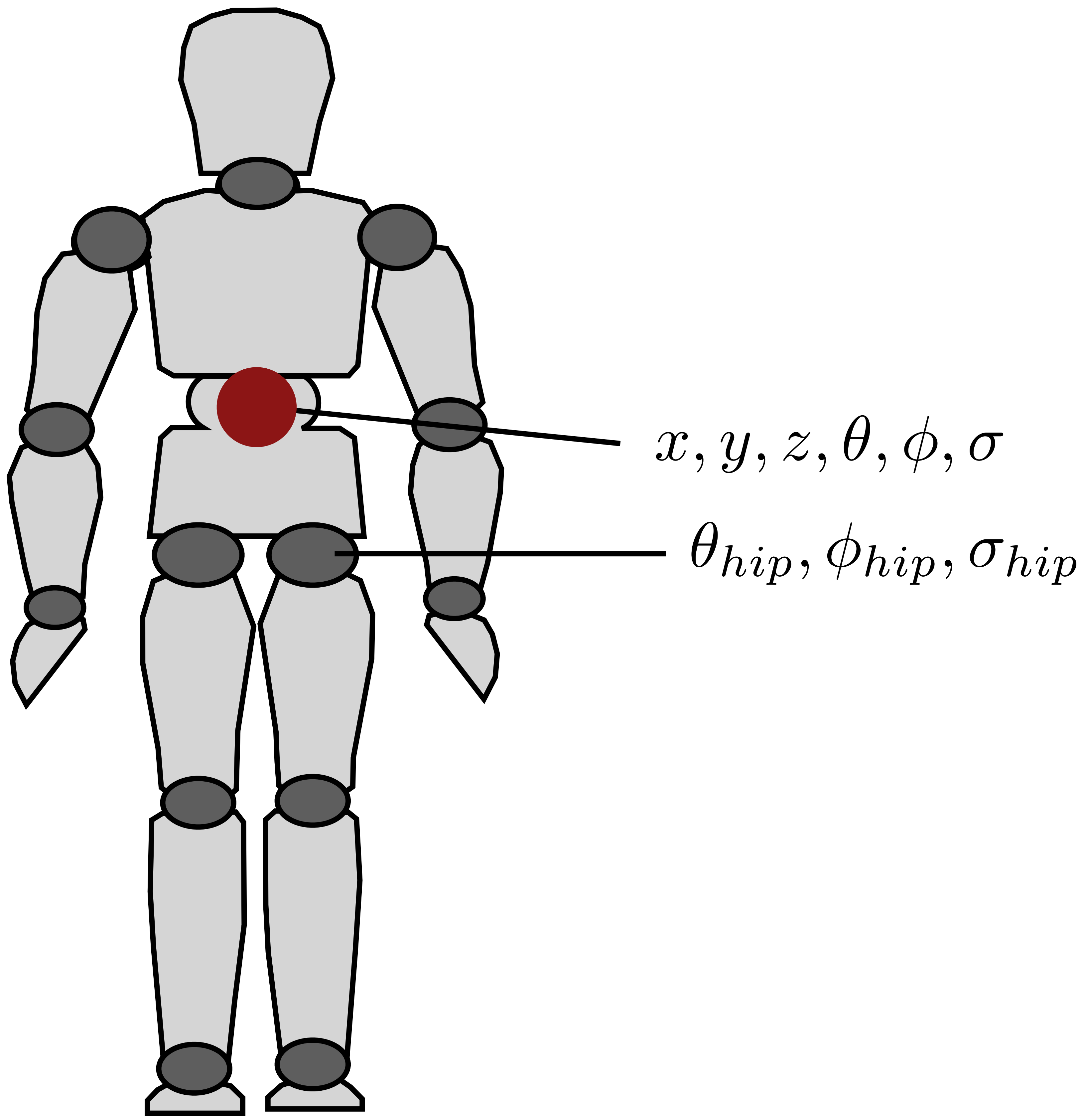

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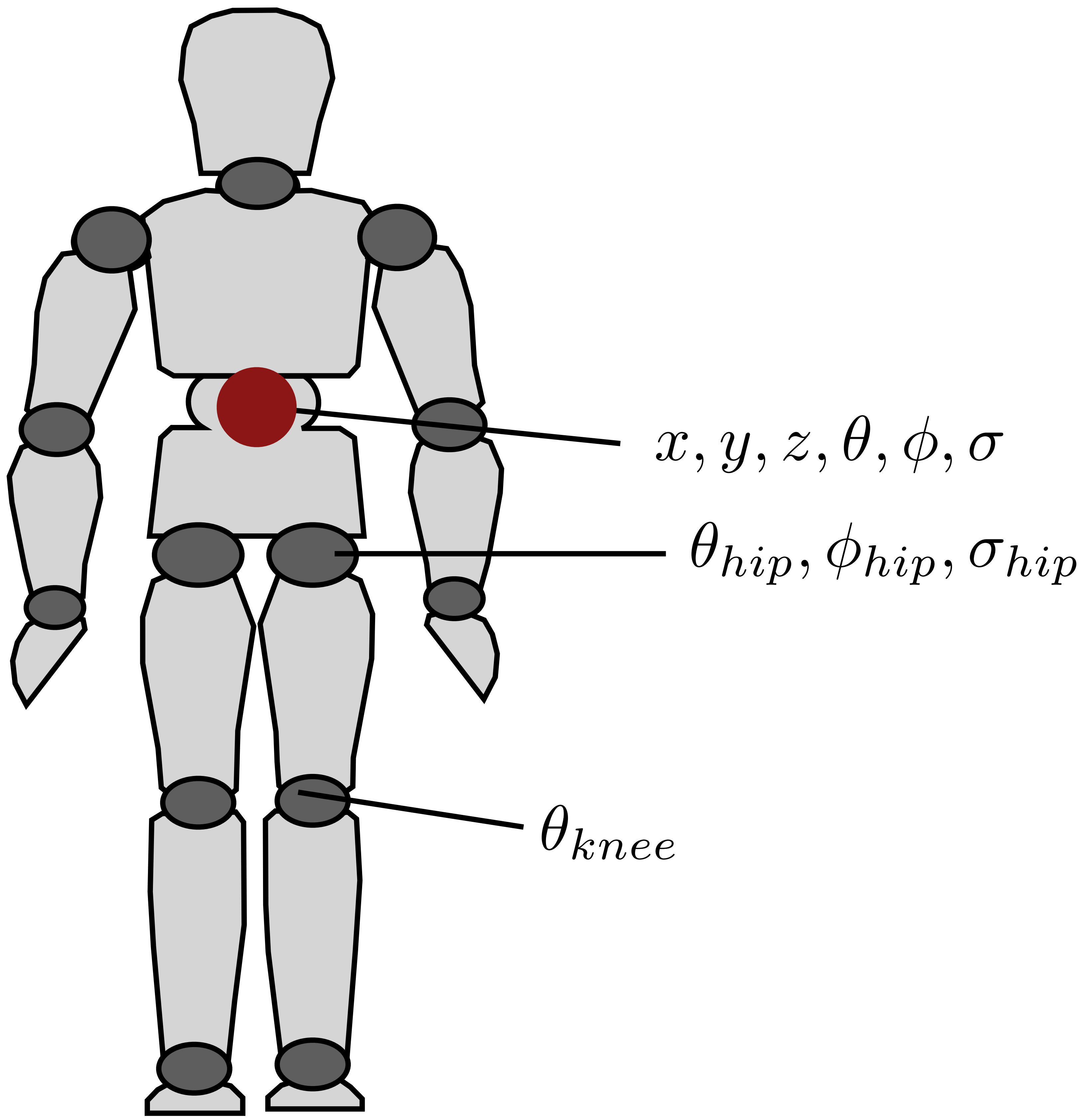

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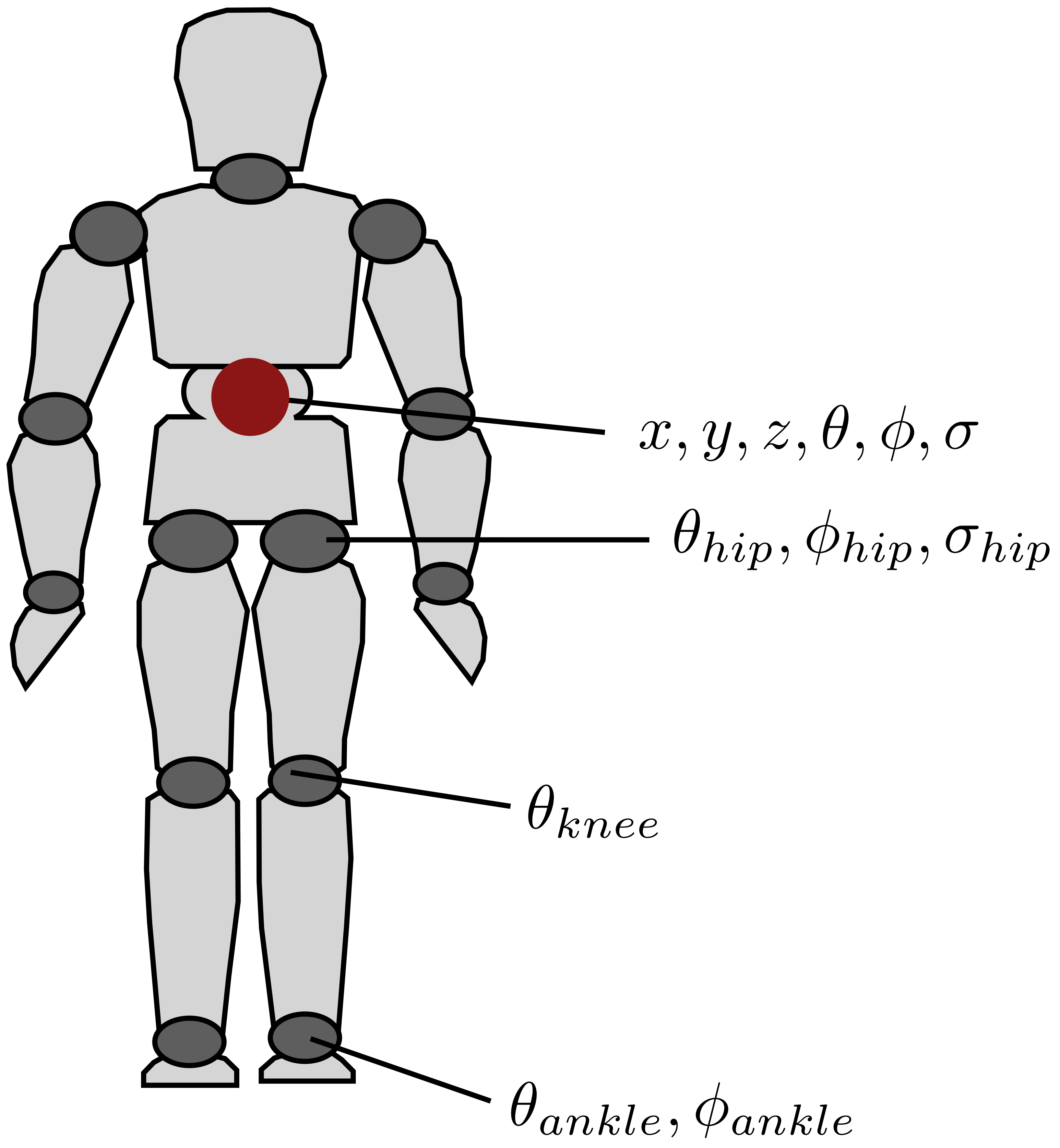

How to represent a pose?

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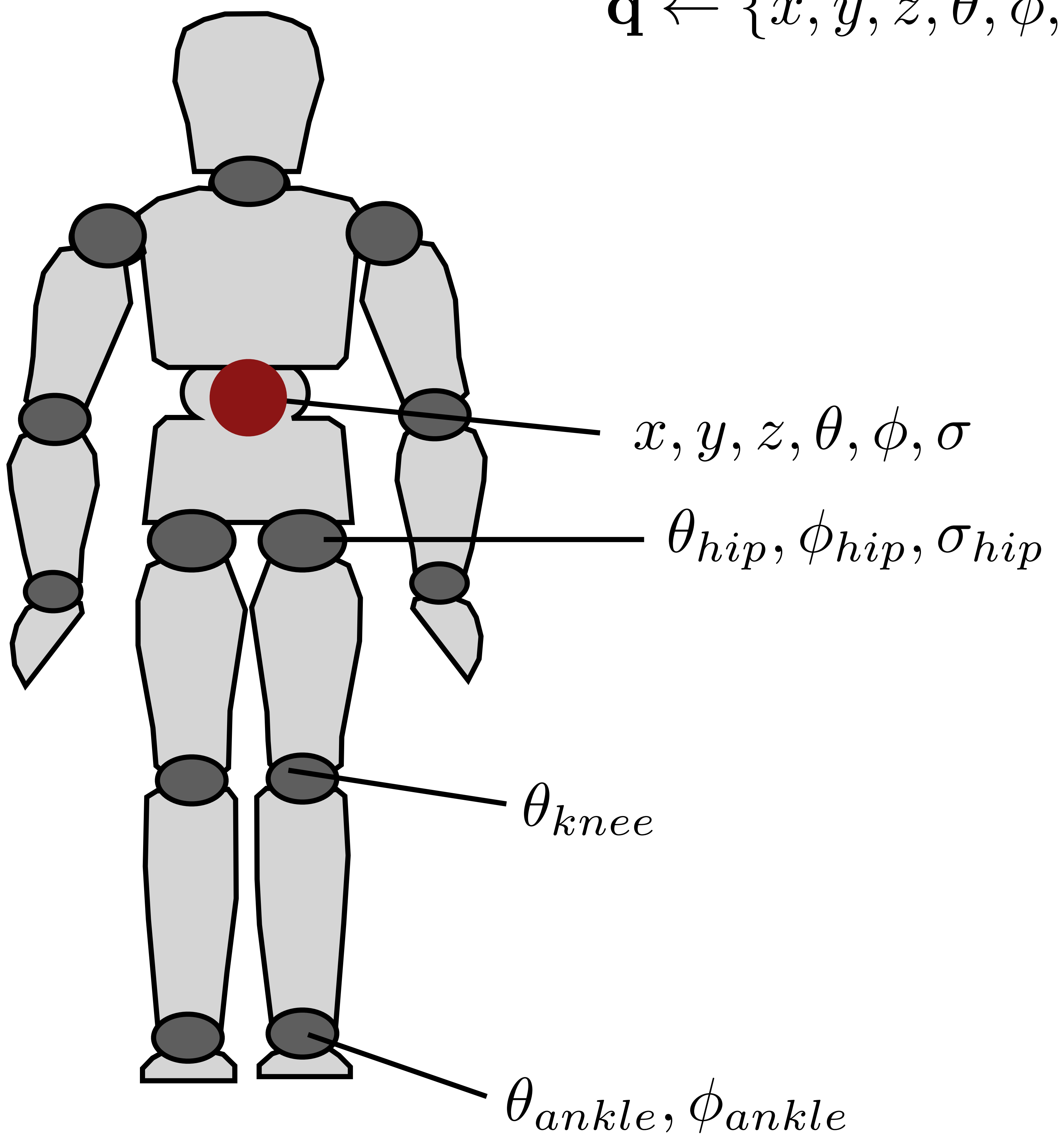
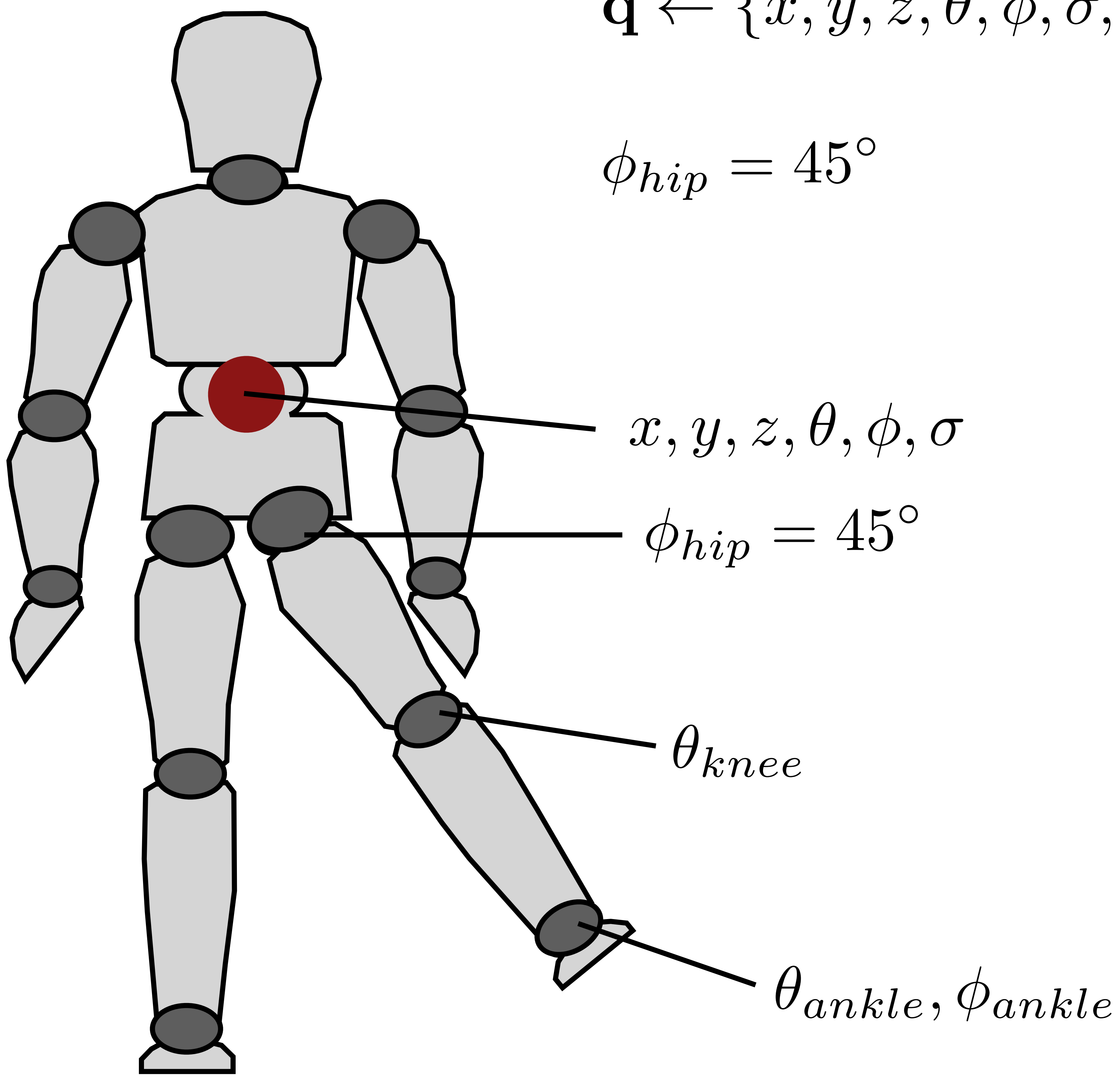


Define hierarchy

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

$$\phi_{hip} = 45^\circ$$



Hierarchical transformations

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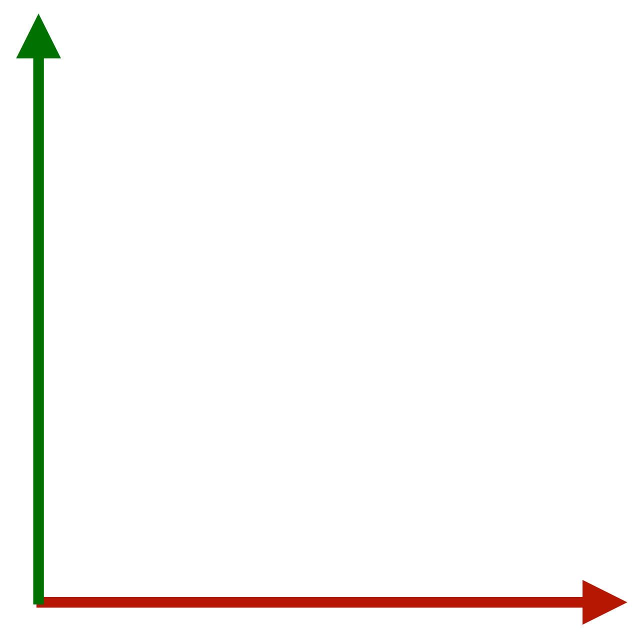

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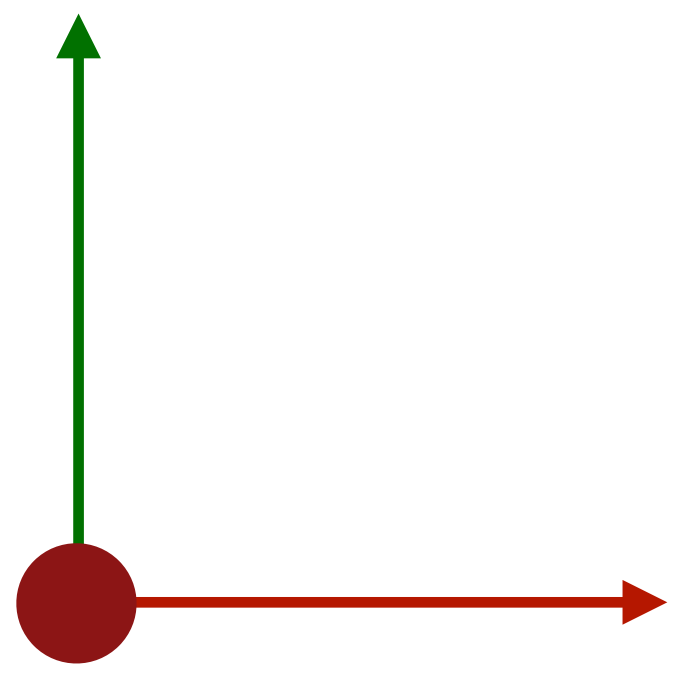

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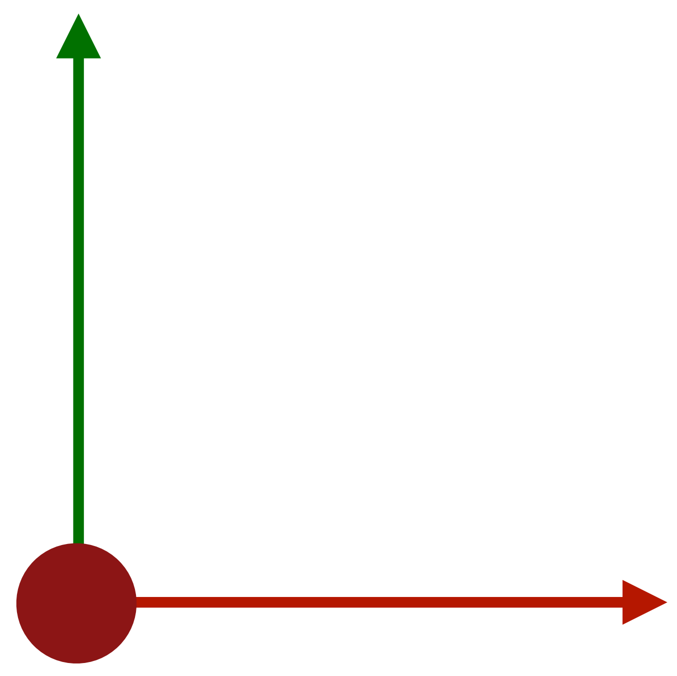
$$\boxed{\mathbf{T}(x, y, z)}$$



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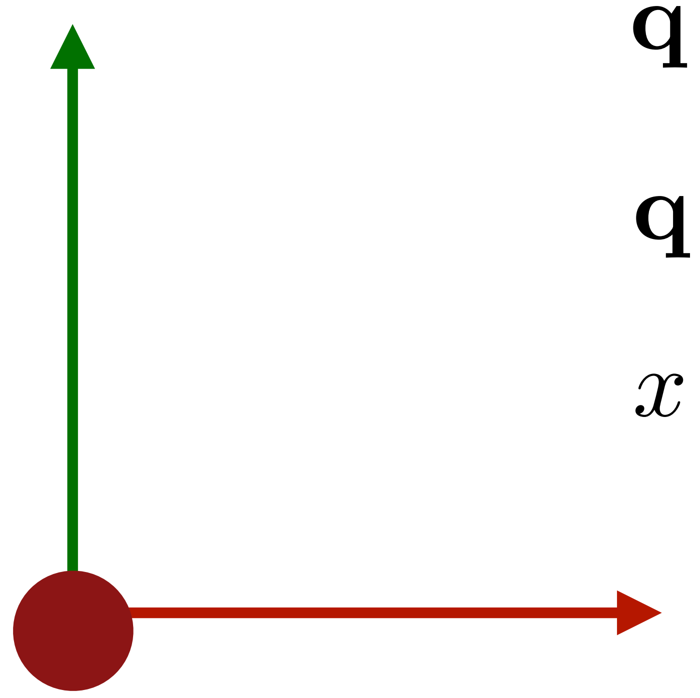

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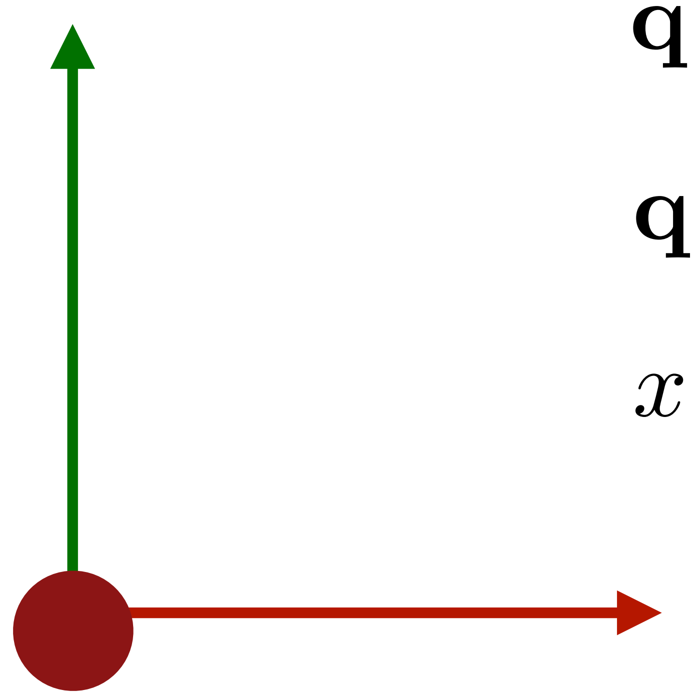

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$$\mathbf{T}(x, y, z)\boxed{\mathbf{R}(\theta, \phi, \sigma)}$$

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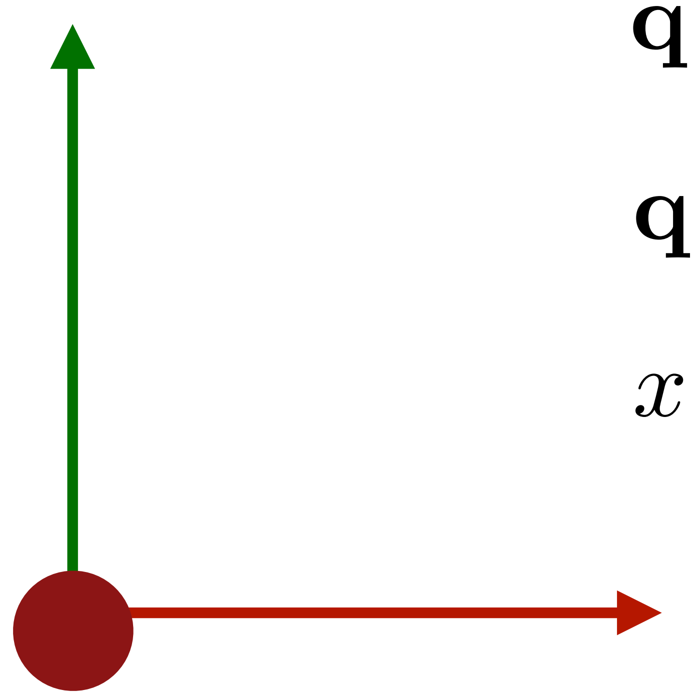

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$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\boxed{\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)}$$

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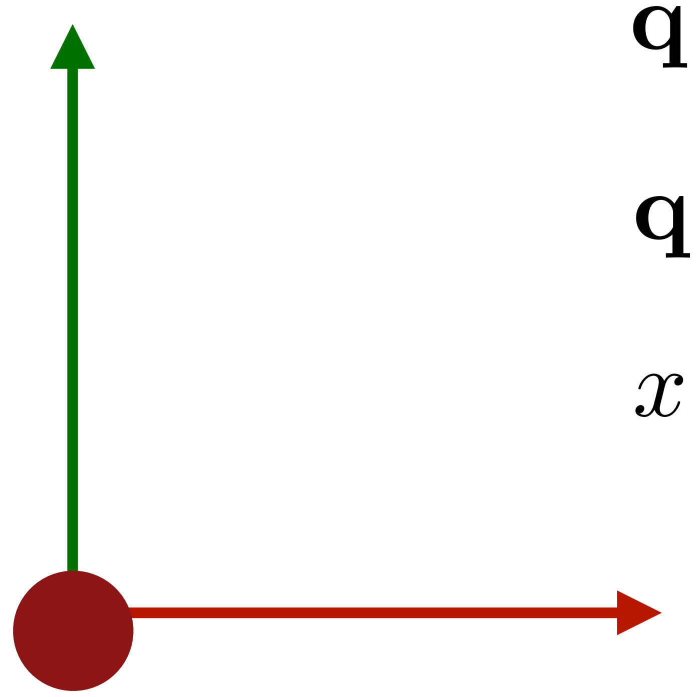

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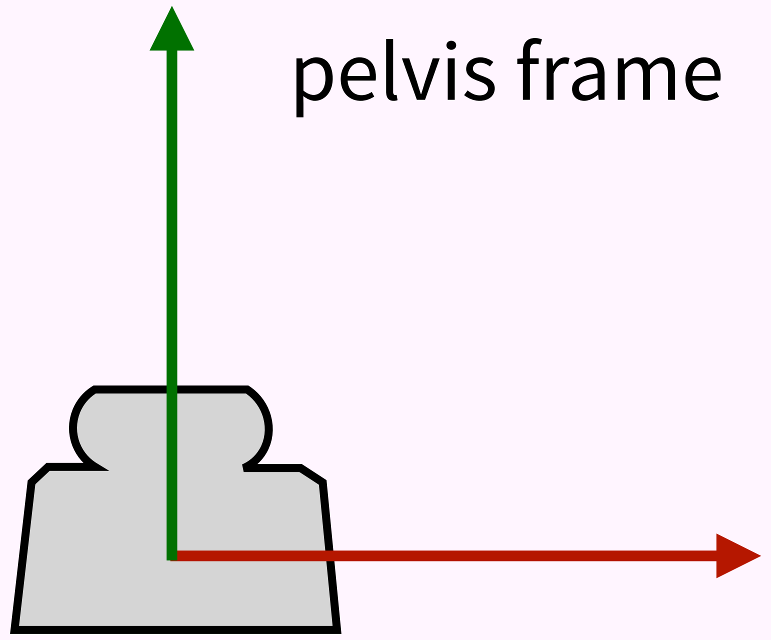
$\mathbf{t}_0$, $\mathbf{r}_0$: translation and rotation from the root to the origin of pelvis frame



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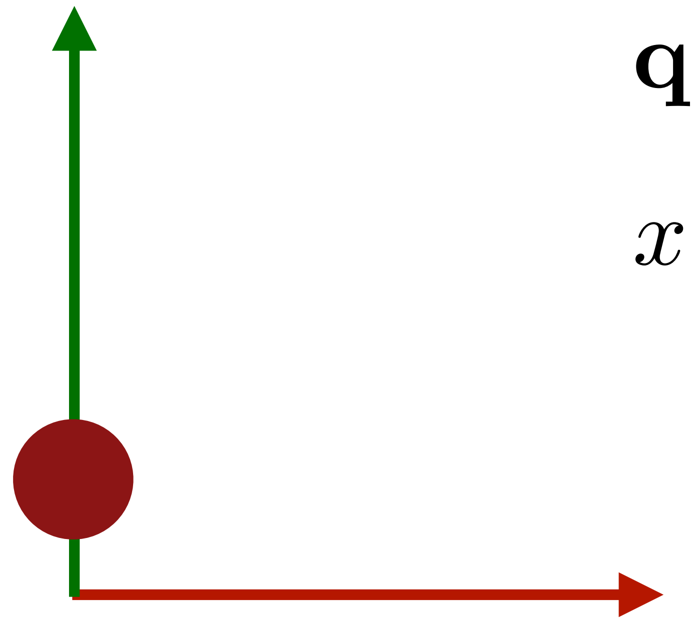

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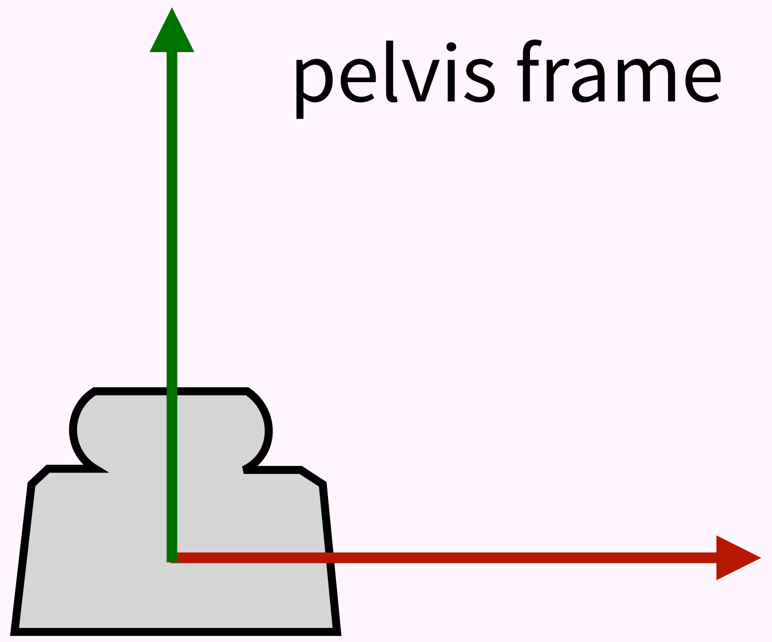

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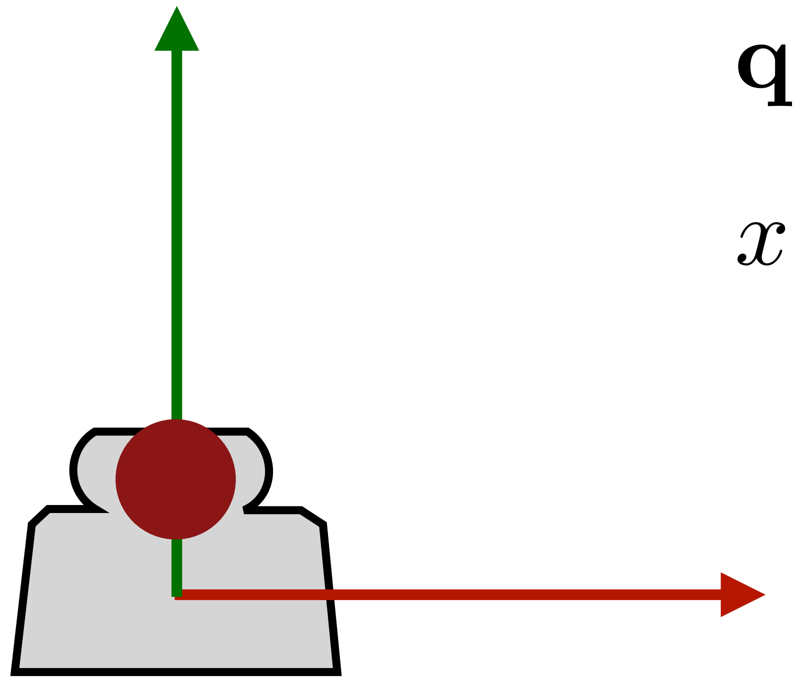

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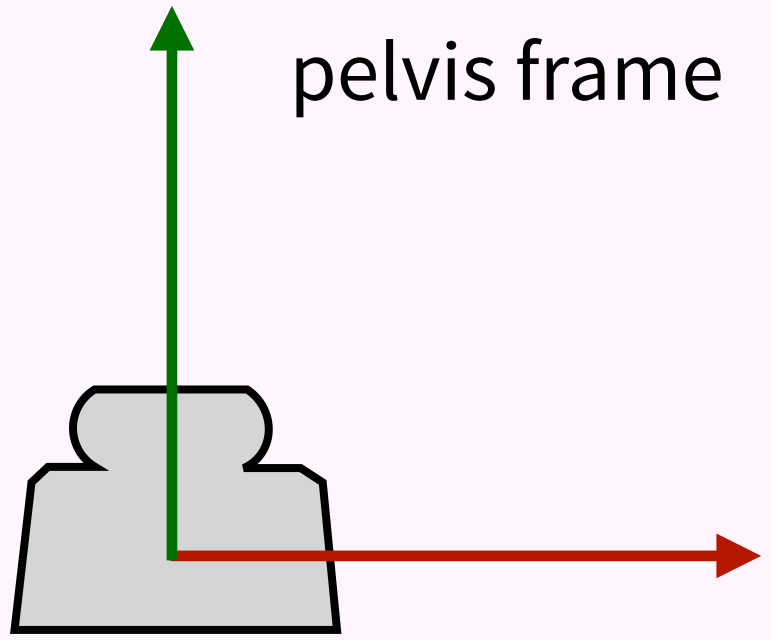

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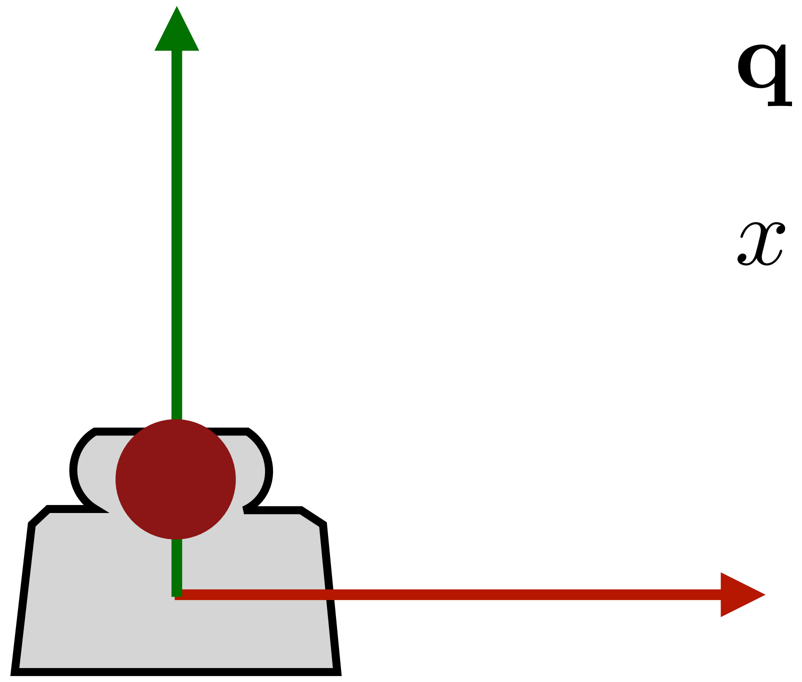

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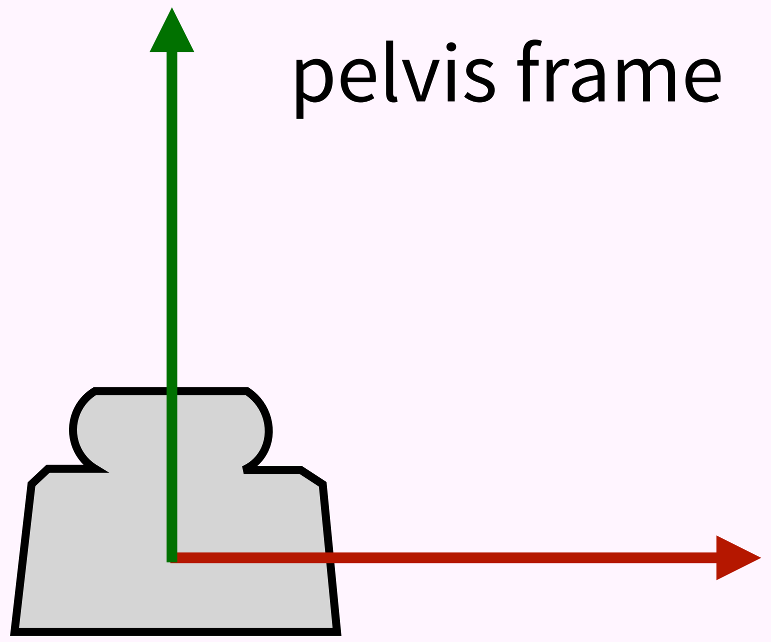


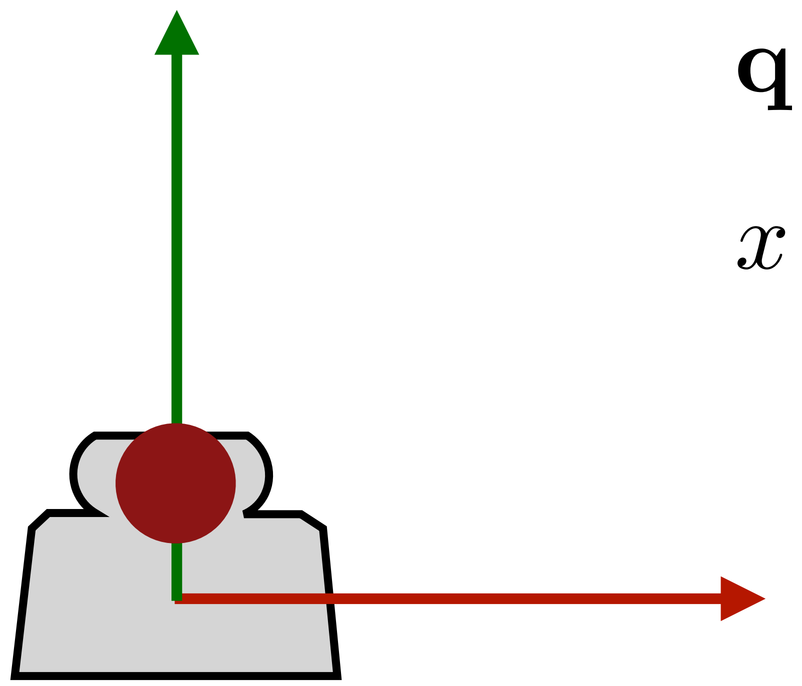
$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\boxed{\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)}$$

Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}$$

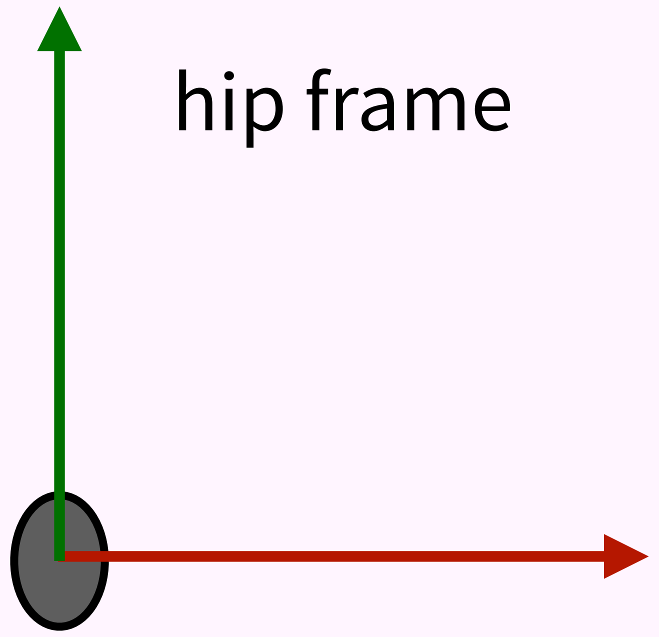
$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$

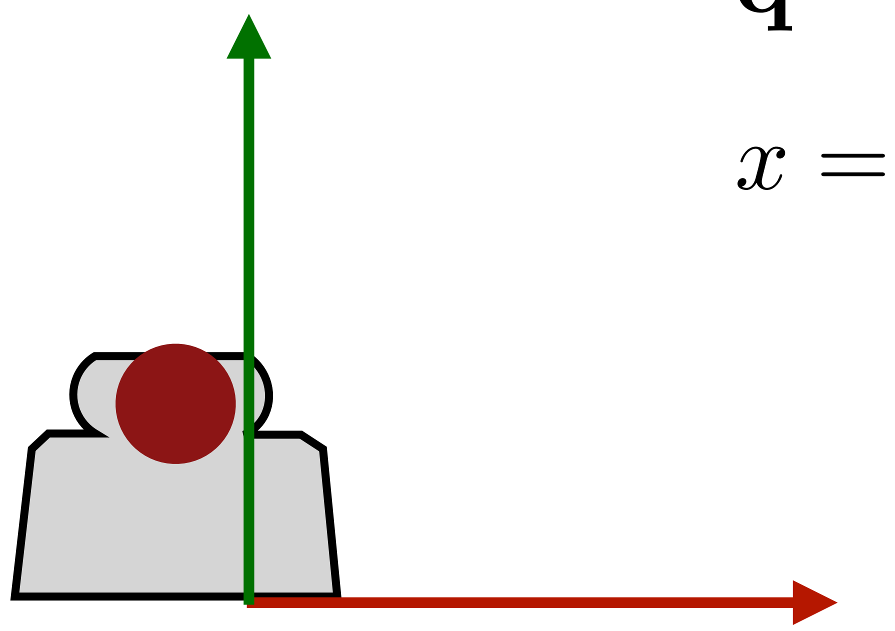


$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\boxed{\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)}$$

$\mathbf{t}_1$ translates from pelvis to hip joint. $\mathbf{r}_1$ rotates -90 degree about z-axis.



Hierarchical transformations



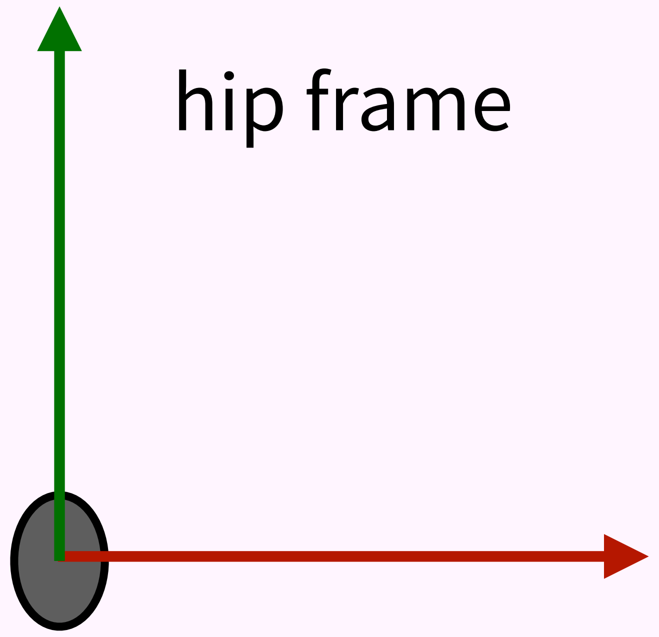
$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}$

$\mathbf{q} = \mathbf{0}$

$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^{\circ}$

$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\boxed{\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)}$$

$\mathbf{t}_1$ translates from pelvis to hip joint. $\mathbf{r}_1$ rotates -90 degree about z-axis.



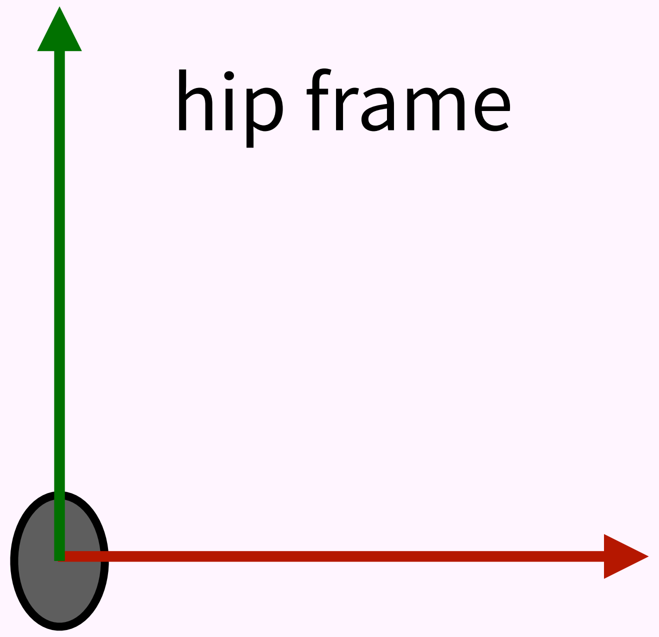
Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}$$

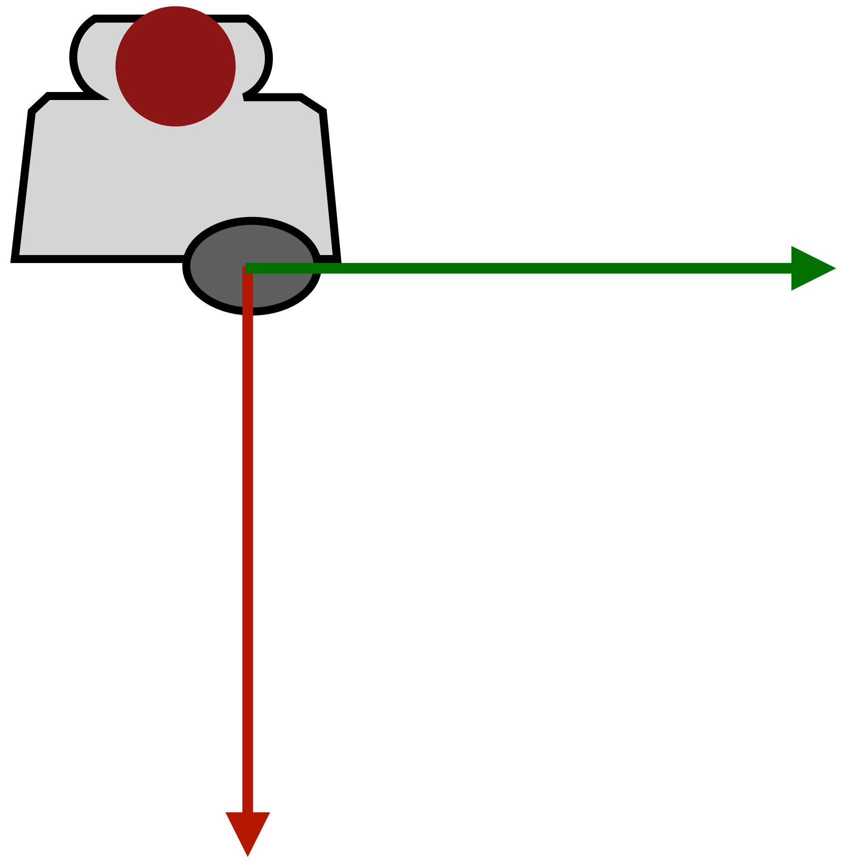
$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$

$\mathbf{t}_1$ translates from pelvis to hip joint. $\mathbf{r}_1$ rotates -90 degree about z-axis.



$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\boxed{\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)}$$



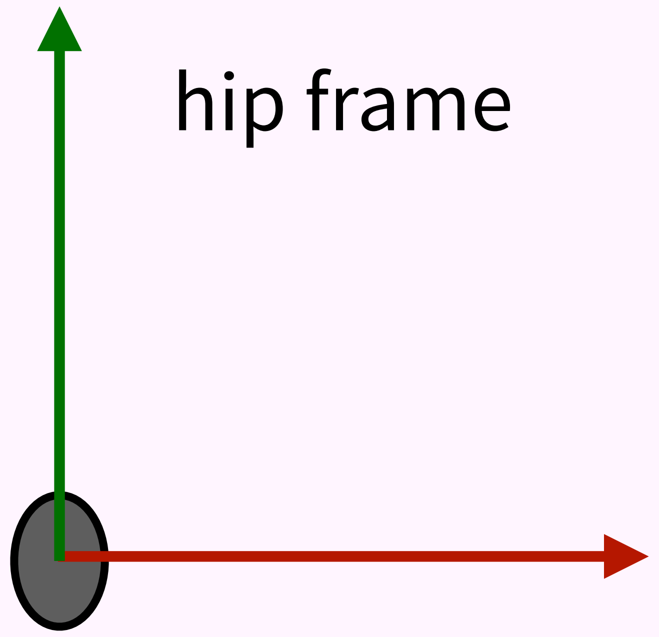
Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}$$

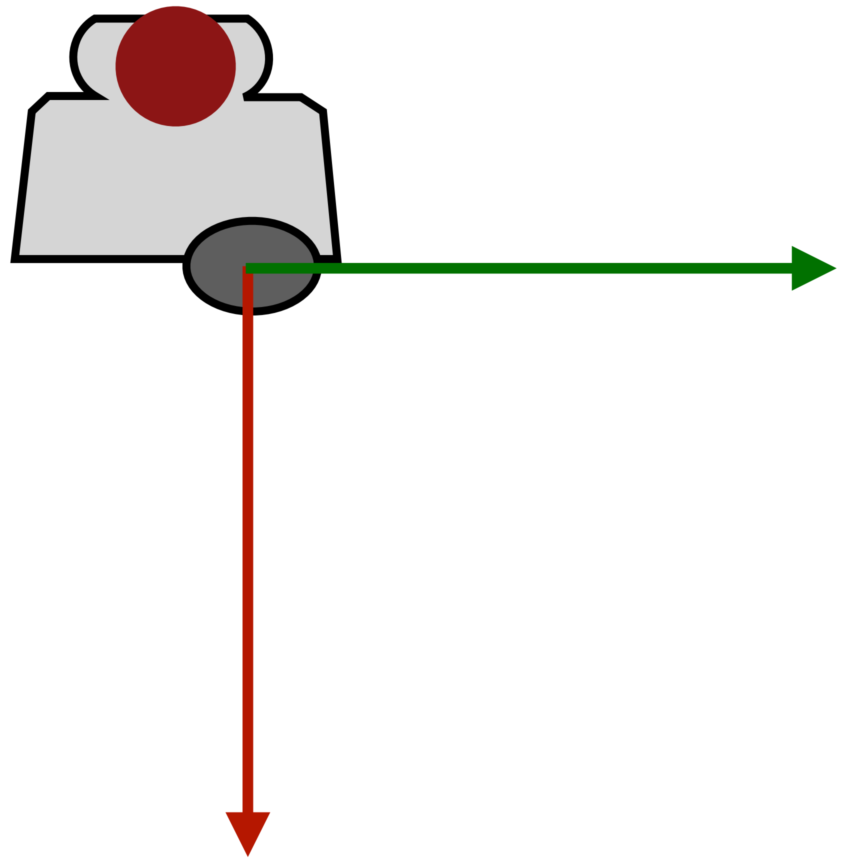
$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$

$\mathbf{t}_1$ translates from pelvis to hip joint. $\mathbf{r}_1$ rotates -90 degree about z-axis.



$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\boxed{\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})}$$



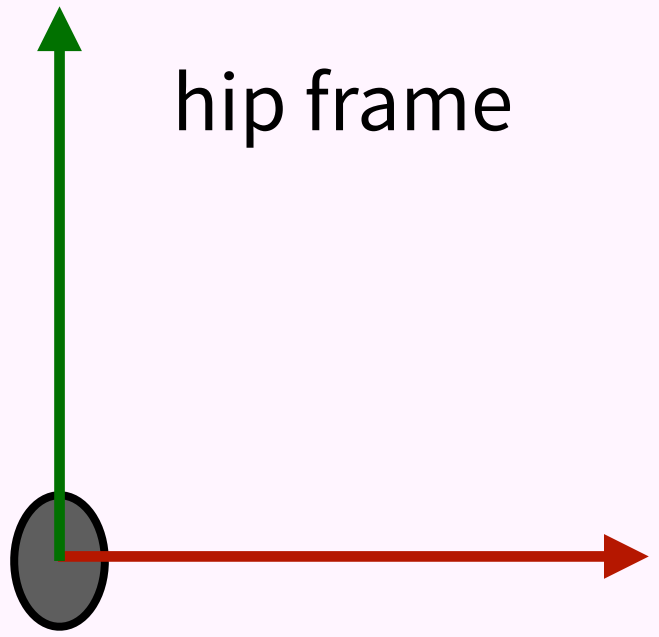
Hierarchical transformations

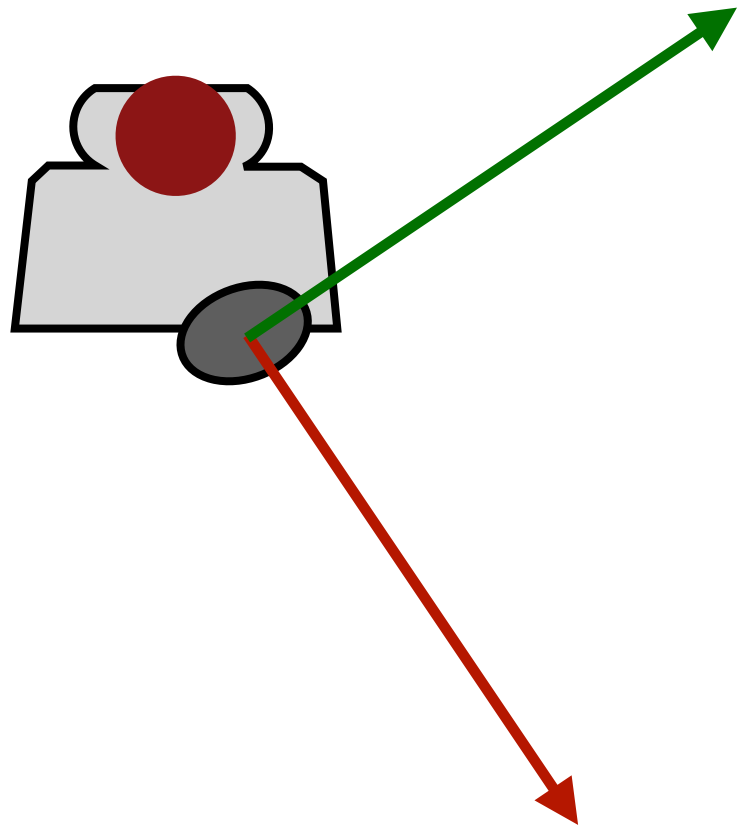
$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}$

$\mathbf{q} = \mathbf{0}$

$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$

$\mathbf{t}_1$ translates from pelvis to hip joint. $\mathbf{r}_1$ rotates -90 degree about z-axis.





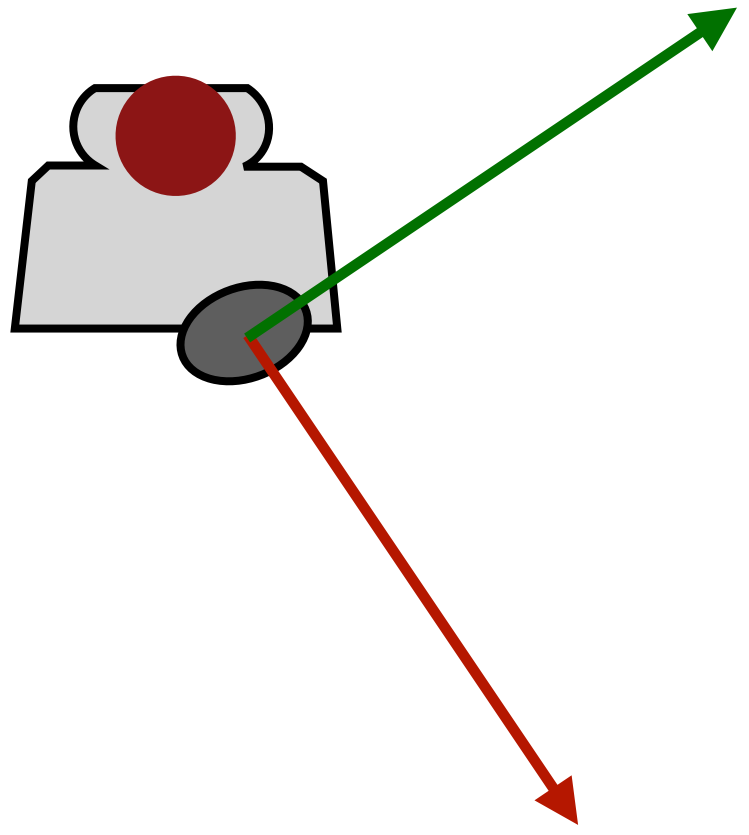
$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\boxed{\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})}$$

Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

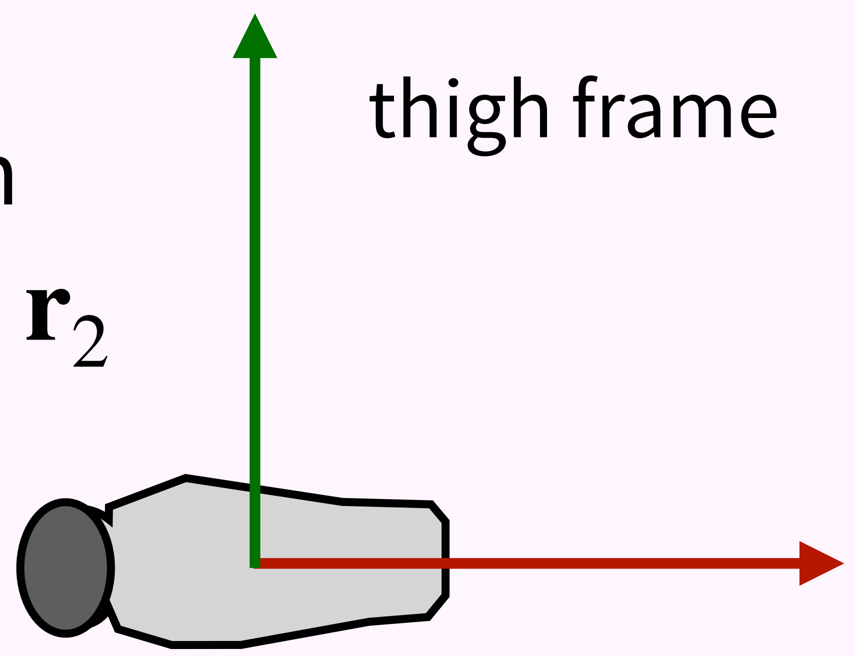
$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\,\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})\,\boxed{\mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)}$$

$\mathbf{t}_2$ translates from hip joint to thigh. $\mathbf{r}_2$ rotates 0 degree.

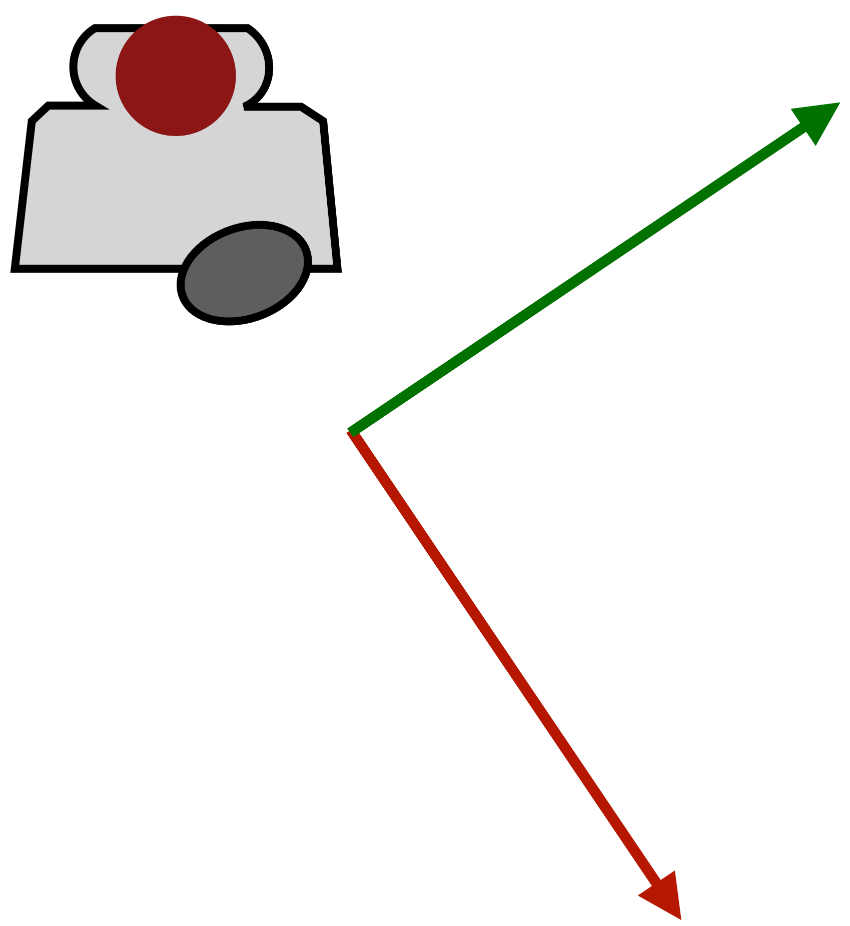


Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

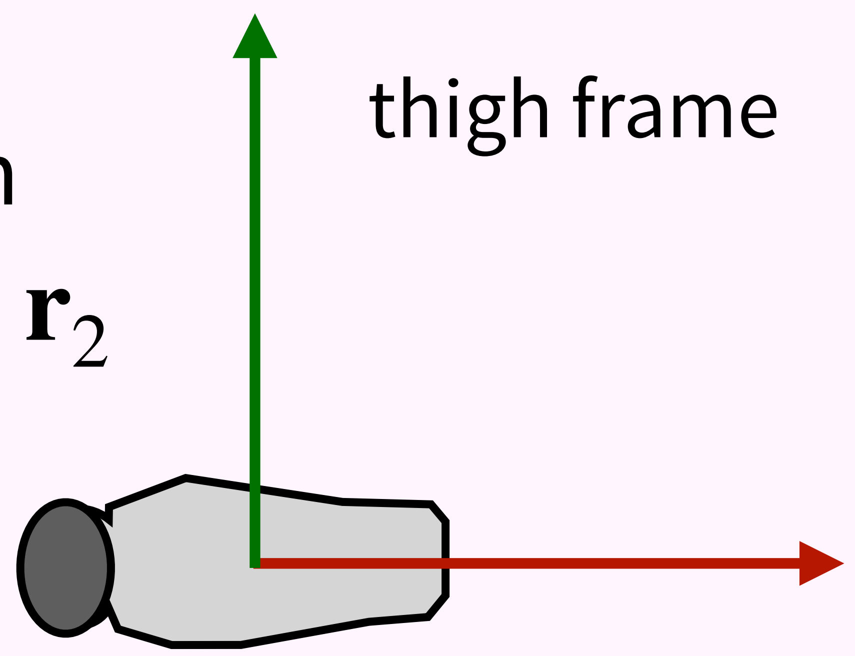
$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\,\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})\,\boxed{\mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)}$$

$\mathbf{t}_2$ translates from hip joint to thigh. $\mathbf{r}_2$ rotates 0 degree.

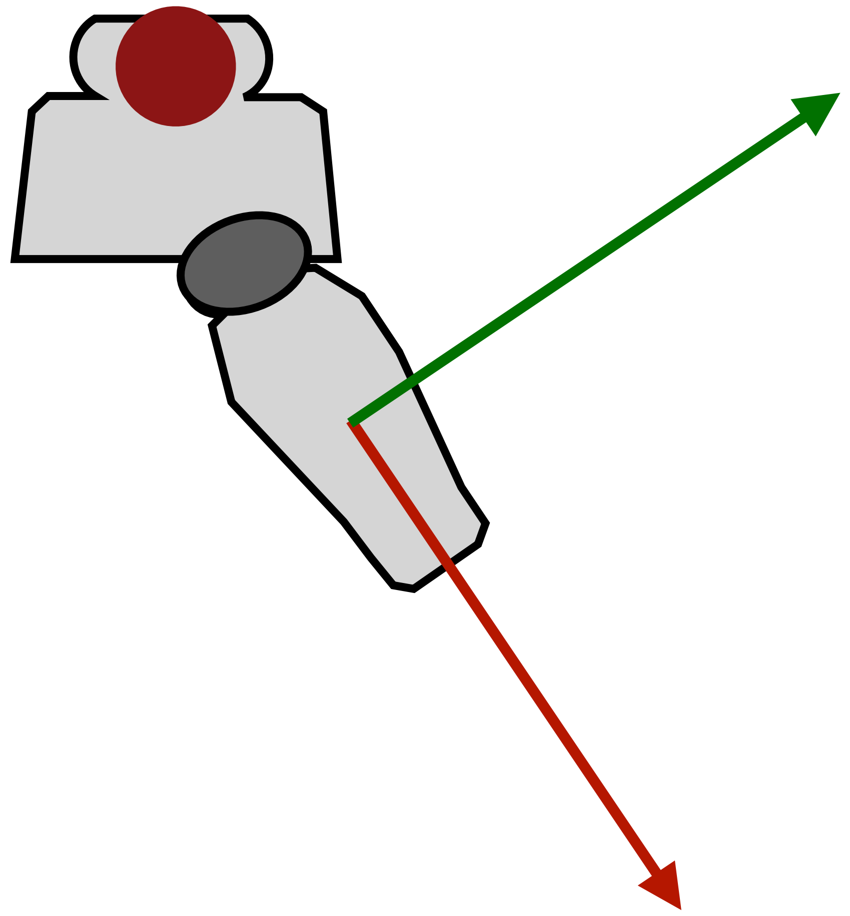


Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

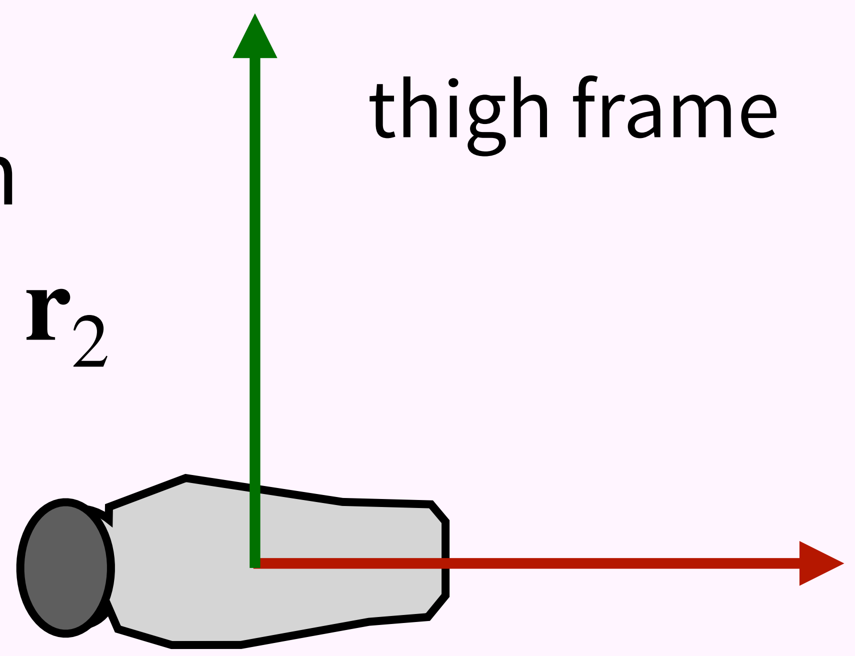
$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\,\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})\,\boxed{\mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)}$$

$\mathbf{t}_2$ translates from hip joint to thigh. $\mathbf{r}_2$ rotates 0 degree.

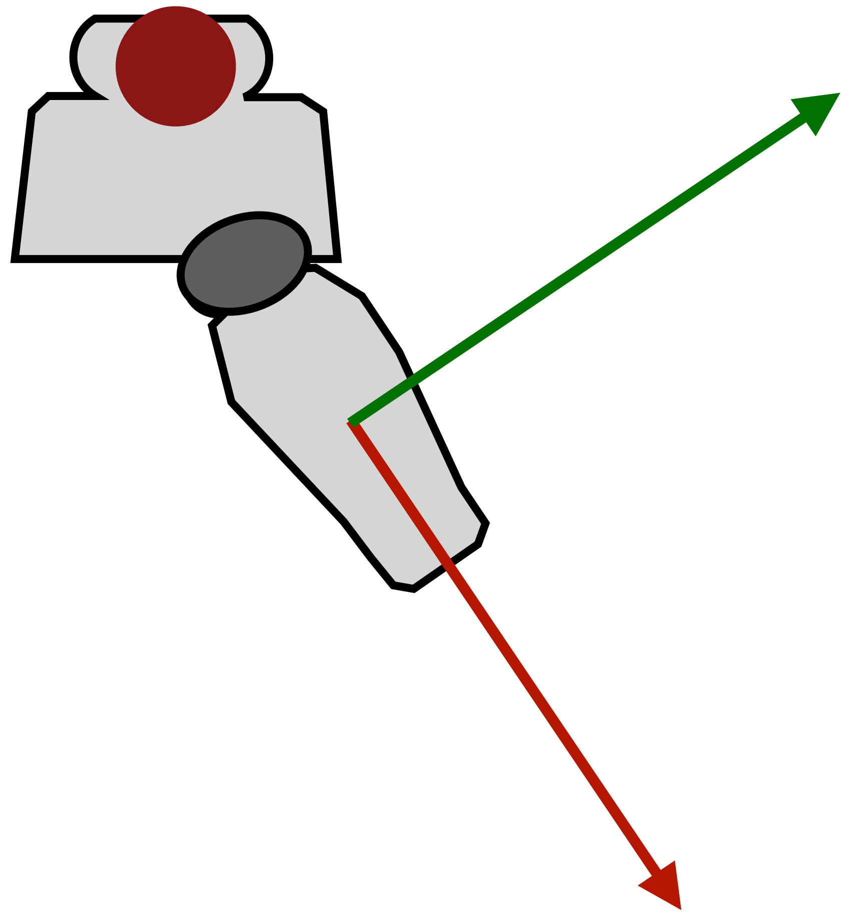


Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\,\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})\,\mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

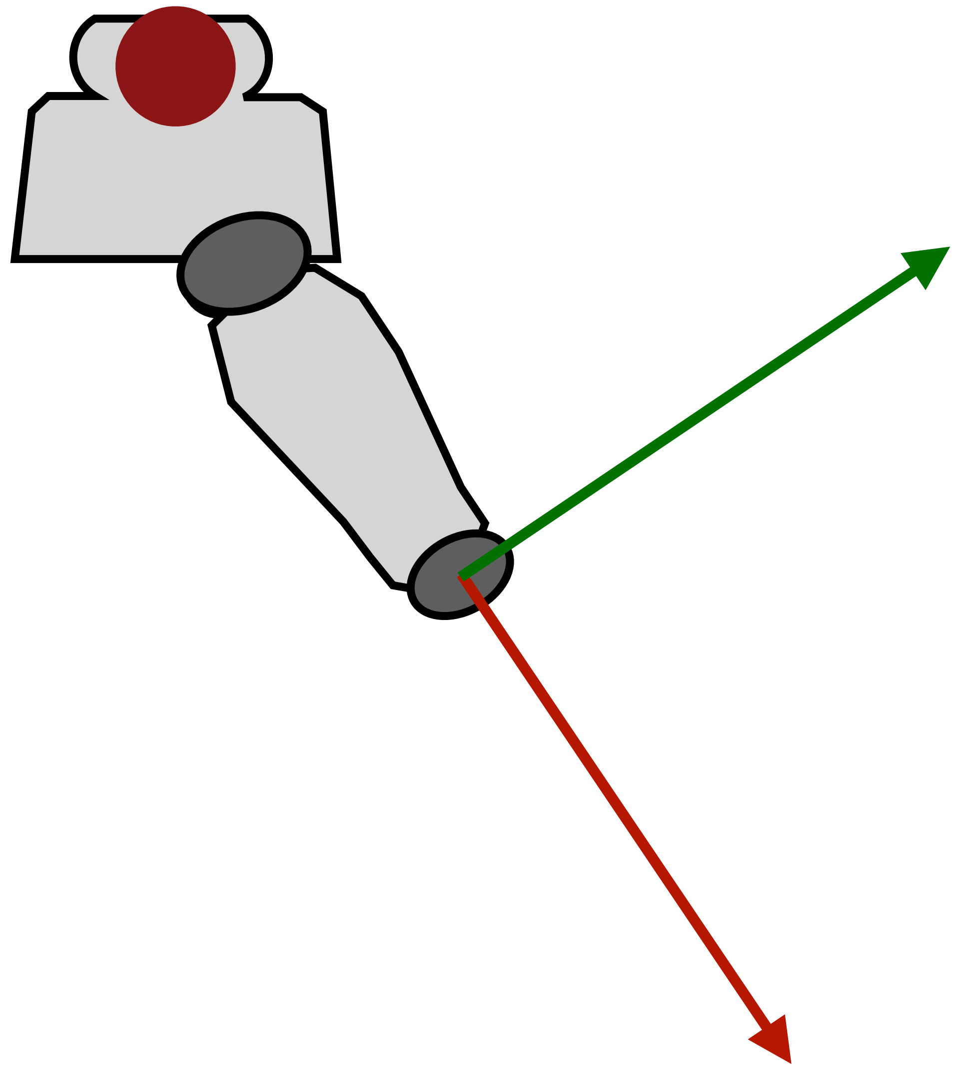
$$\boxed{\mathbf{F}(\mathbf{t}_3, \mathbf{r}_3)}$$

Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\,\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})\,\mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

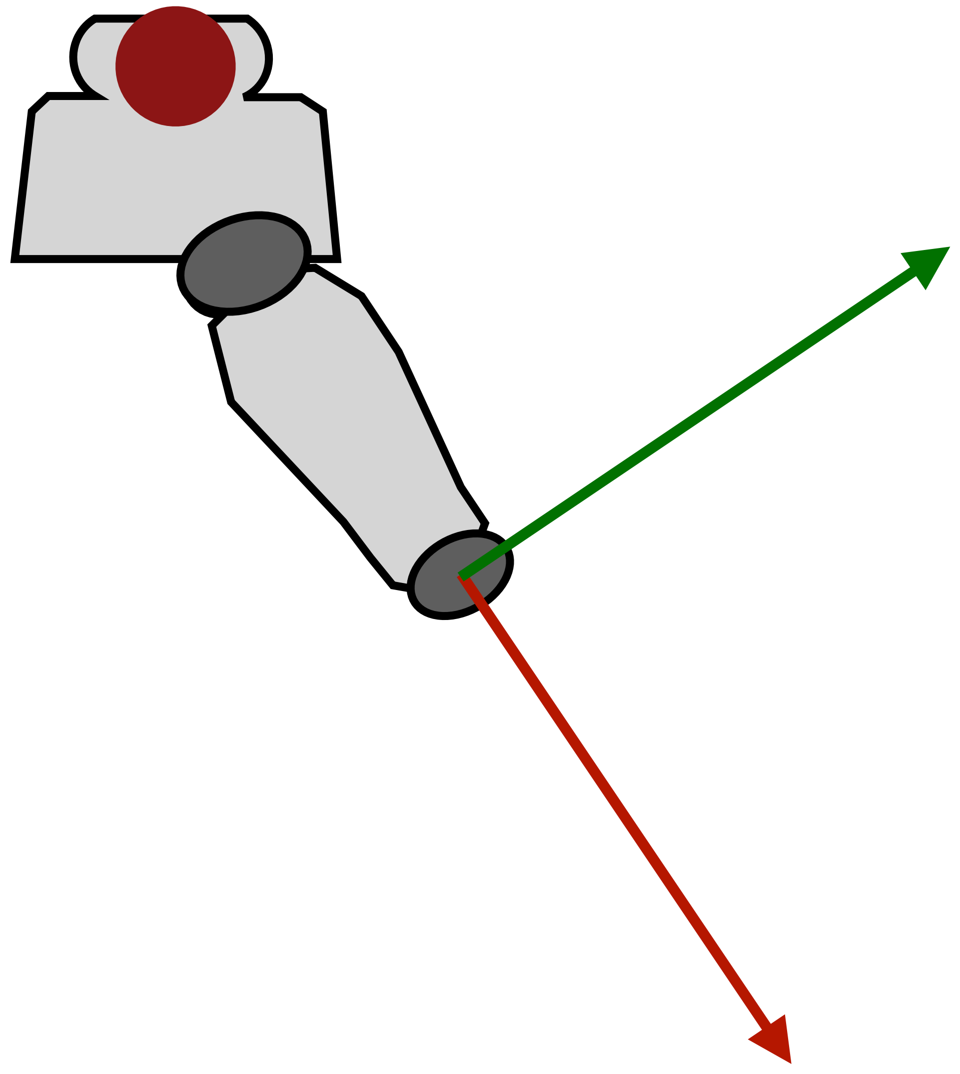
$$\boxed{\mathbf{F}(\mathbf{t}_3, \mathbf{r}_3)}$$

Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\,\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})\,\mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

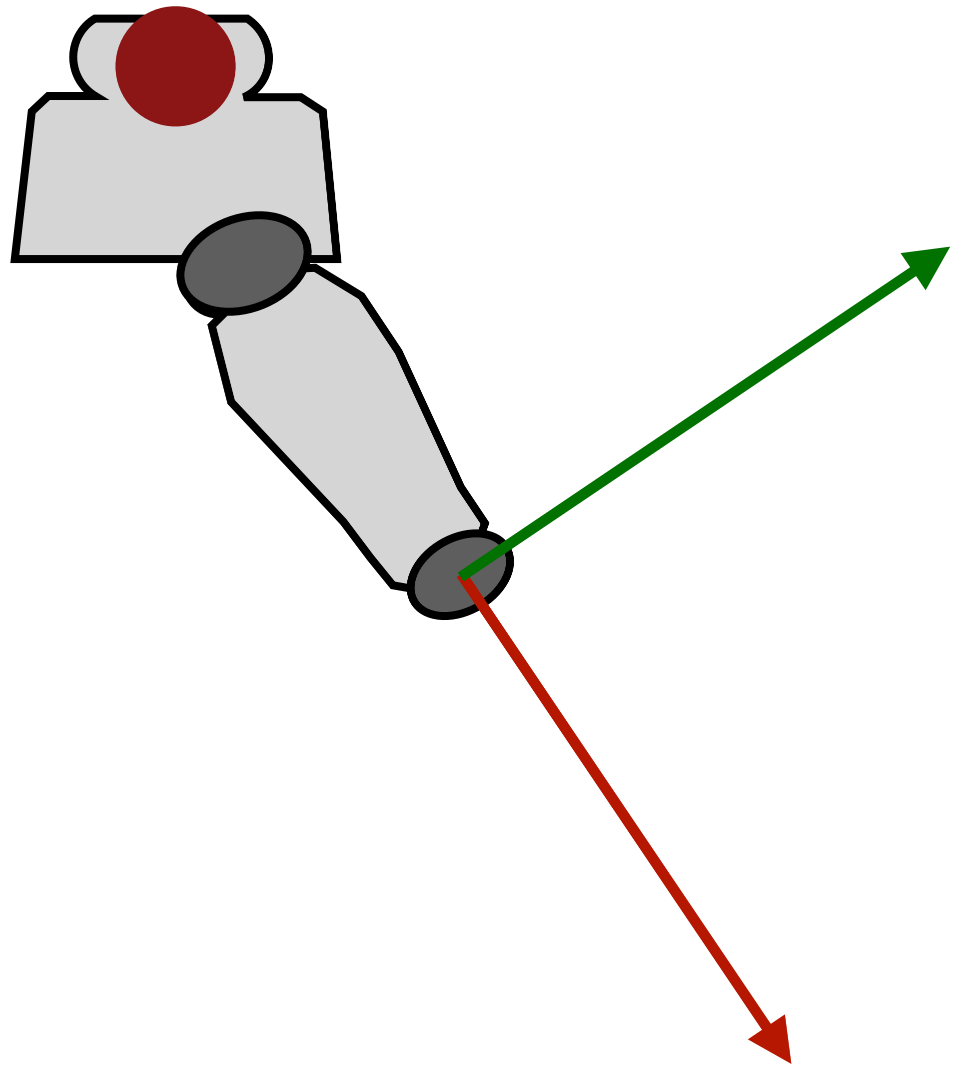
$$\mathbf{F}(\mathbf{t}_3, \mathbf{r}_3)\boxed{\mathbf{R}(\theta_{knee})}$$

Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\,\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})\,\mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

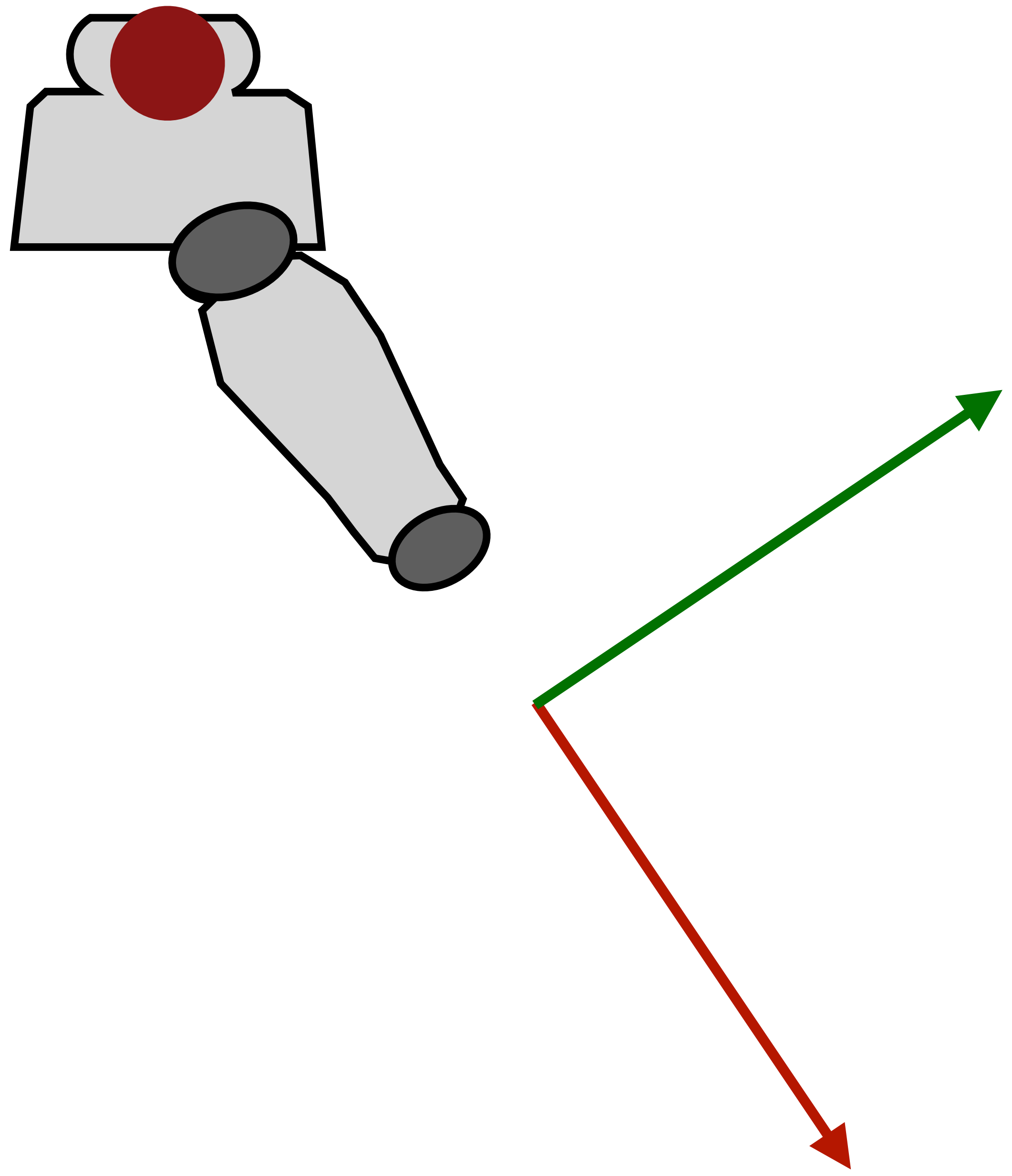
$$\mathbf{F}(\mathbf{t}_3, \mathbf{r}_3)\mathbf{R}(\theta_{knee})\boxed{\mathbf{F}(\mathbf{t}_4, \mathbf{r}_4)}$$

Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\,\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})\,\mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

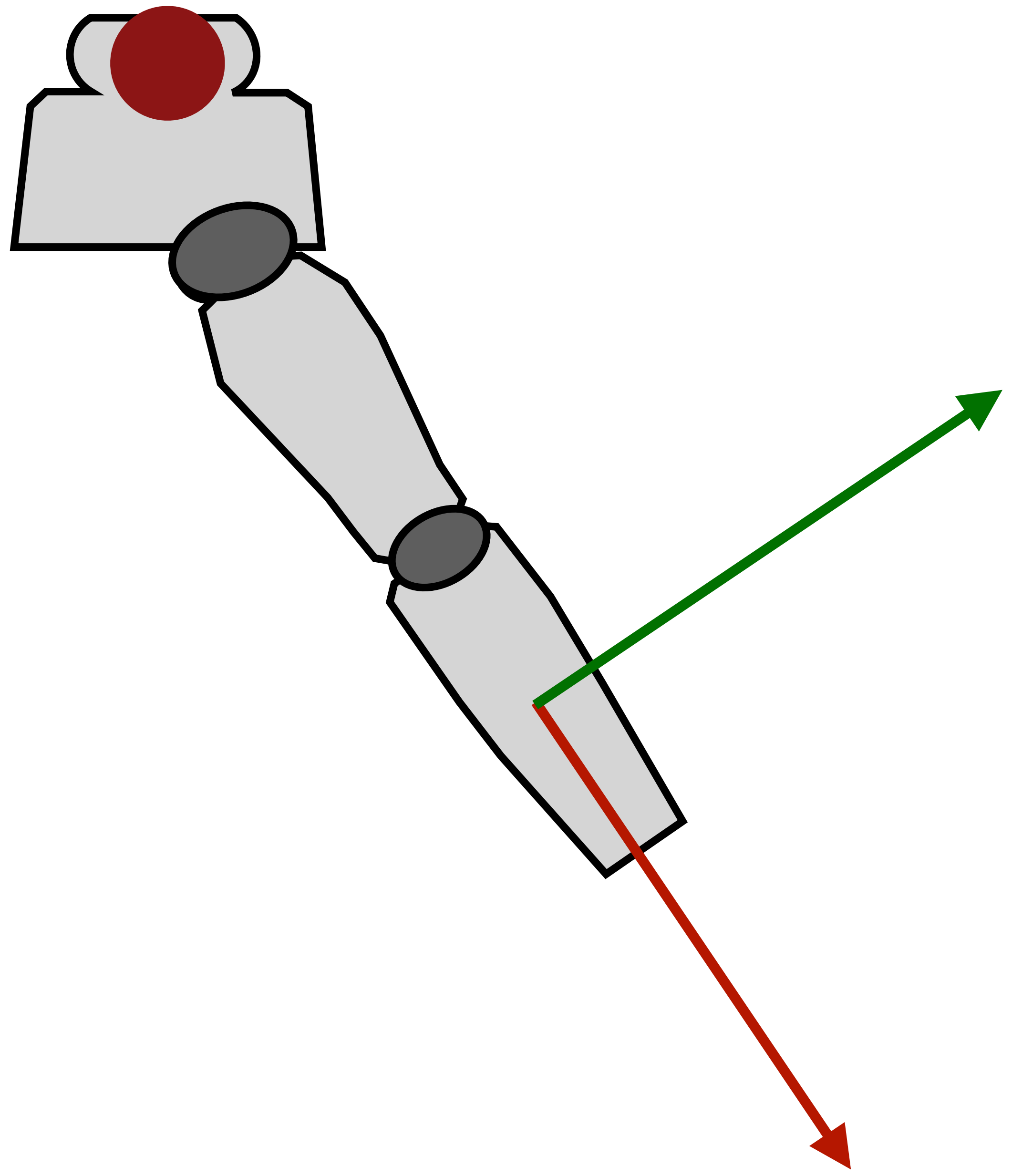
$$\mathbf{F}(\mathbf{t}_3, \mathbf{r}_3)\mathbf{R}(\theta_{knee})\boxed{\mathbf{F}(\mathbf{t}_4, \mathbf{r}_4)}$$

Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

$$\mathbf{q} = \mathbf{0}$$

$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\,\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})\,\mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

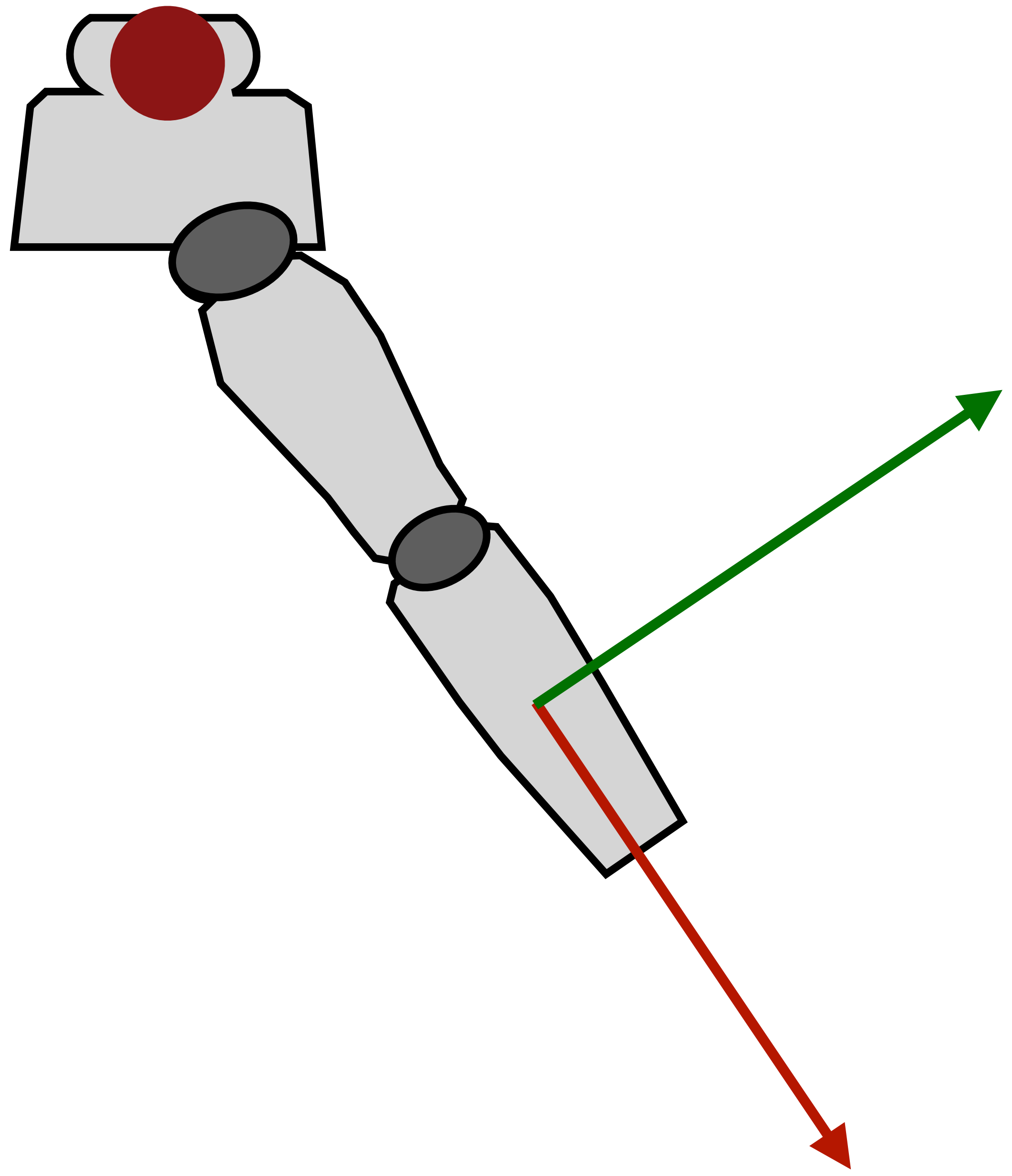
$$\mathbf{F}(\mathbf{t}_3, \mathbf{r}_3)\mathbf{R}(\theta_{knee})\boxed{\mathbf{F}(\mathbf{t}_4, \mathbf{r}_4)}$$

Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

$$\mathbf{q} = \mathbf{0}$$

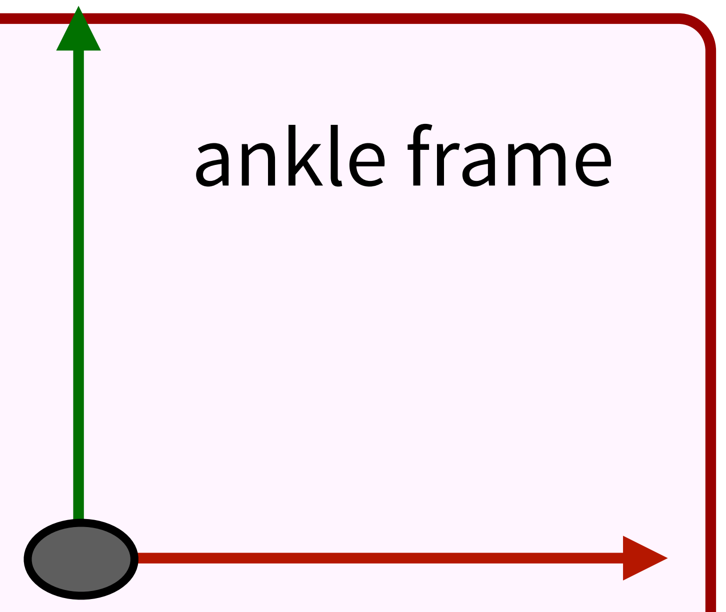
$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x,y,z)\mathbf{R}(\theta,\phi,\sigma)\mathbf{F}(\mathbf{t}_0,\mathbf{r}_0)\mathbf{F}(\mathbf{t}_1,\mathbf{r}_1)\,\mathbf{R}(\theta_{hip},\phi_{hip},\sigma_{hip})\,\mathbf{F}(\mathbf{t}_2,\mathbf{r}_2)$$

$$\mathbf{F}(\mathbf{t}_3,\mathbf{r}_3)\mathbf{R}(\theta_{knee})\mathbf{F}(\mathbf{t}_4,\mathbf{r}_4)\,\boxed{\mathbf{F}(\mathbf{t}_5,\mathbf{r}_5)}$$

$\mathbf{t}_5$ translates from shank to ankle joint.
$\mathbf{r}_5$ rotates +90 degree about z-axis.

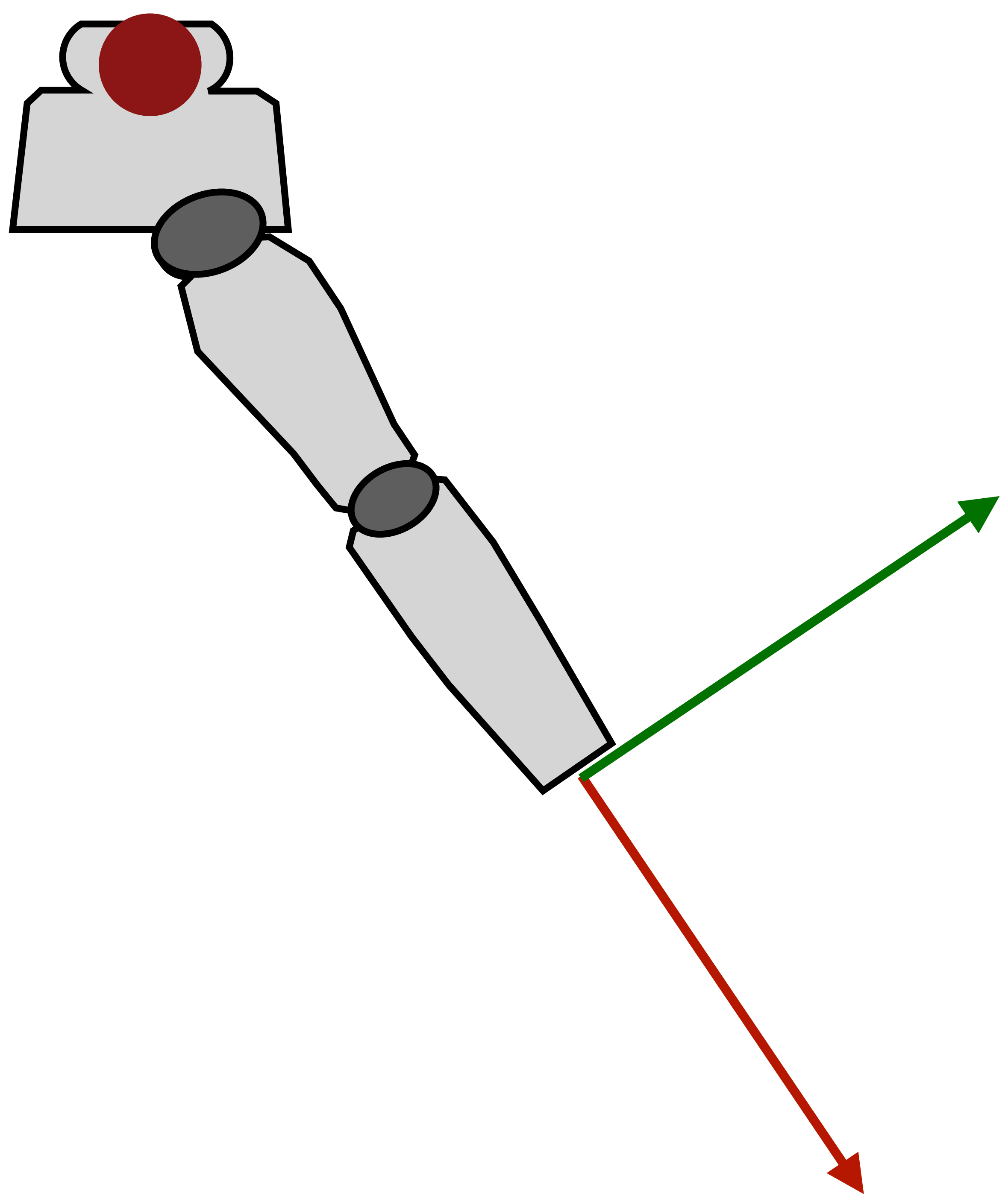


Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

$$\mathbf{q} = \mathbf{0}$$

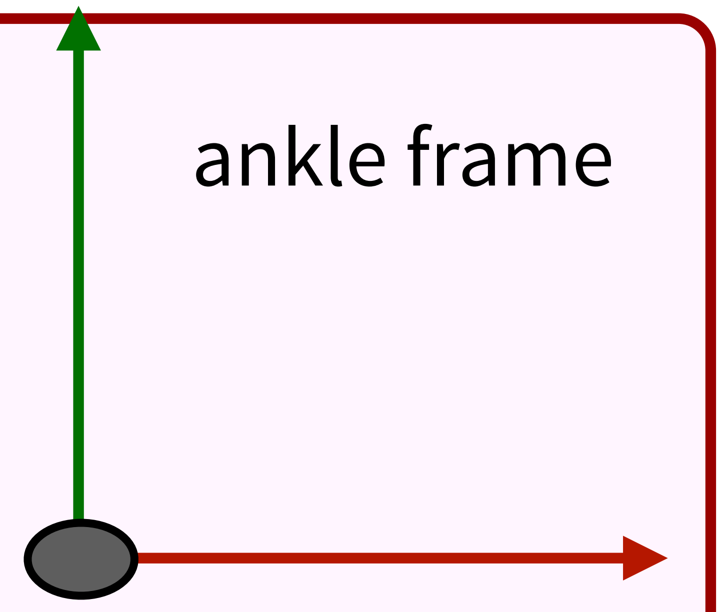
$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\,\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})\,\mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

$$\mathbf{F}(\mathbf{t}_3, \mathbf{r}_3)\mathbf{R}(\theta_{knee})\mathbf{F}(\mathbf{t}_4, \mathbf{r}_4)\,\boxed{\mathbf{F}(\mathbf{t}_5, \mathbf{r}_5)}$$

$\mathbf{t}_5$ translates from shank to ankle joint.
$\mathbf{r}_5$ rotates +90 degree about z-axis.

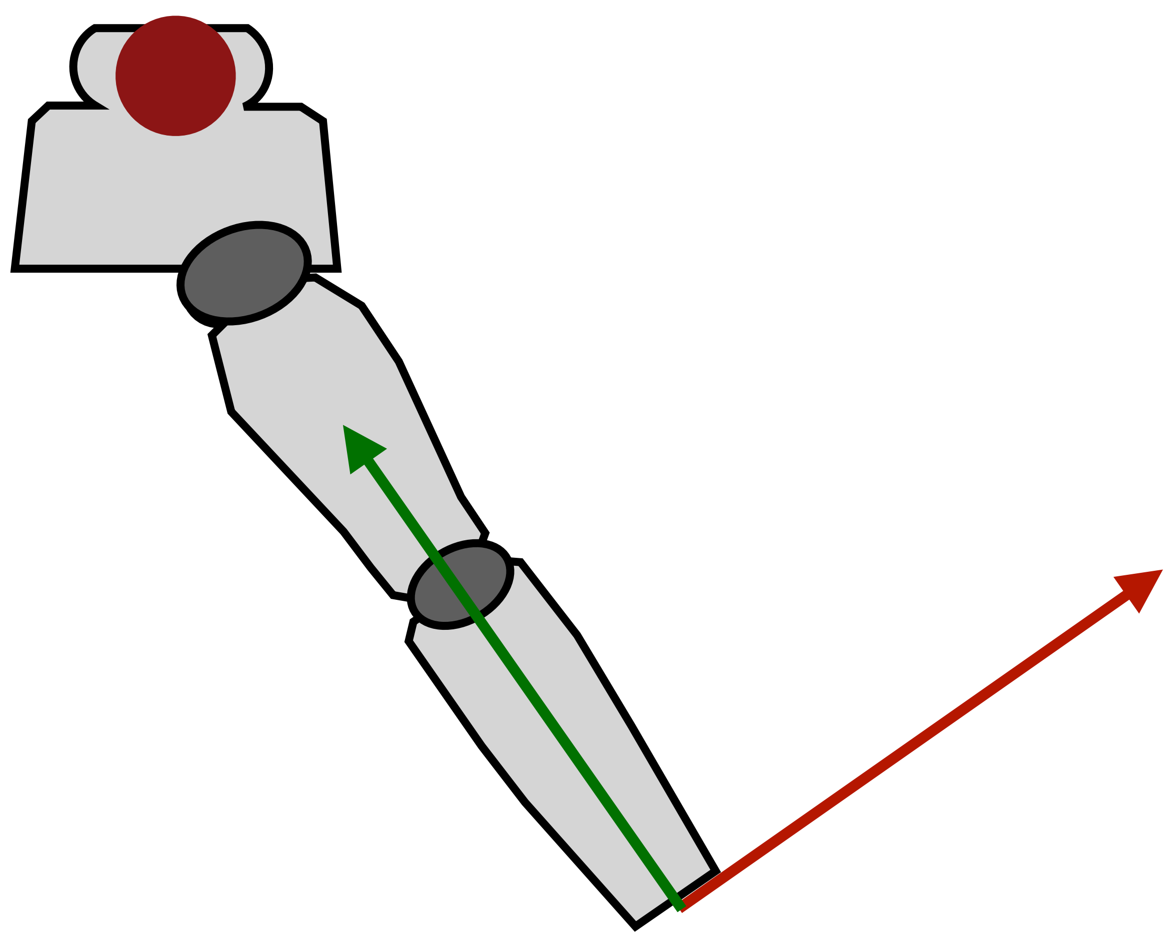


Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

$$\mathbf{q} = \mathbf{0}$$

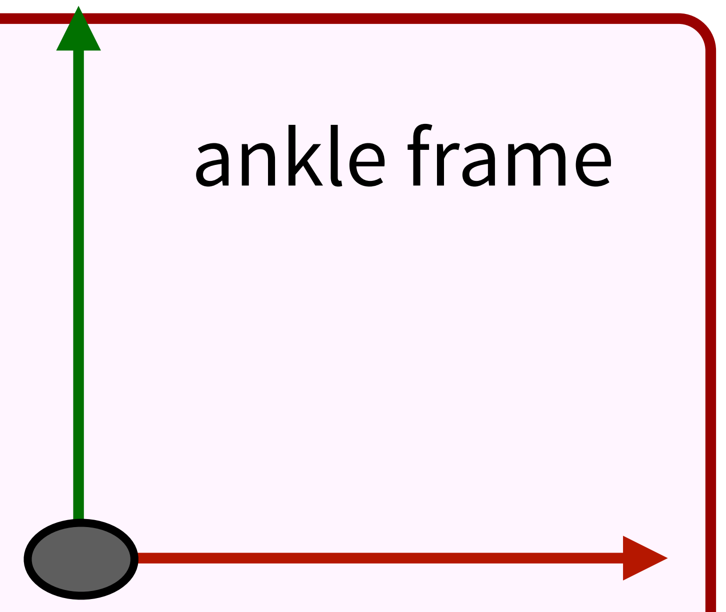
$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\,\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})\,\mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

$$\mathbf{F}(\mathbf{t}_3, \mathbf{r}_3)\mathbf{R}(\theta_{knee})\mathbf{F}(\mathbf{t}_4, \mathbf{r}_4)\,\boxed{\mathbf{F}(\mathbf{t}_5, \mathbf{r}_5)}$$

$\mathbf{t}_5$ translates from shank to ankle joint.
$\mathbf{r}_5$ rotates +90 degree about z-axis.

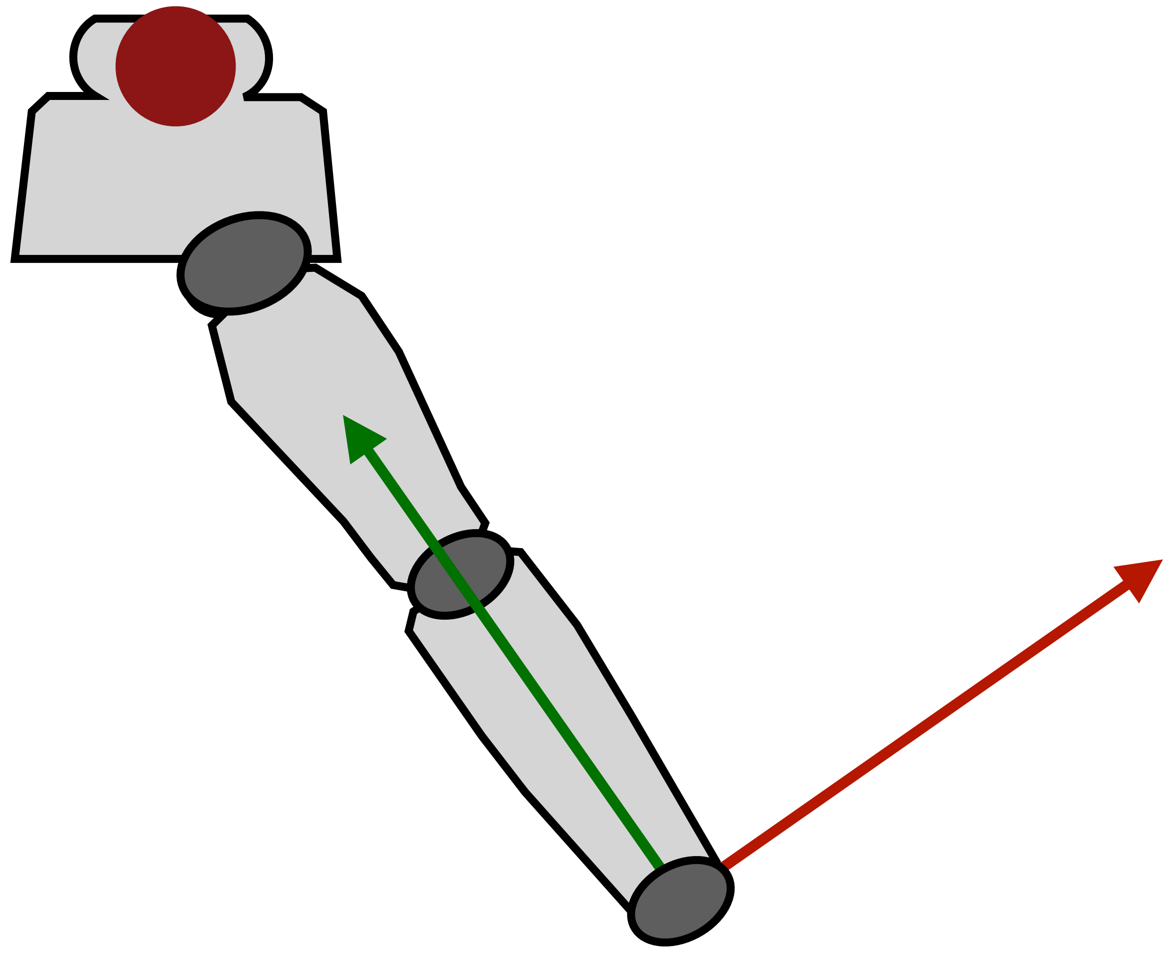


Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

$$\mathbf{q} = \mathbf{0}$$

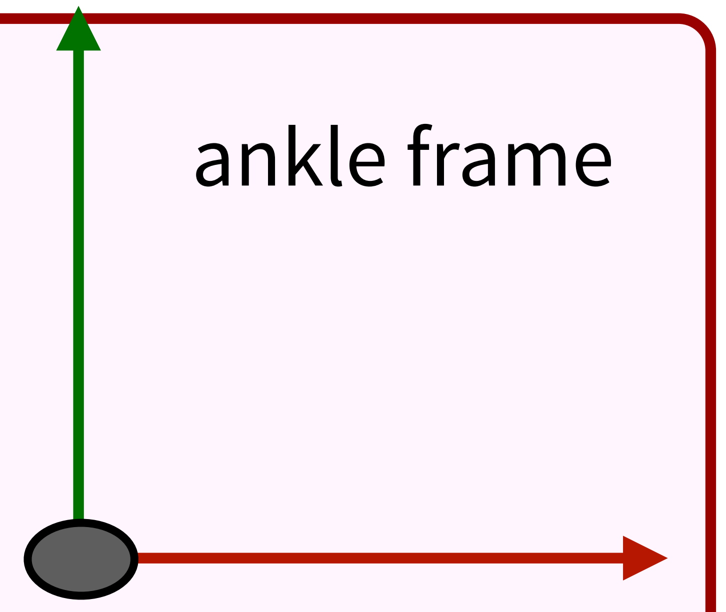
$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\,\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})\,\mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

$$\mathbf{F}(\mathbf{t}_3, \mathbf{r}_3)\mathbf{R}(\theta_{knee})\mathbf{F}(\mathbf{t}_4, \mathbf{r}_4)\,\boxed{\mathbf{F}(\mathbf{t}_5, \mathbf{r}_5)}$$

$\mathbf{t}_5$ translates from shank to ankle joint.
$\mathbf{r}_5$ rotates +90 degree about z-axis.

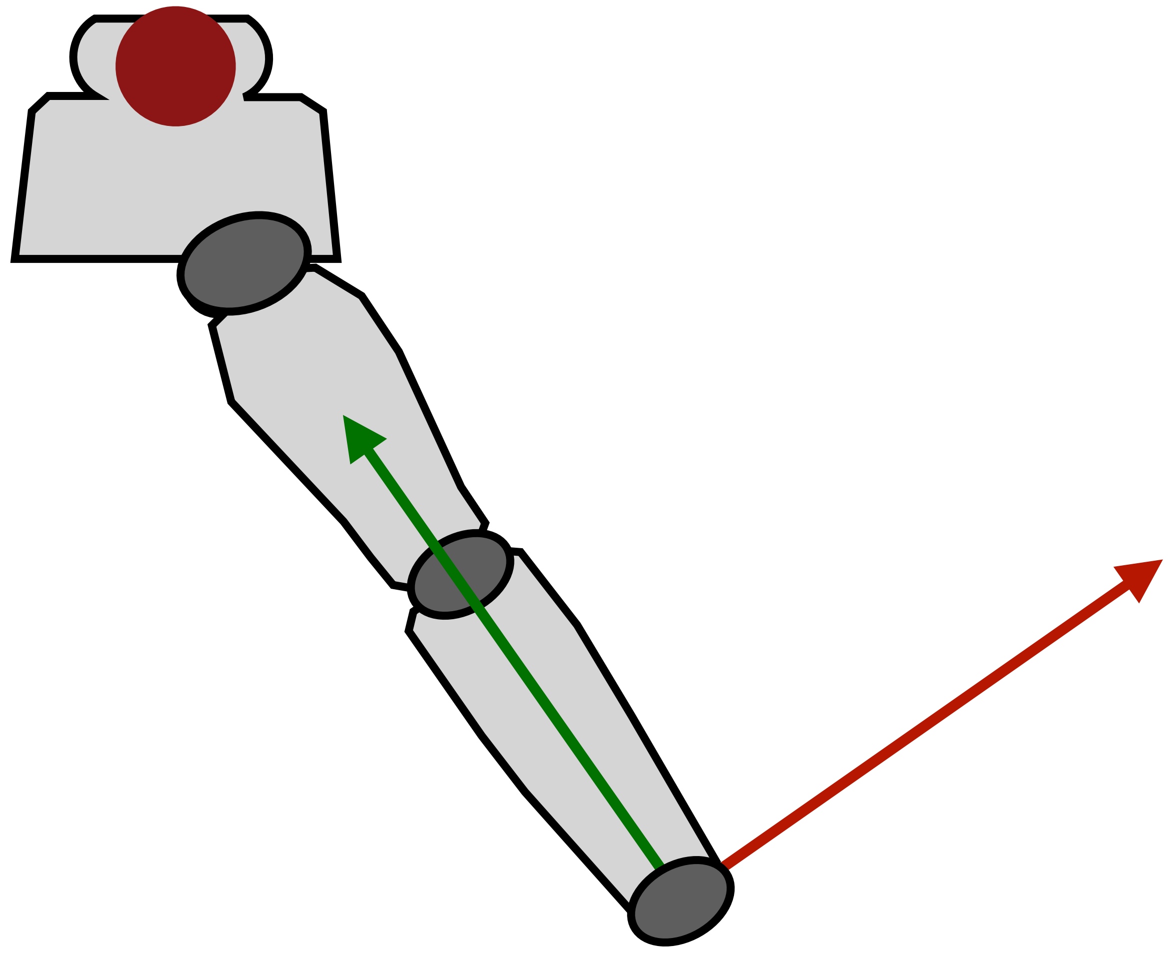


Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

$$\mathbf{q} = \mathbf{0}$$

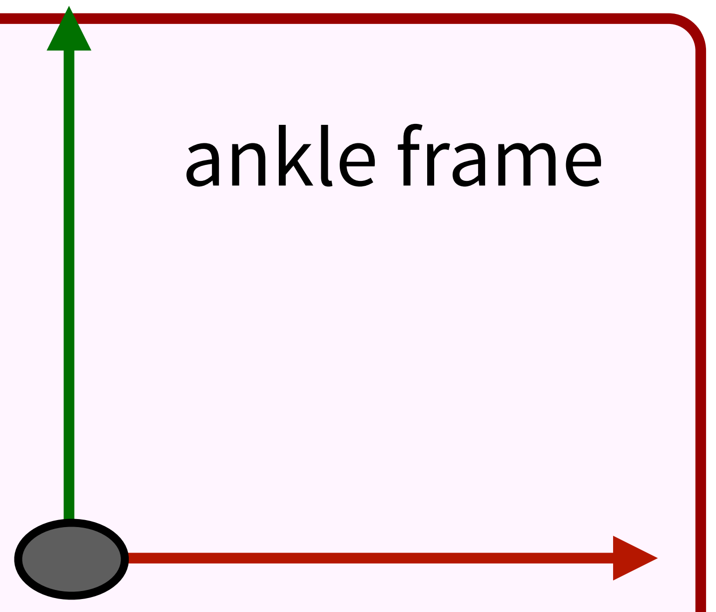
$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\,\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})\,\mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

$$\mathbf{F}(\mathbf{t}_3, \mathbf{r}_3)\mathbf{R}(\theta_{knee})\mathbf{F}(\mathbf{t}_4, \mathbf{r}_4)\,\mathbf{F}(\mathbf{t}_5, \mathbf{r}_5)\boxed{\mathbf{R}(\theta_{ankle}, \phi_{ankel})}$$

$\mathbf{t}_5$ translates from shank to ankle joint.
$\mathbf{r}_5$ rotates +90 degree about z-axis.

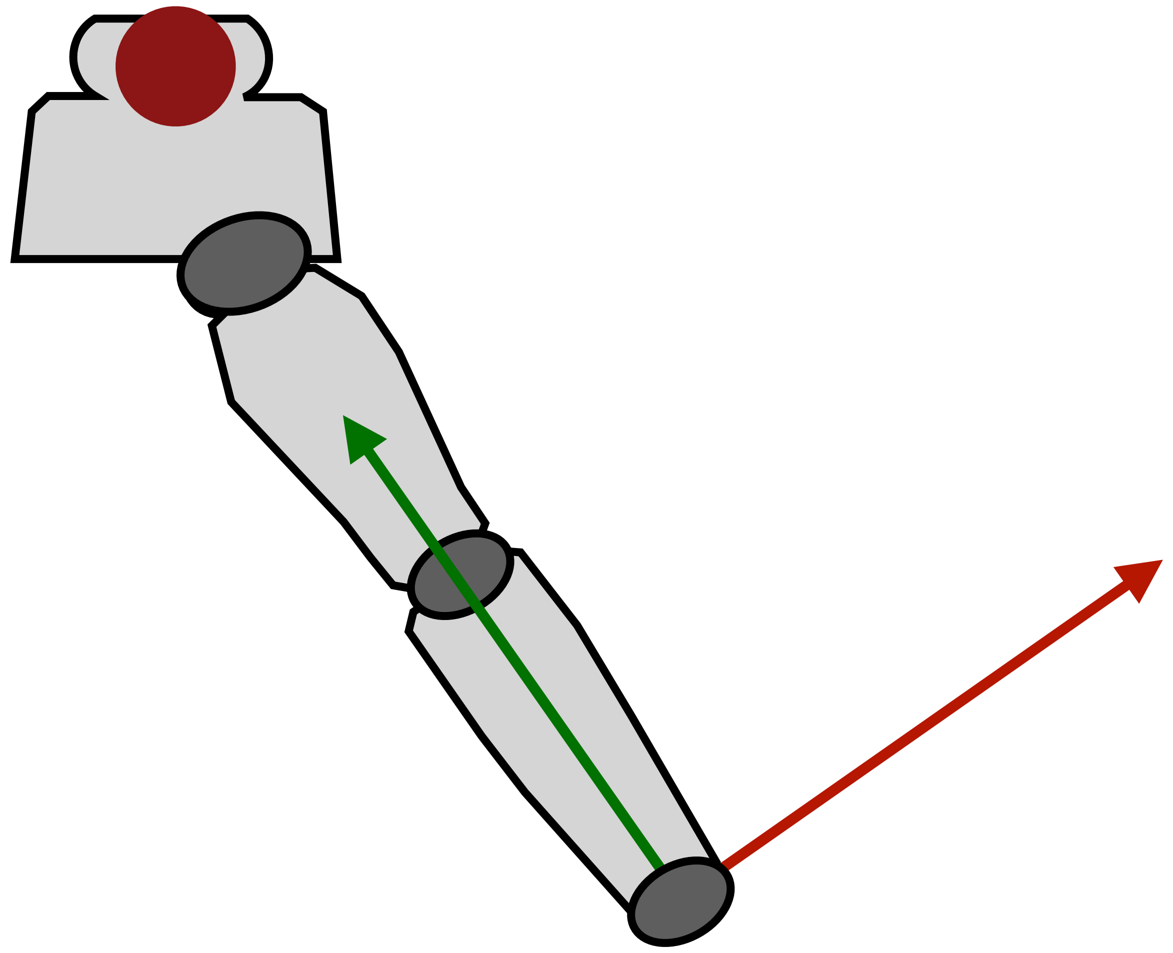


Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

$$\mathbf{q} = \mathbf{0}$$

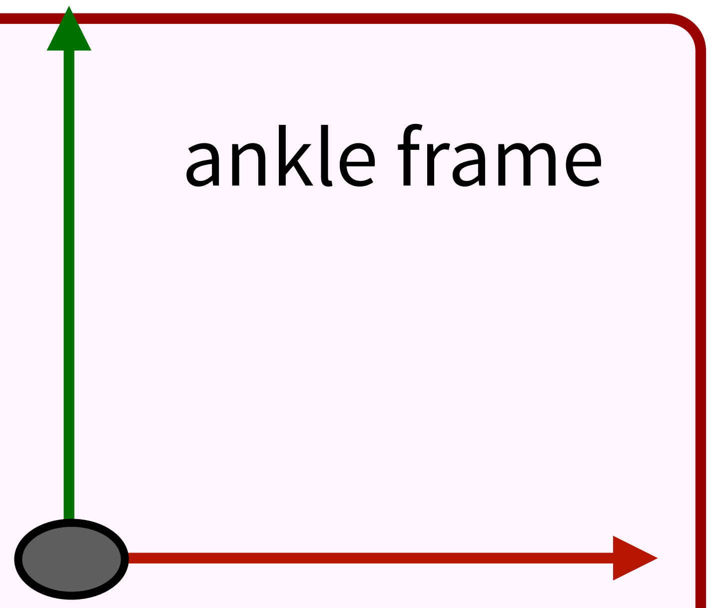
$$x = -1.5 \quad y = 1.5 \quad \phi_{hip} = 45^\circ$$



$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\,\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})\,\mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

$$\mathbf{F}(\mathbf{t}_3, \mathbf{r}_3)\mathbf{R}(\theta_{knee})\mathbf{F}(\mathbf{t}_4, \mathbf{r}_4)\,\mathbf{F}(\mathbf{t}_5, \mathbf{r}_5)\mathbf{R}(\theta_{ankle}, \phi_{ankel})\boxed{\mathbf{F}(\mathbf{t}_6, \mathbf{r}_6)}$$

$\mathbf{t}_5$ translates from shank to ankle joint.
$\mathbf{r}_5$ rotates +90 degree about z-axis.



Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

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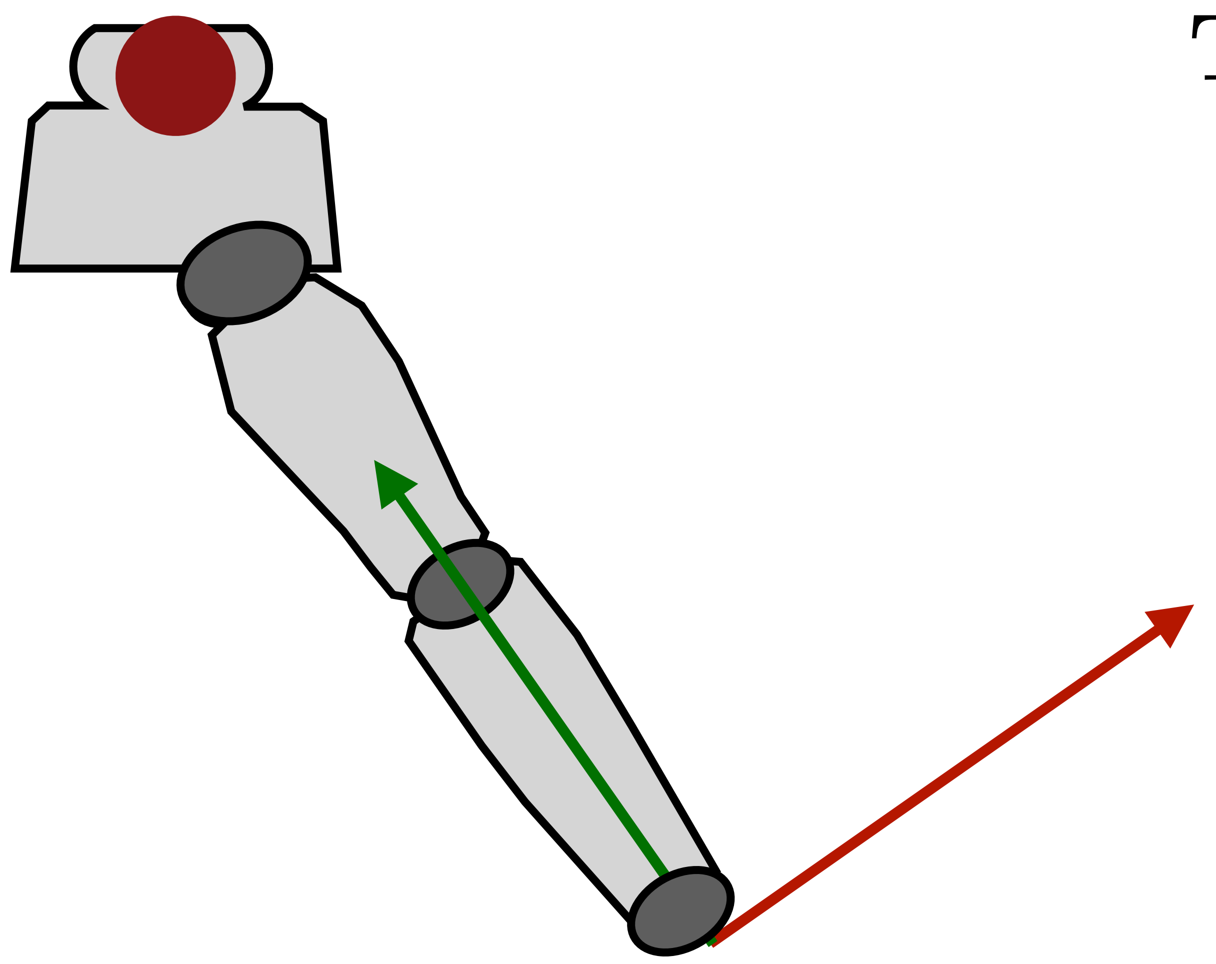

$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\,\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})\,\mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

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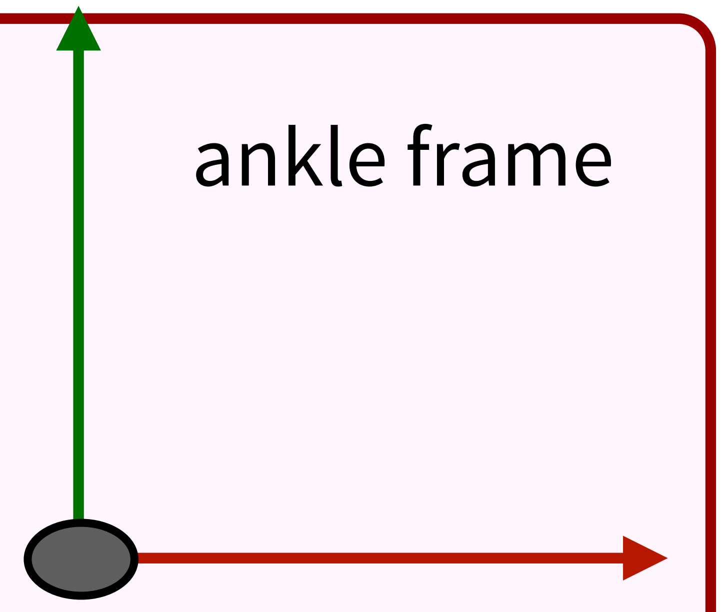

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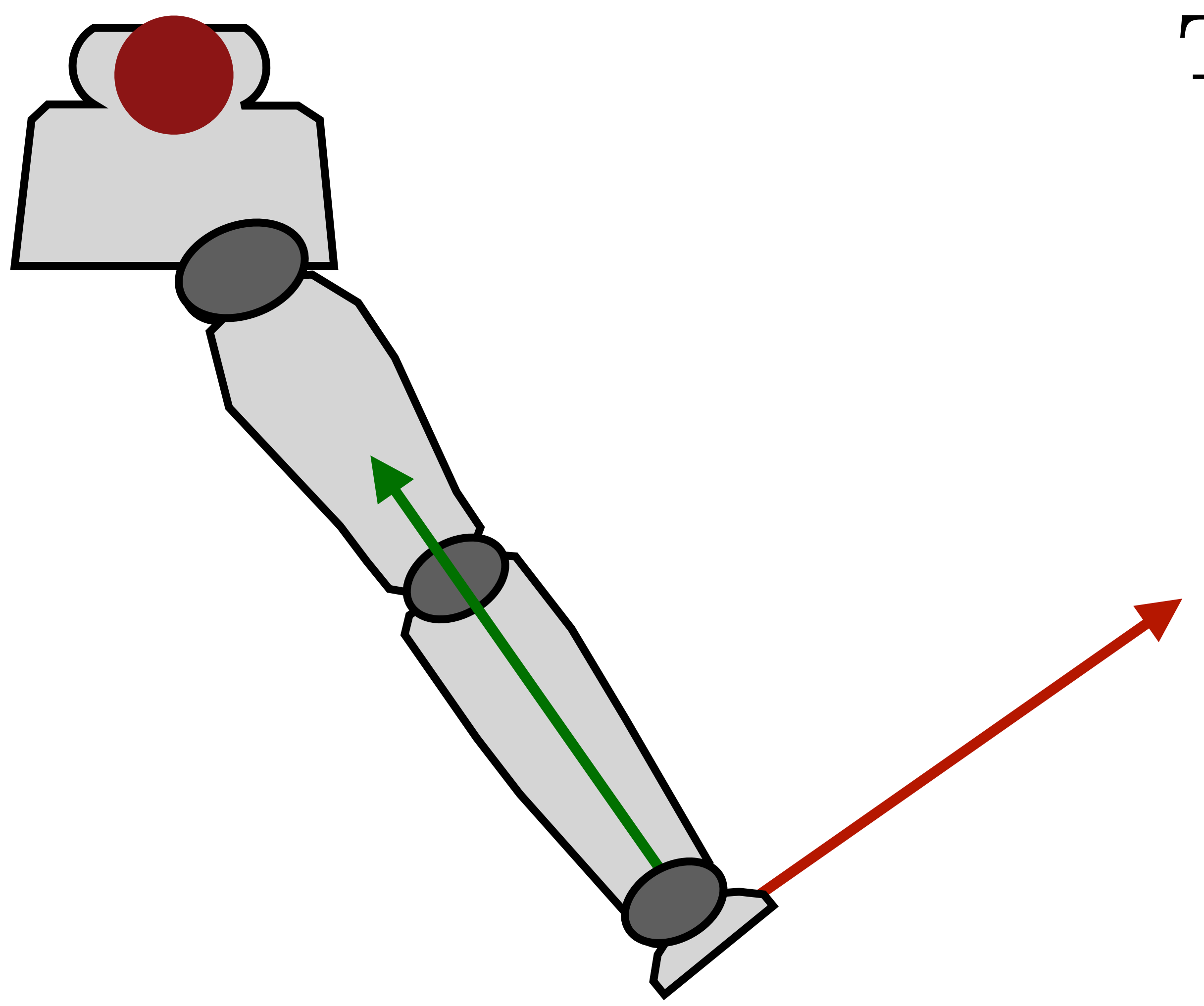

$$\mathbf{T}(x,y,z)\mathbf{R}(\theta,\phi,\sigma)\mathbf{F}(\mathbf{t}_0,\mathbf{r}_0)\mathbf{F}(\mathbf{t}_1,\mathbf{r}_1)\,\mathbf{R}(\theta_{hip},\phi_{hip},\sigma_{hip})\,\mathbf{F}(\mathbf{t}_2,\mathbf{r}_2)$$

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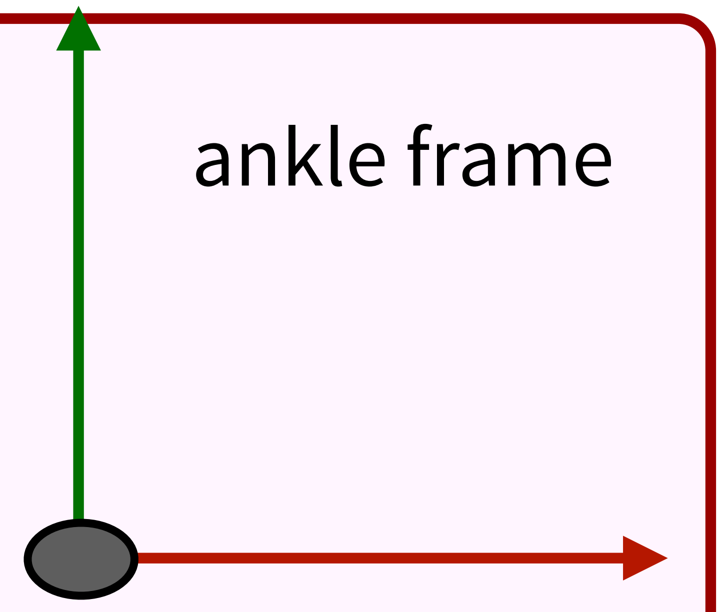

Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

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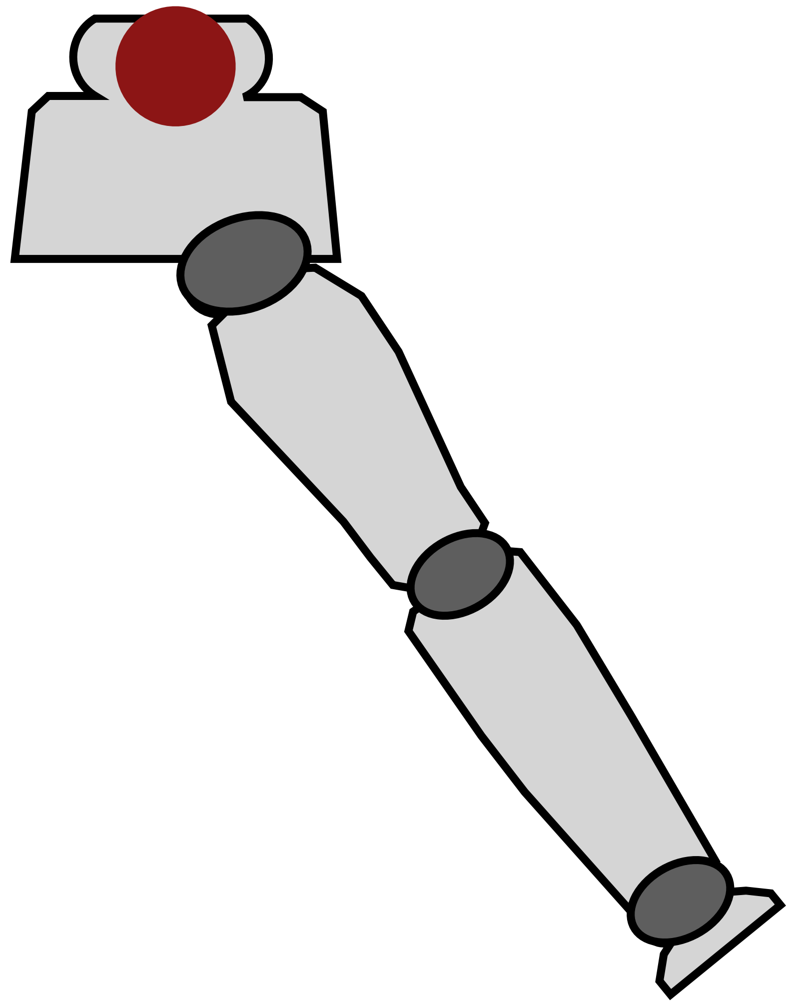

$$\mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\,\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})\,\mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

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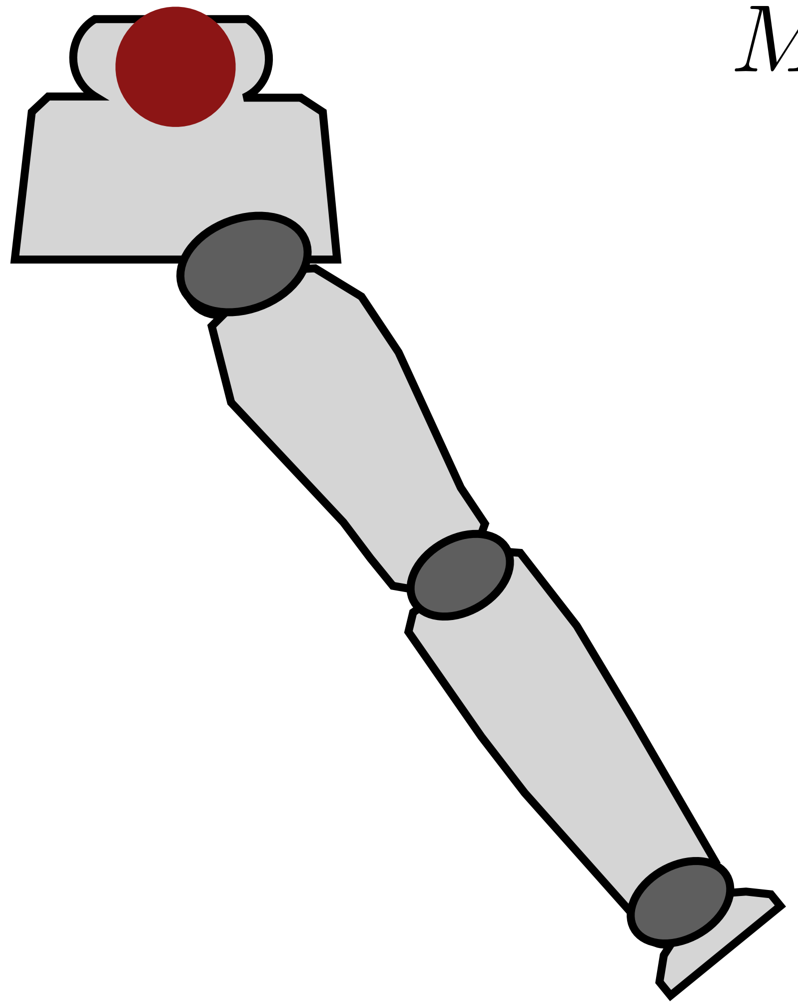

$$M_{foot}^{world} = \mathbf{T}(x, y, z)\mathbf{R}(\theta, \phi, \sigma)\mathbf{F}(\mathbf{t}_0, \mathbf{r}_0)\mathbf{F}(\mathbf{t}_1, \mathbf{r}_1)\,\mathbf{R}(\theta_{hip}, \phi_{hip}, \sigma_{hip})\,\mathbf{F}(\mathbf{t}_2, \mathbf{r}_2)$$

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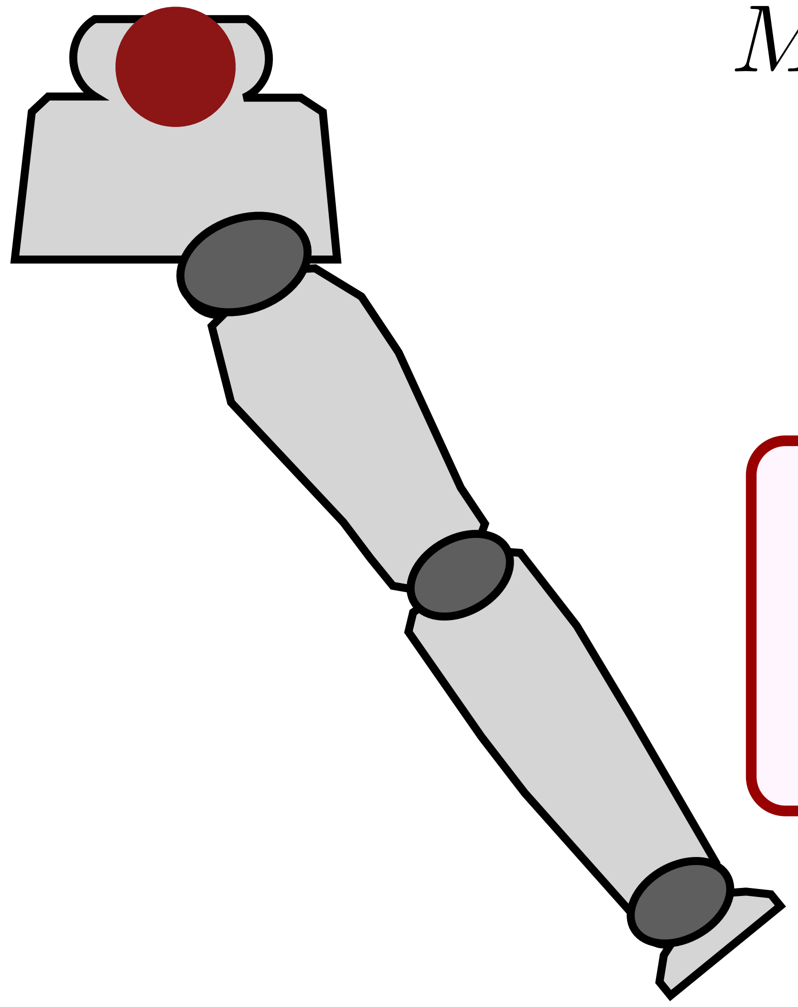

$$M^{world}_{foot} = \mathbf{T}(x,y,z)\mathbf{R}(\theta,\phi,\sigma)\mathbf{F}(\mathbf{t}_0,\mathbf{r}_0)\mathbf{F}(\mathbf{t}_1,\mathbf{r}_1)\,\mathbf{R}(\theta_{hip},\phi_{hip},\sigma_{hip})\,\mathbf{F}(\mathbf{t}_2,\mathbf{r}_2)$$
$$\mathbf{F}(\mathbf{t}_3,\mathbf{r}_3)\mathbf{R}(\theta_{knee})\mathbf{F}(\mathbf{t}_4,\mathbf{r}_4)\,\mathbf{F}(\mathbf{t}_5,\mathbf{r}_5)\mathbf{R}(\theta_{ankle},\phi_{ankel})\mathbf{F}(\mathbf{t}_6,\mathbf{r}_6)$$

kinematic chain from world to left foot

Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

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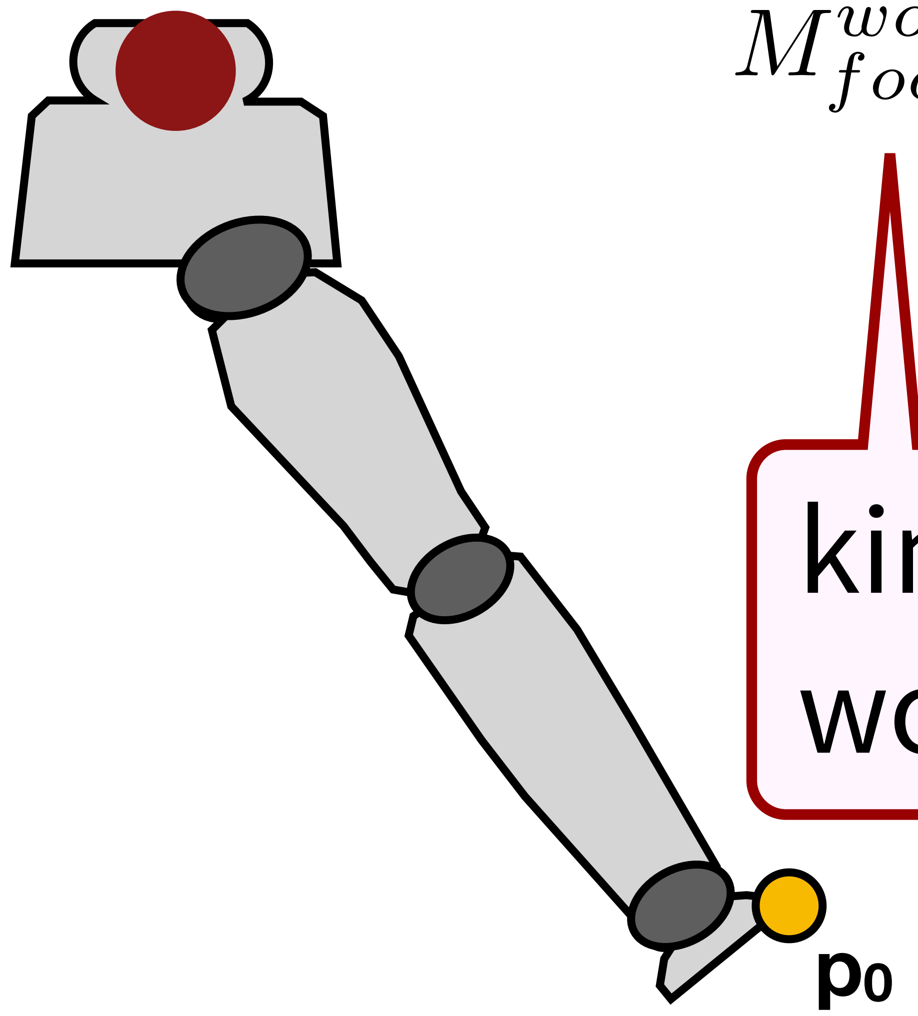

kinematic chain from world to left foot

Hierarchical transformations

$$\mathbf{q} \leftarrow \{x, y, z, \theta, \phi, \sigma, \theta_{hip}, \phi_{hip}, \sigma_{hip}, \theta_{knee}, \theta_{ankle}, \phi_{ankle}, \cdots\}$$

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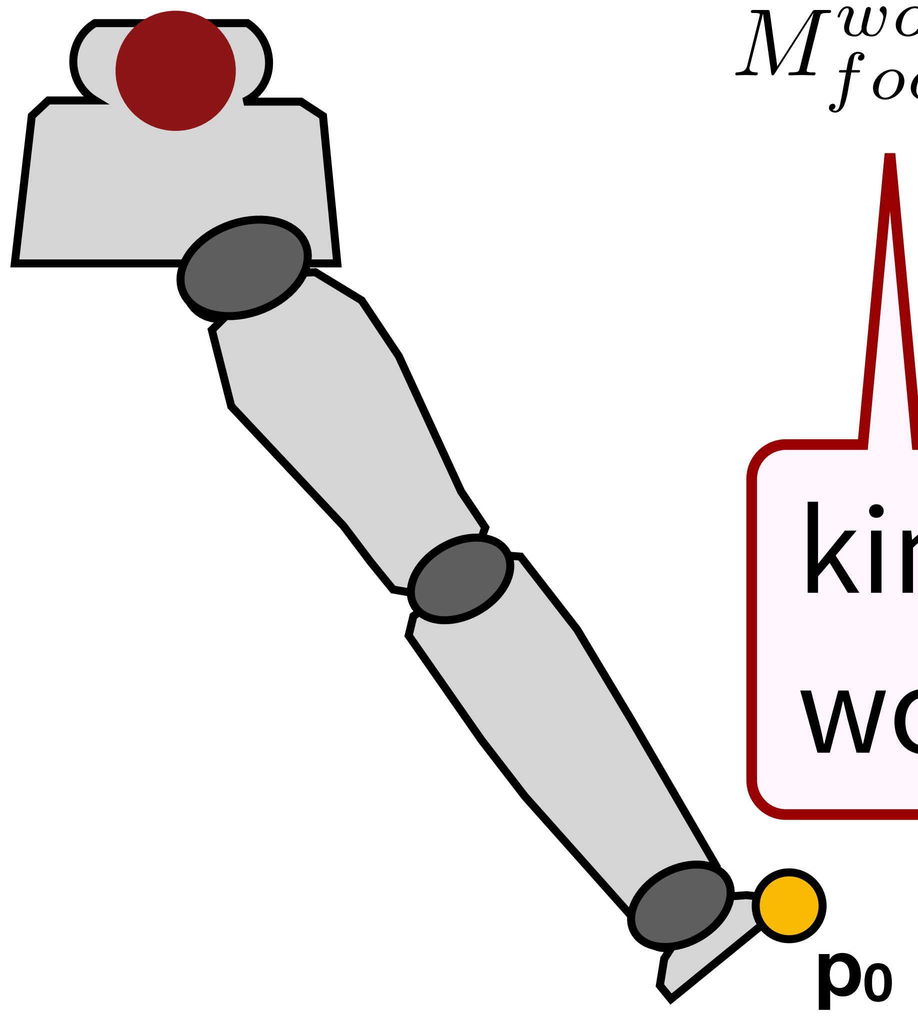
kinematic chain from world to left foot



$$\mathbf{p} = M_{foot}^{world}\mathbf{p}_0$$

Hierarchical transformations

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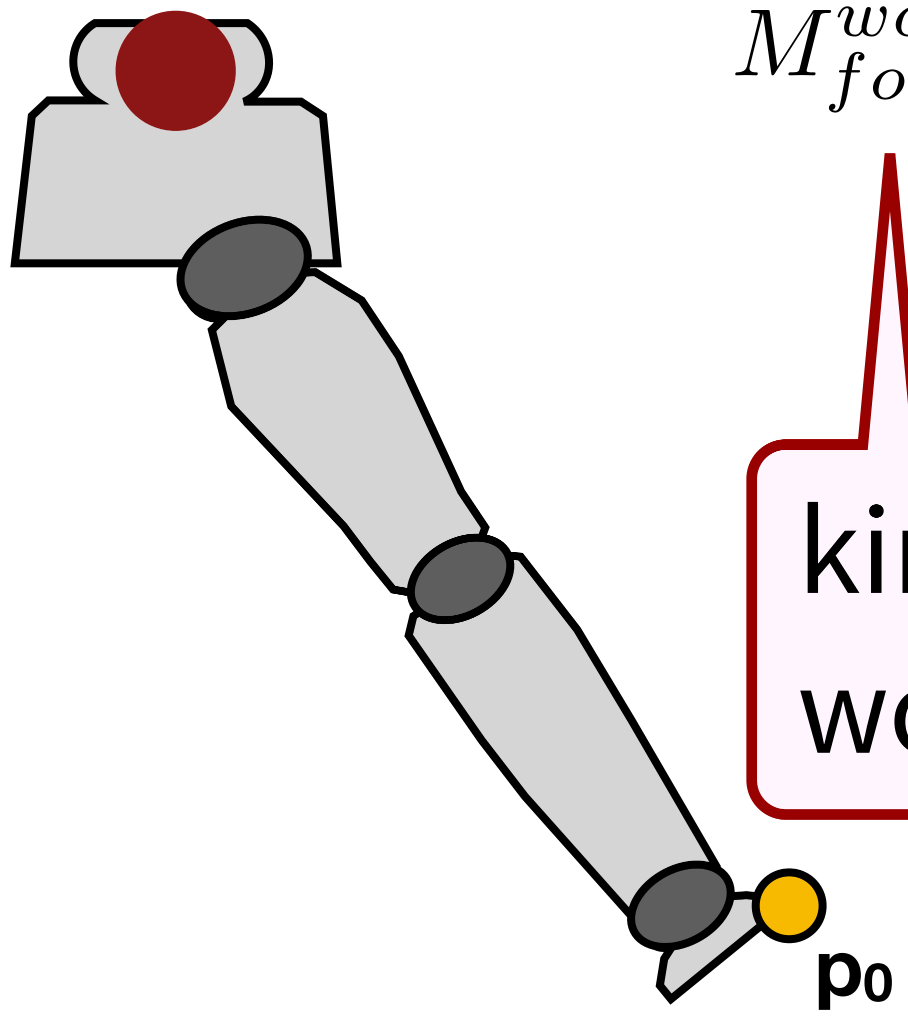
kinematic chain from world to left foot



a point in foot coordinate frame

$$\mathbf{p} = M_{foot}^{world}\mathbf{p}_0$$

Hierarchical transformations

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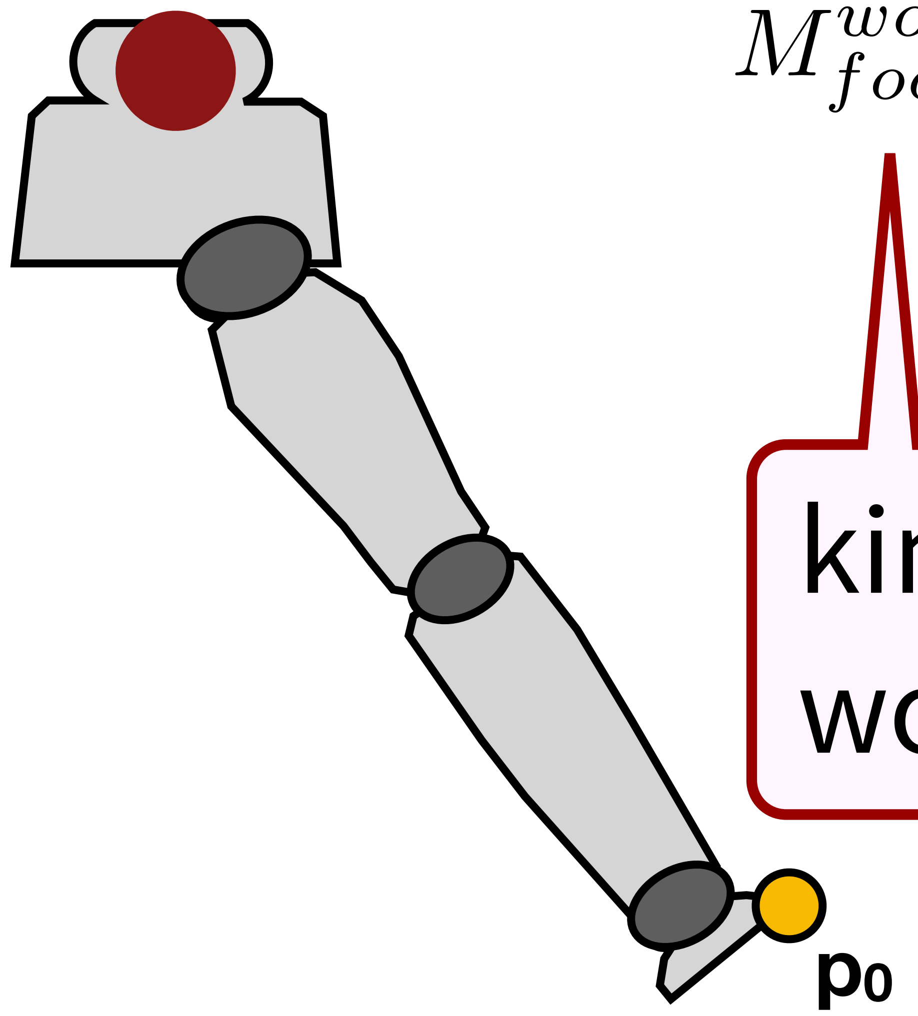

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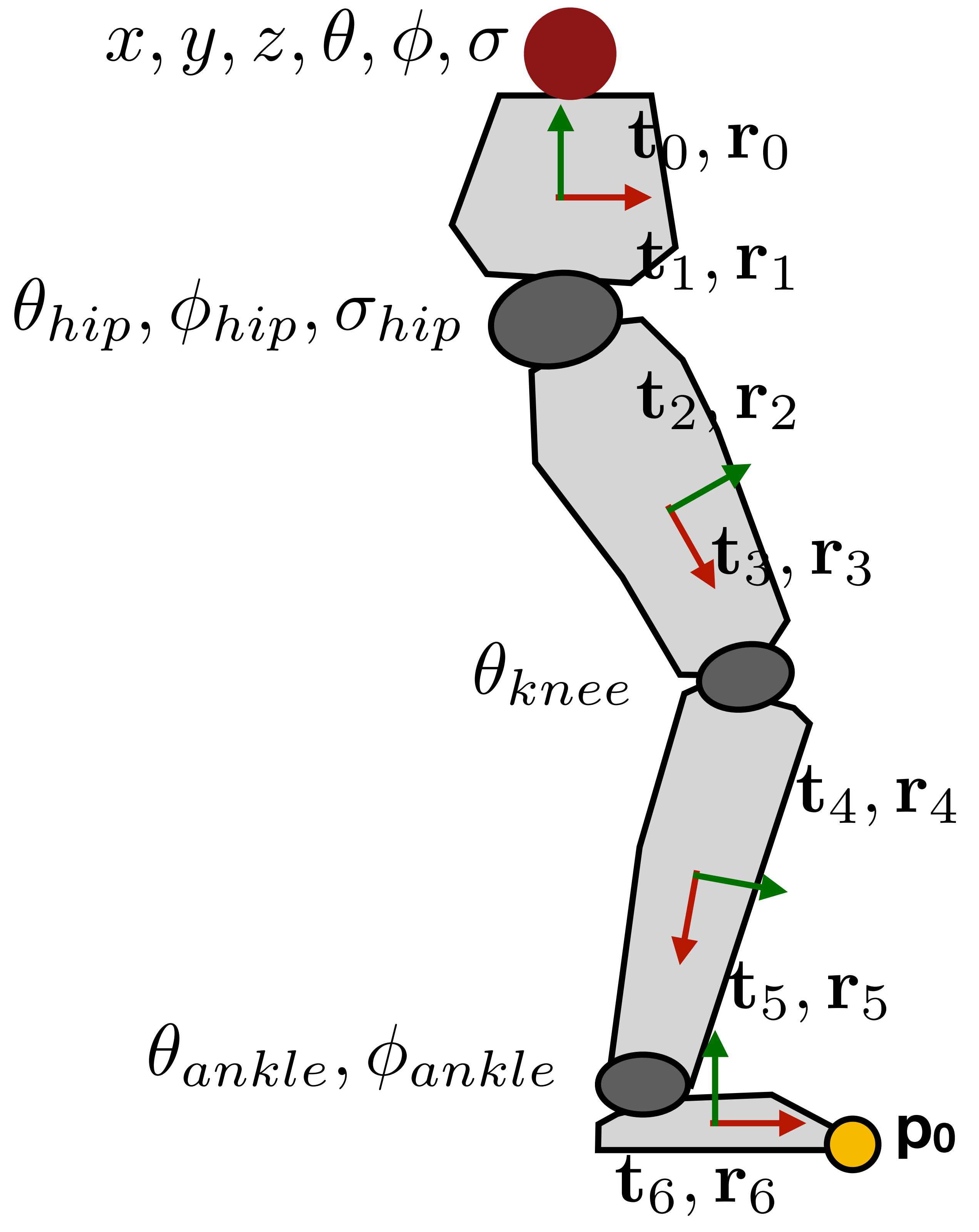
kinematic chain from world to left foot

a point in foot coordinate frame

$$\mathbf{p} = M_{foot}^{world}\mathbf{p}_0$$

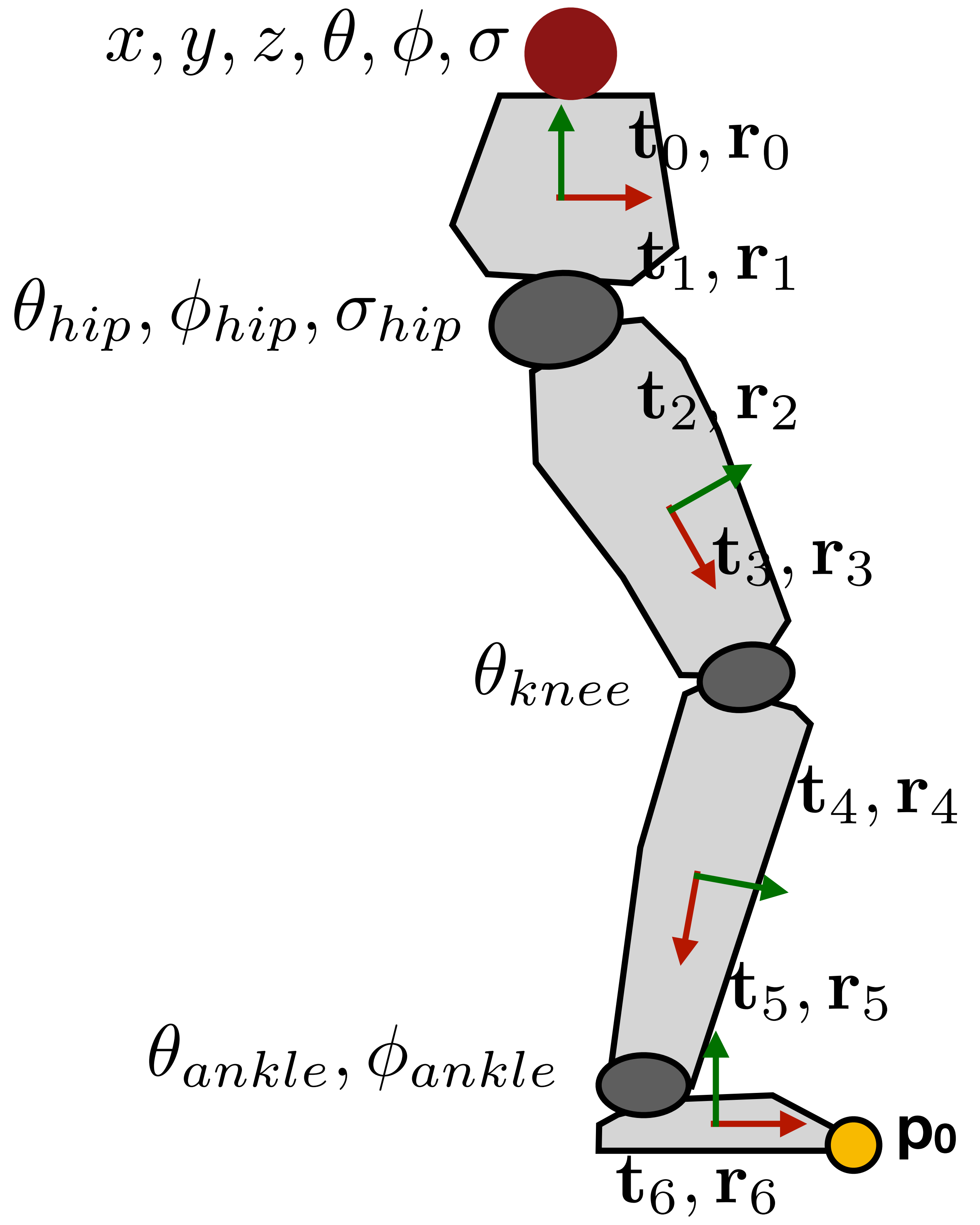
The same point in world frame

A point in a local frame



A point in a local frame

- We now know how to refer a point in its own local frame and in the world frame. For example,



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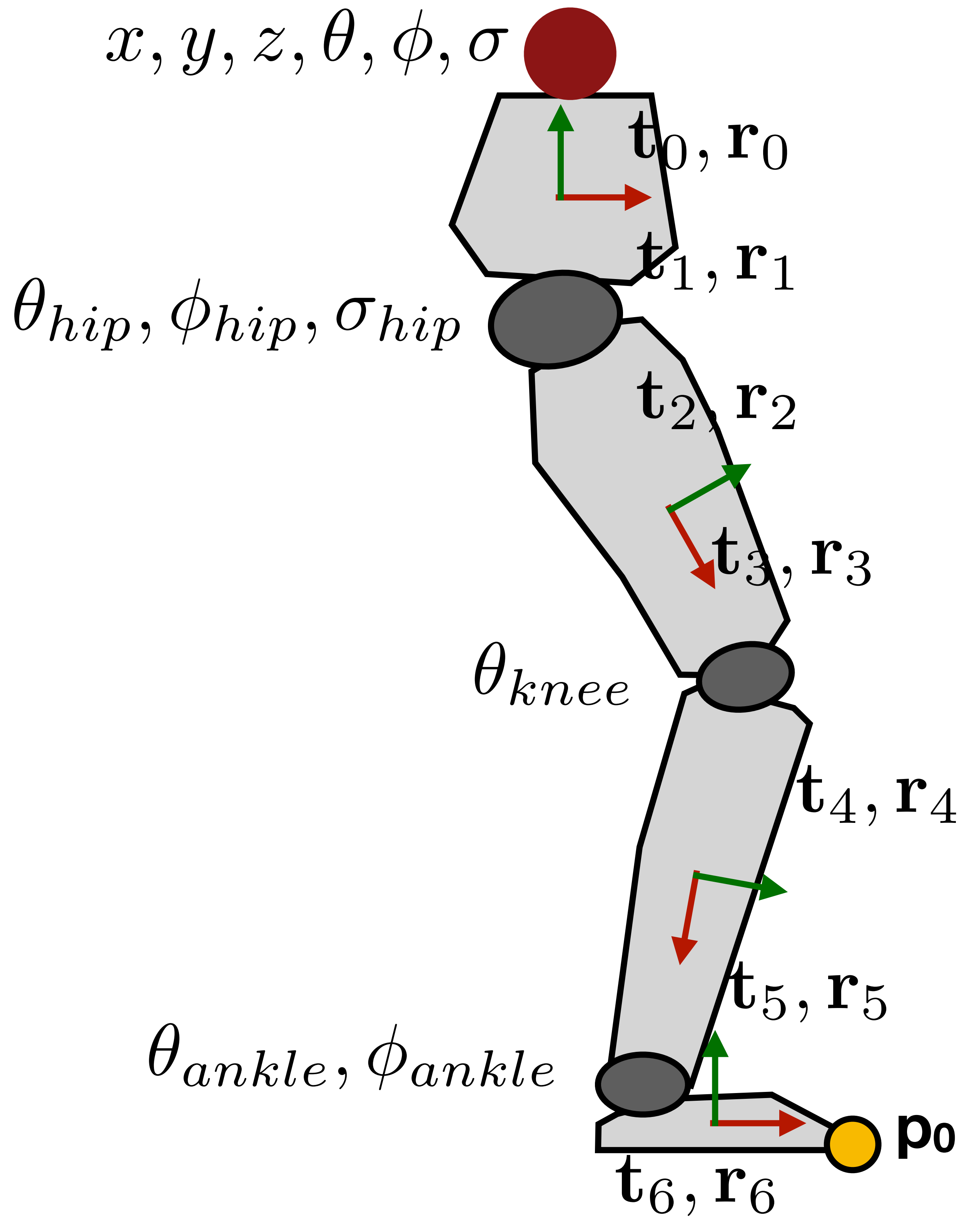

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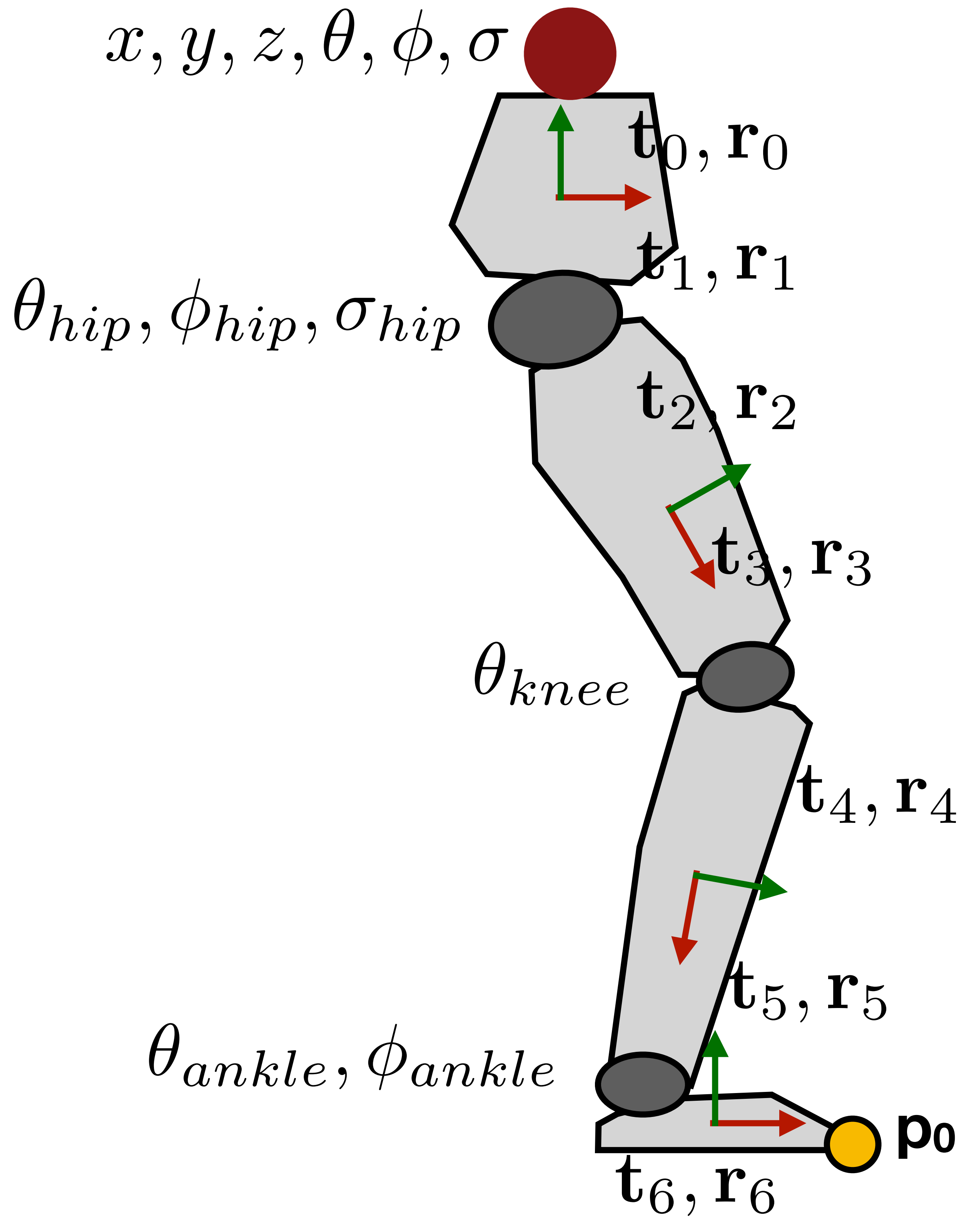
- We now know how to refer a point in its own local frame and in the world frame. For example,

$$\mathbf{p} = M_{foot}^{world}\mathbf{p}_0$$

- With the kinematic transformation chain, we can refer to any point in any coordinate frame.



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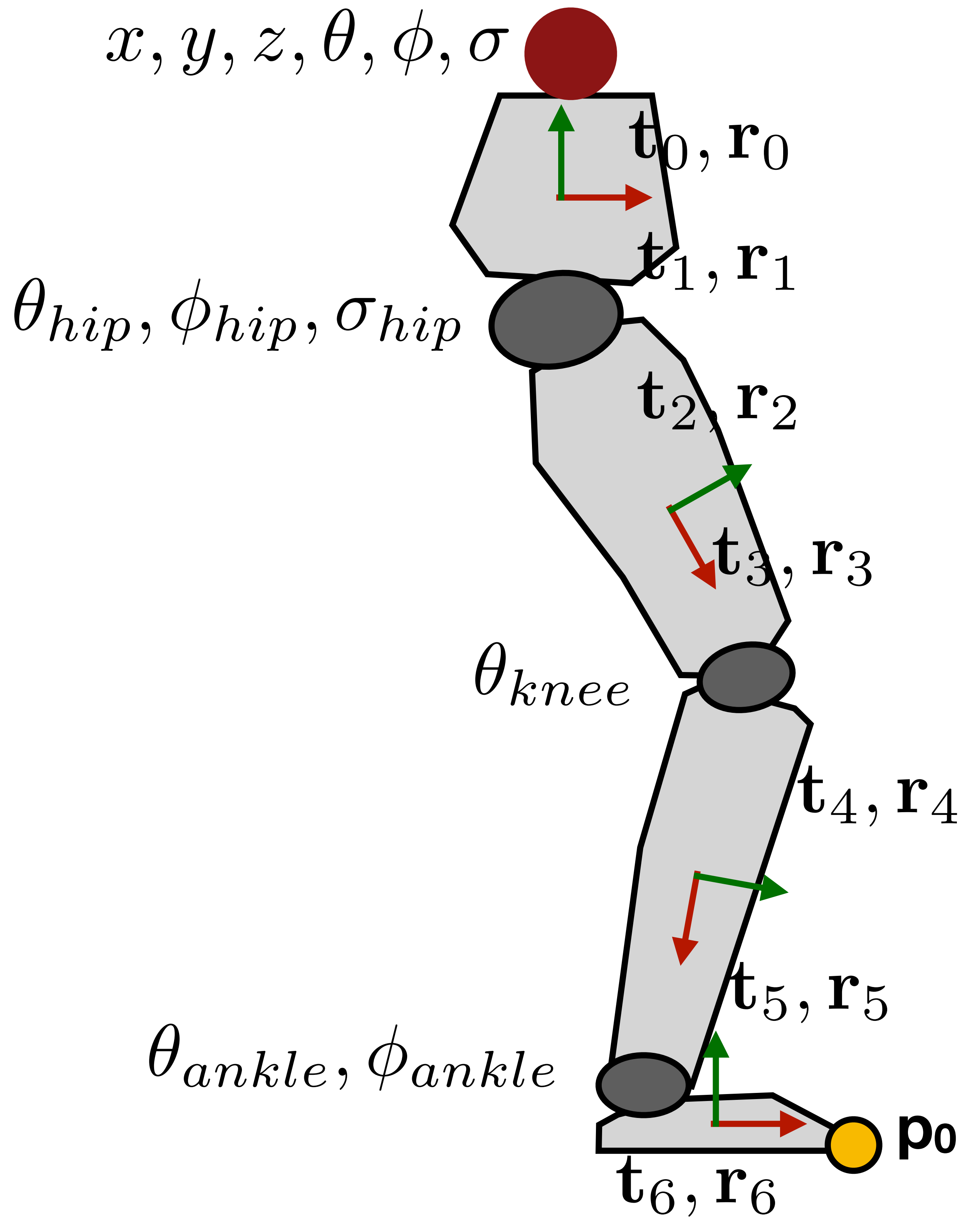
- For example, the toe in the coordinate frame of the thigh is



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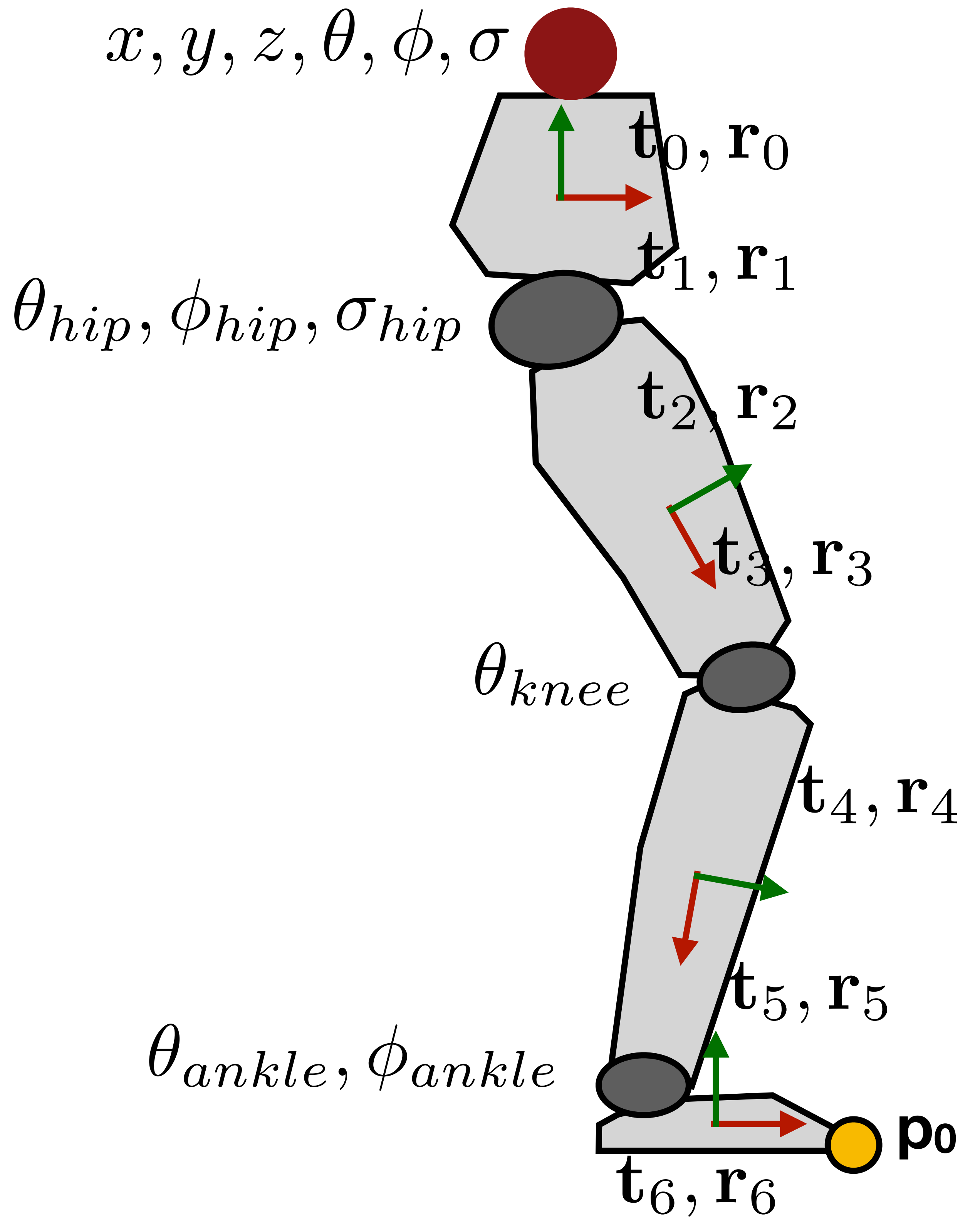
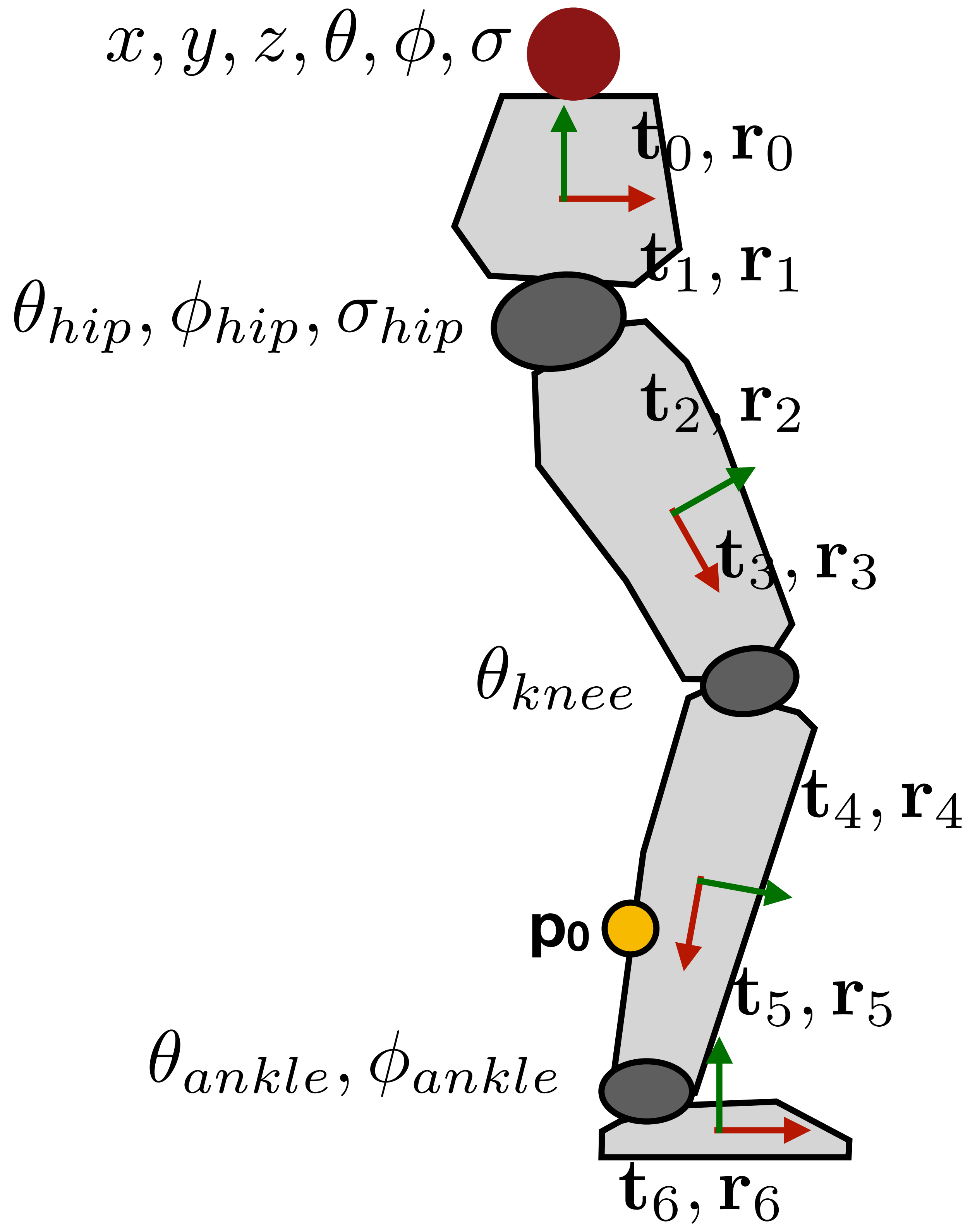
$$\mathbf{F}(\mathbf{t}_3, \mathbf{r}_3)\mathbf{R}(\theta_{knee})\mathbf{F}(\mathbf{t}_4, \mathbf{r}_4)\mathbf{F}(\mathbf{t}_5, \mathbf{r}_5)\mathbf{R}(\theta_{ankle})\mathbf{R}(\phi_{ankle})\mathbf{F}(\mathbf{t}_6, \mathbf{r}_6)\mathbf{p}_0$$



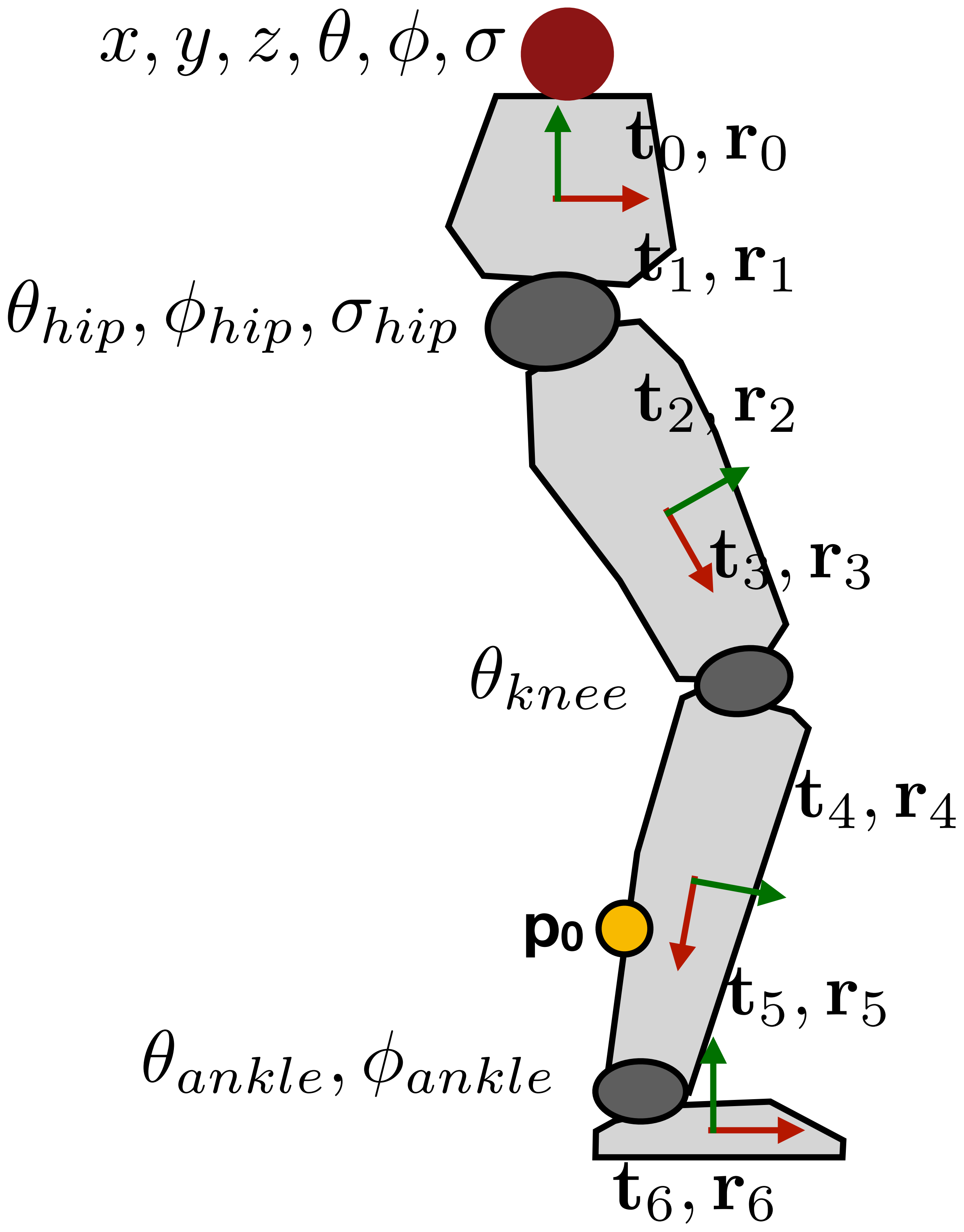
Quiz

- **What is the coordinate of the yellow point in the foot frame?**

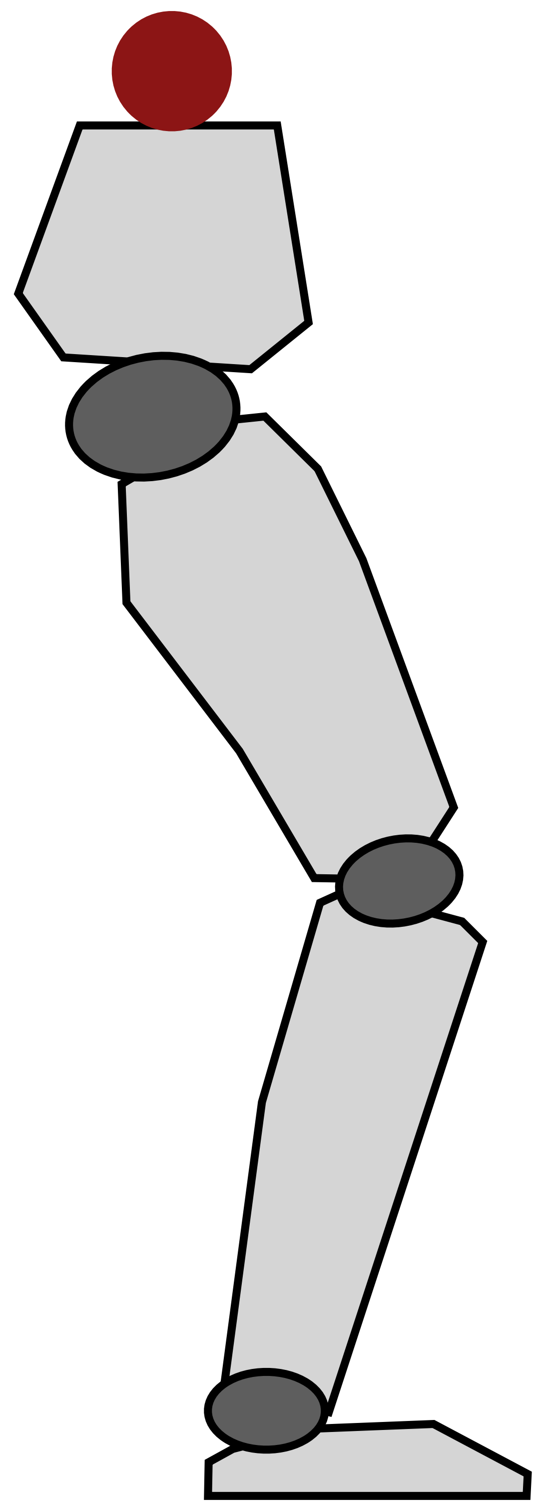


Quiz

- What is the relative position of the yellow point to the hip joint in the world frame?

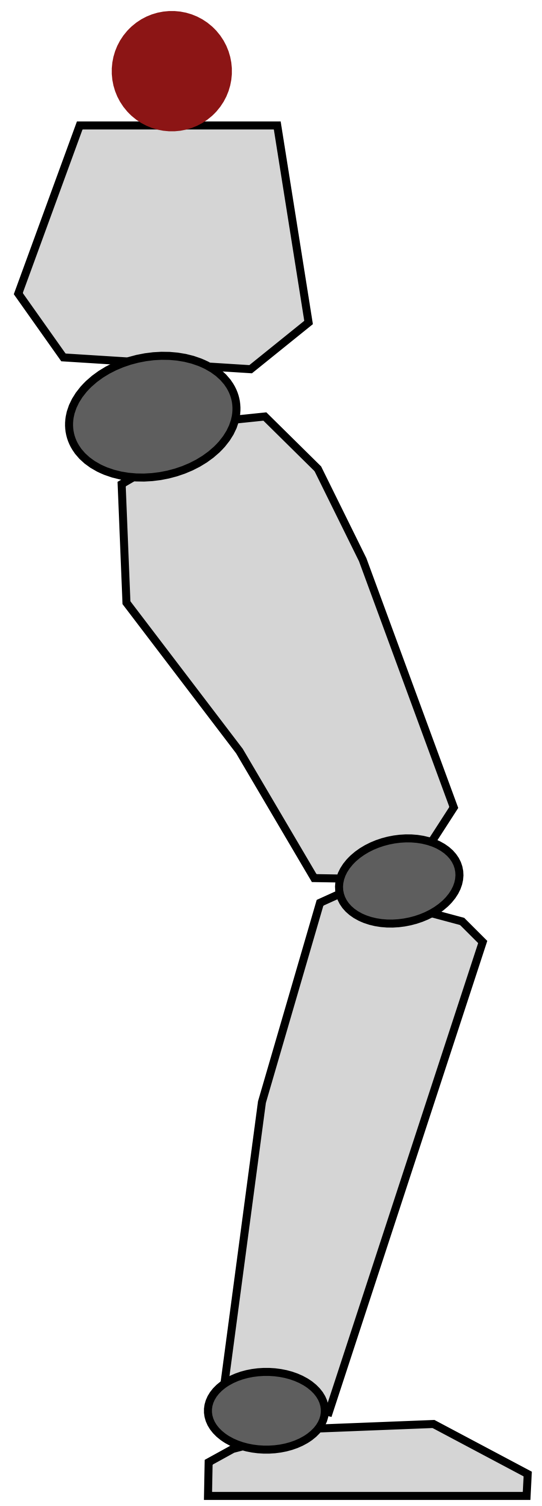


Center of mass



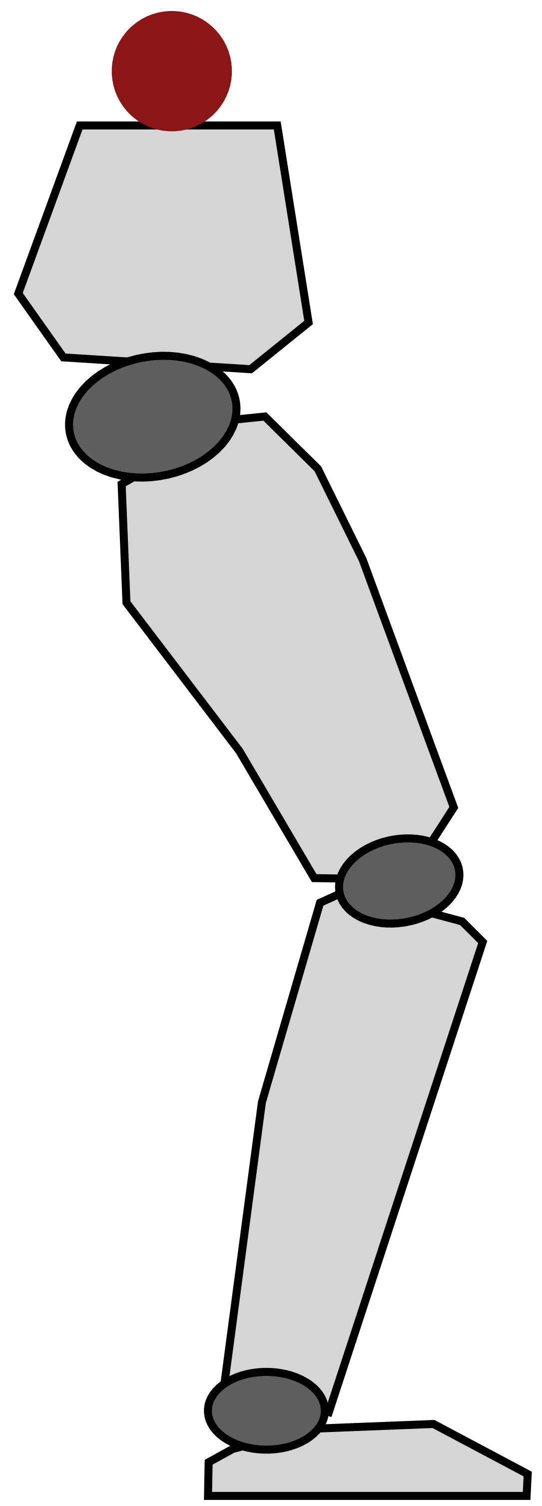
Center of mass

- Center of mass (COM) is an important kinematic concept for simulation and control, such as calculating angular motion or evaluating balance.



Center of mass

- Center of mass (COM) is an important kinematic concept for simulation and control, such as calculating angular motion or evaluating balance.
- COM is a 3D point that depends on the pose. For example, the COM of the leg is given by the COM of each body segment weighted by its mass.



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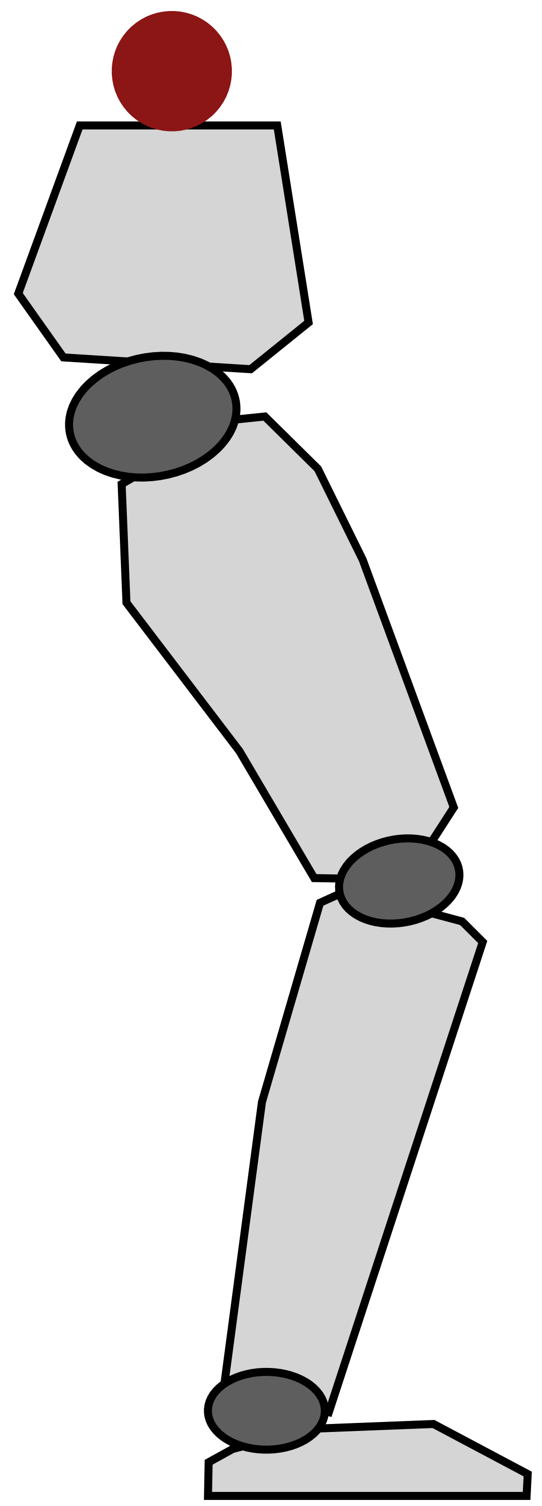
$$M_{pelv}^{world}\begin{bmatrix}0\\0\\0\\1\end{bmatrix} \qquad M_{thig}^{world}\begin{bmatrix}0\\0\\0\\1\end{bmatrix} \qquad M_{shan}^{world}\begin{bmatrix}0\\0\\0\\1\end{bmatrix} \qquad M_{foot}^{world}\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$



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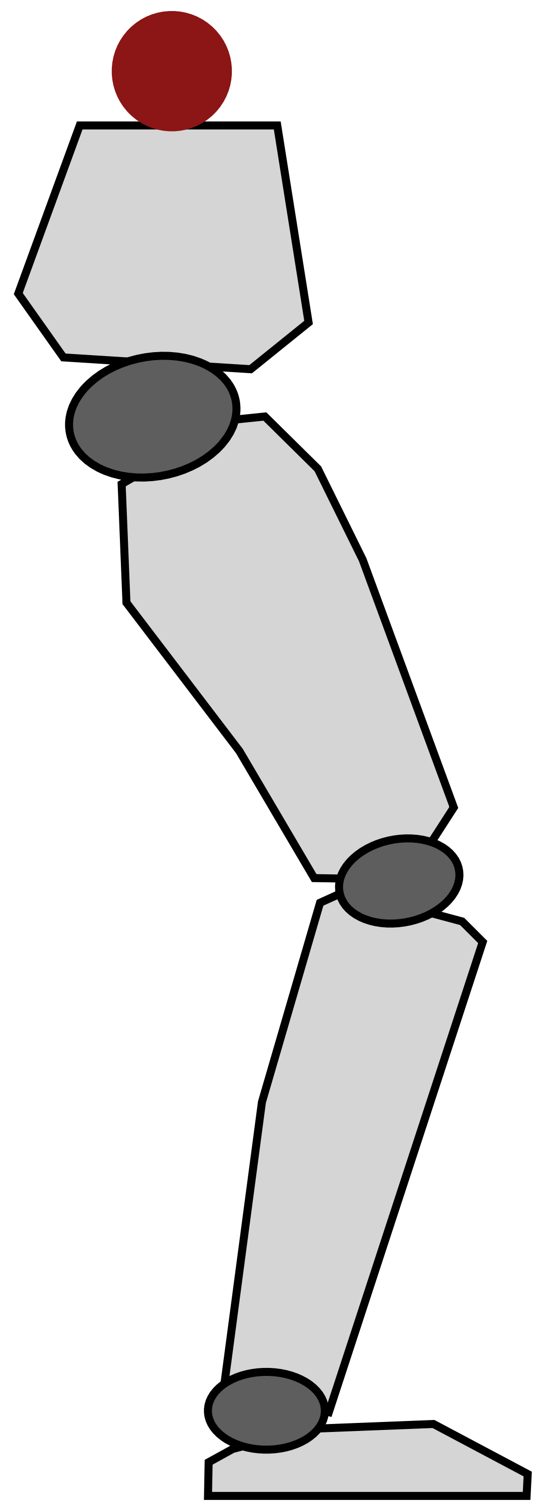

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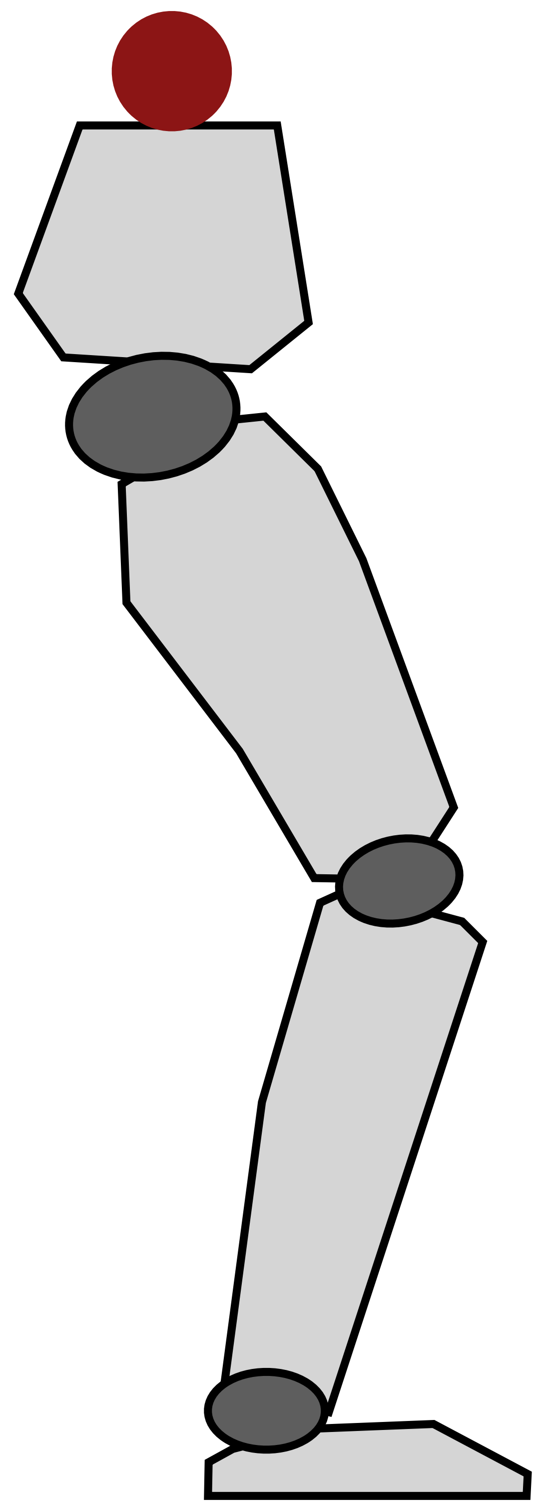

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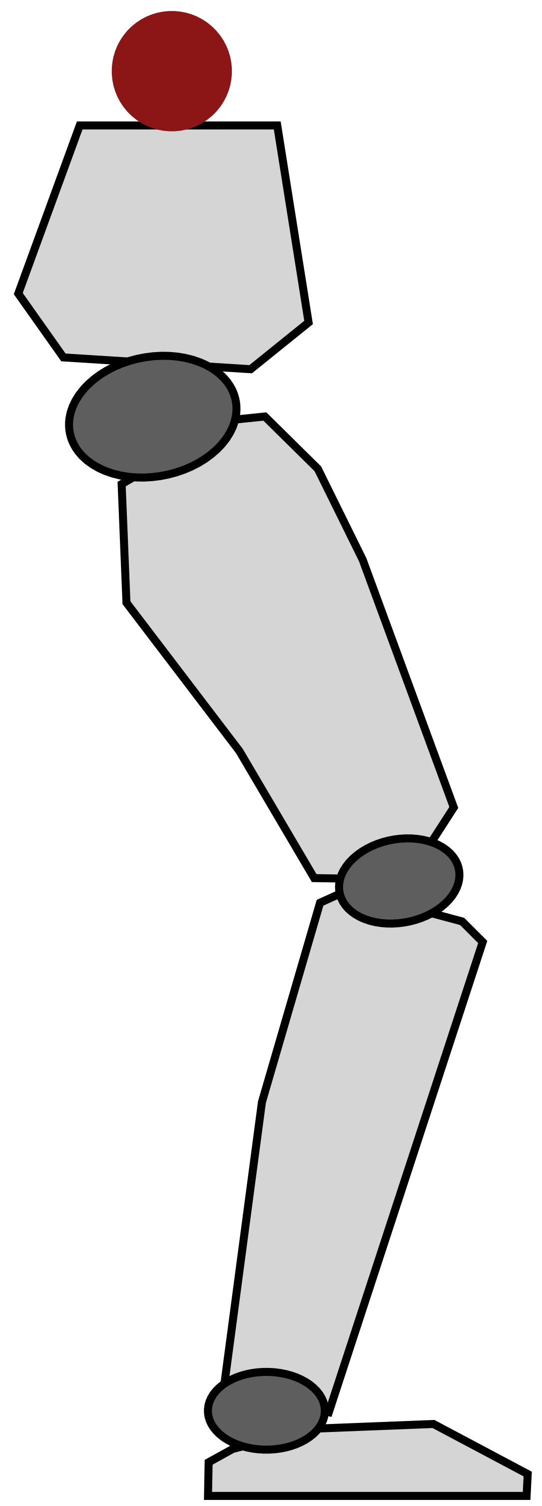

$$\frac{m_{pelv} M_{pelv}^{world} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + m_{thig} M_{thig}^{world} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + m_{shan} M_{shan}^{world} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + m_{foot} M_{foot}^{world} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}{m_{pelv} + m_{thig} + m_{shan} + m_{foot}}$$

What assumptions are we making here?

Center of mass

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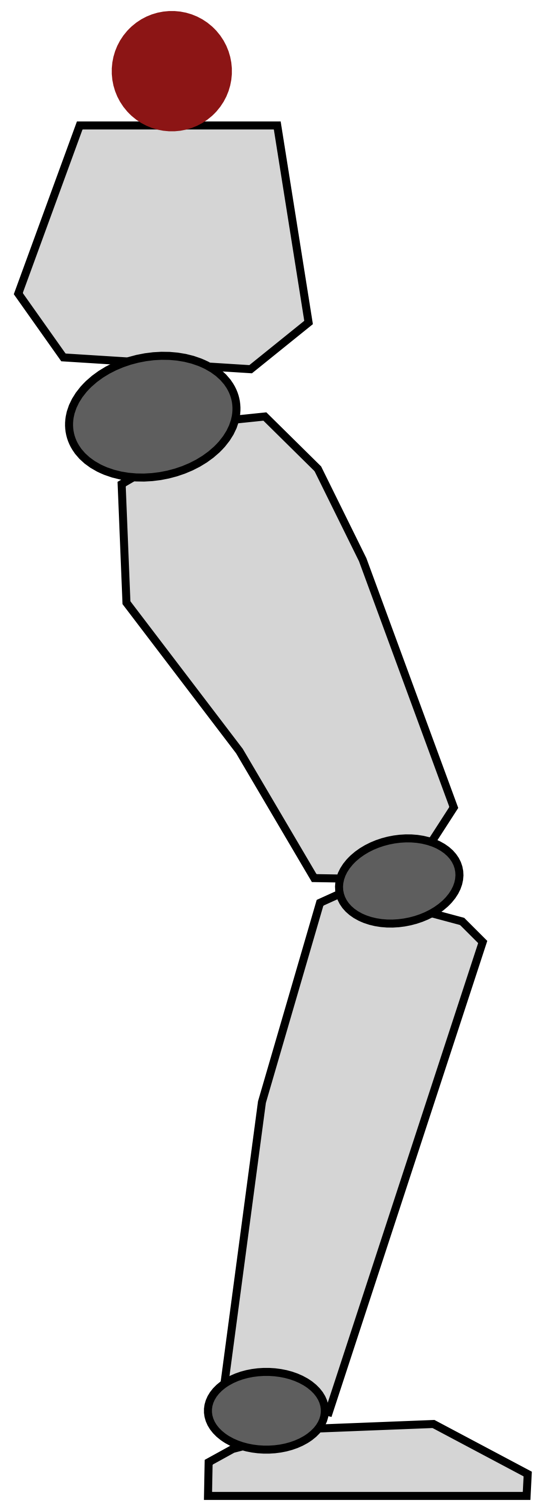

What assumptions are we making here?

The body has uniform density!

Center of mass

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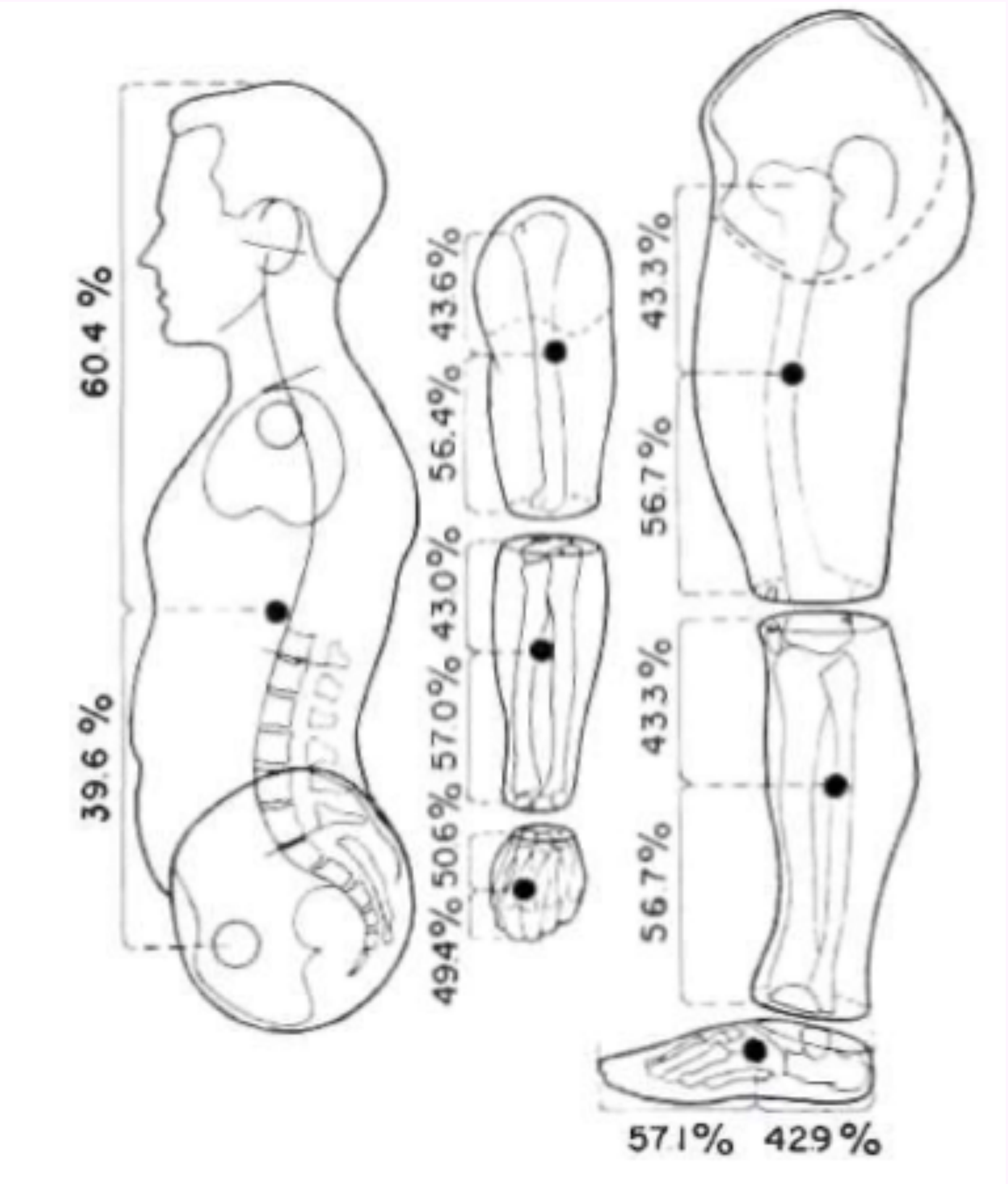
- **COM is a 3D point that depends on the pose. For example, the C… leg is given by the COM of each body segment weighted by its m…**

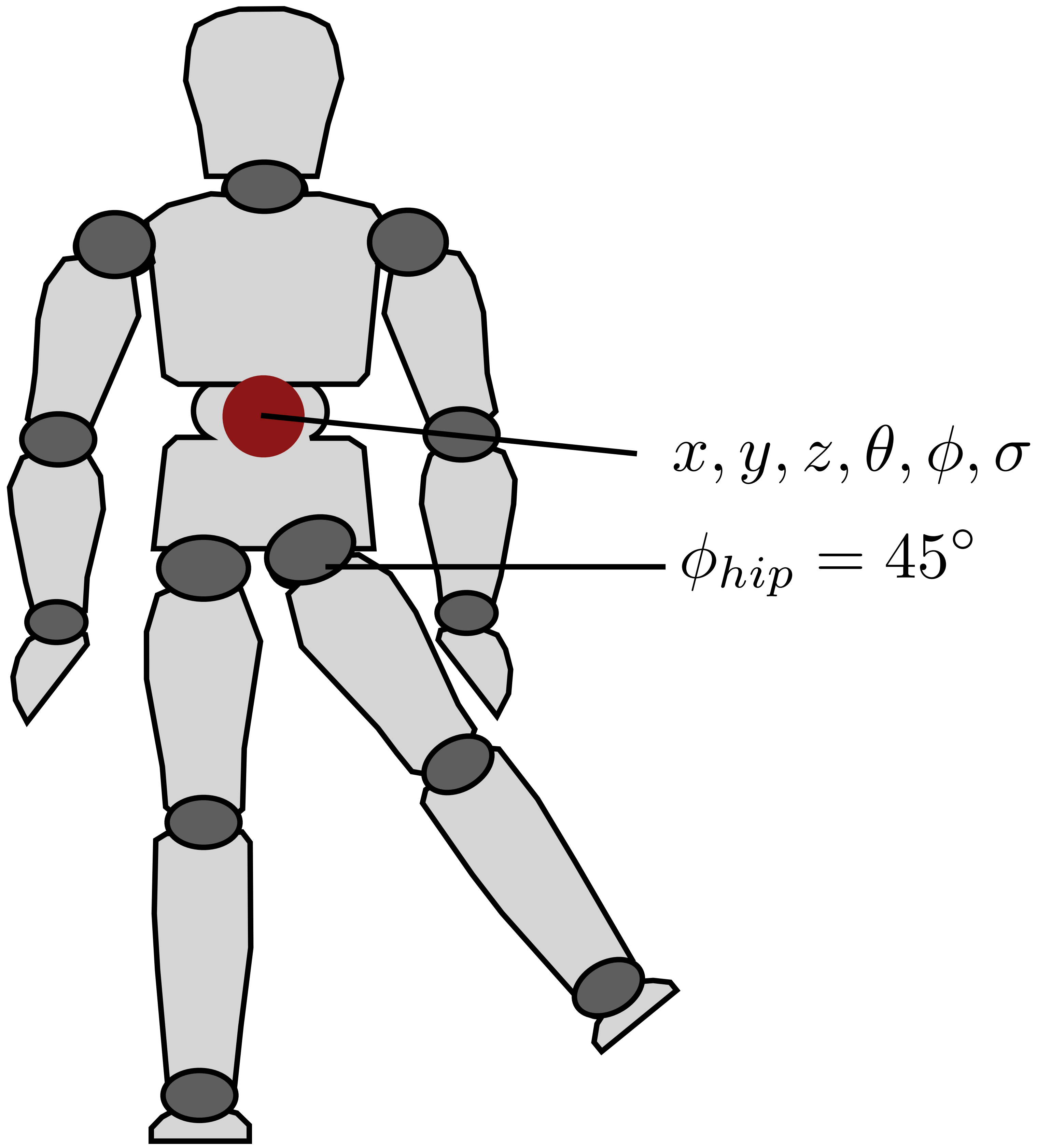
$$\frac{m_{pelv} M_{pelv}^{world} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + m_{thig} M_{thig}^{world} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + m_{shan} M_{shan}^{world} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + m_{foot} M_{foot}^{world} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}{m_{pelv} + m_{thig} + m_{shan} + m_{foot}}$$



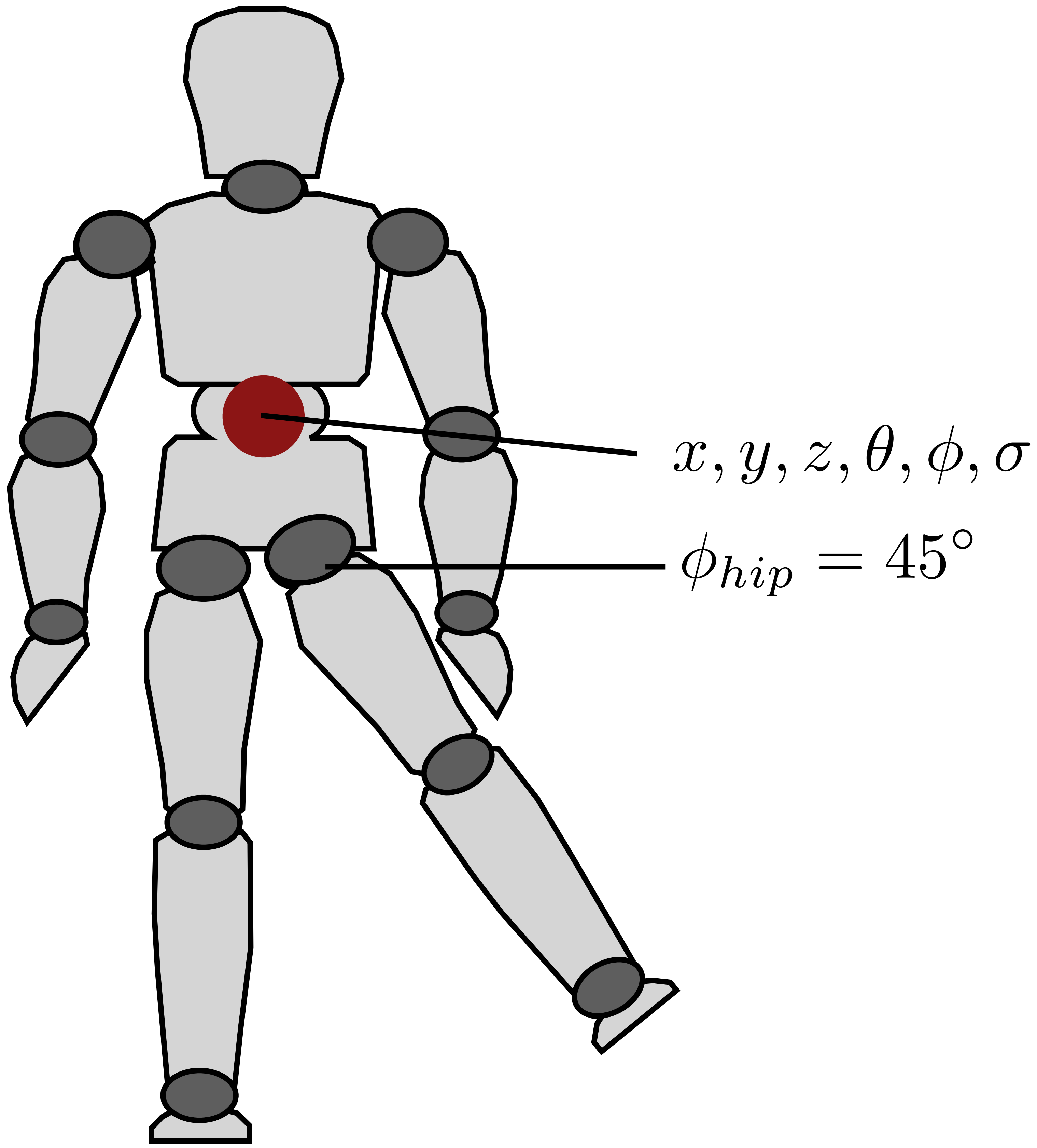
What assumptions are we making here?

The body has uniform density!

Root location

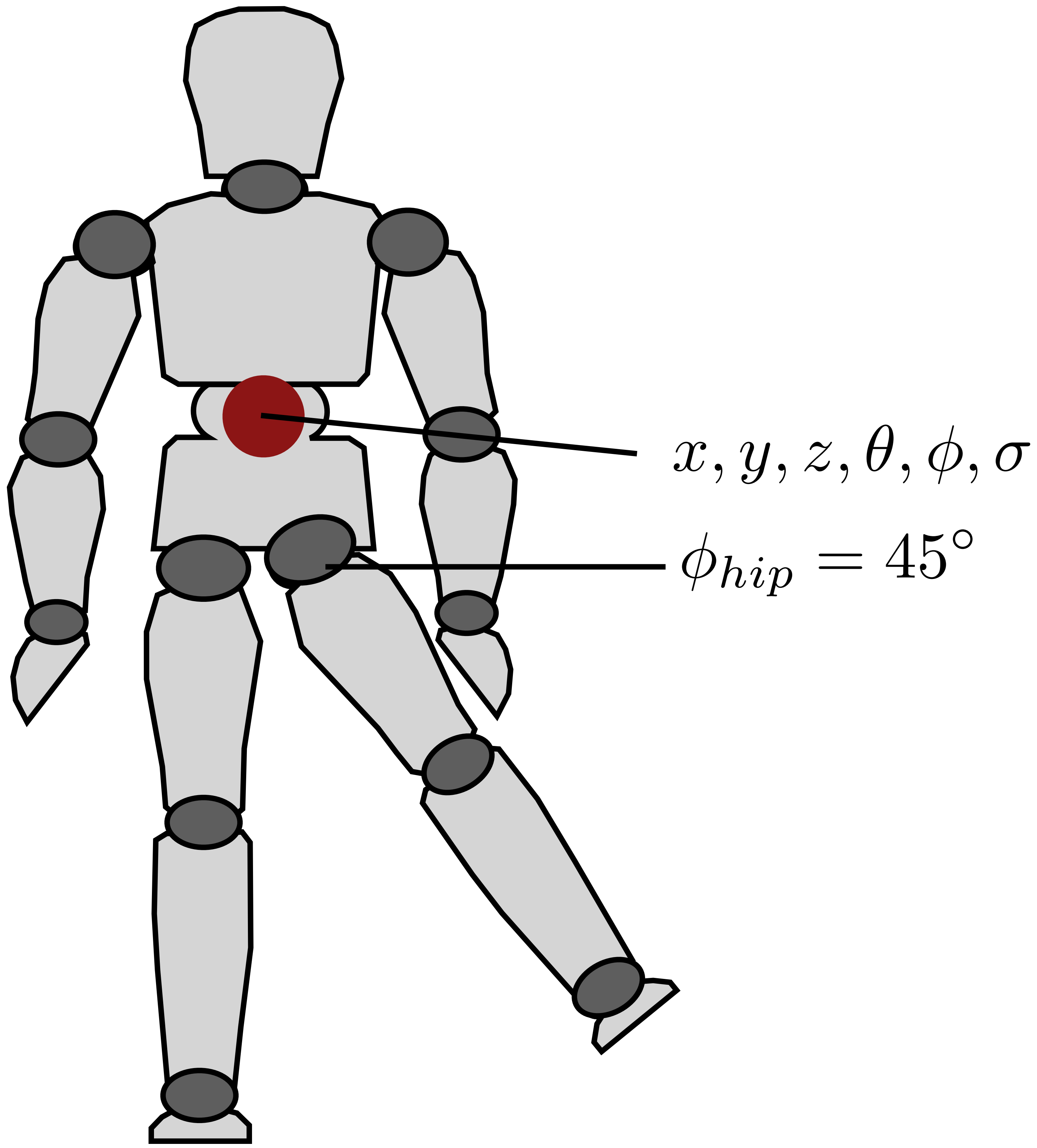


Root location

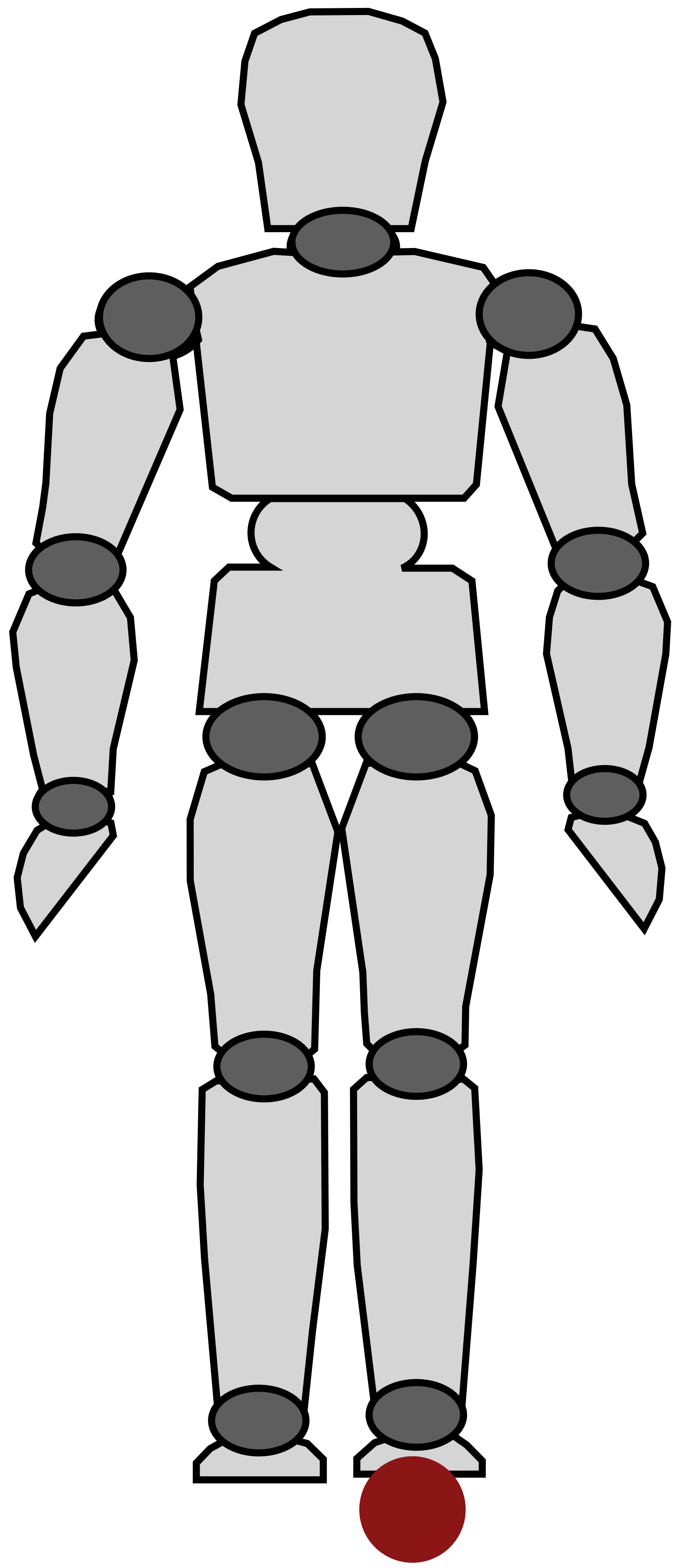


root: abdomen

Root location

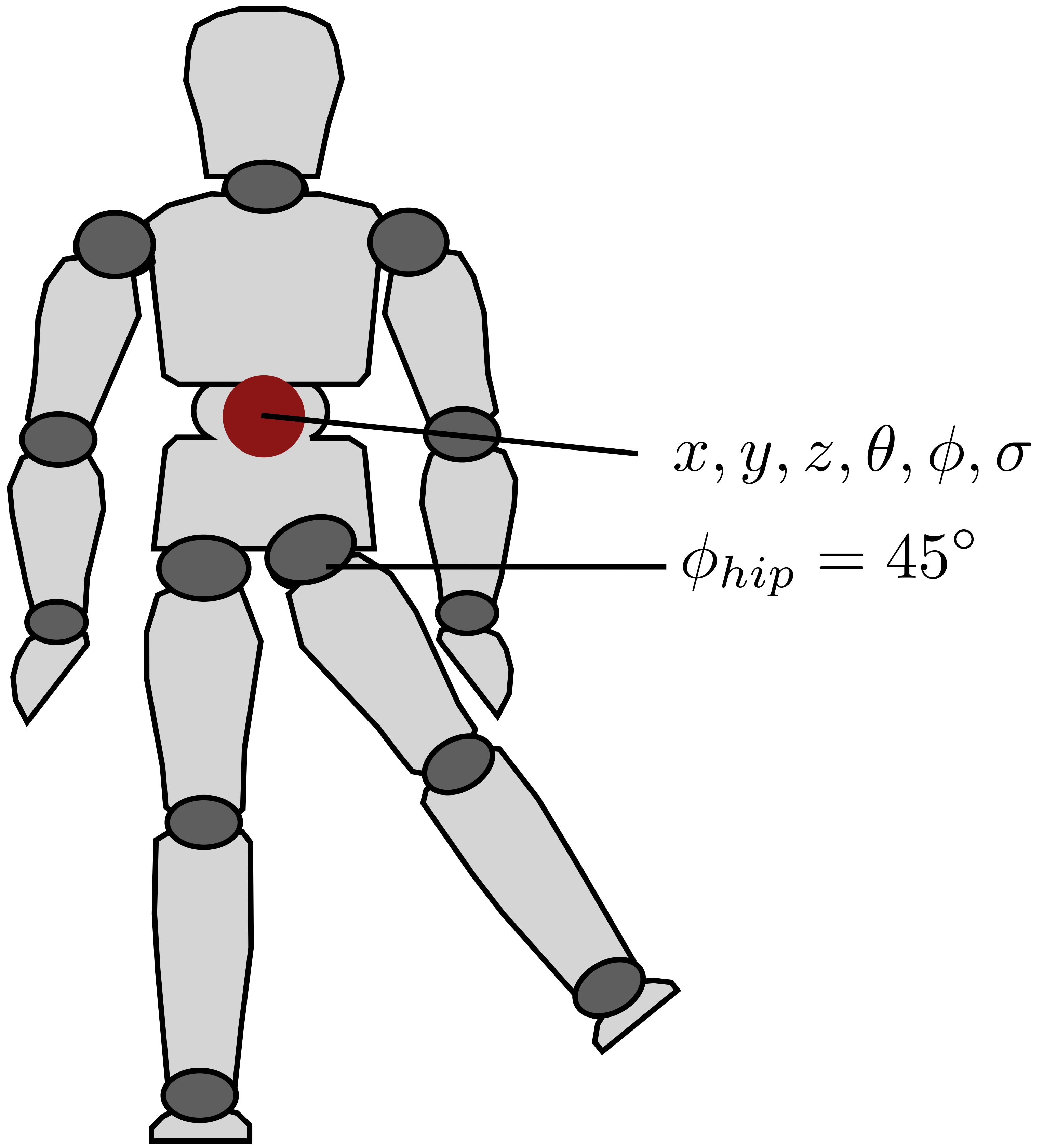


root: abdomen

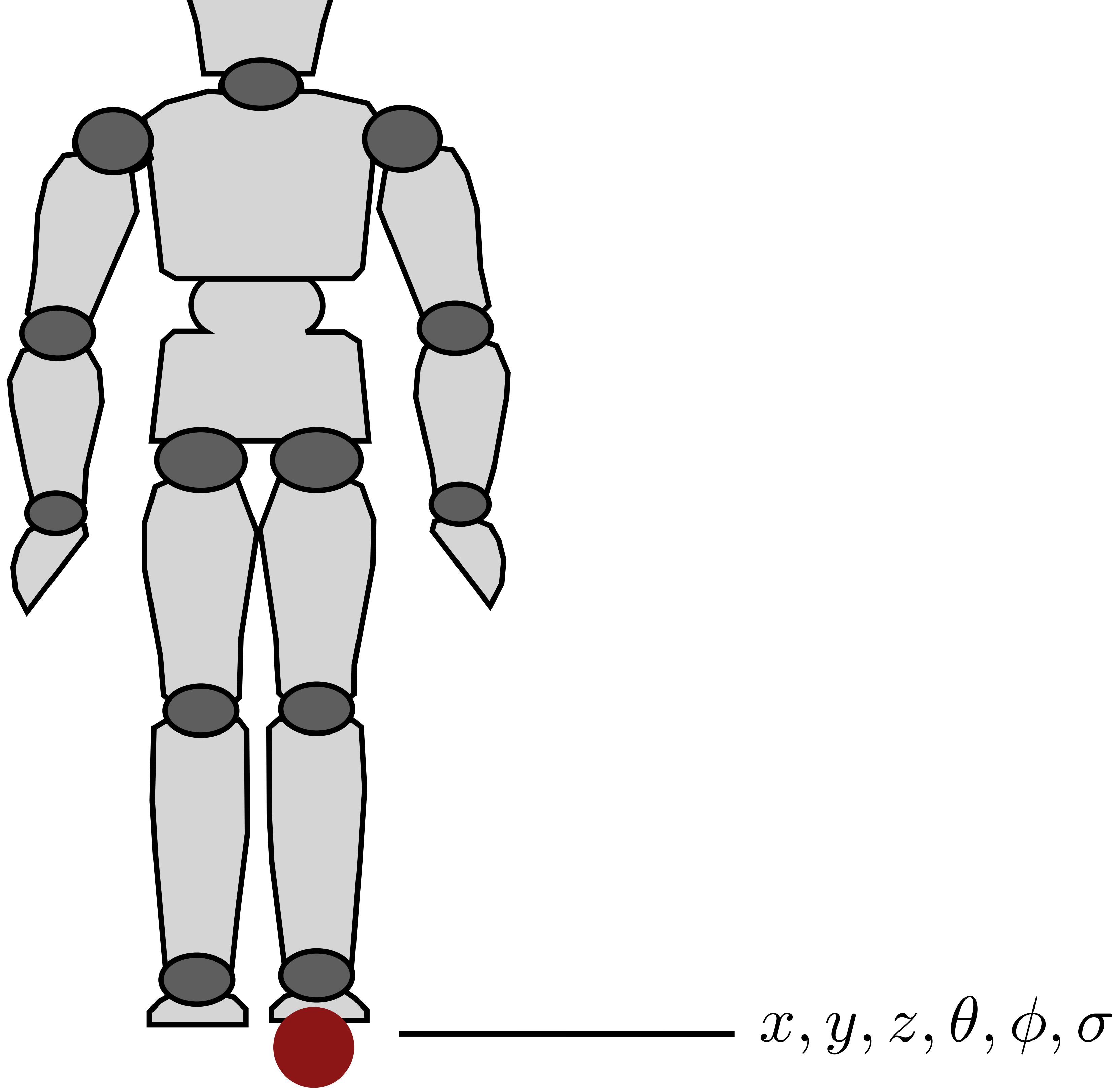


root: left foot

Root location

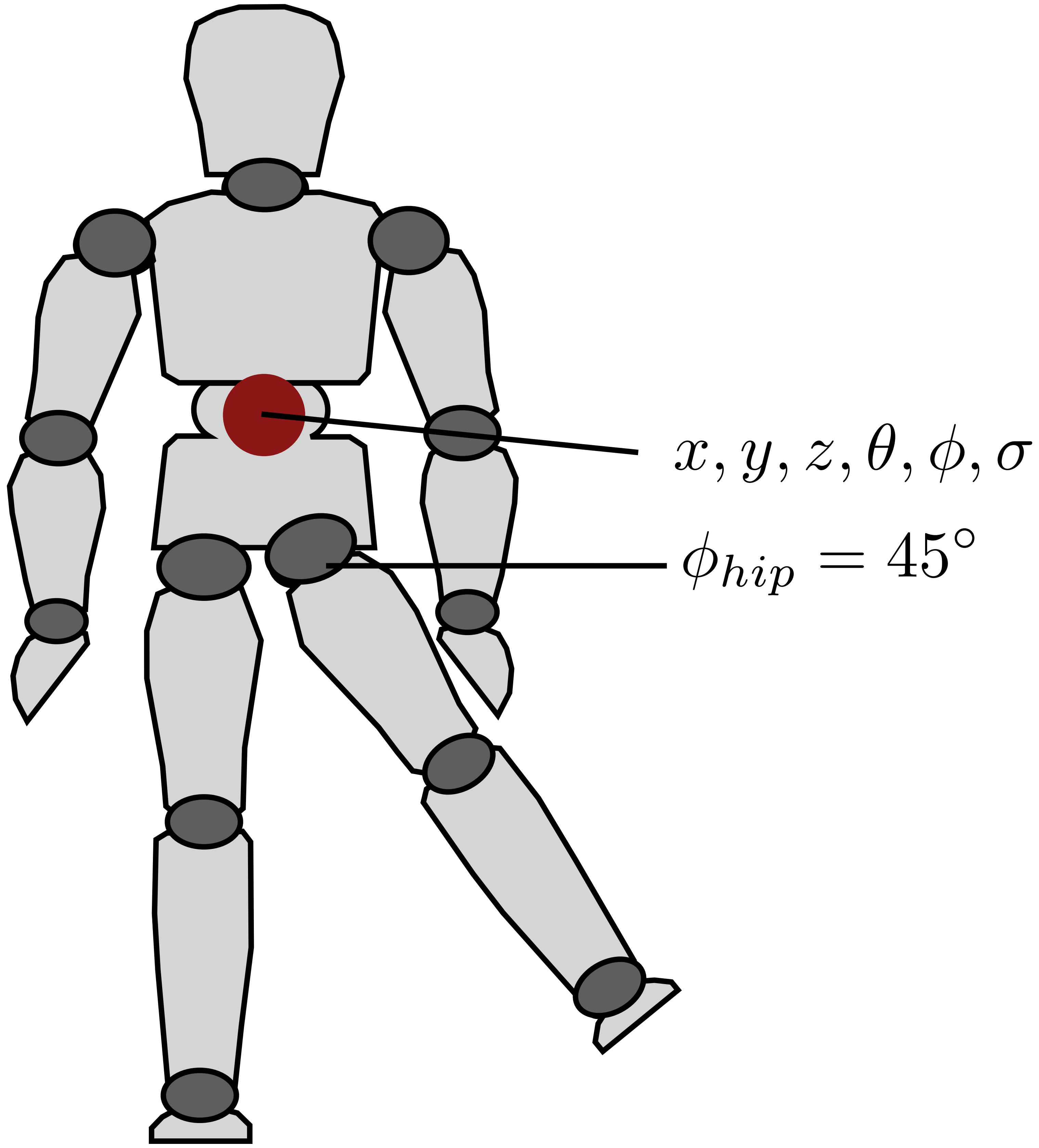


root: abdomen

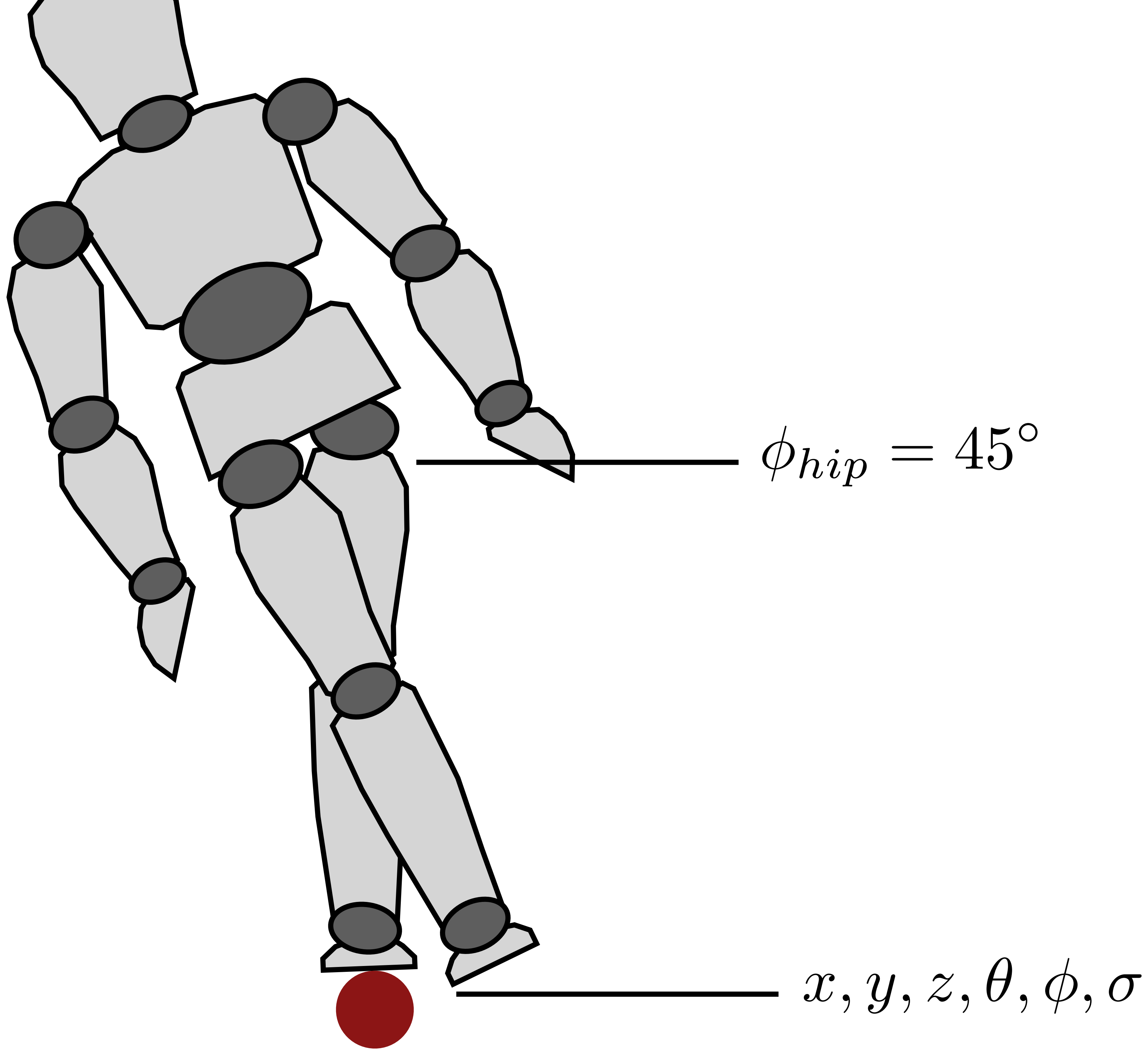


root: left foot

Root location

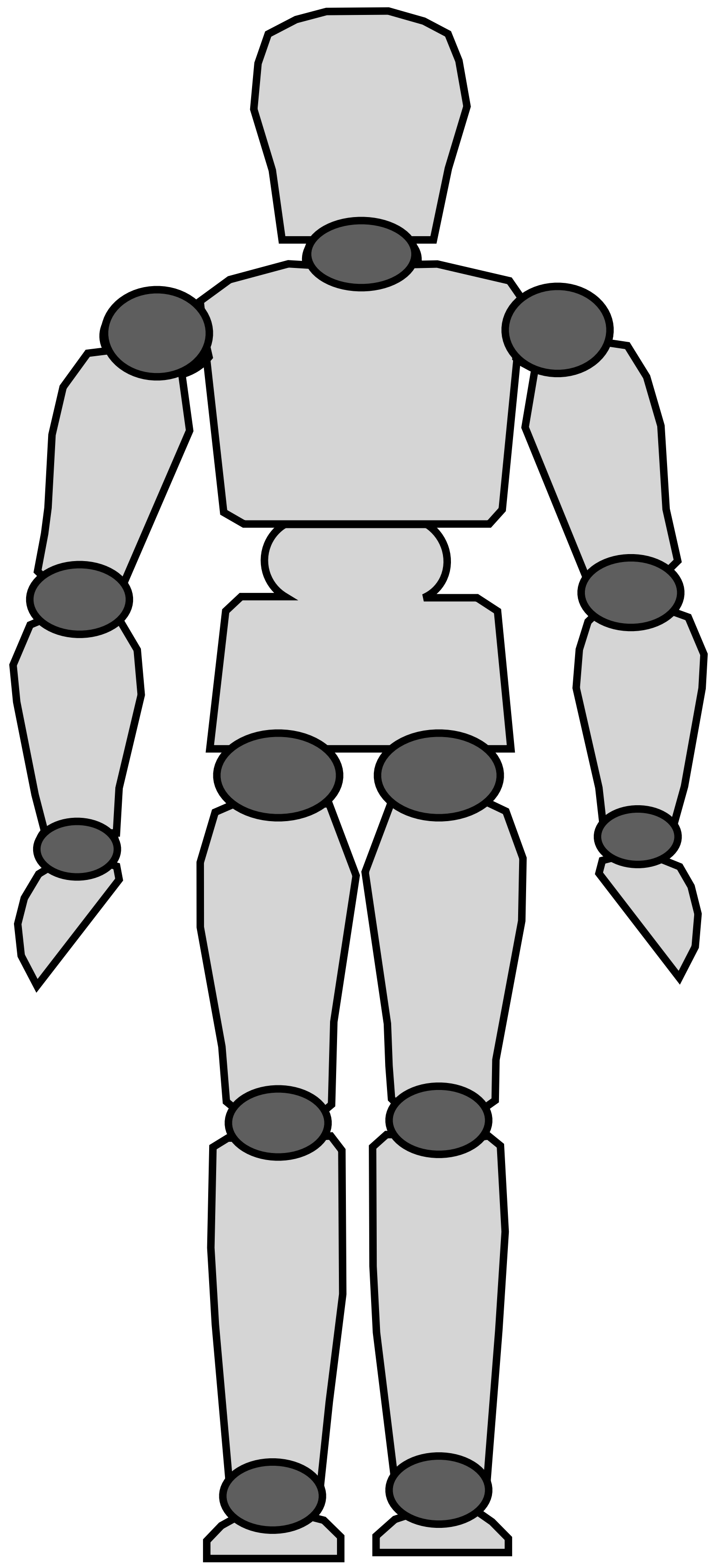


root: abdomen

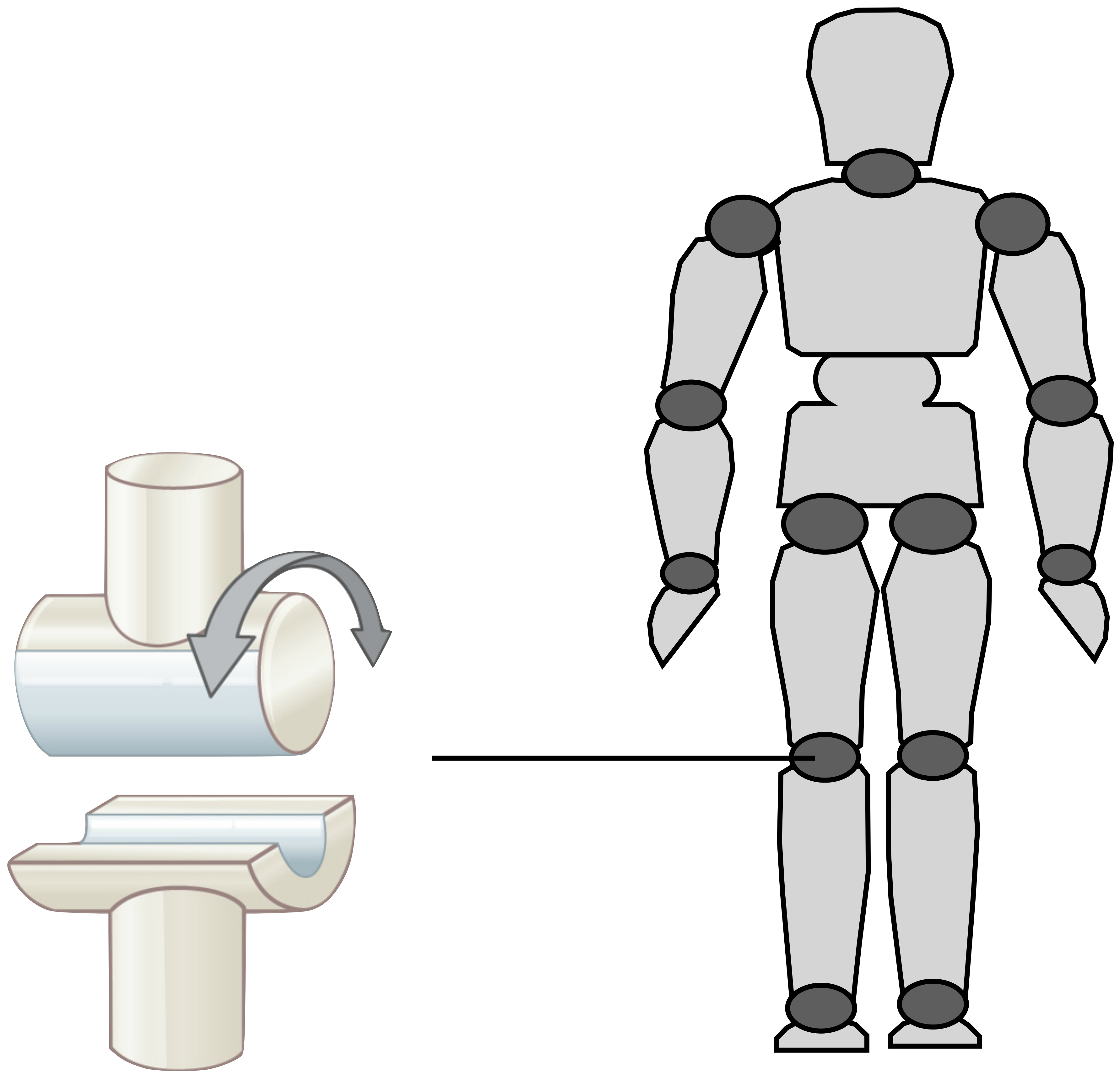


root: left foot

Parameterizing human model

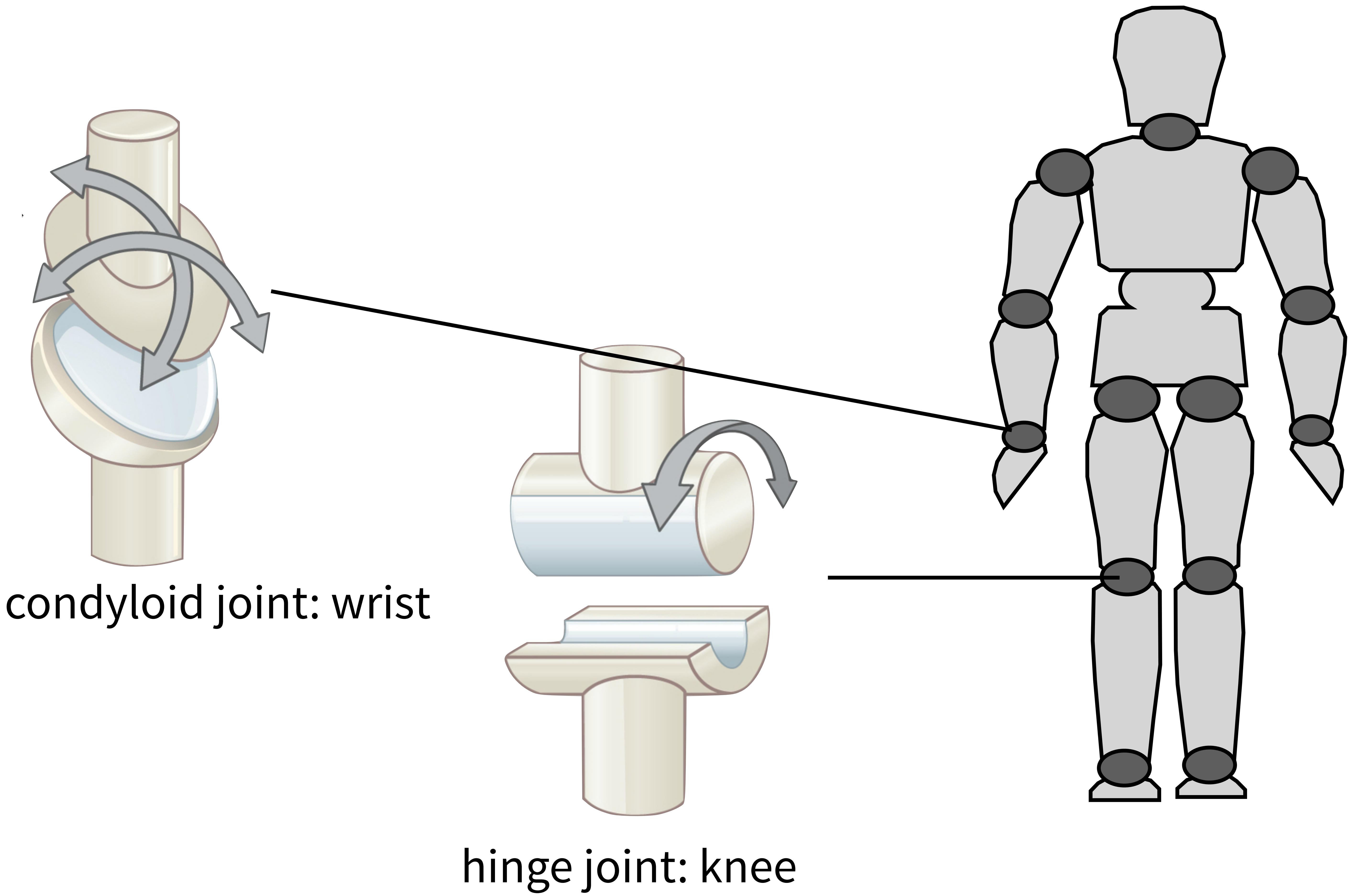


Parameterizing human model

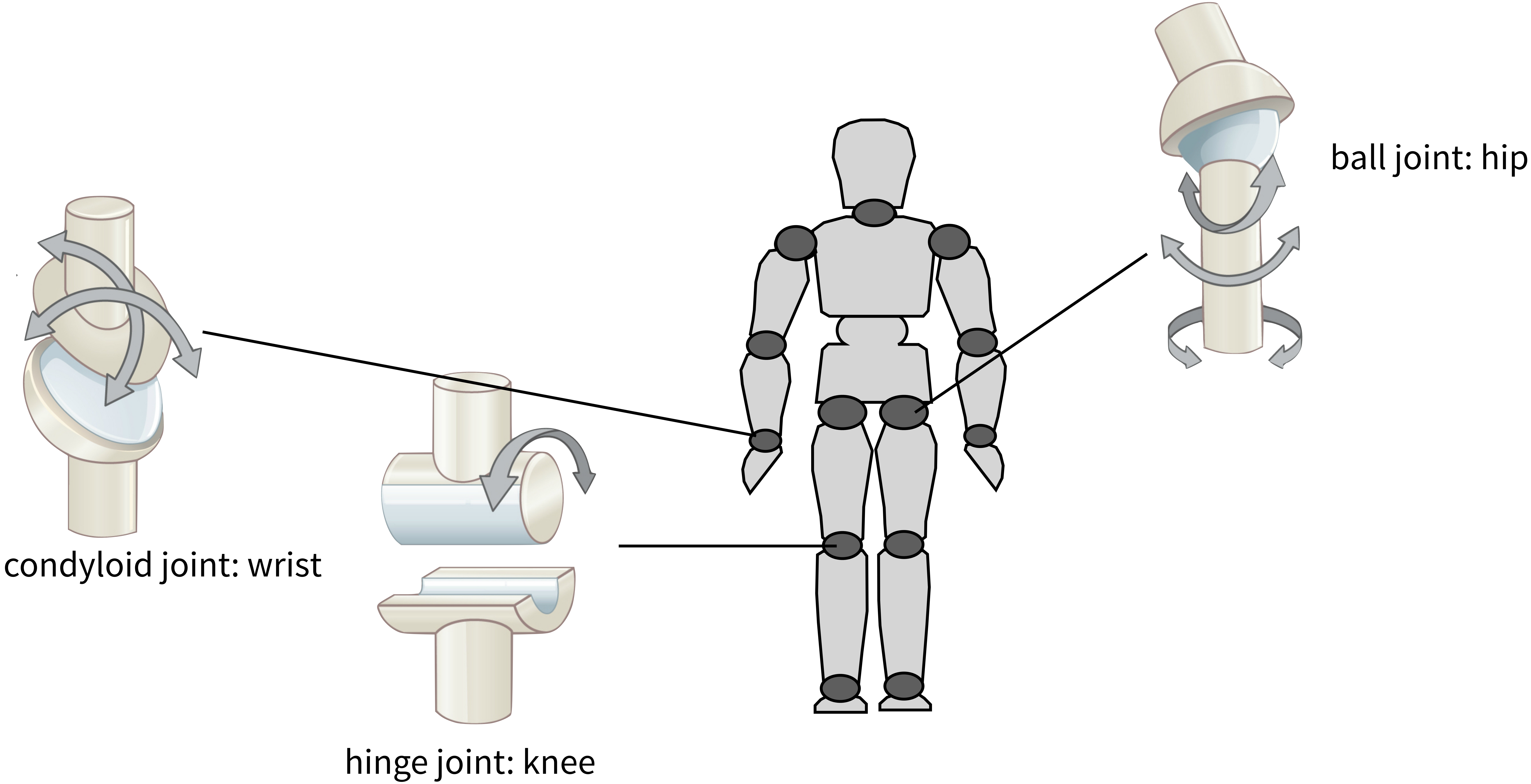


hinge joint: knee

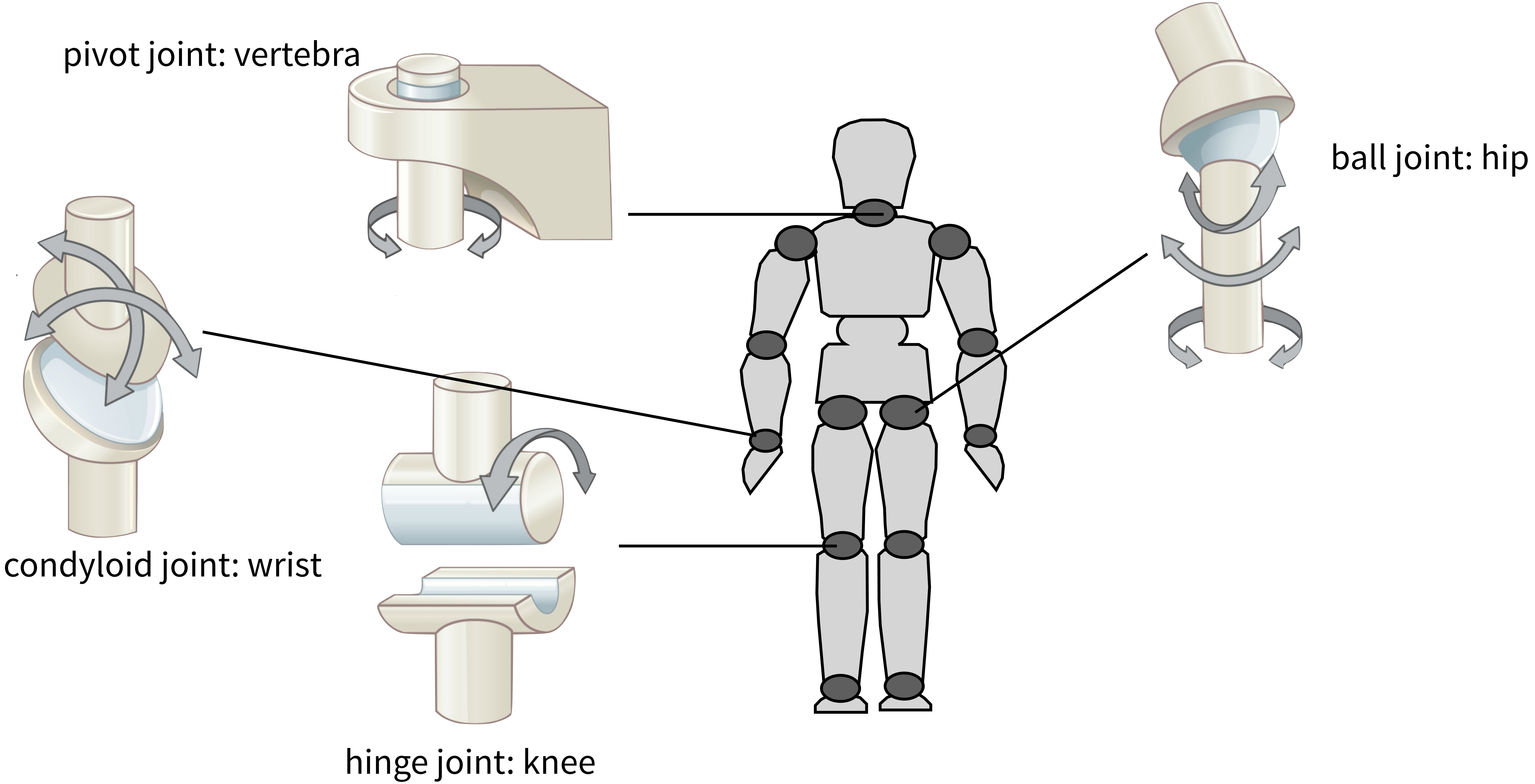
Parameterizing human model



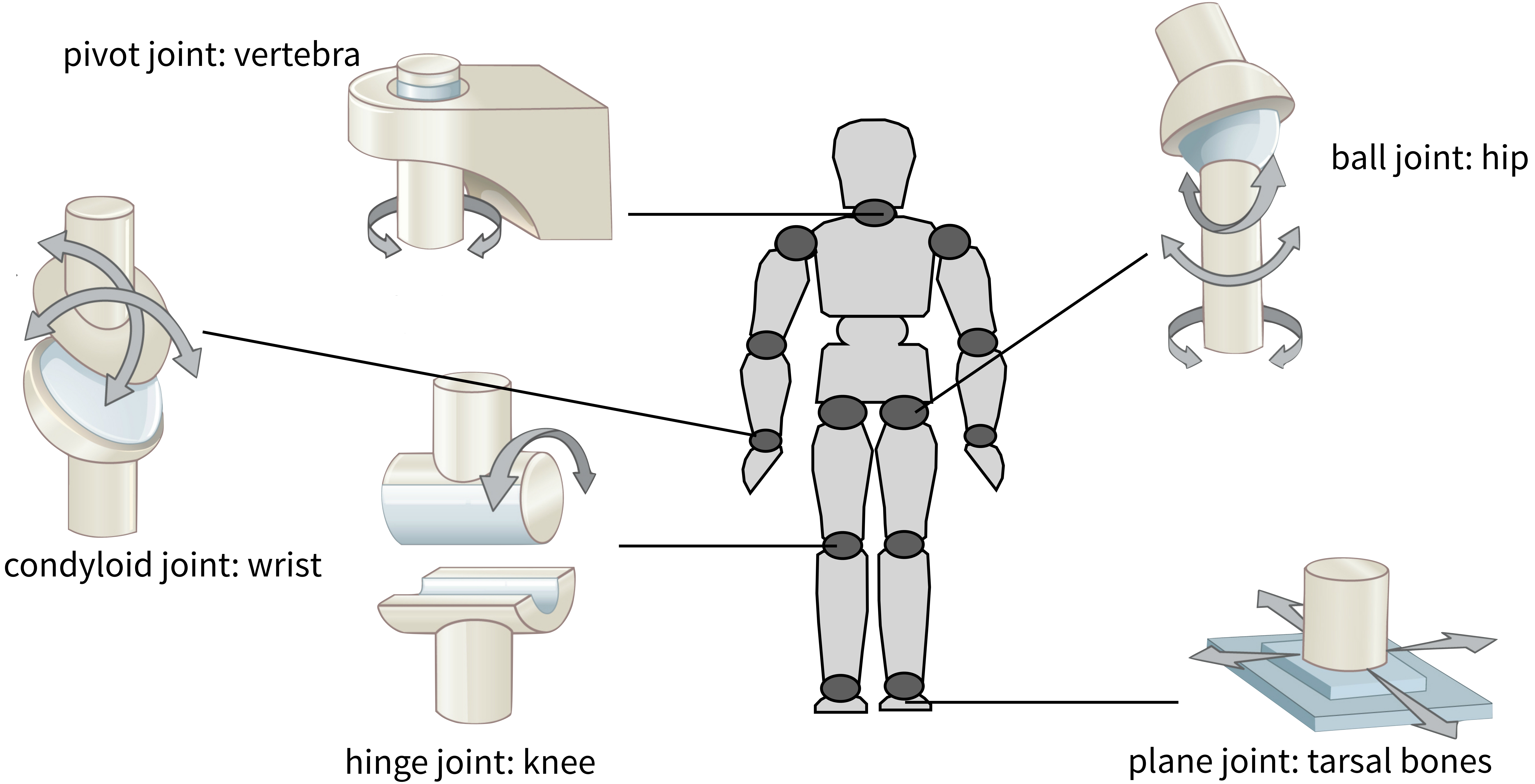
Parameterizing human model



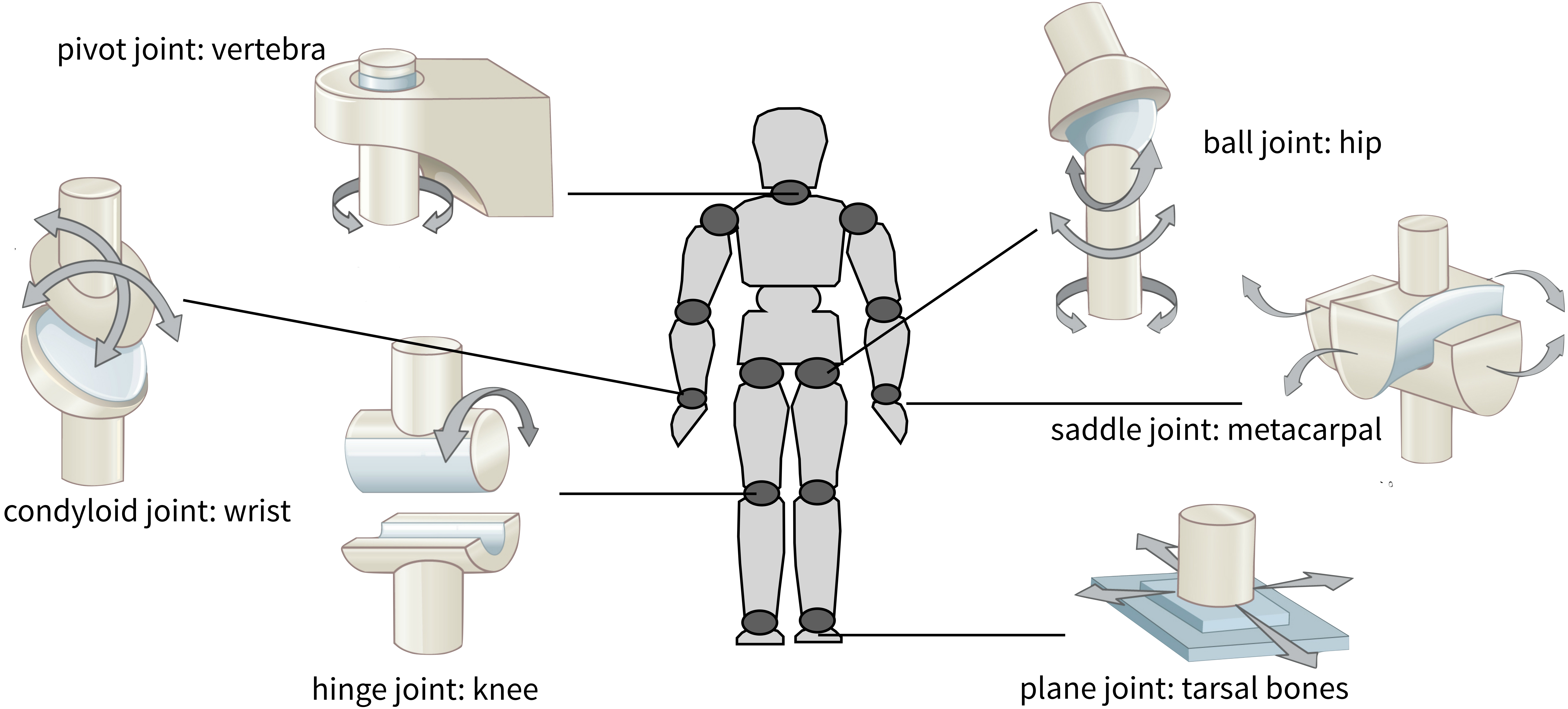
Parameterizing human model



Parameterizing human model



Parameterizing human model



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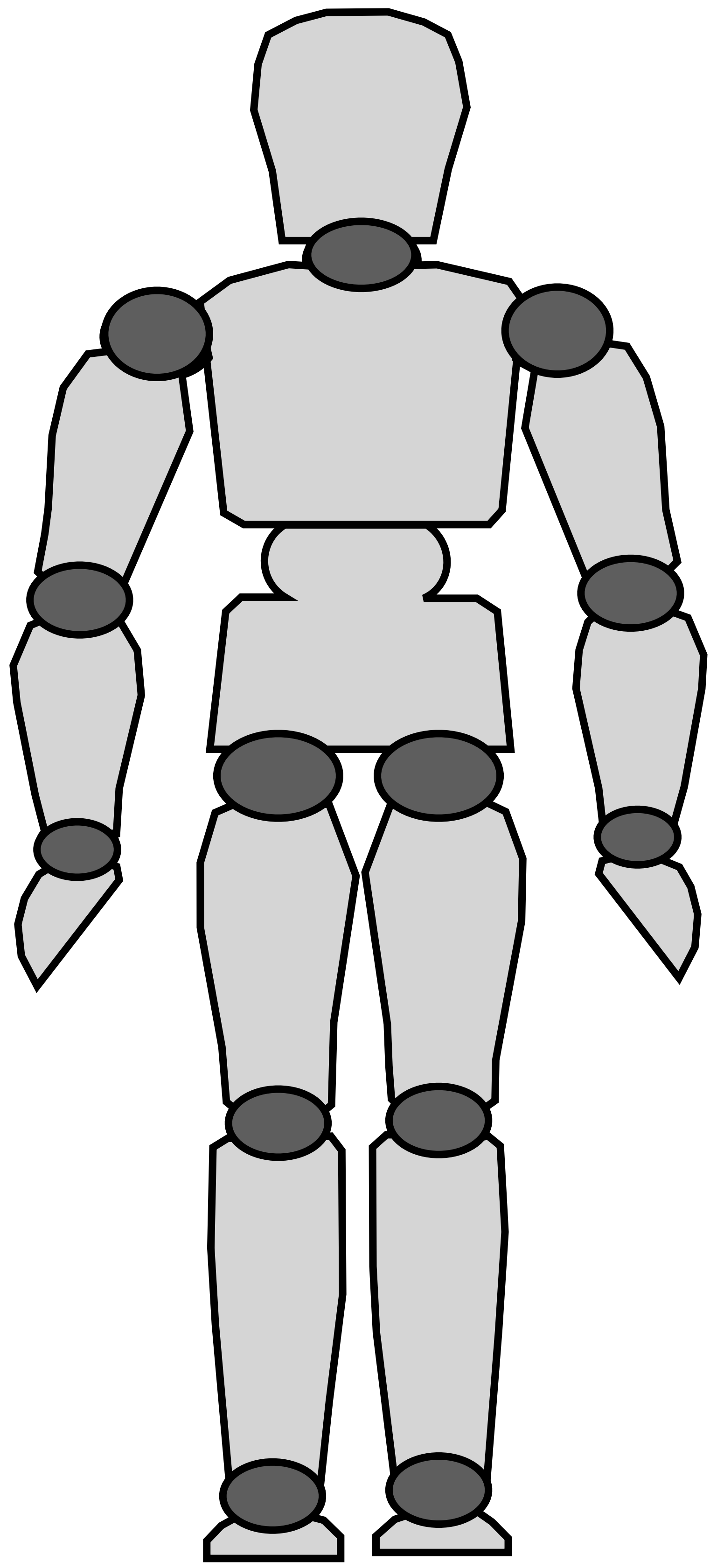
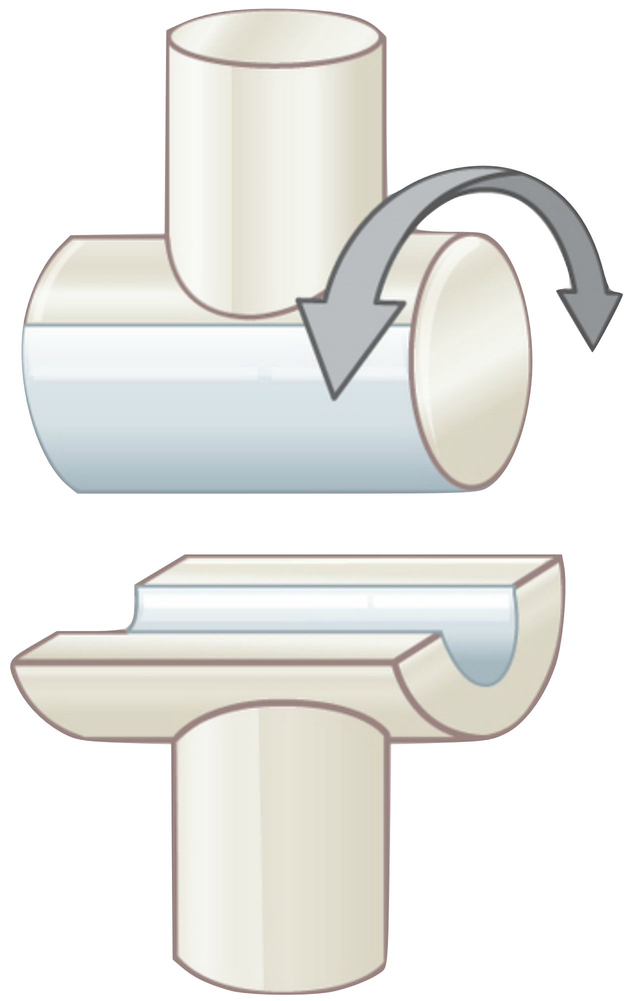
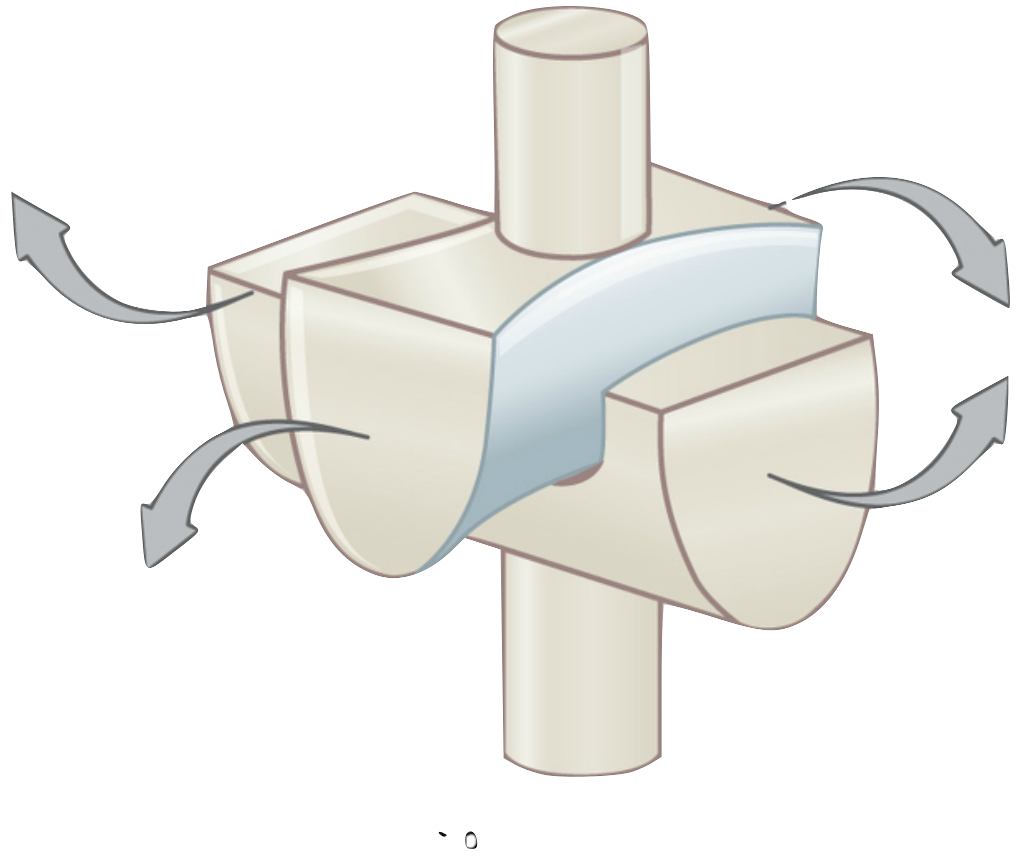
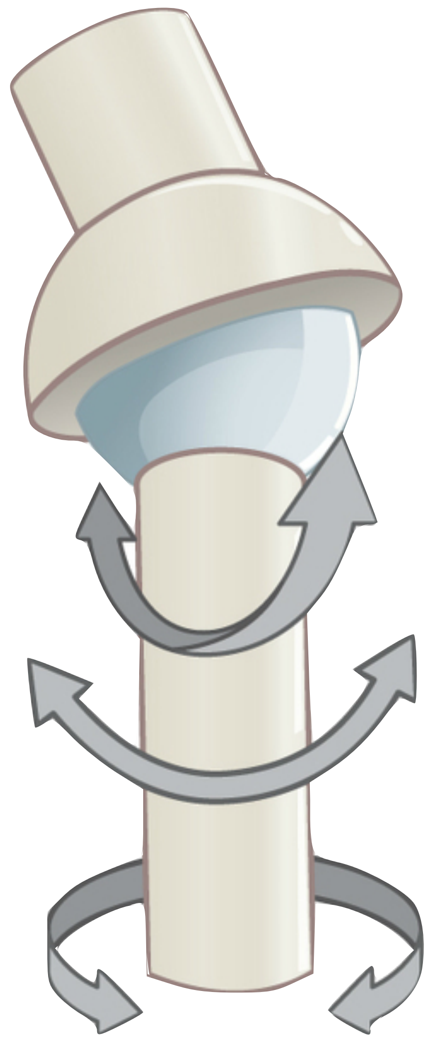




1-DOF joint

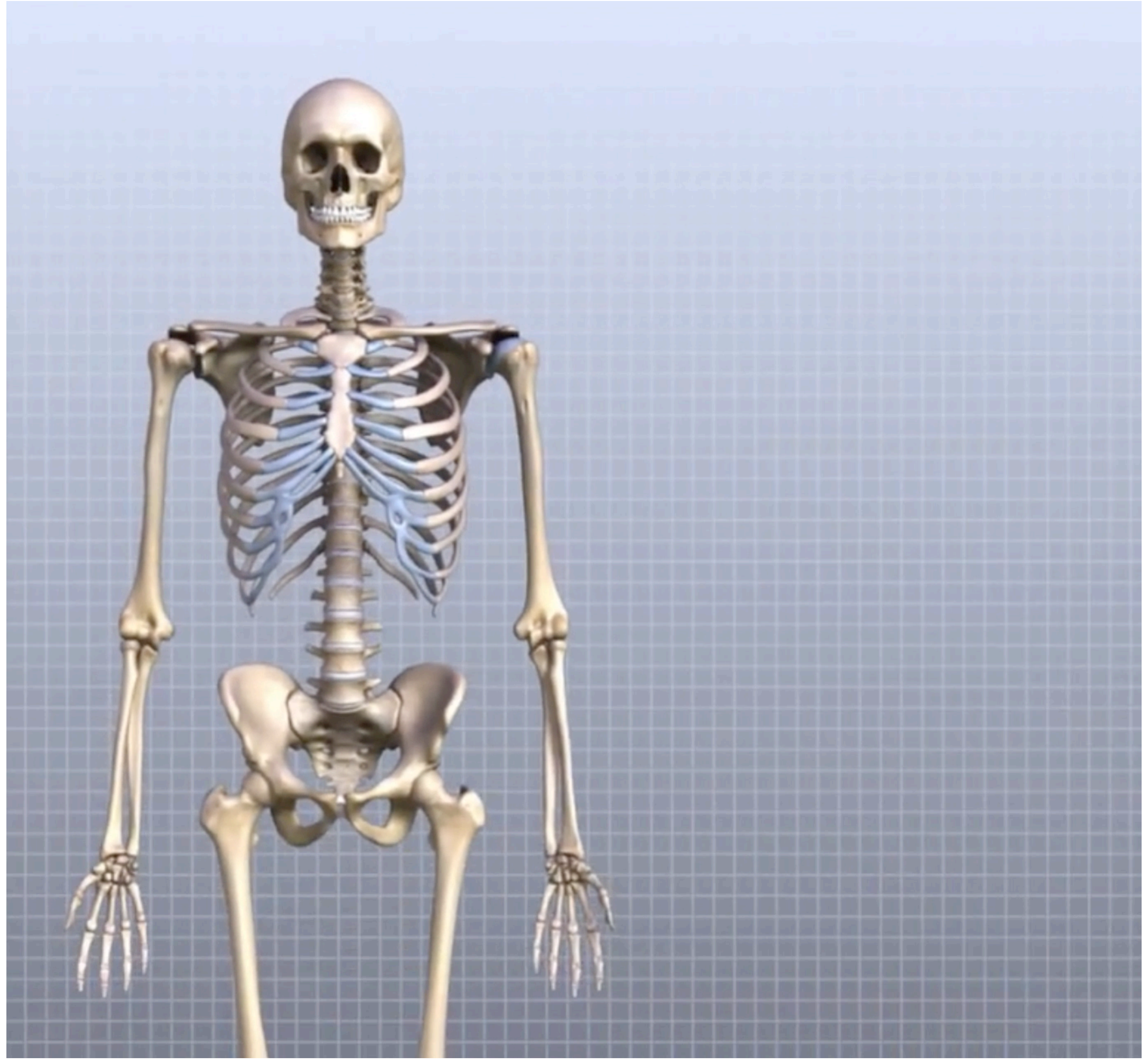


2-DOF joint



3-DOF joint

Case study: How to model human shoulder?



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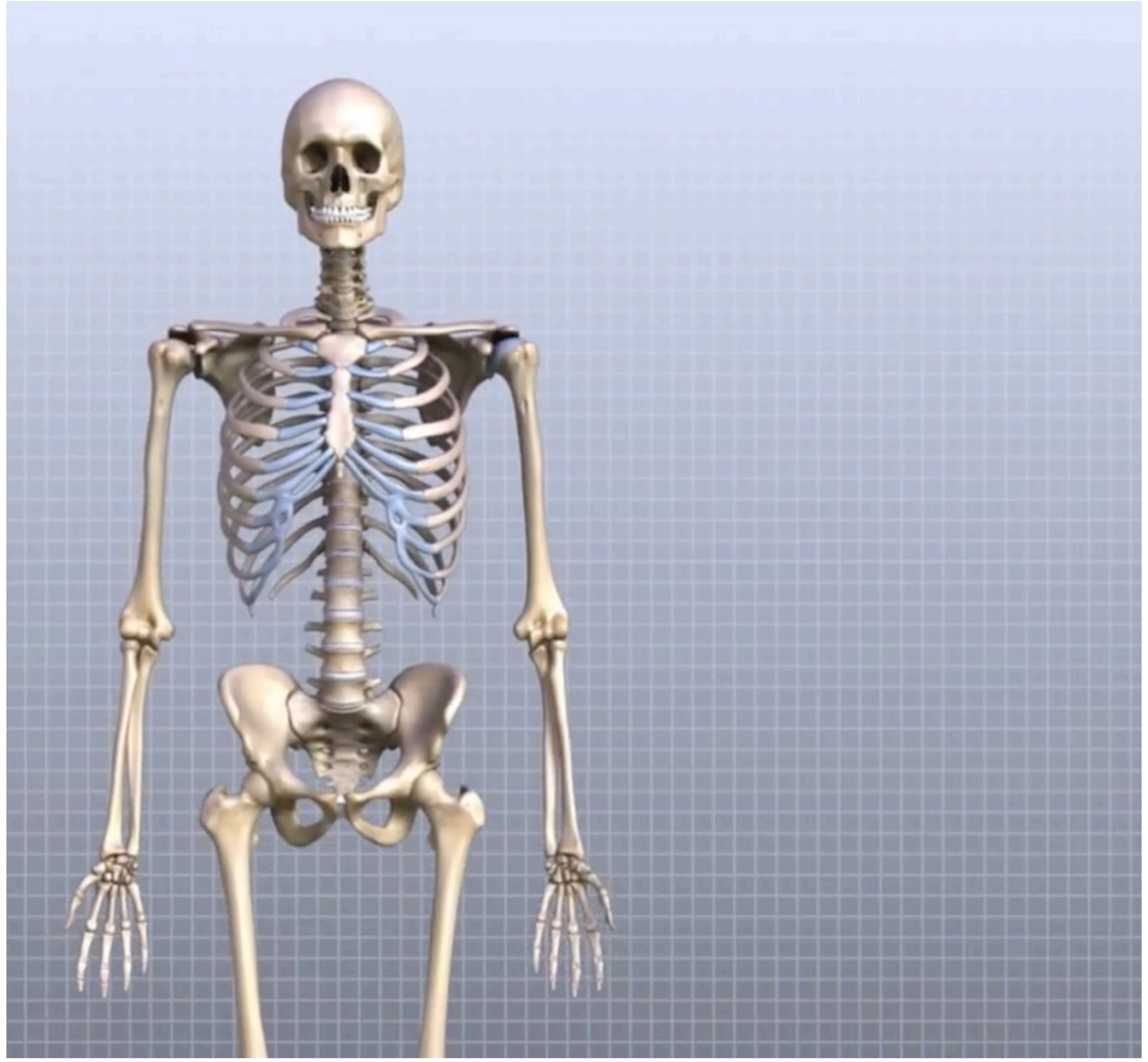

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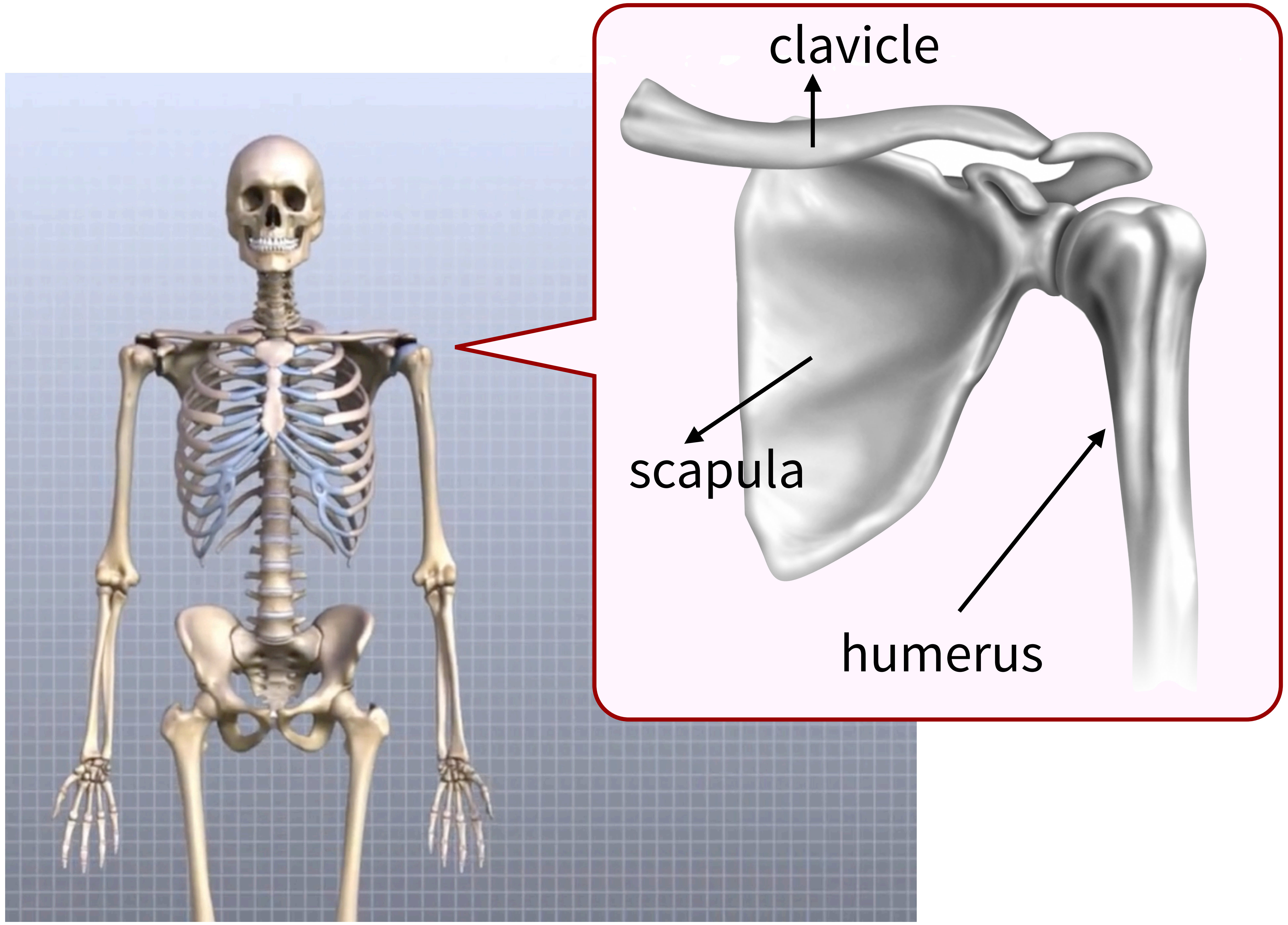

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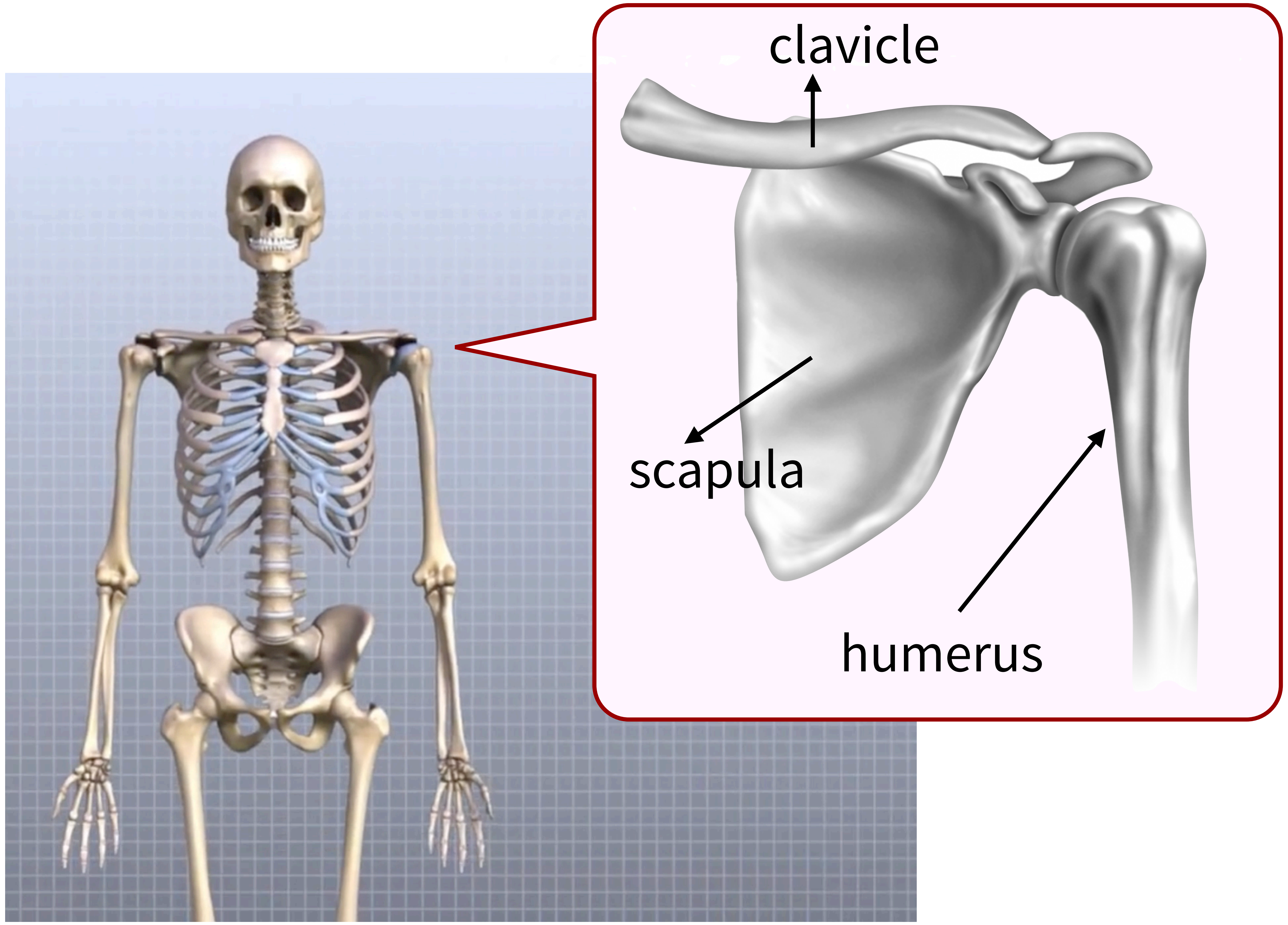
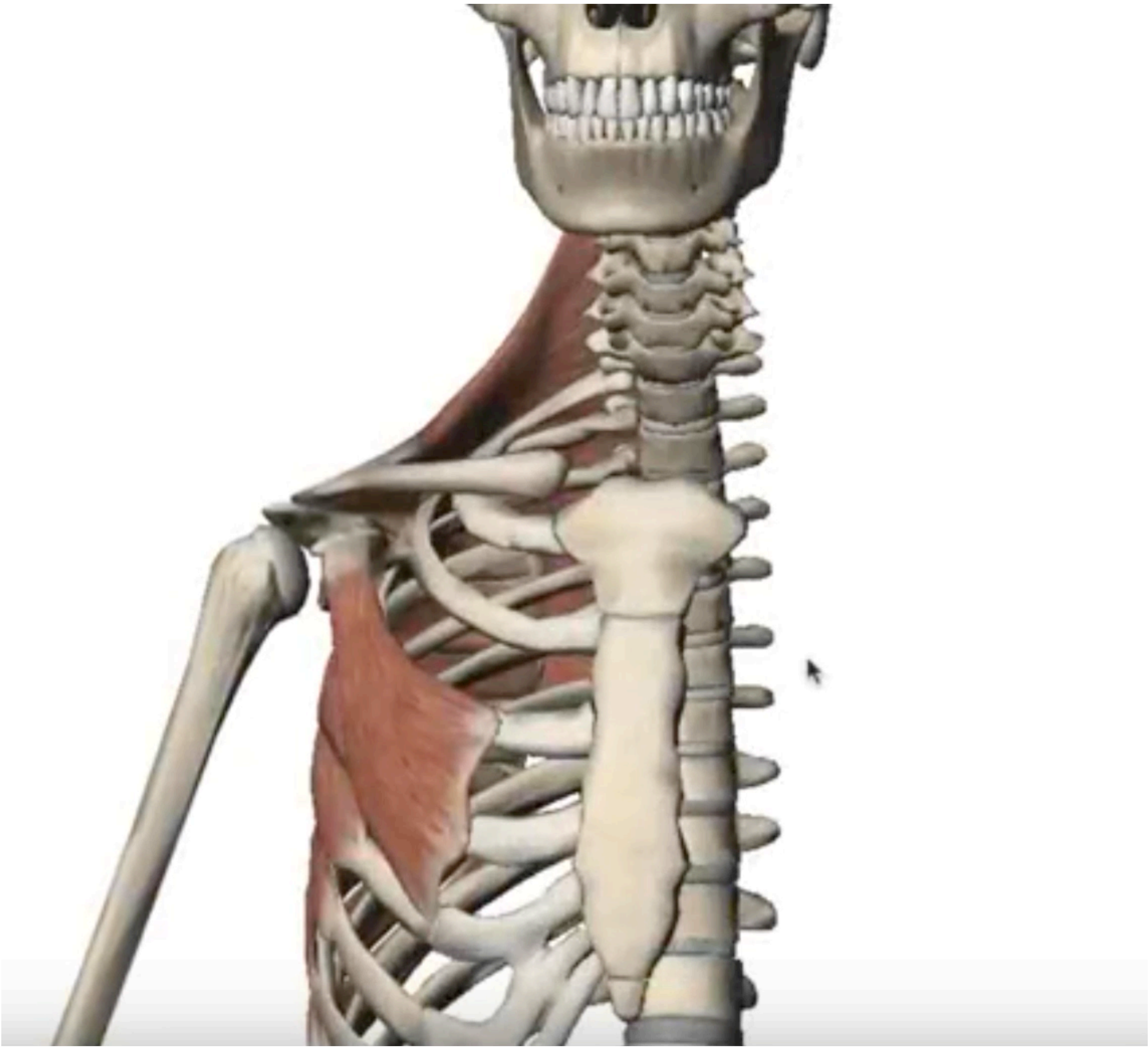
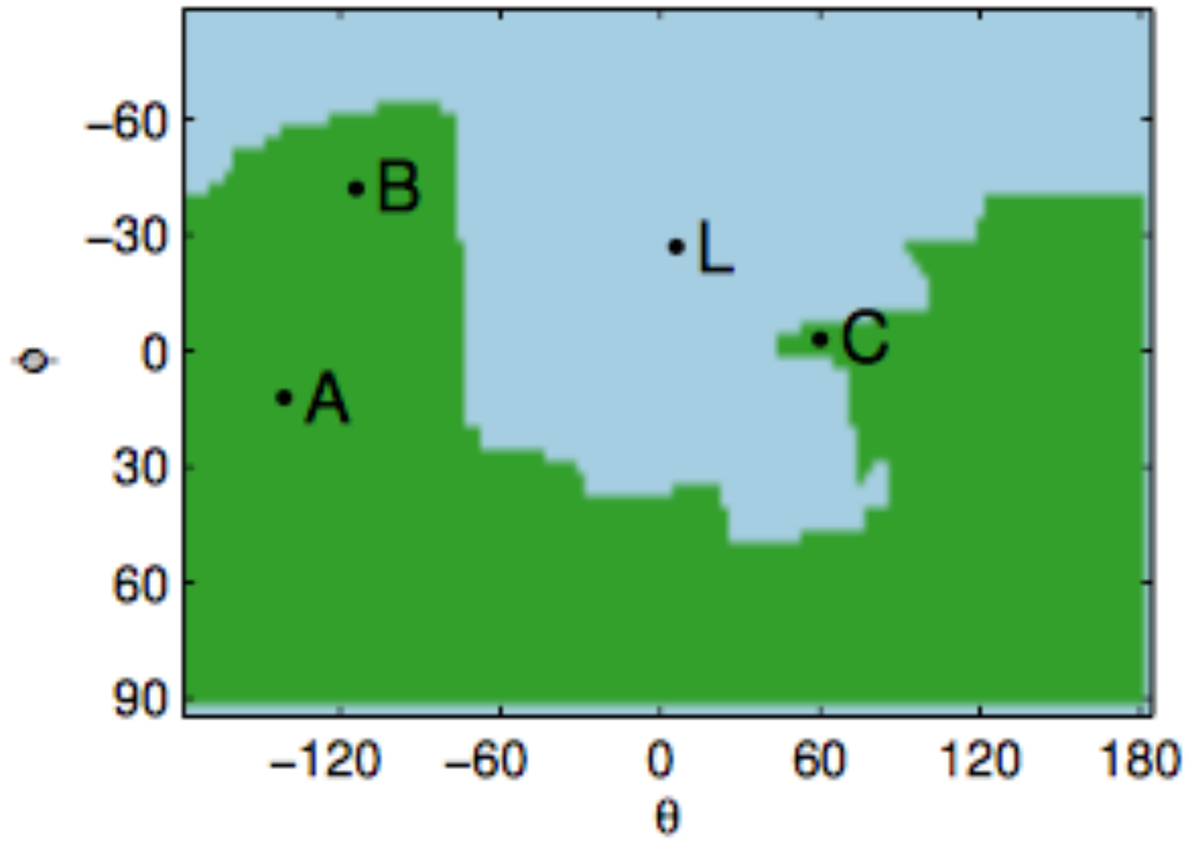




Joint limits

Valid region of shoulder configurations



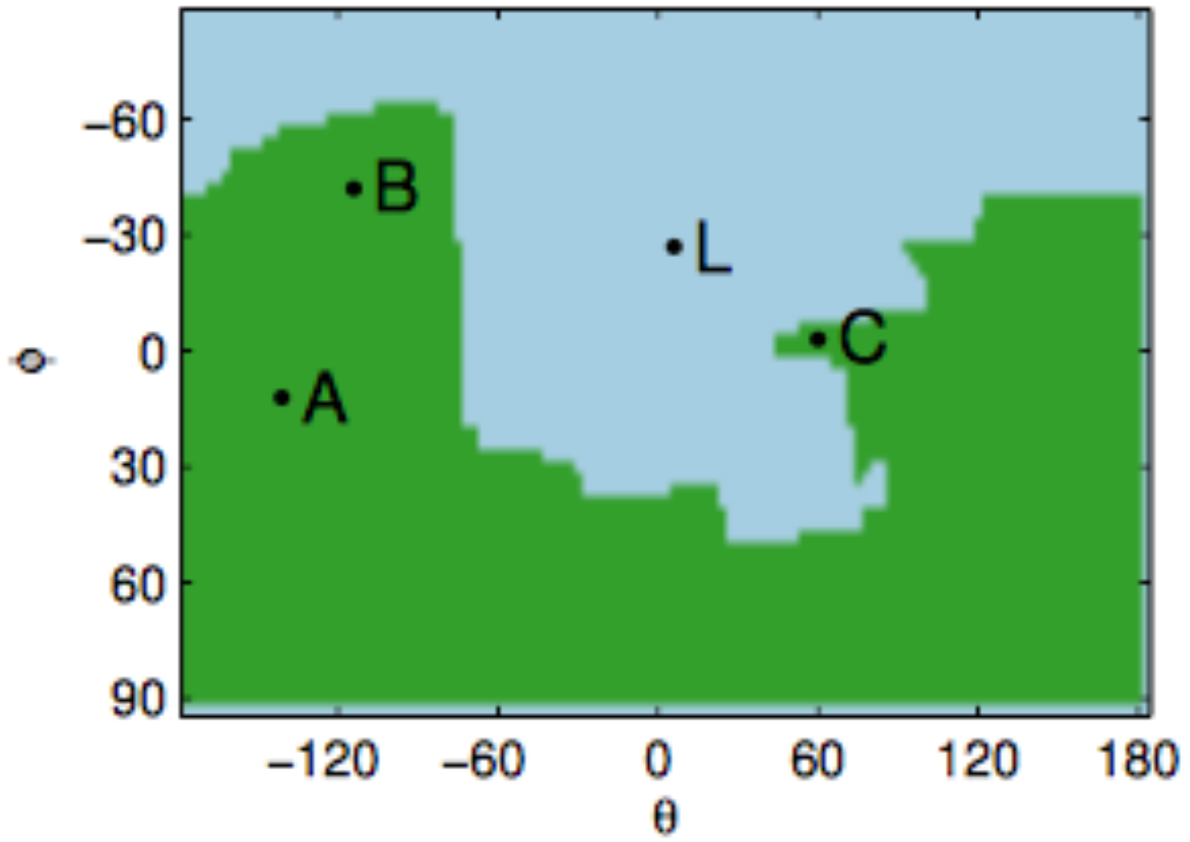
recorded data

typically modeled

Image from Akhter and Black CVPR 2015

Joint limits

Valid region of shoulder configurations



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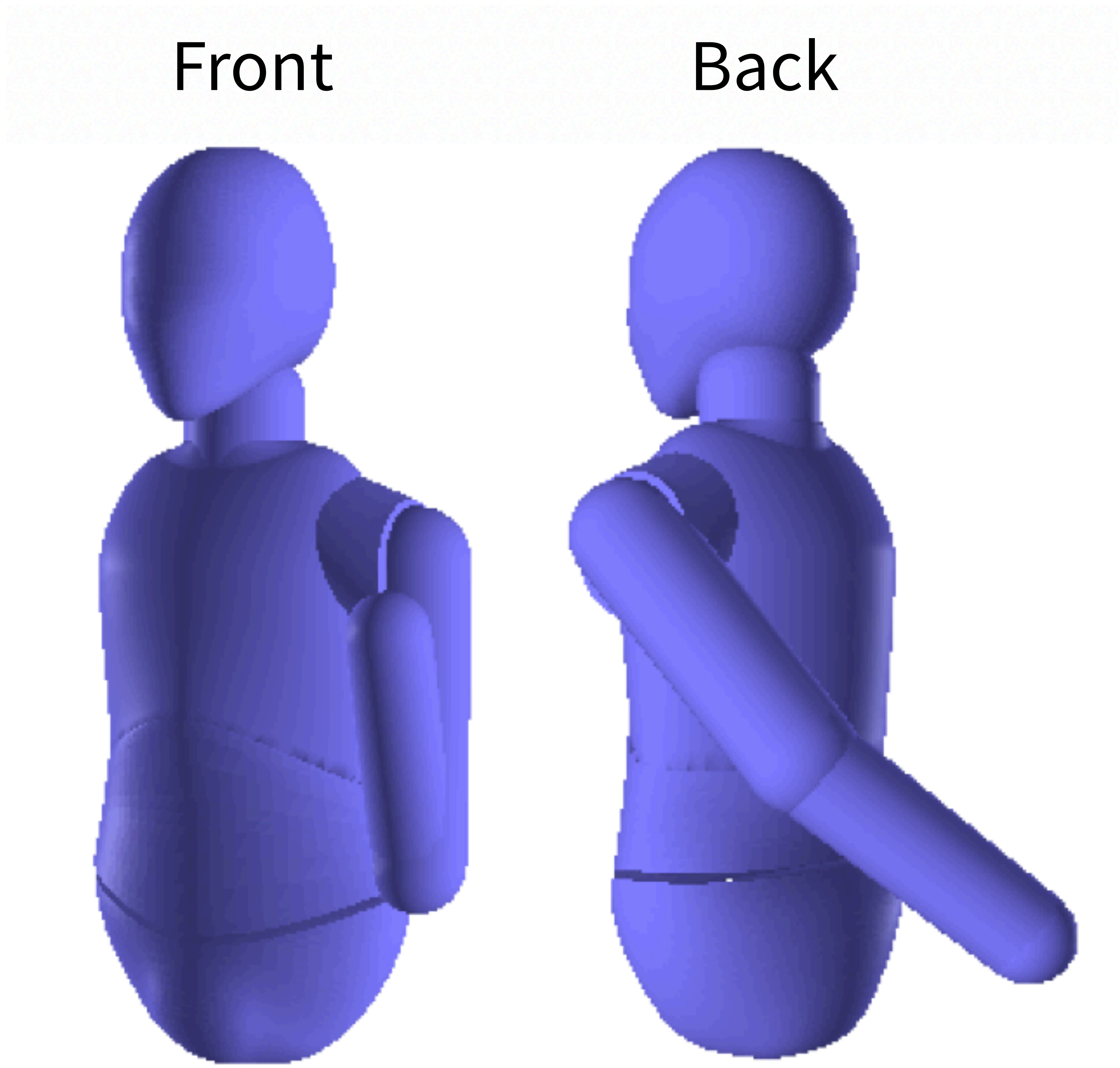
Observation #1

Human joint limit is very complex

Image from Akhter and Black CVPR 2015

Joint limits



<u>Observation #1</u>

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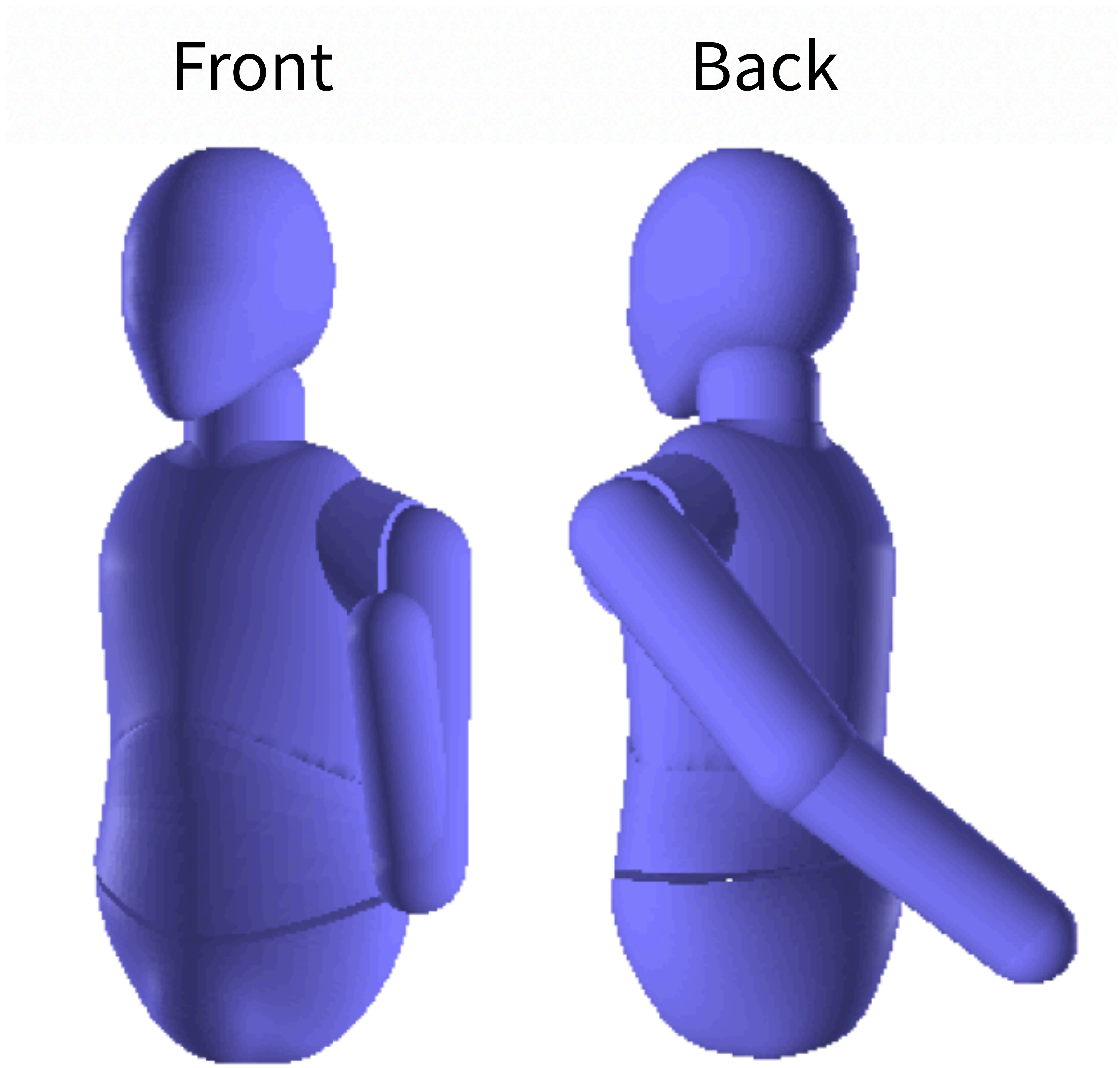
Joint limits



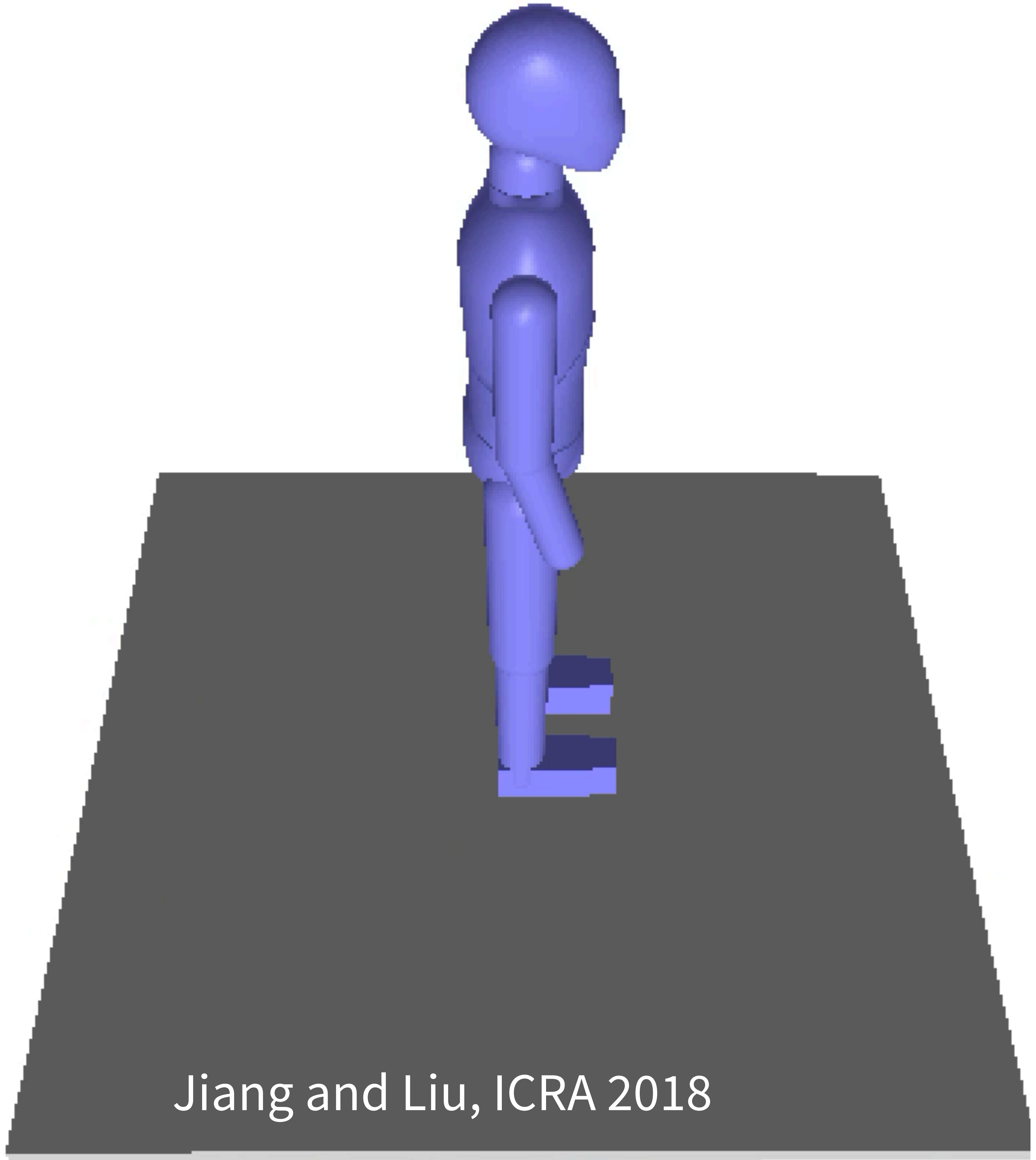
<u>Observation #1</u>

Human joint limit is very complex

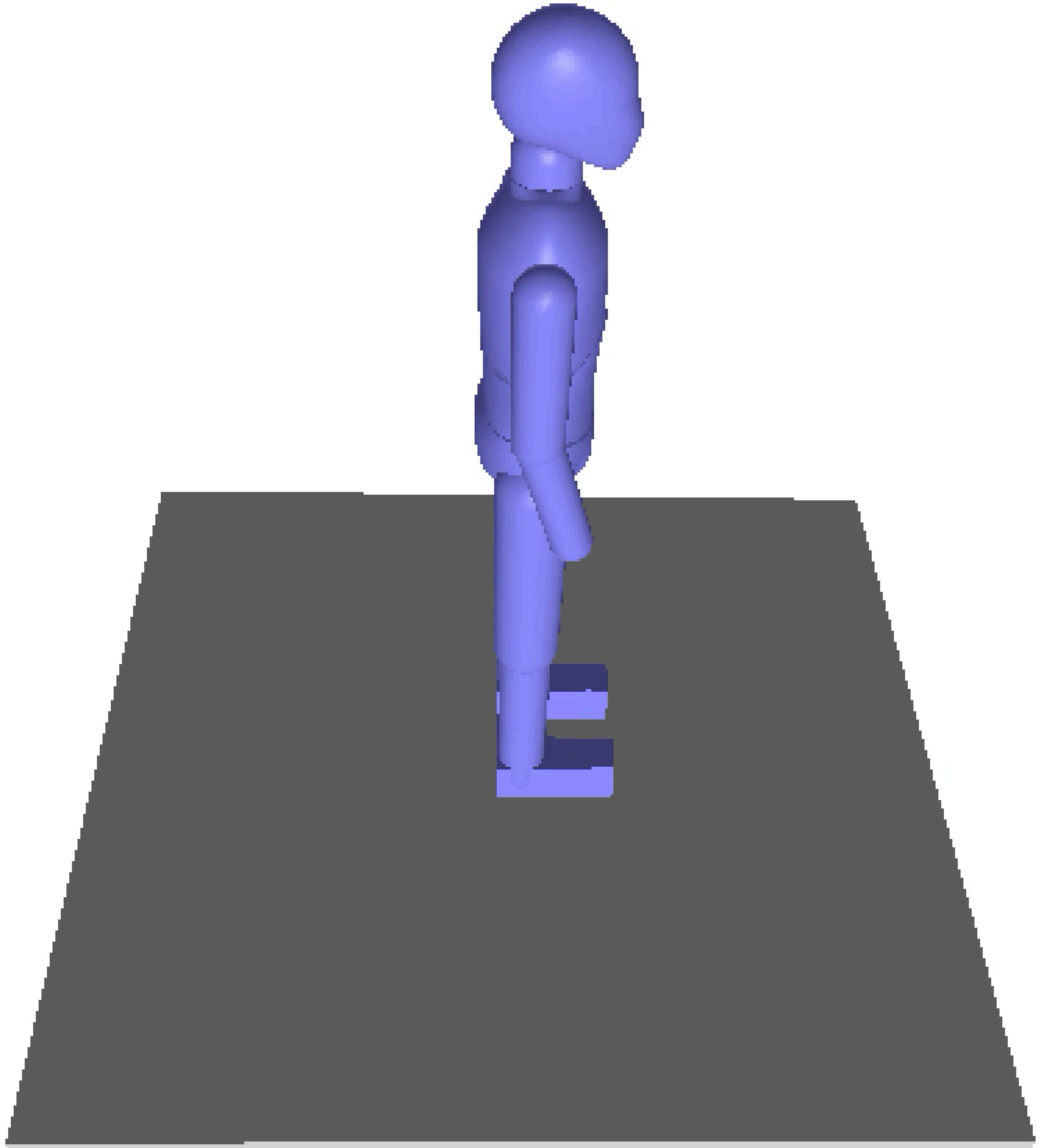
<u>Observation #2</u>

Human joint limit is pose-dependent

Learned joint limit $D(\boldsymbol{q})$



Box Constraints



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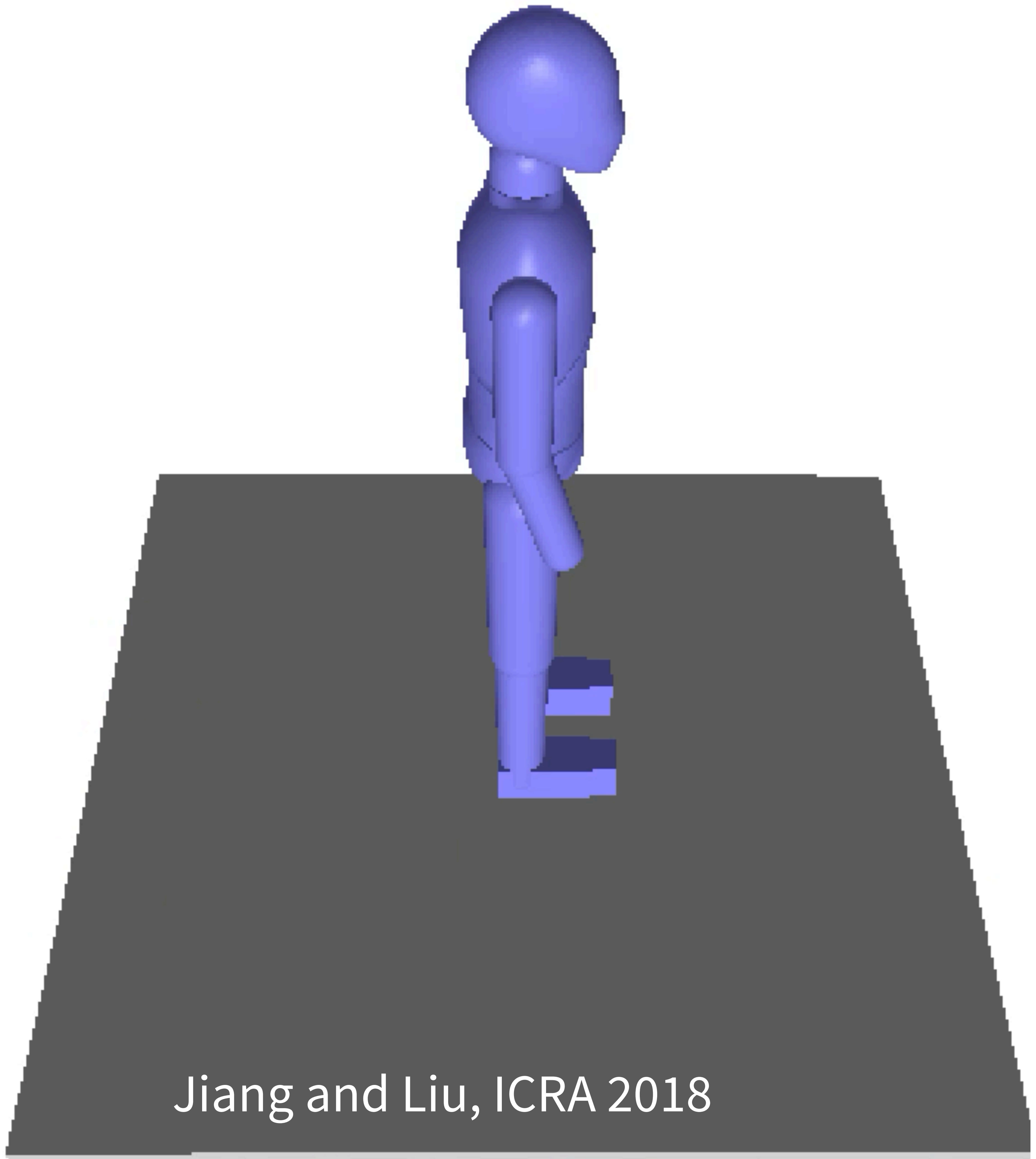
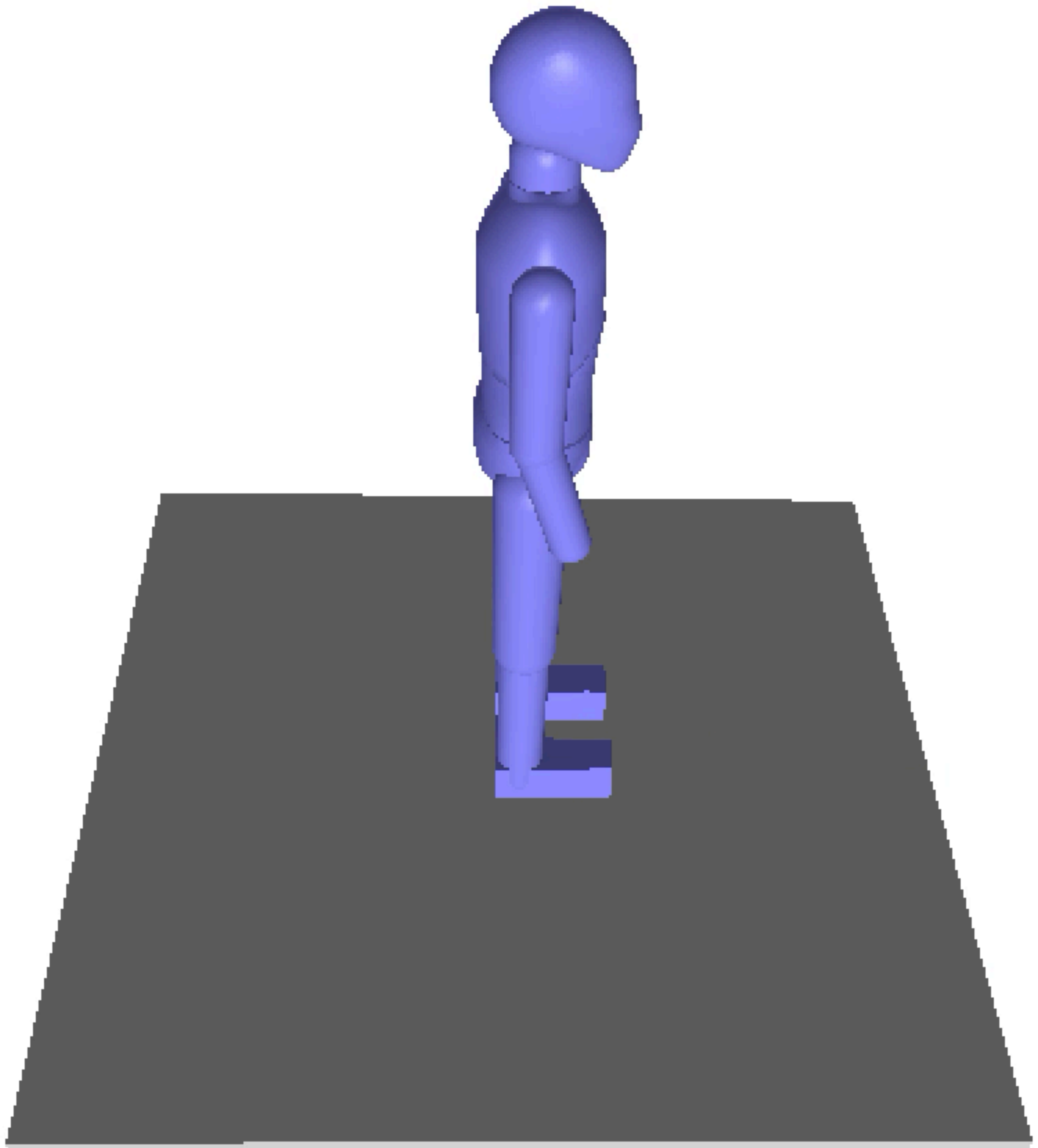


Box Constraints



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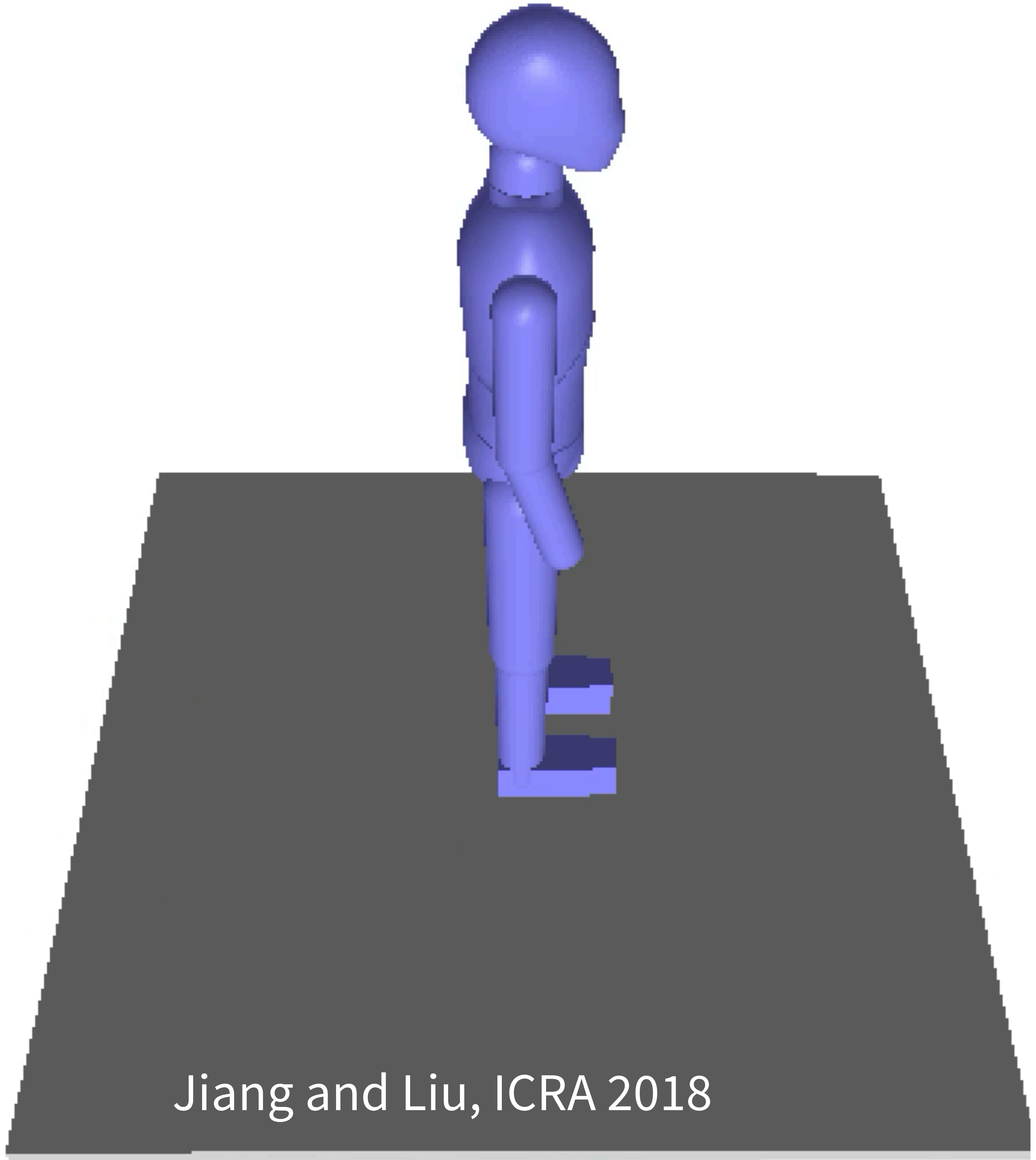

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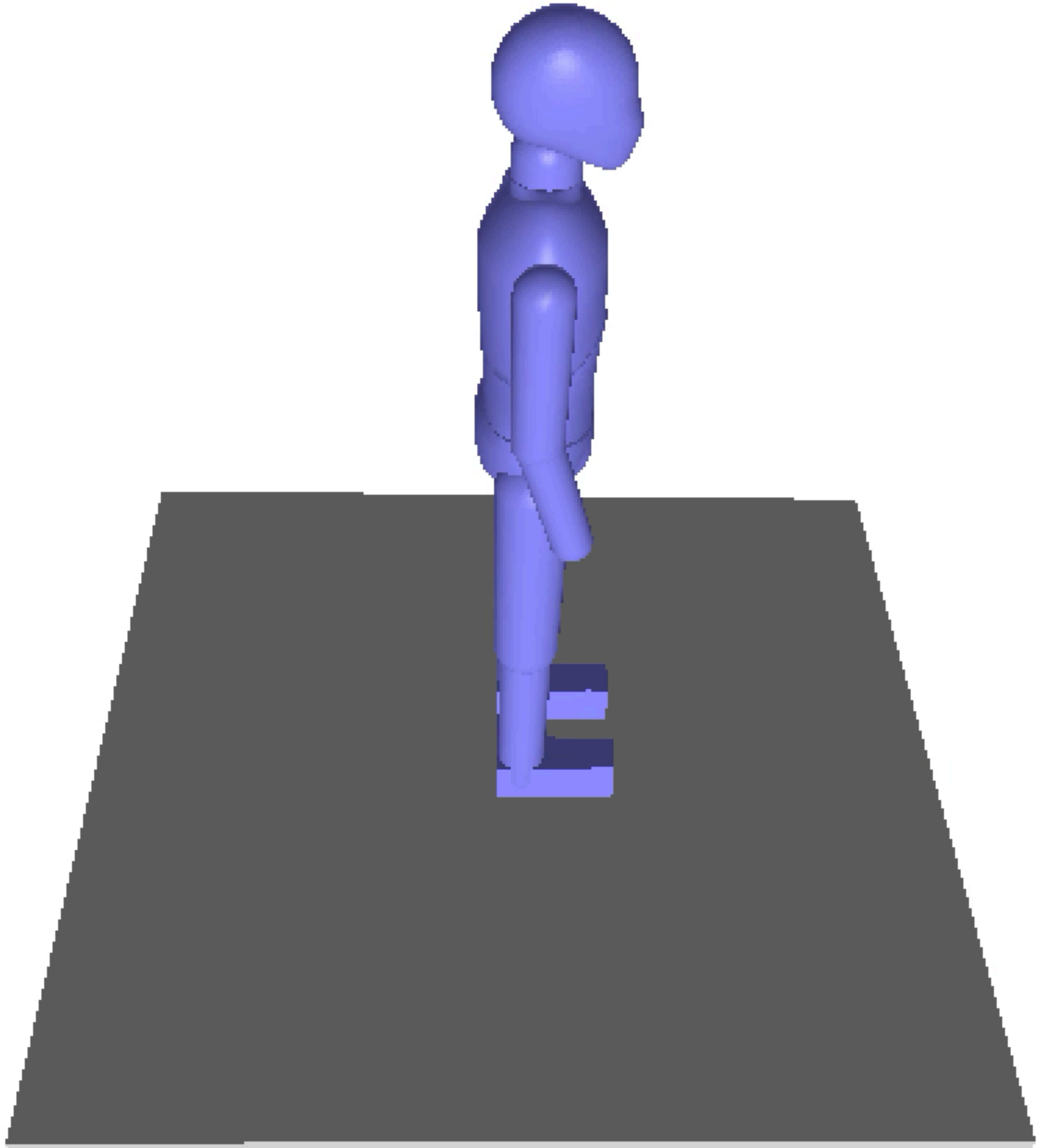

Limitations

- Directly controlling joints is not always what we want (e.g. hard to avoid foot- skating when animating human gait).

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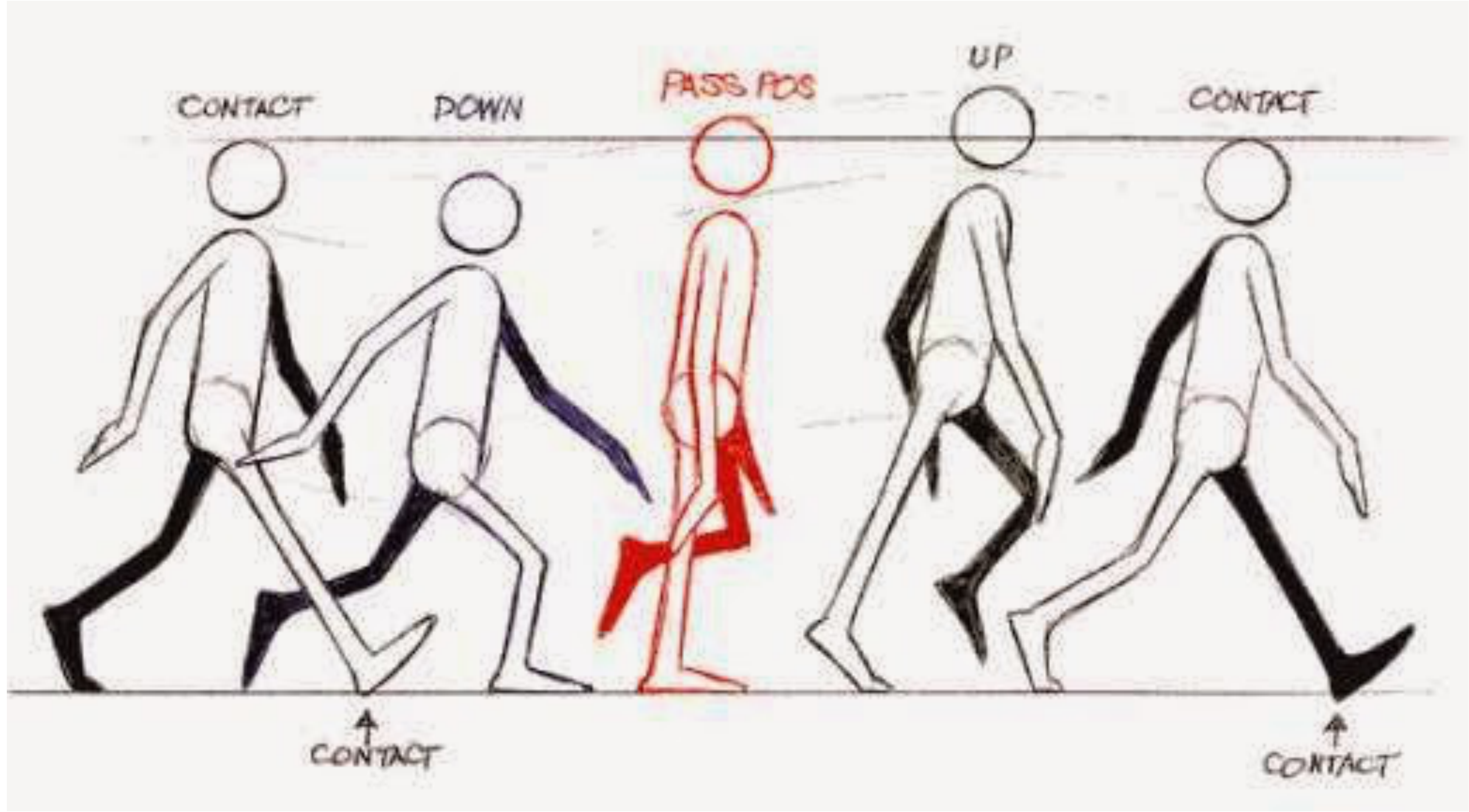


FK vs IK

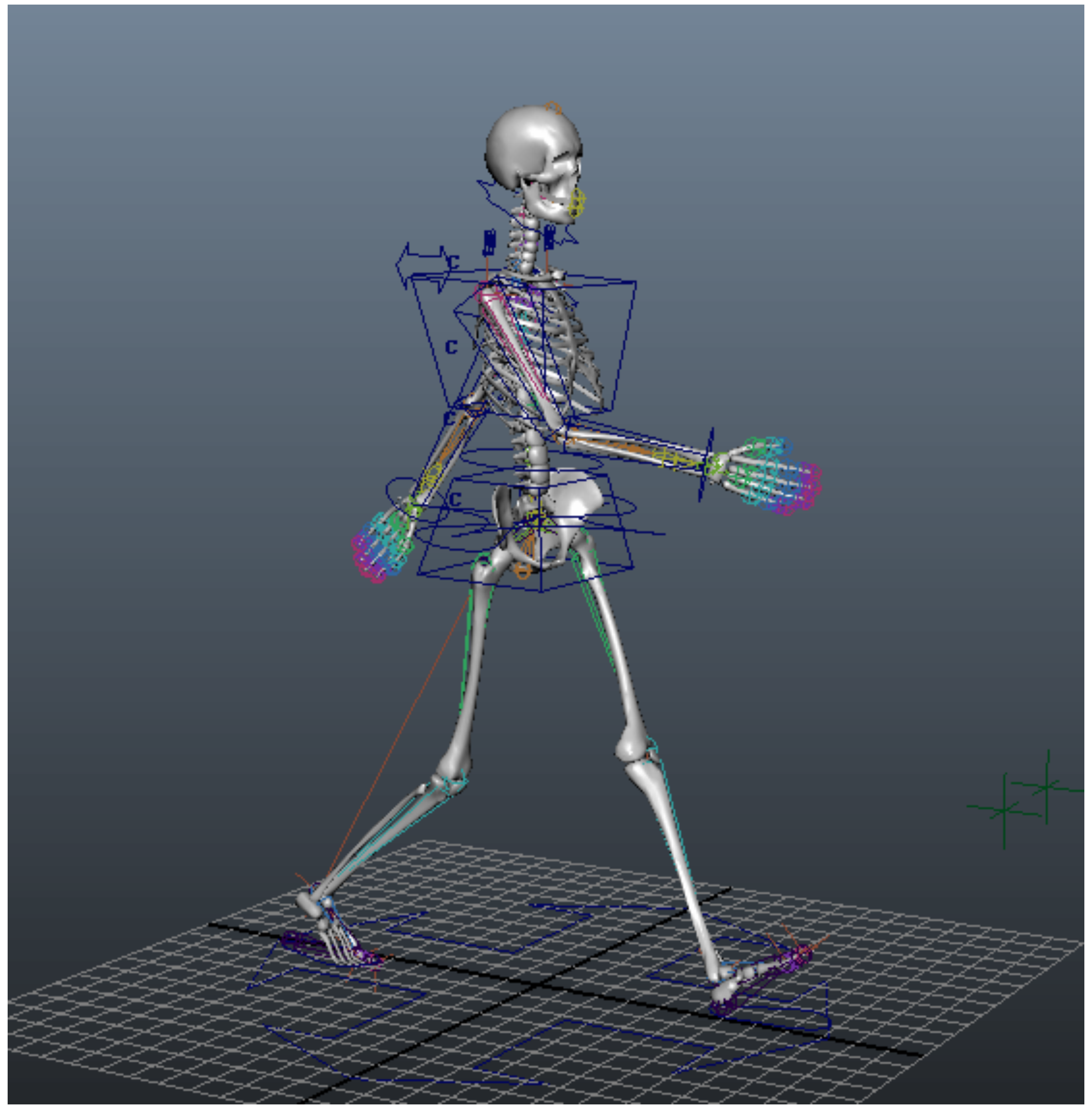
- **Forward kinematics**
  - Given a joint configuration, what is the 3D position of a point on the structure?
- **Inverse kinematics**
  - Given a target position for a point on the structure, what angles do the joints need to be to achieve that target point?

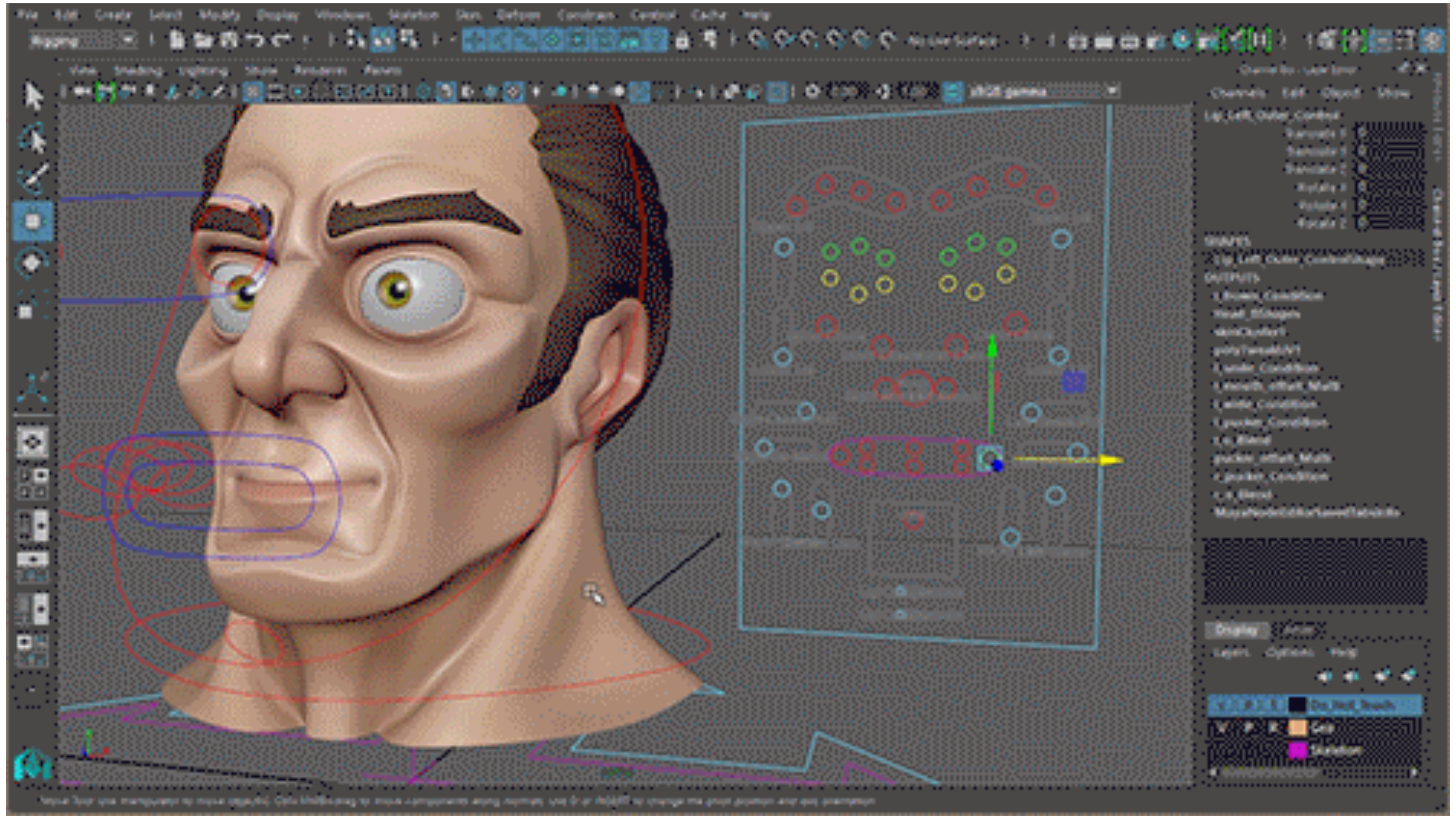
Limitations

- Directly controlling joints is not always what we want (e.g. hard to avoid foot- skating when animating human gait).
- Assigning the value of each DOF individually is tedious for a complex model.

Rigging

- The process of defining (meta) degrees of freedom on an animatable character.
- Rigging simplifies the animation process and improves production efficiency.





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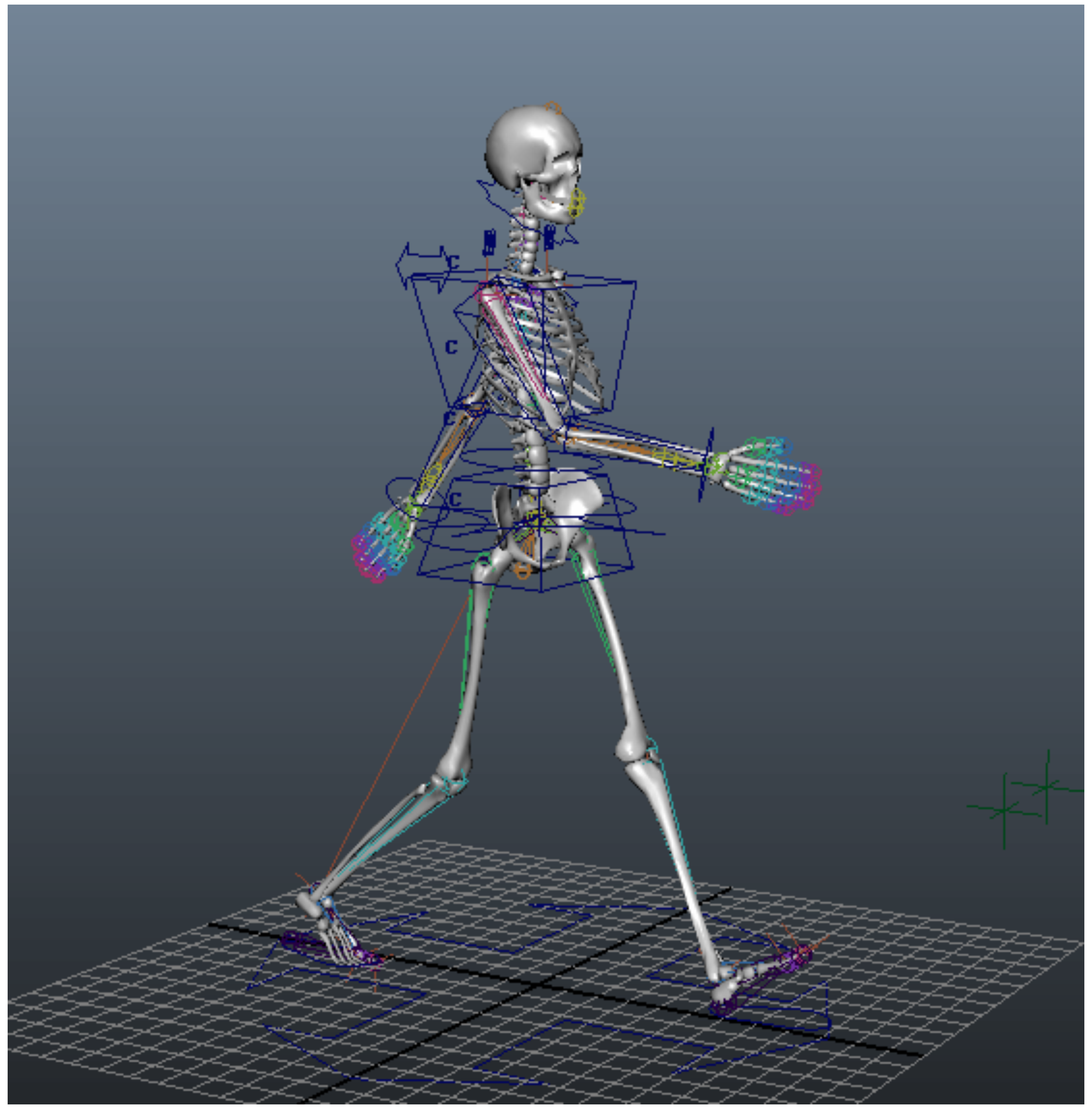

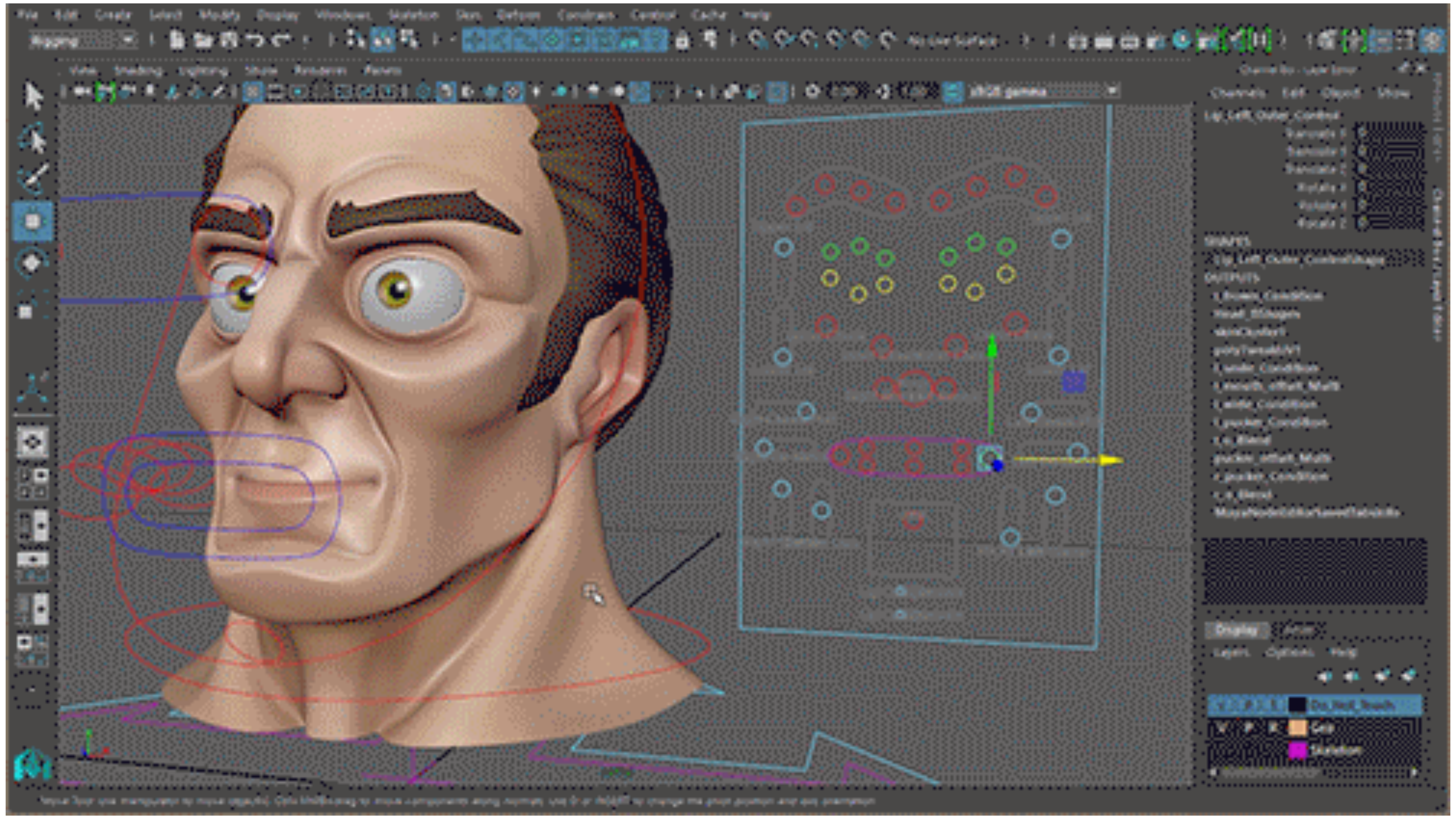

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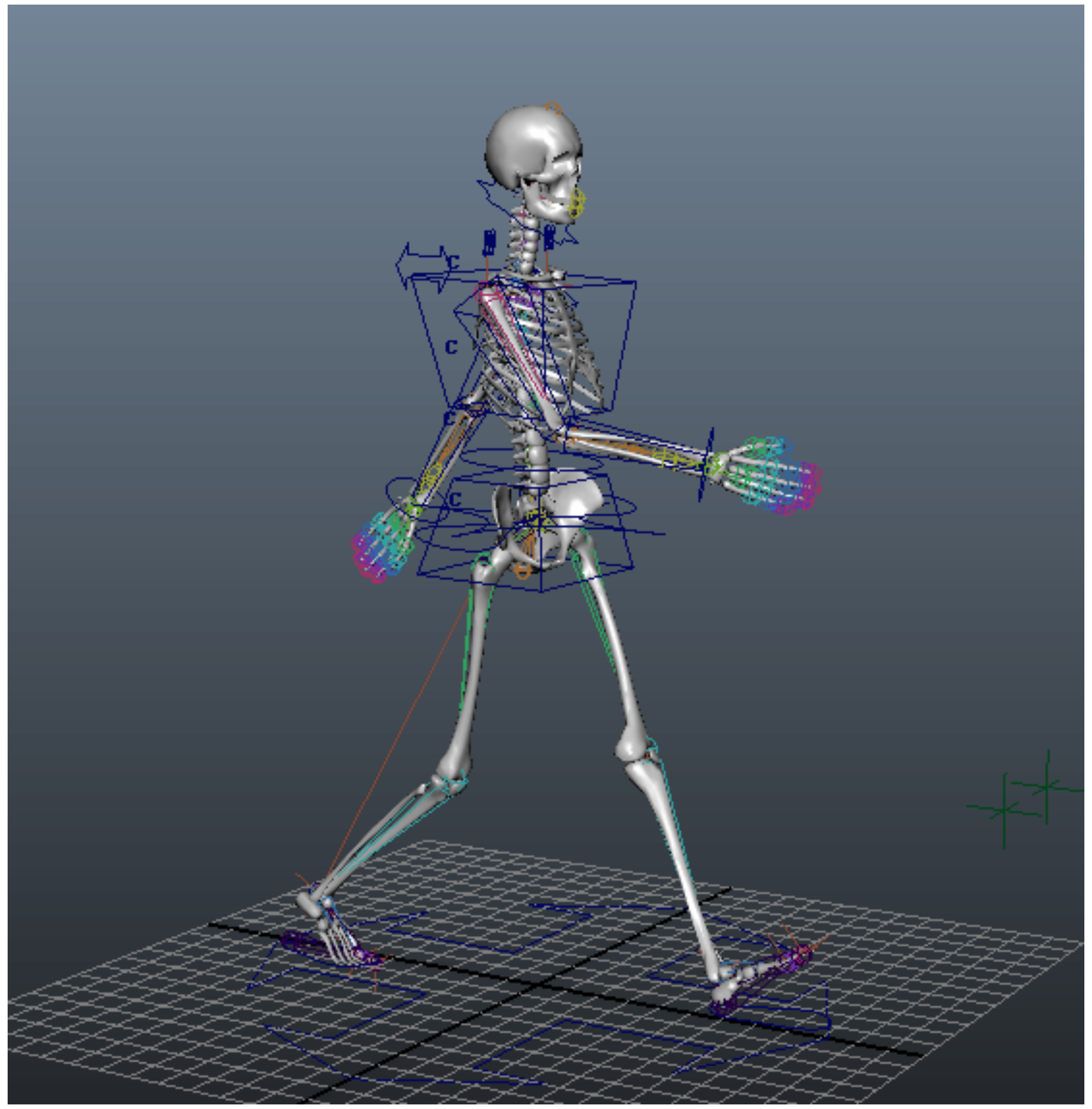

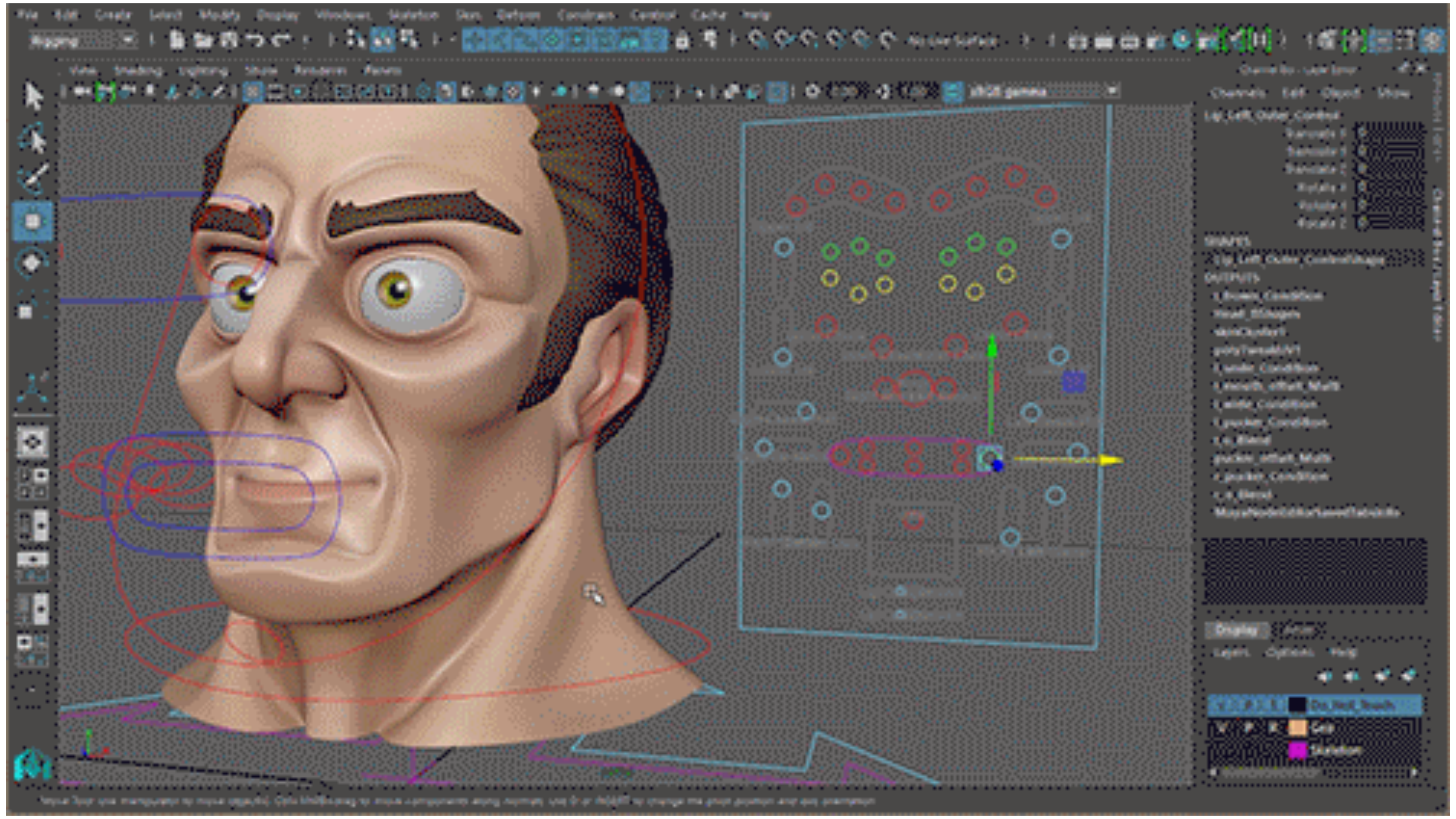

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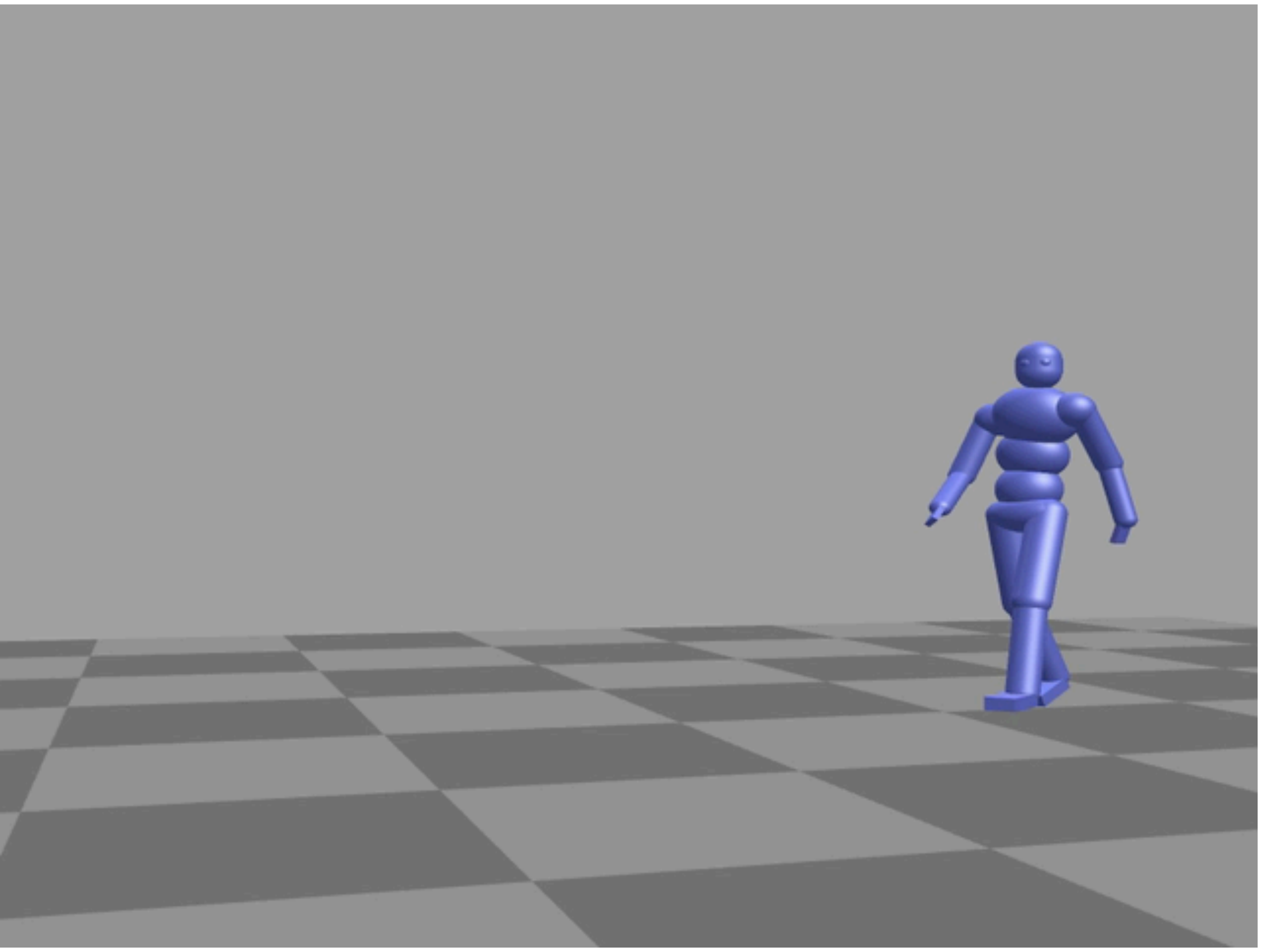
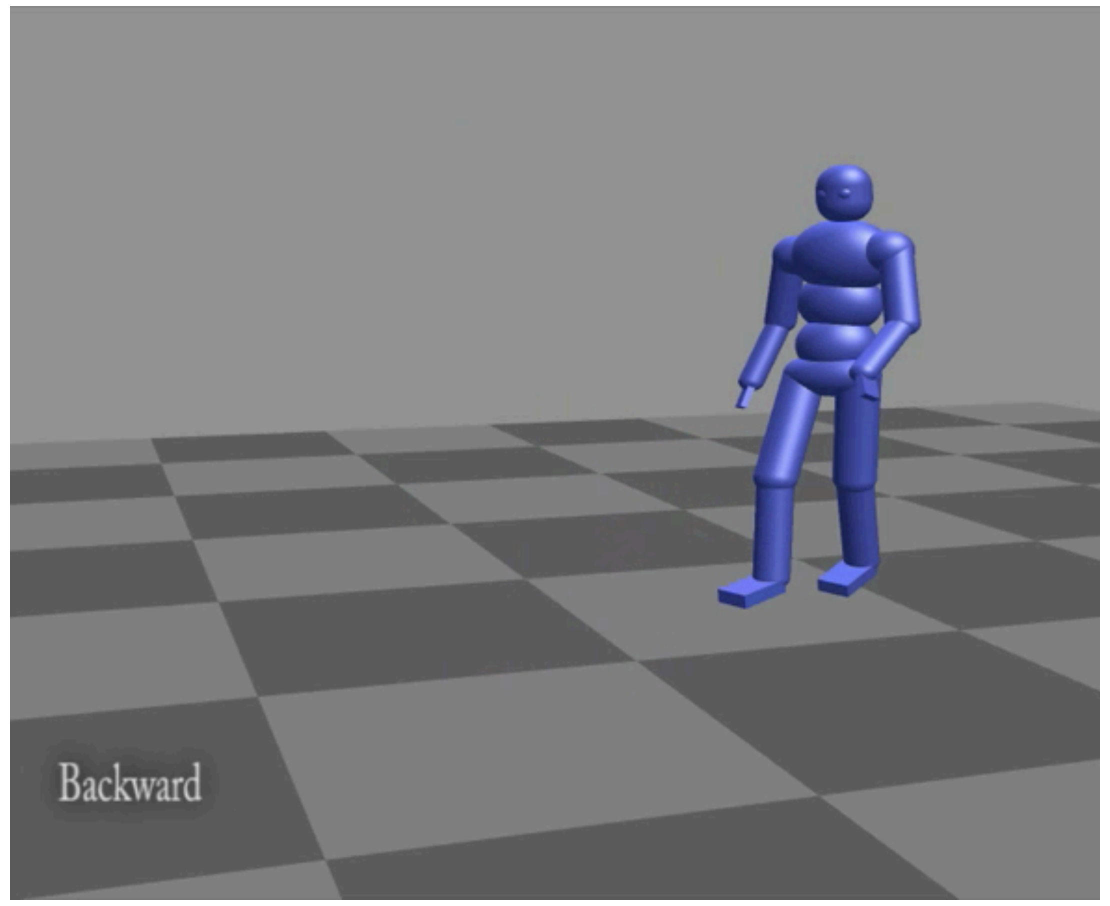
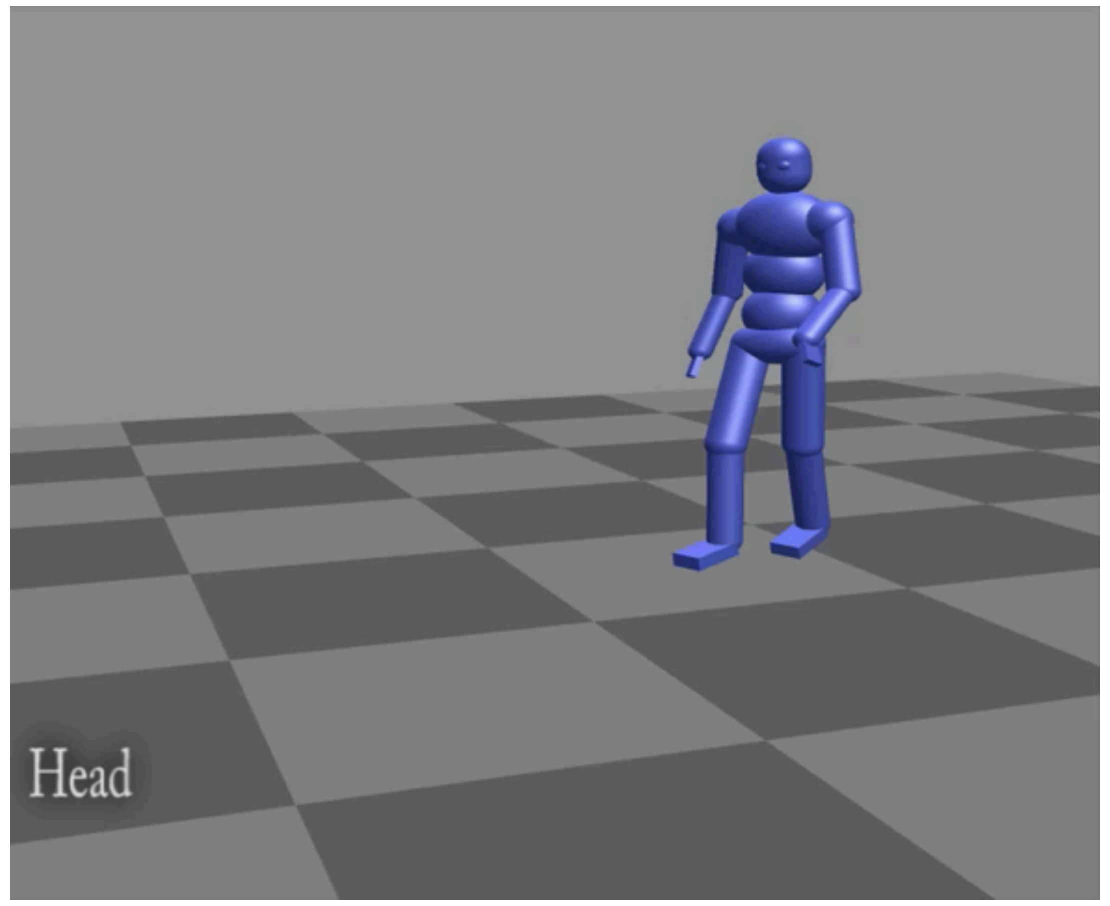
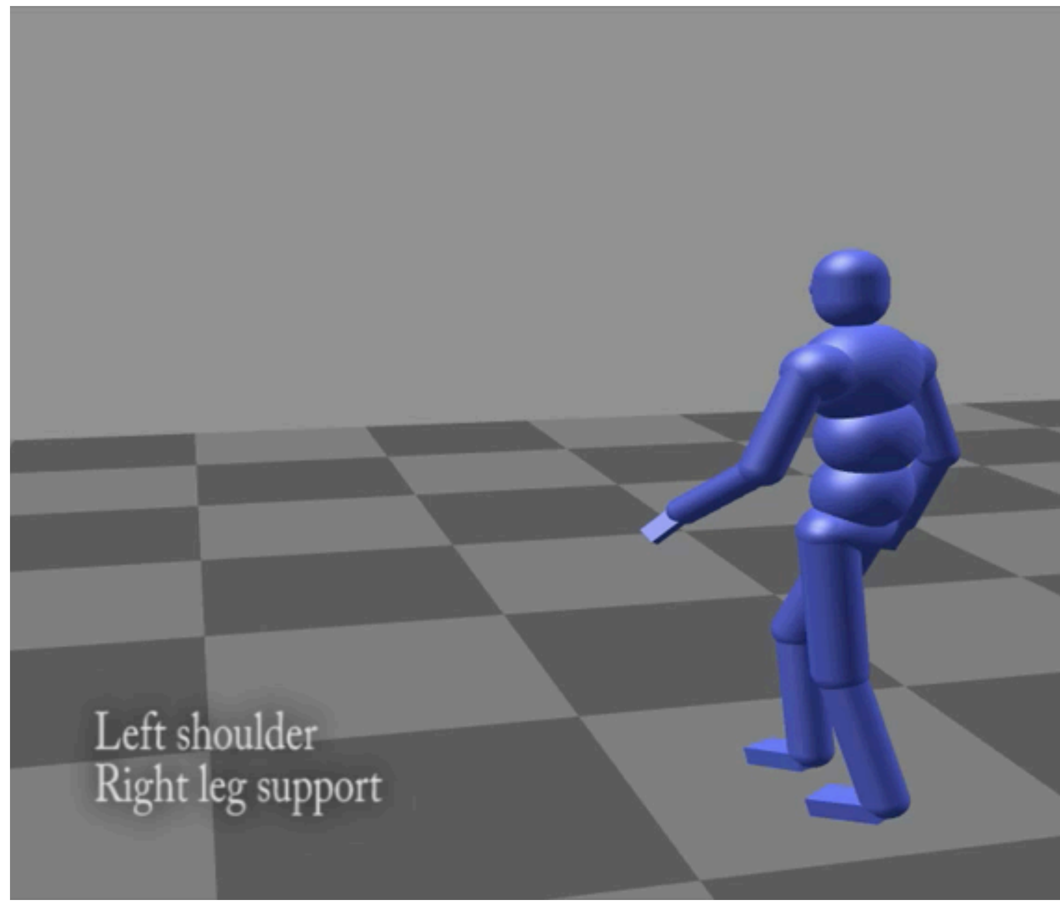
Physics

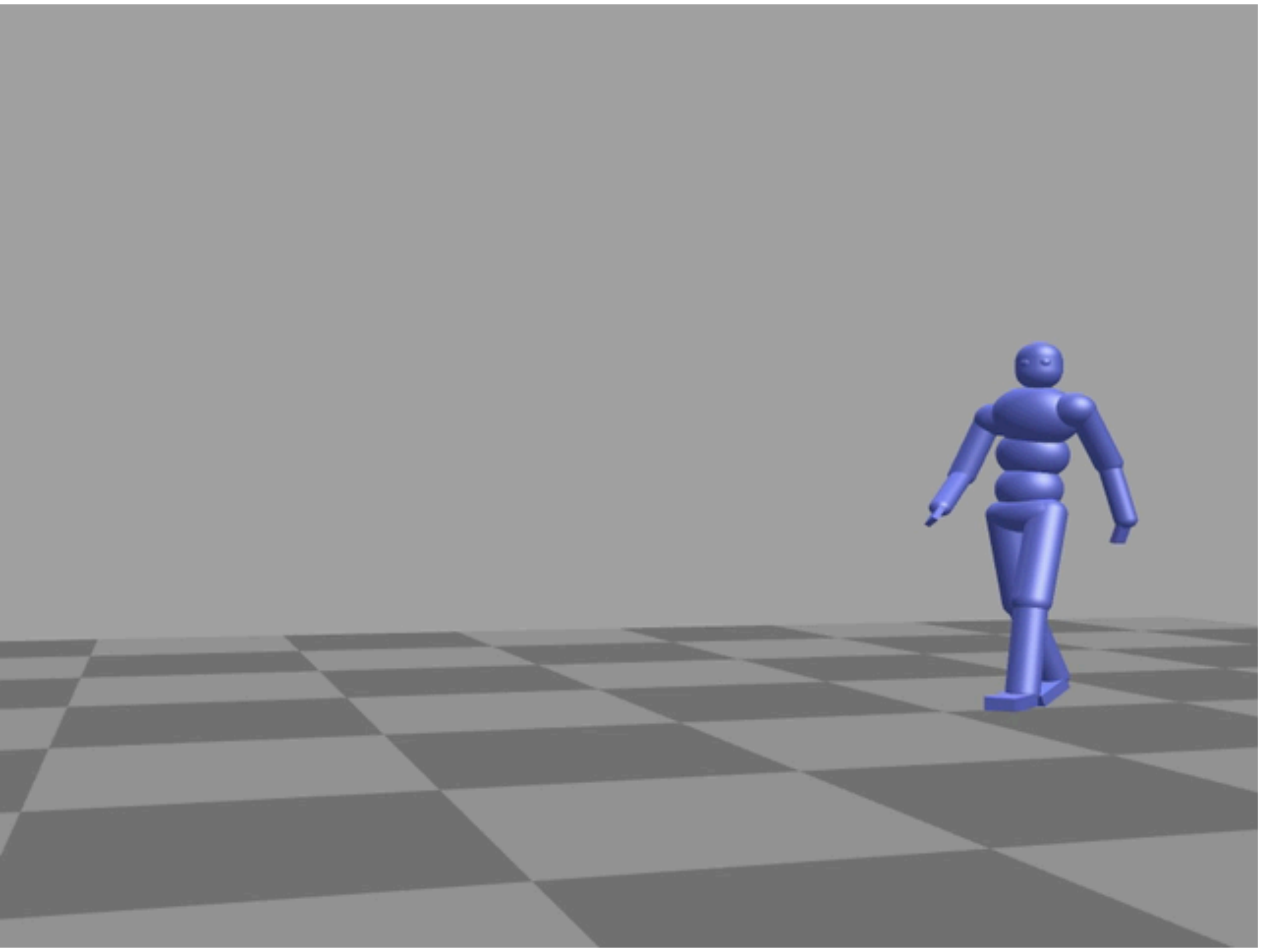


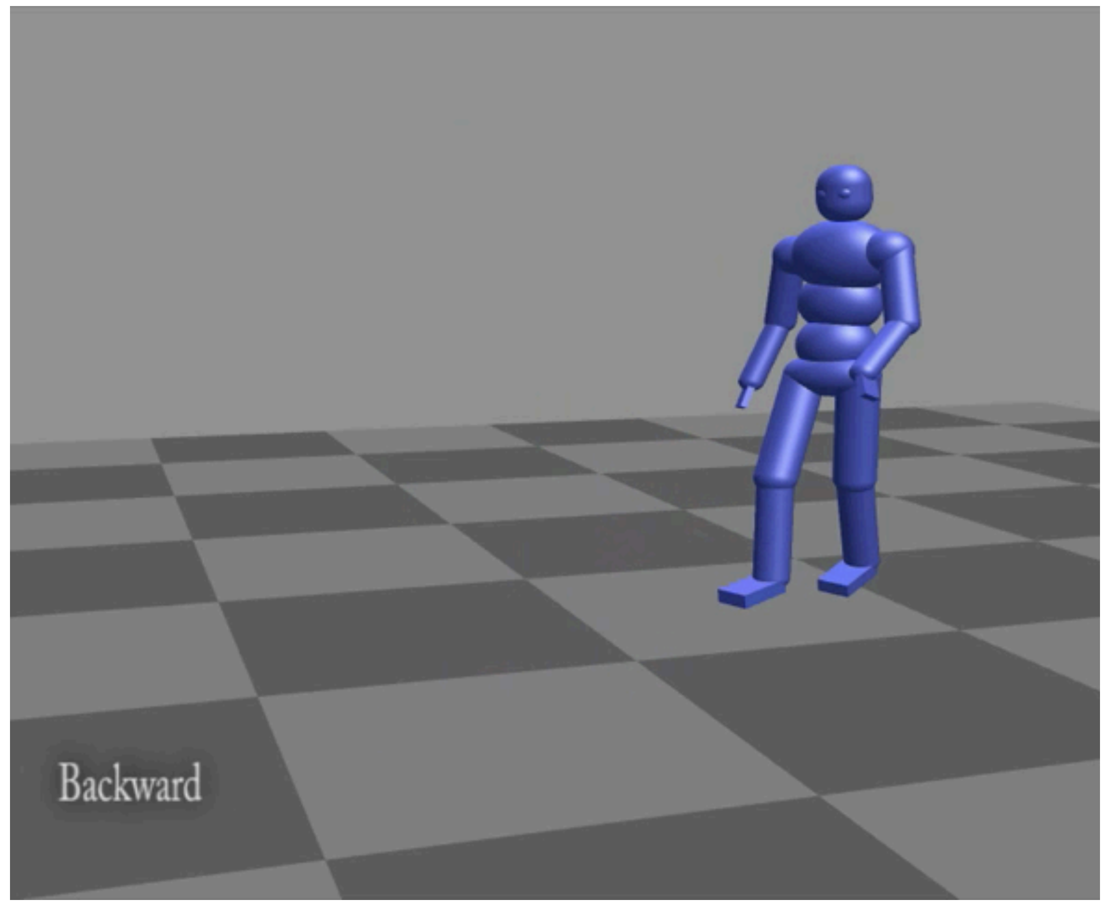


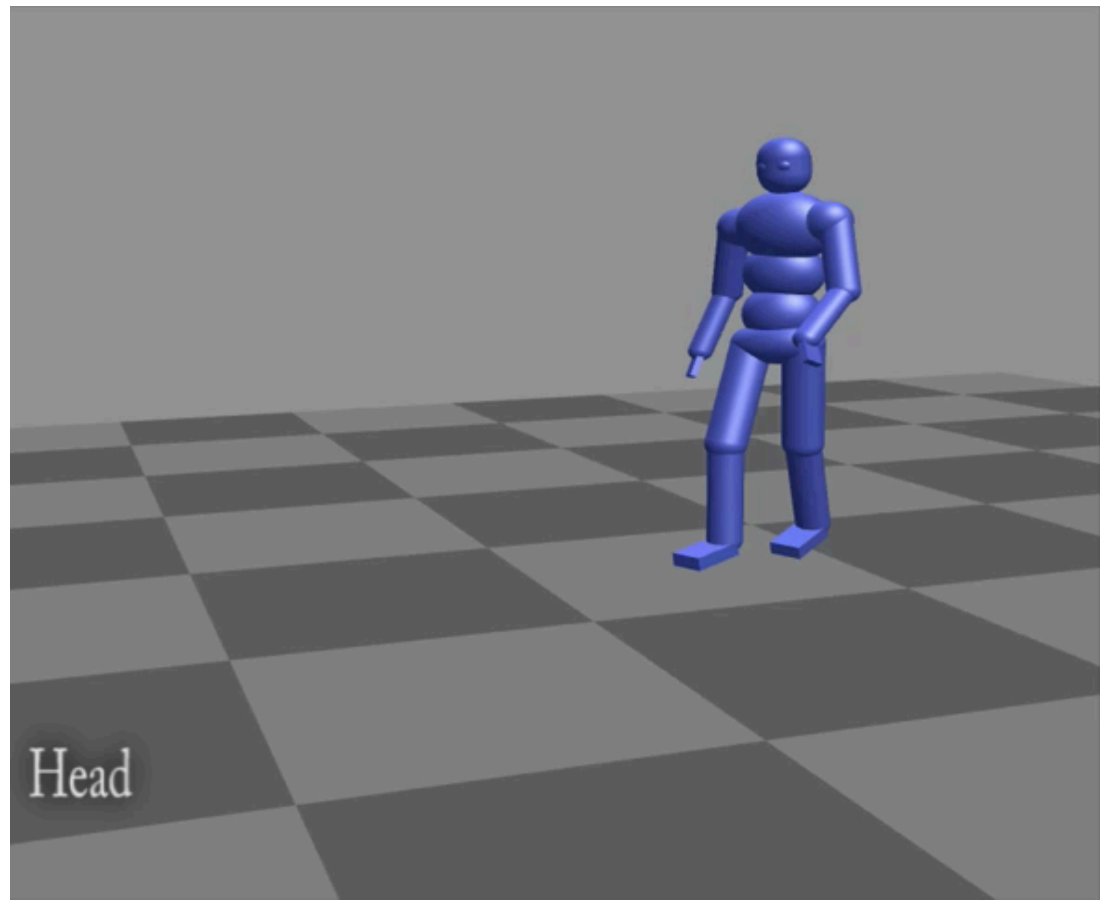


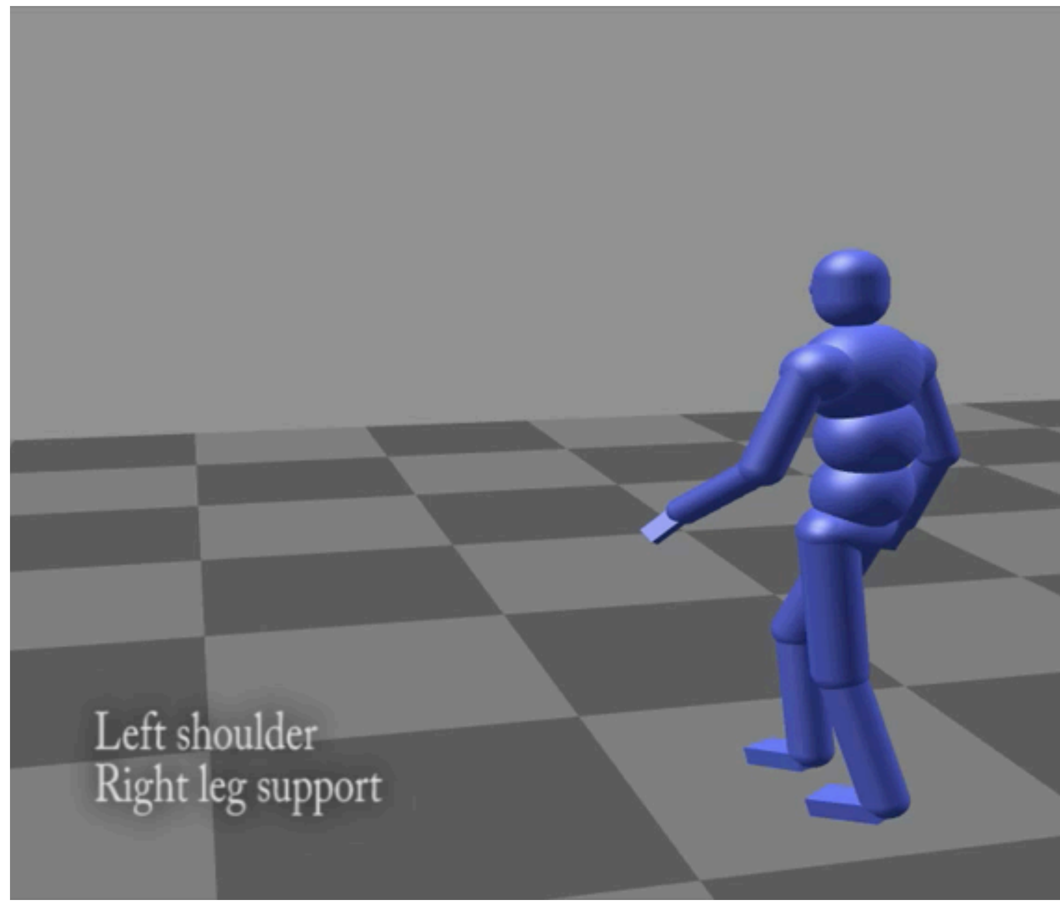


Physics









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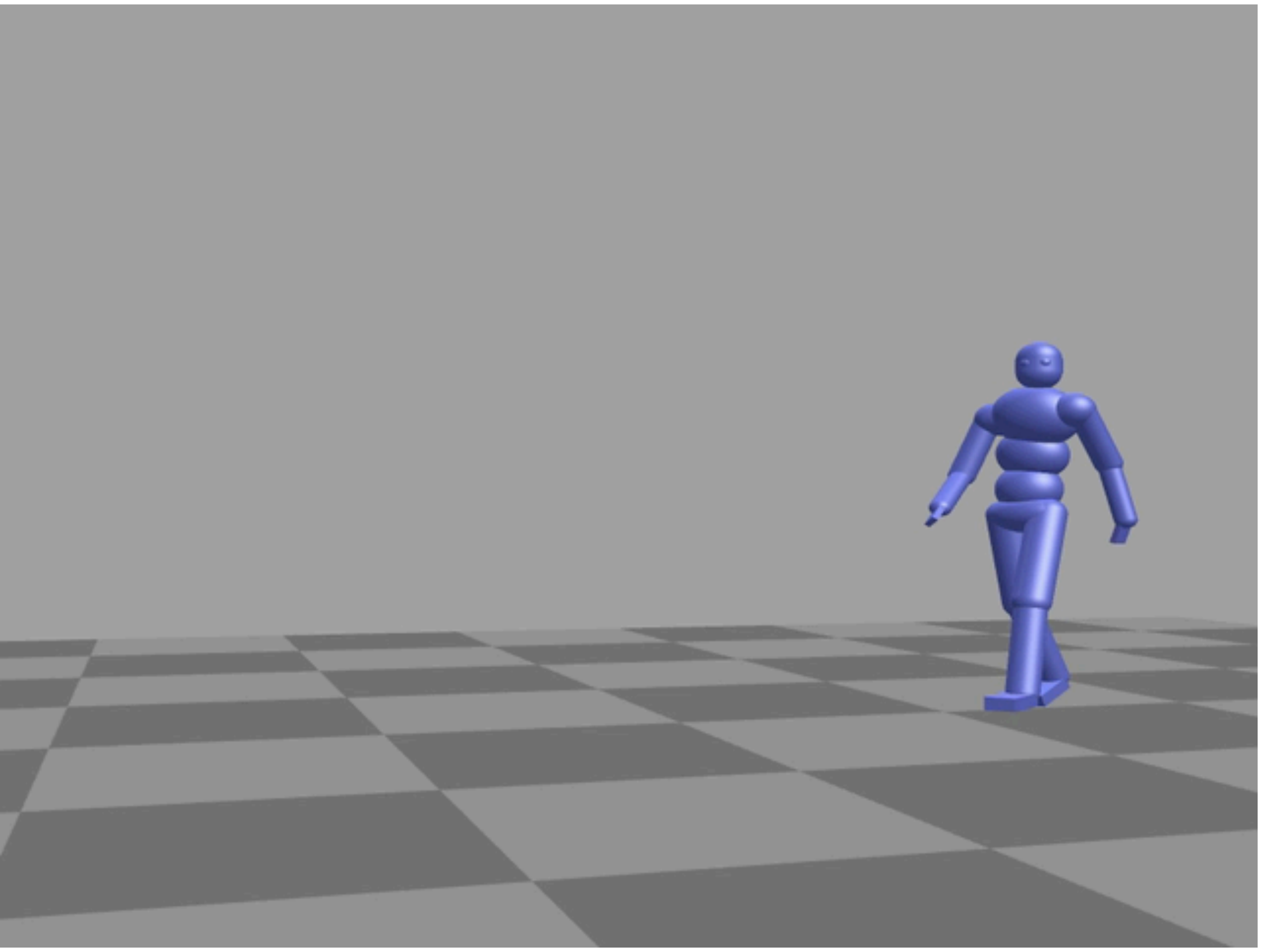
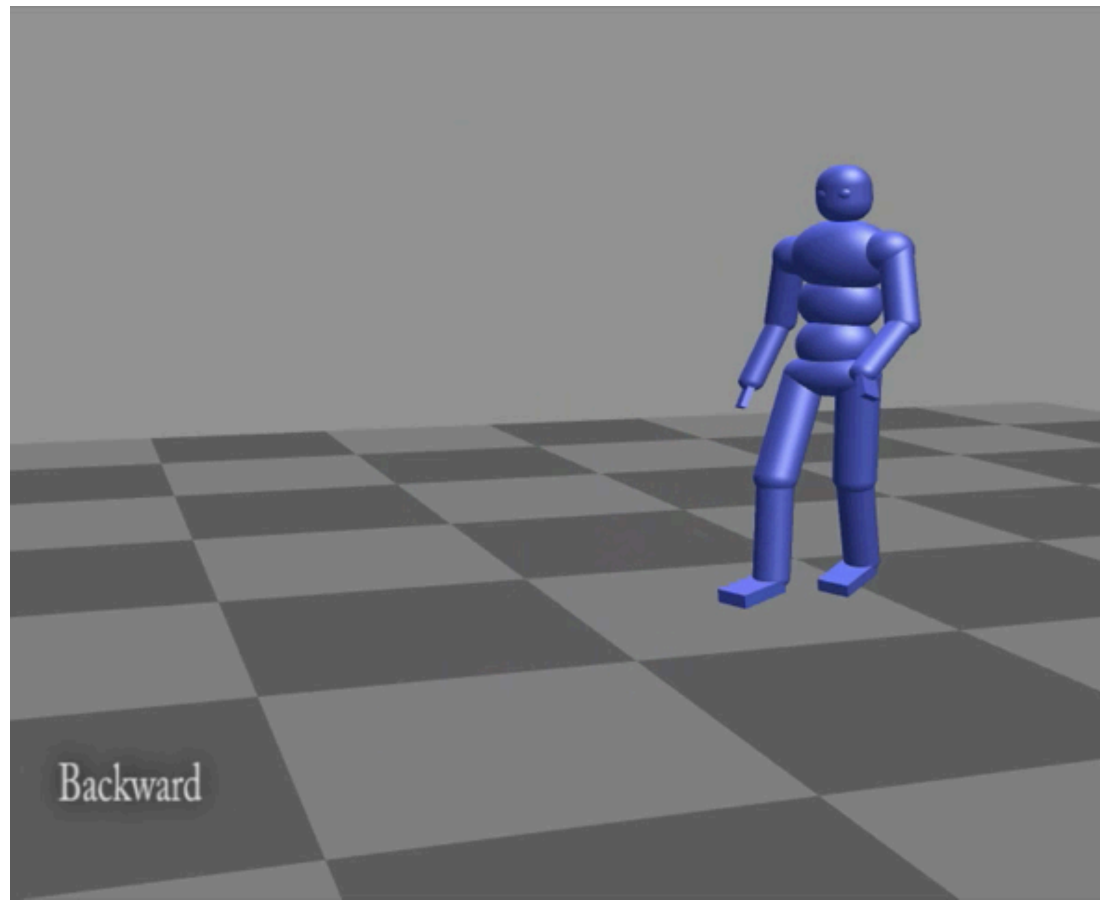
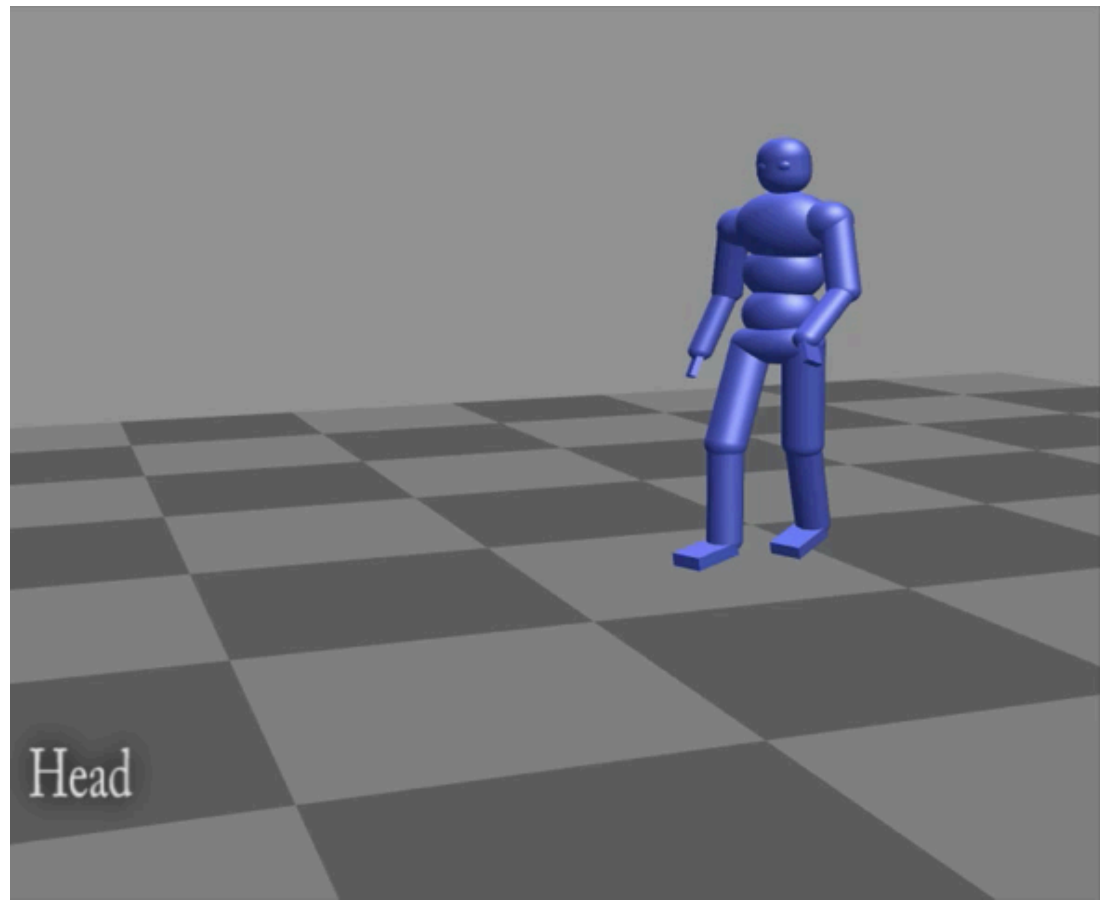
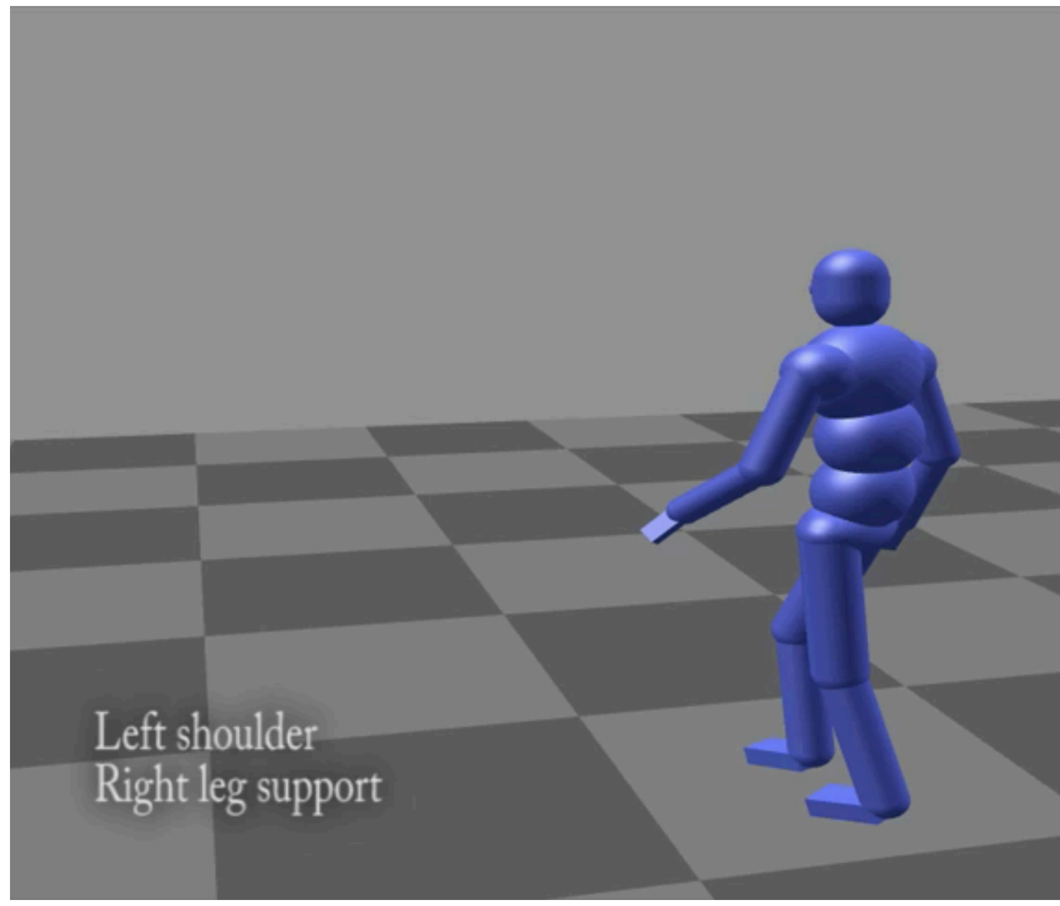


Character responding to various external pushes







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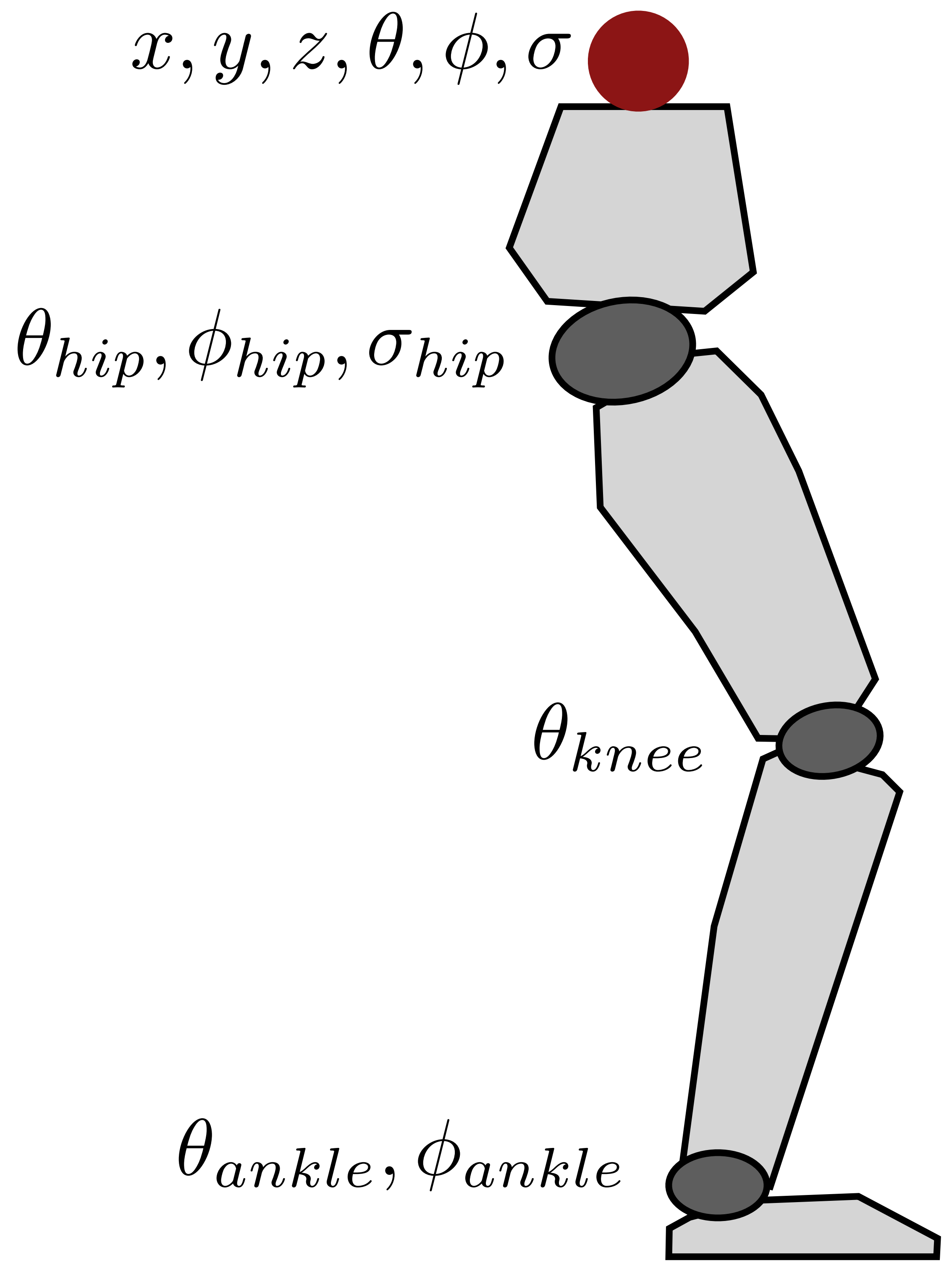
- Assigning the value of each DOF individually is tedious for a complex model.
- Character is not affected by physics (e.g. going through wall).
- It's difficult to change the hierarchical structure on-the-fly. For example, dismembering the human figure (usually in video games).

Maximum vs generalized coordinates

Generalized (reduced) coordinates



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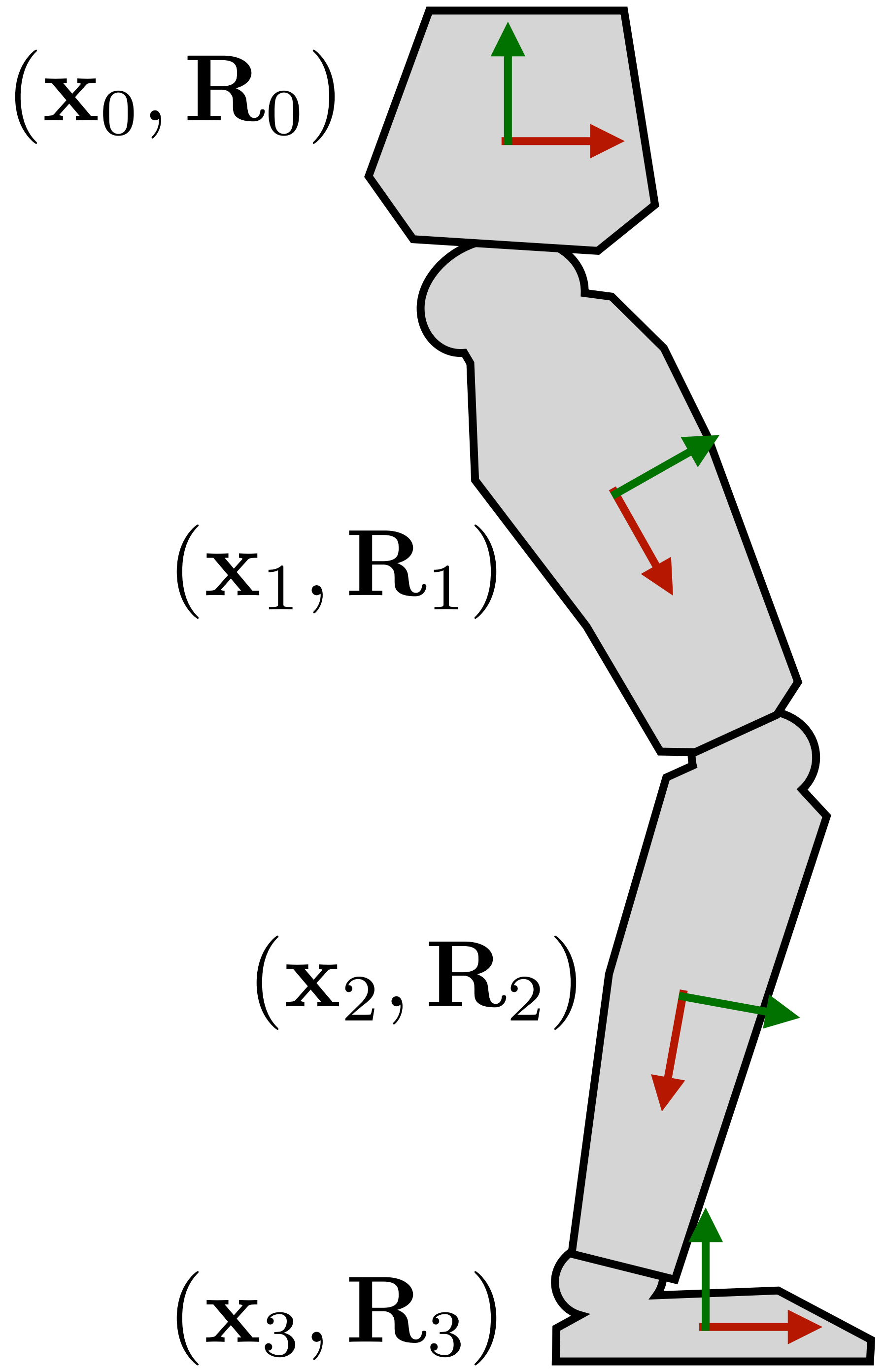

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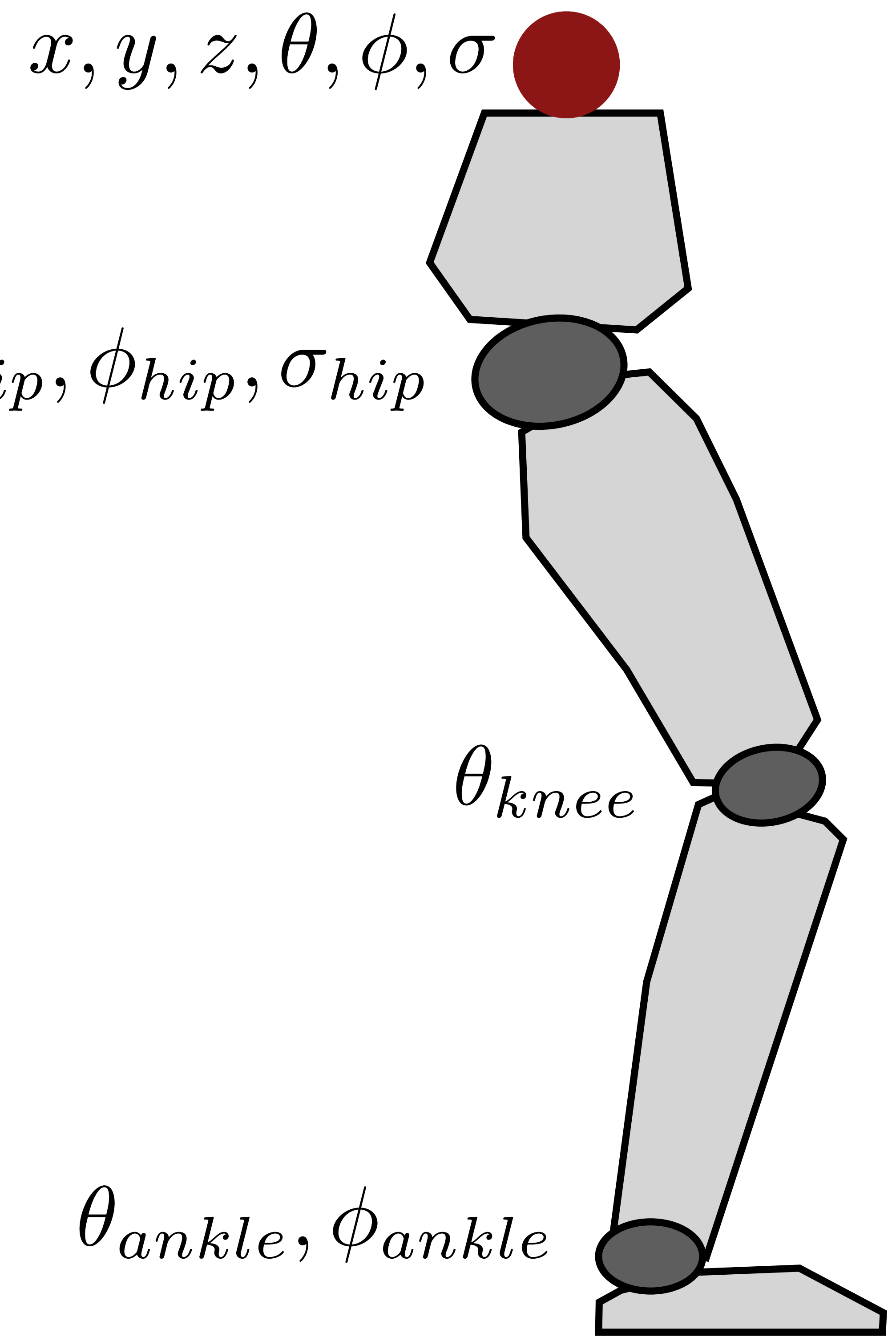

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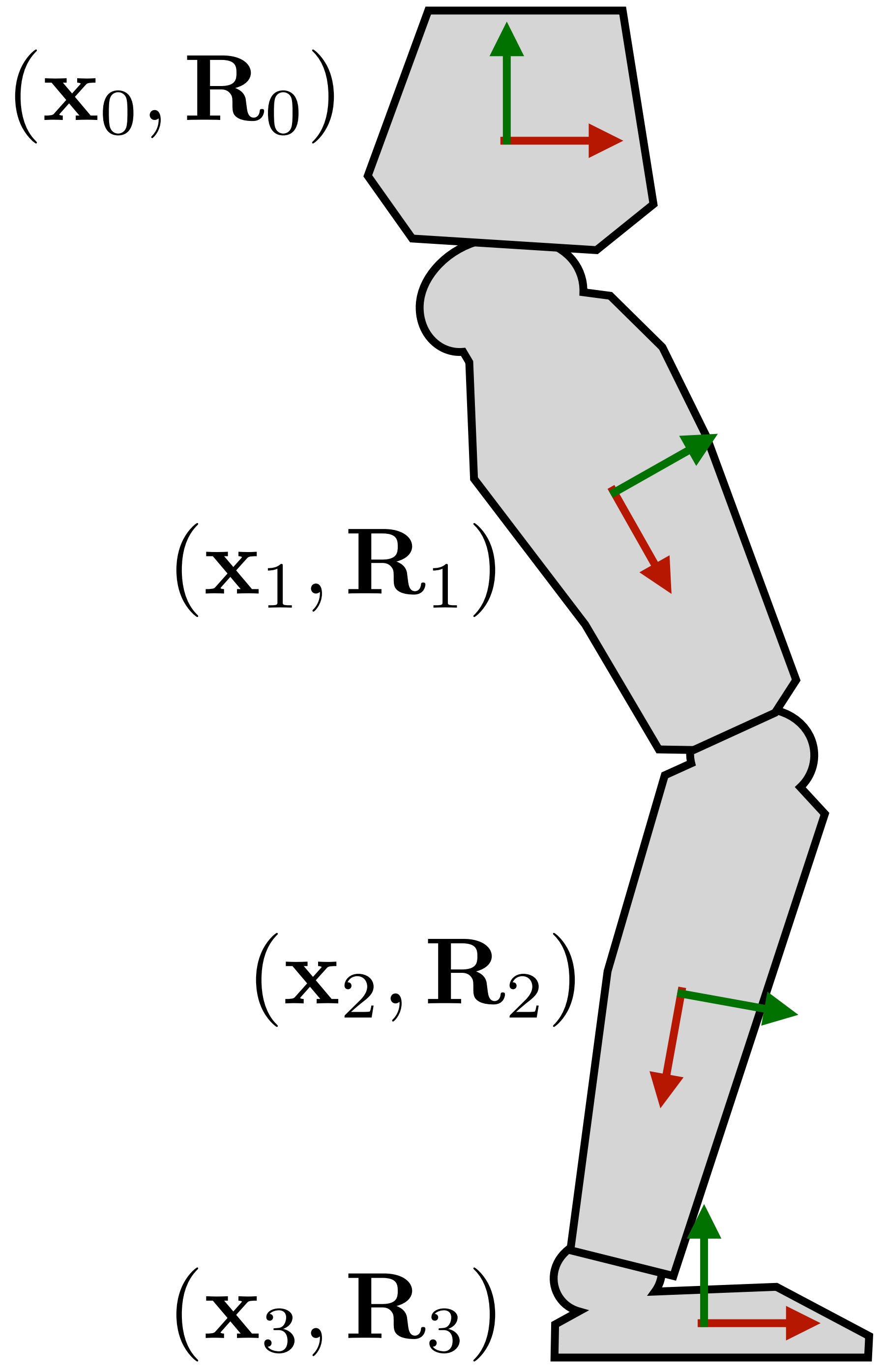

degree of freedom: 24

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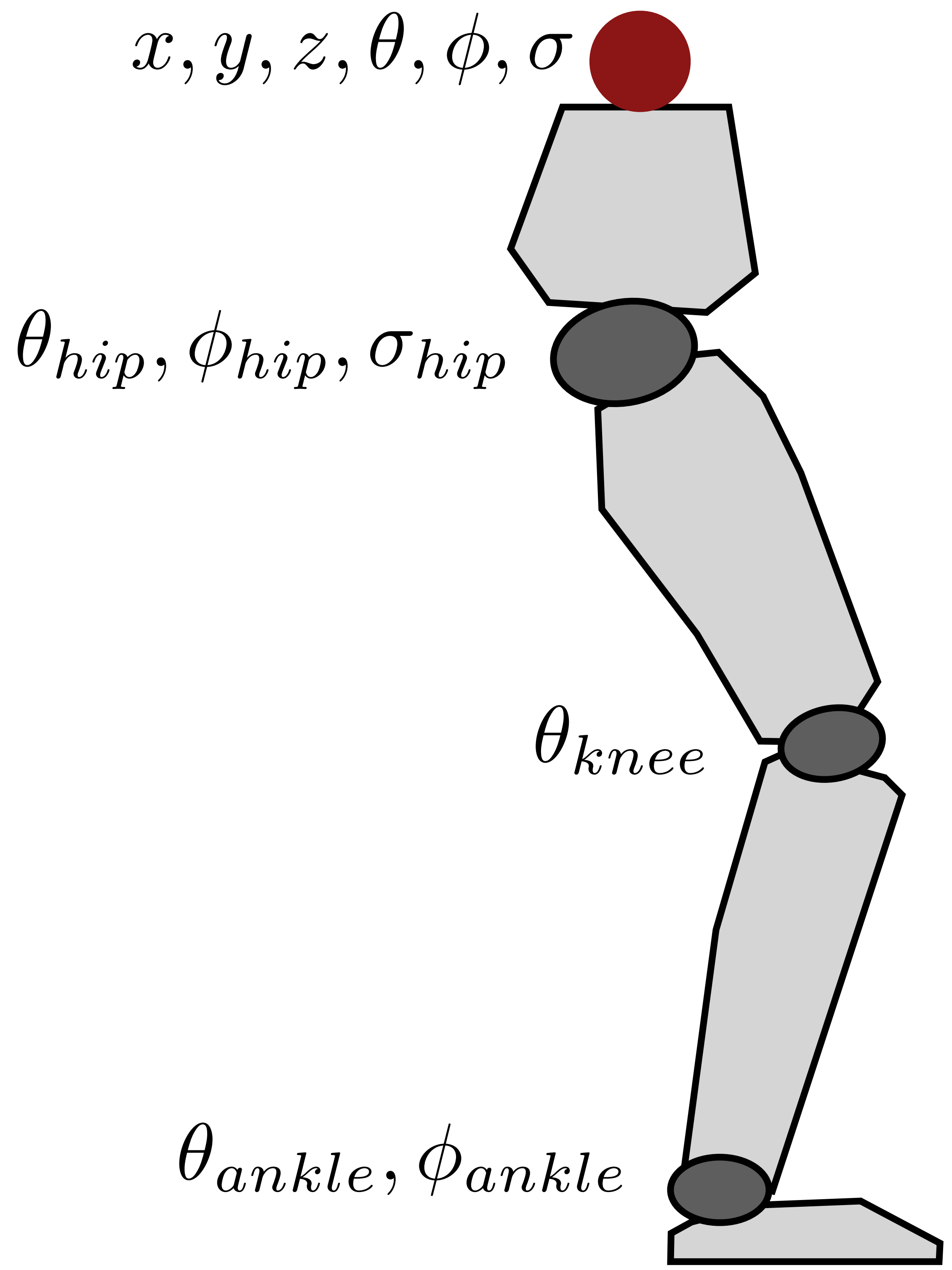

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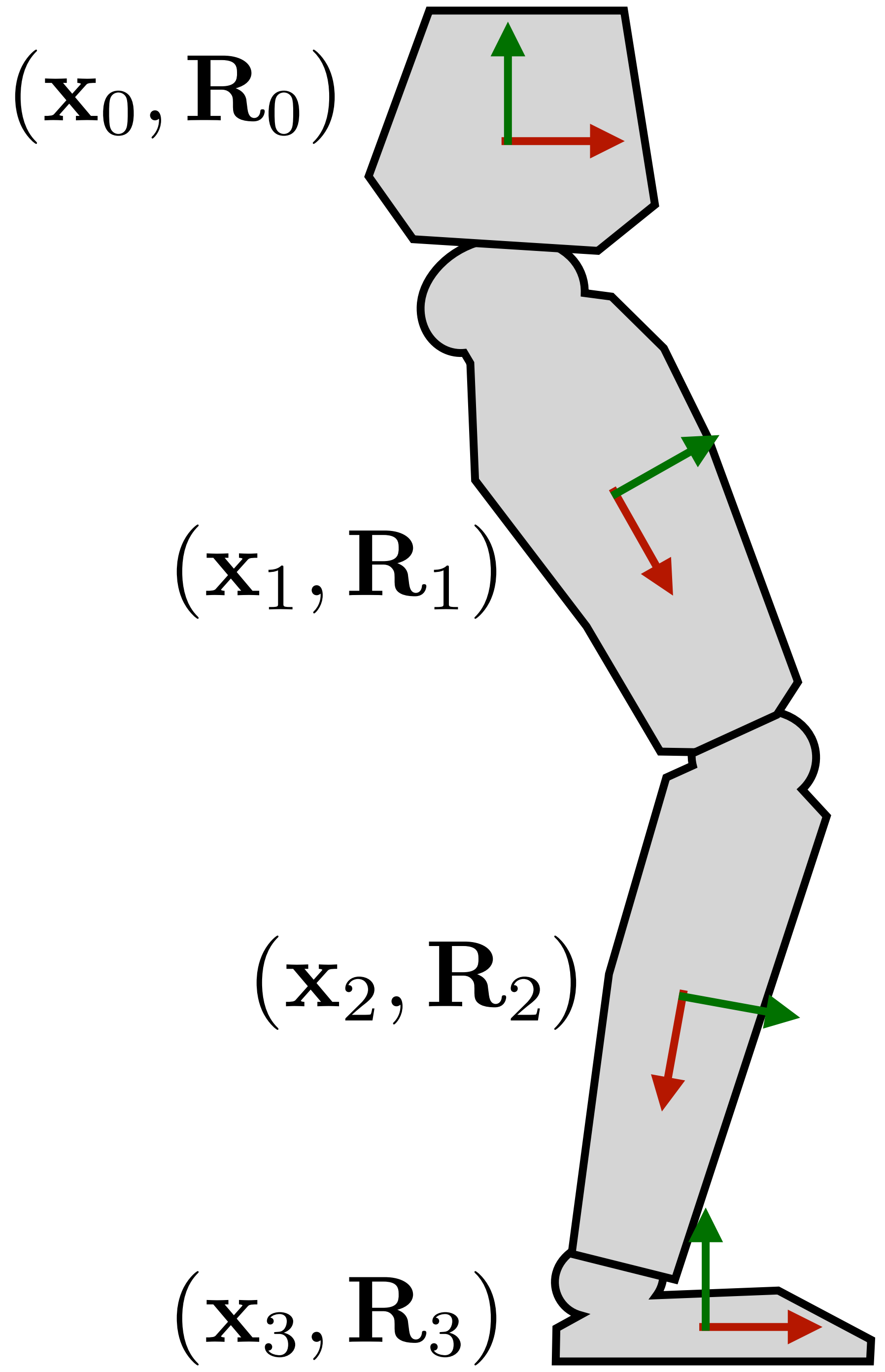

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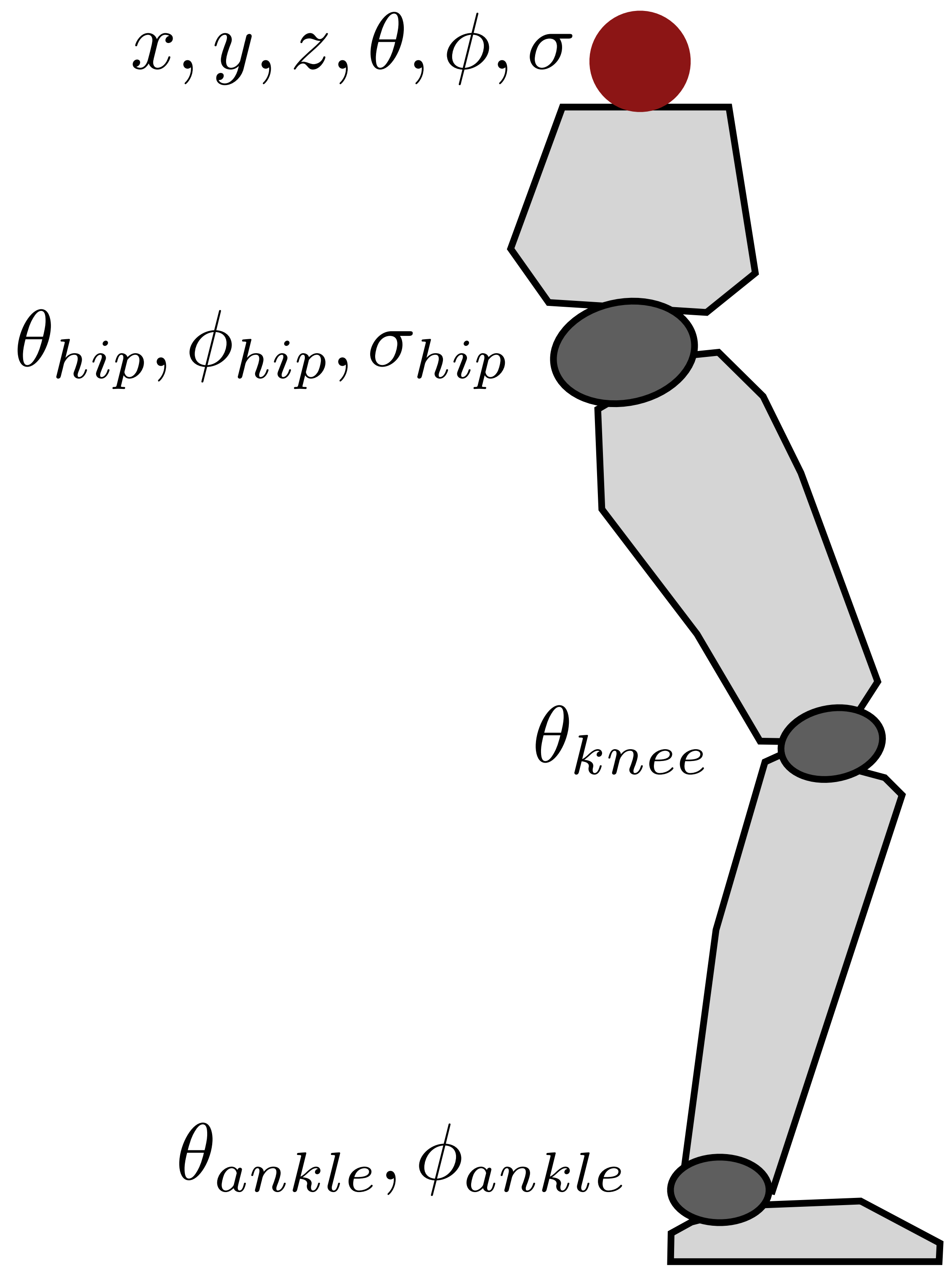
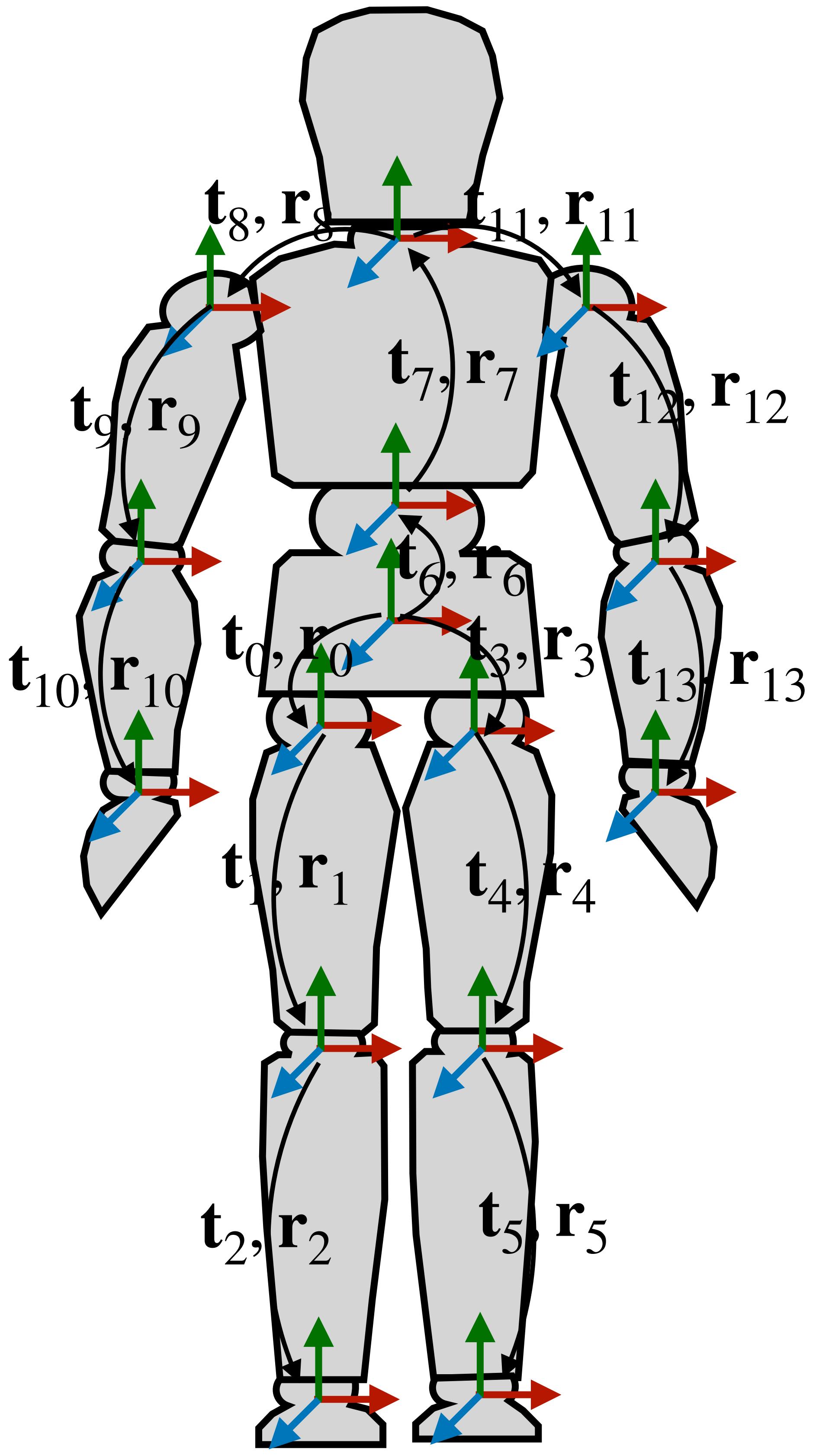
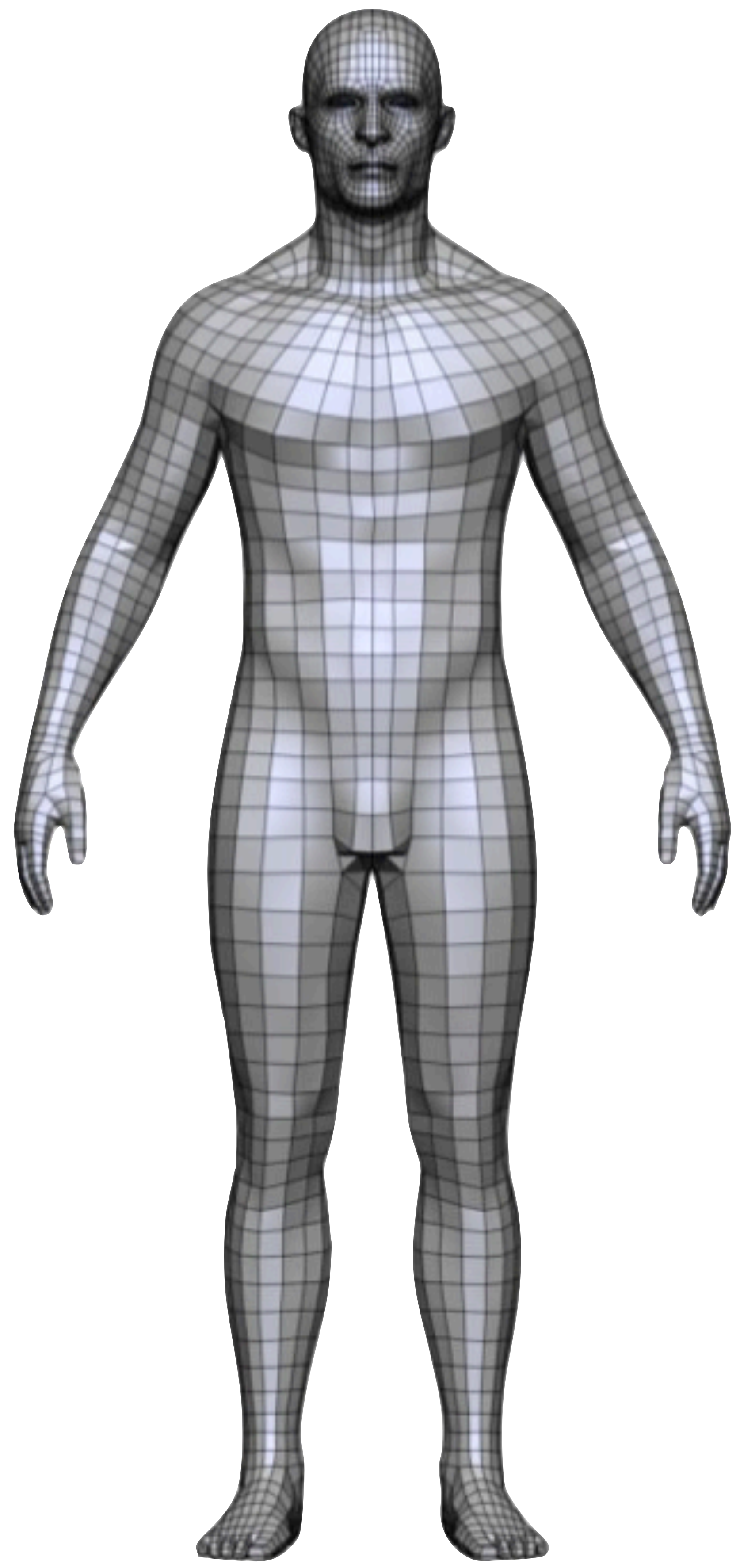


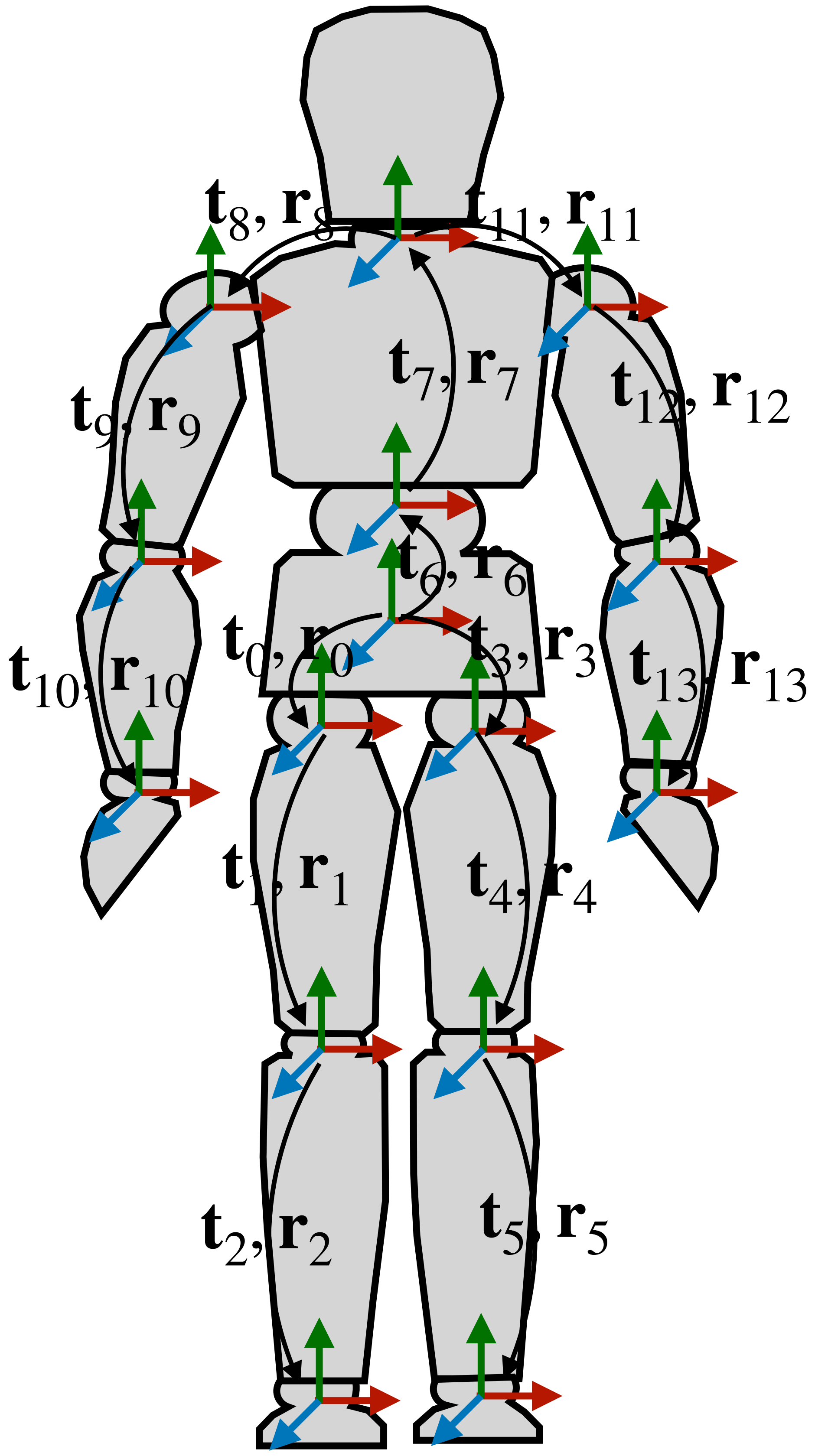
degree of freedom: 12

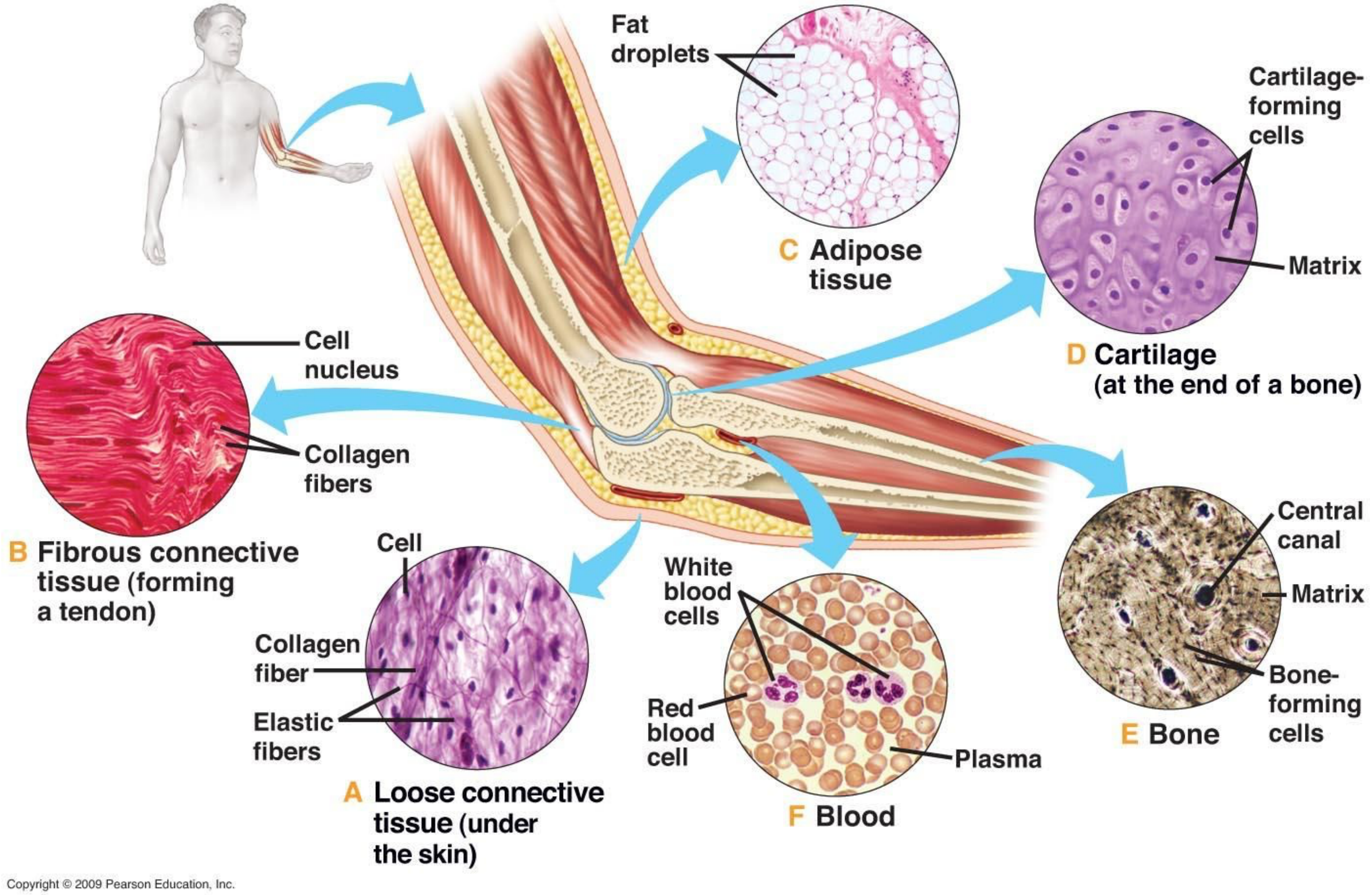
Visualization of kinematics

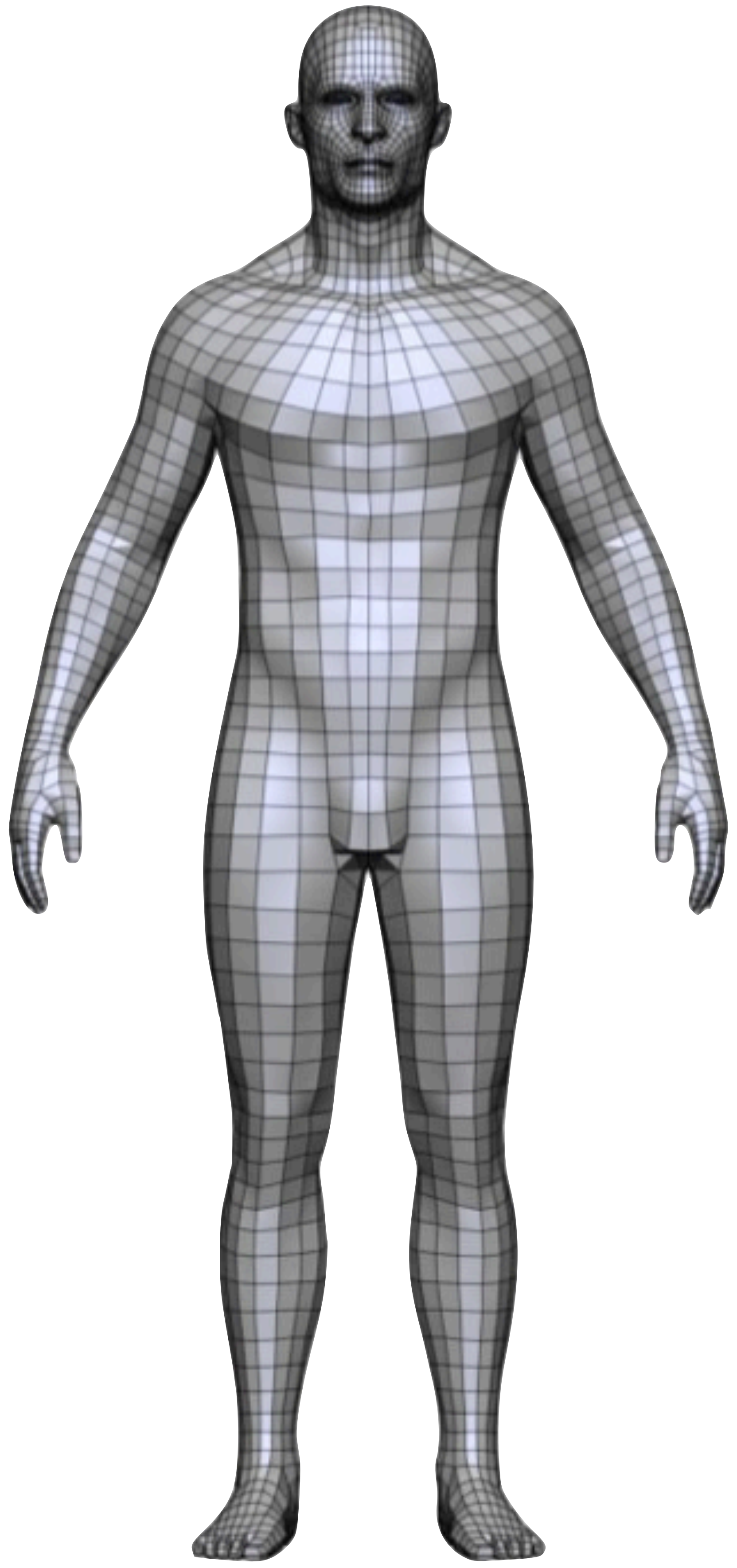




Physics approach

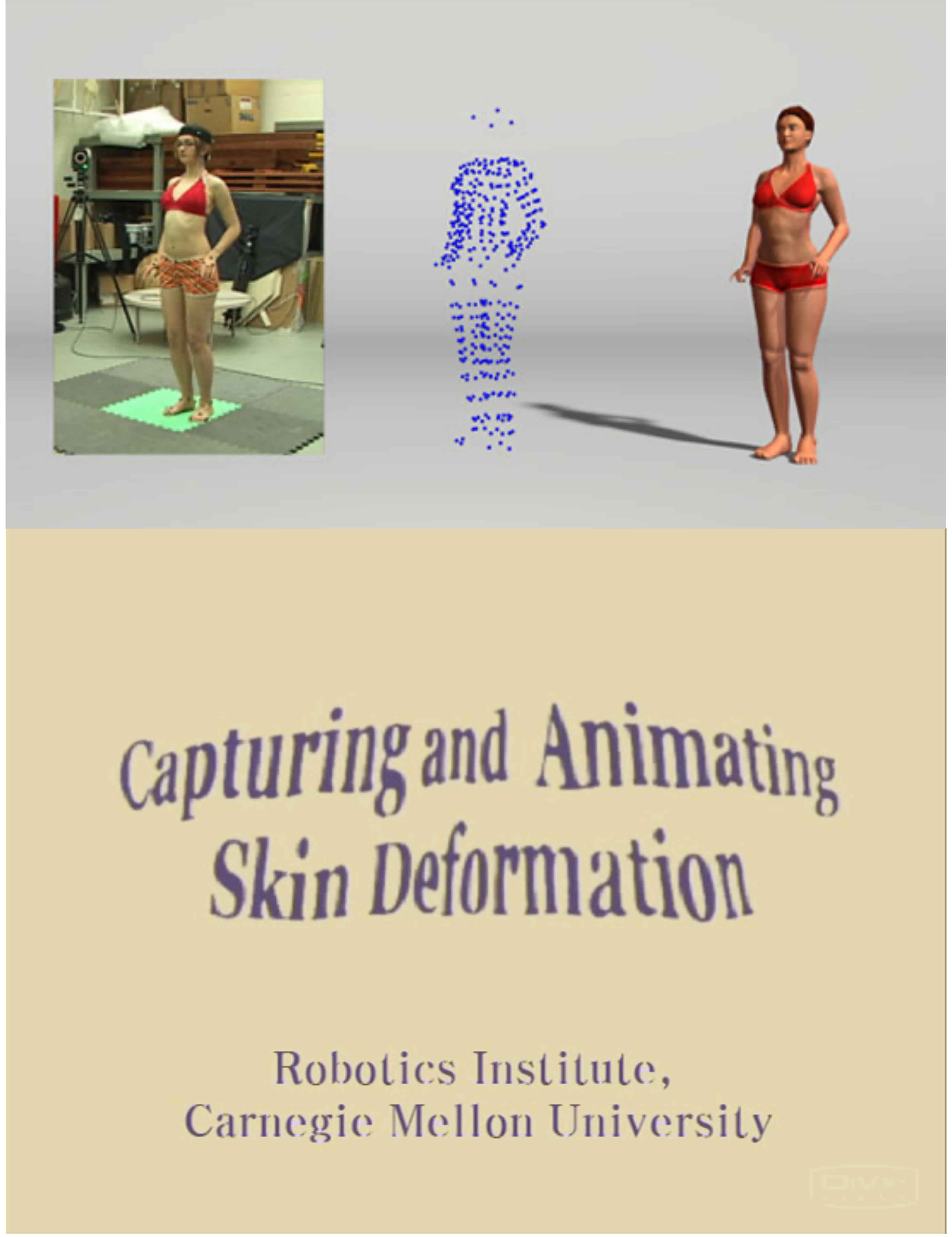






Real-data approach

- Use a commercial motion capture system with approximately 450 markers.
- Still need to build an actor-specific surface model using rigid transformation plus residual deformation.
- Can model muscles budging, jiggling, stretching and high-frequency motion



Park et al SIGGRAPH 2006

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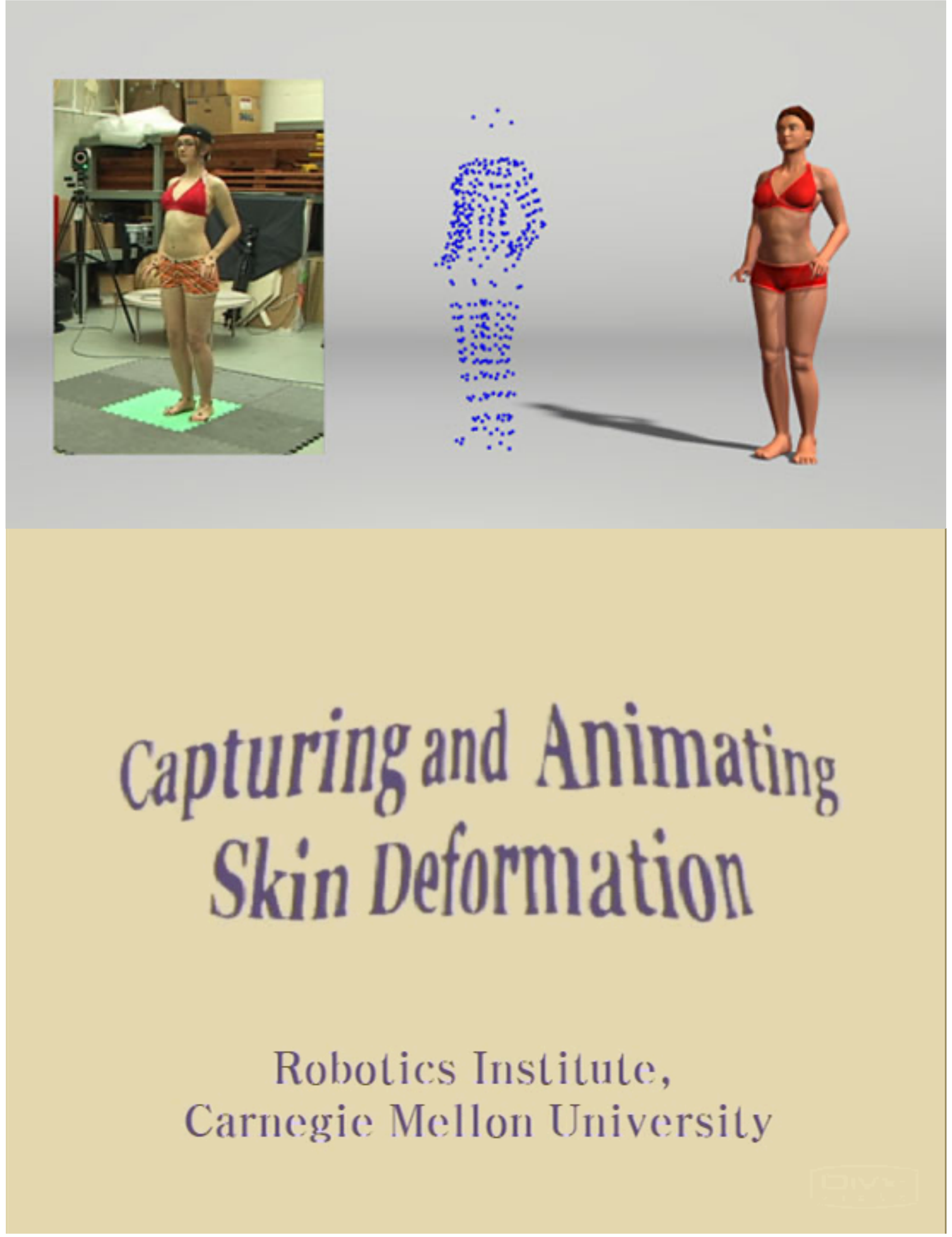
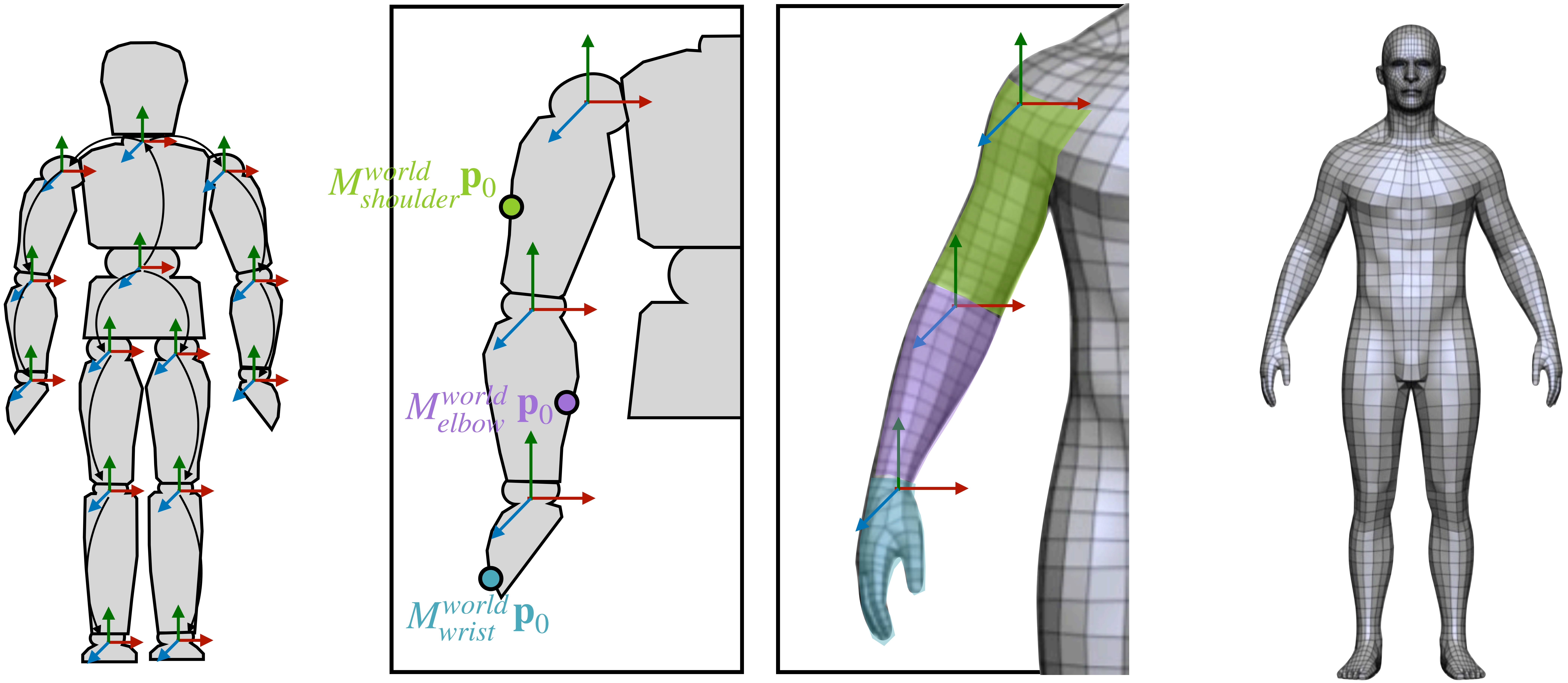
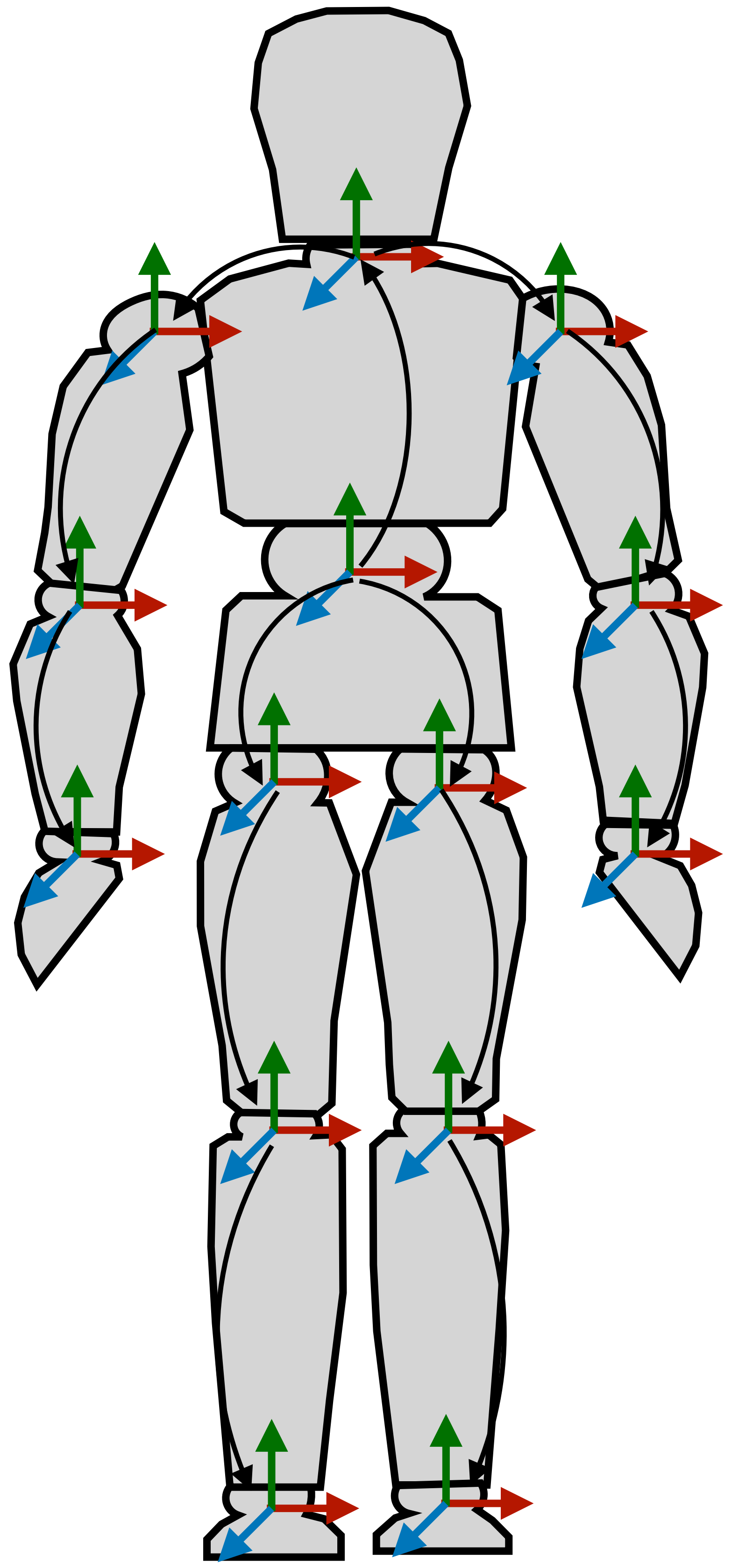


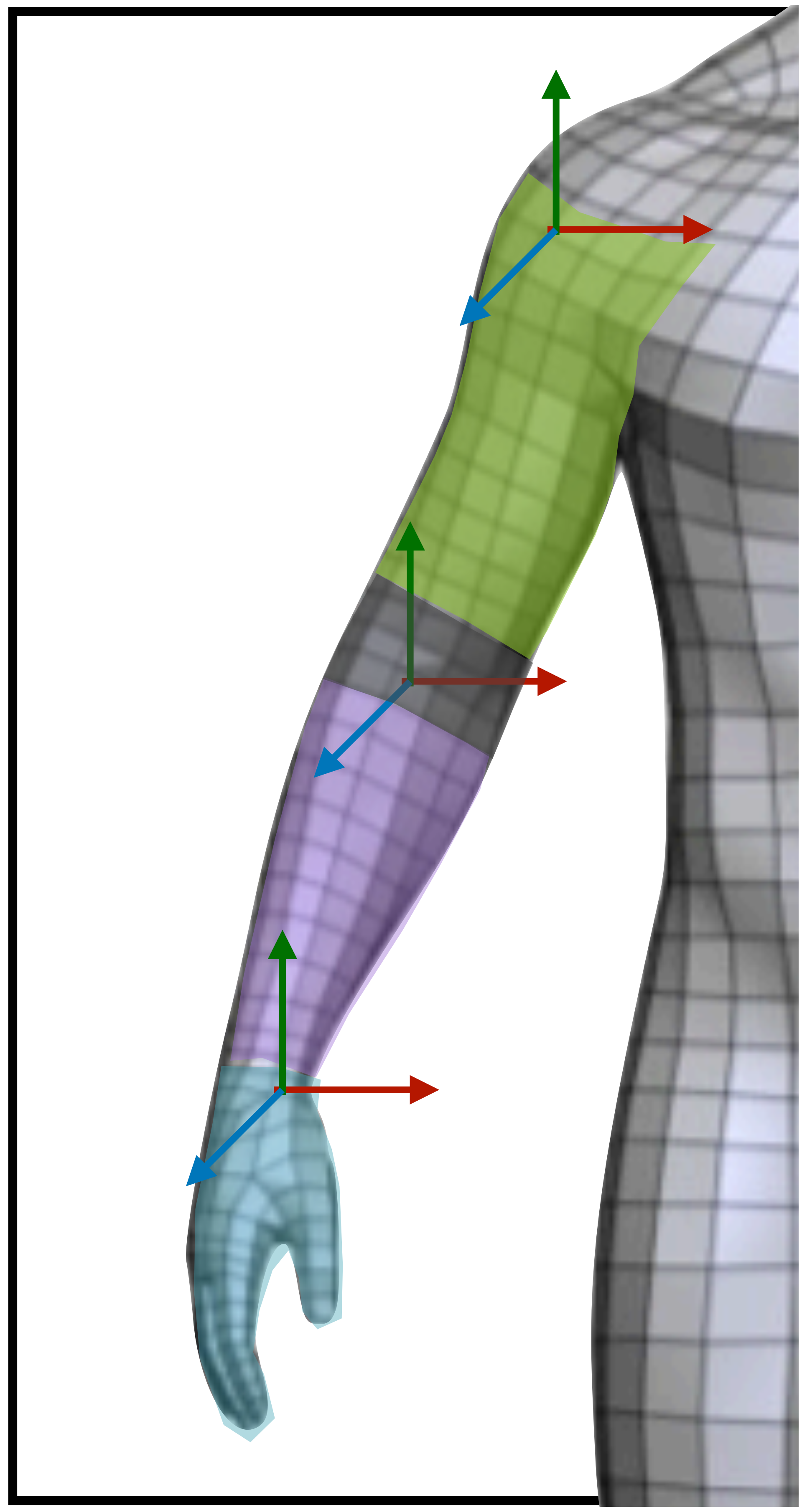
Park et al SIGGRAPH 2006

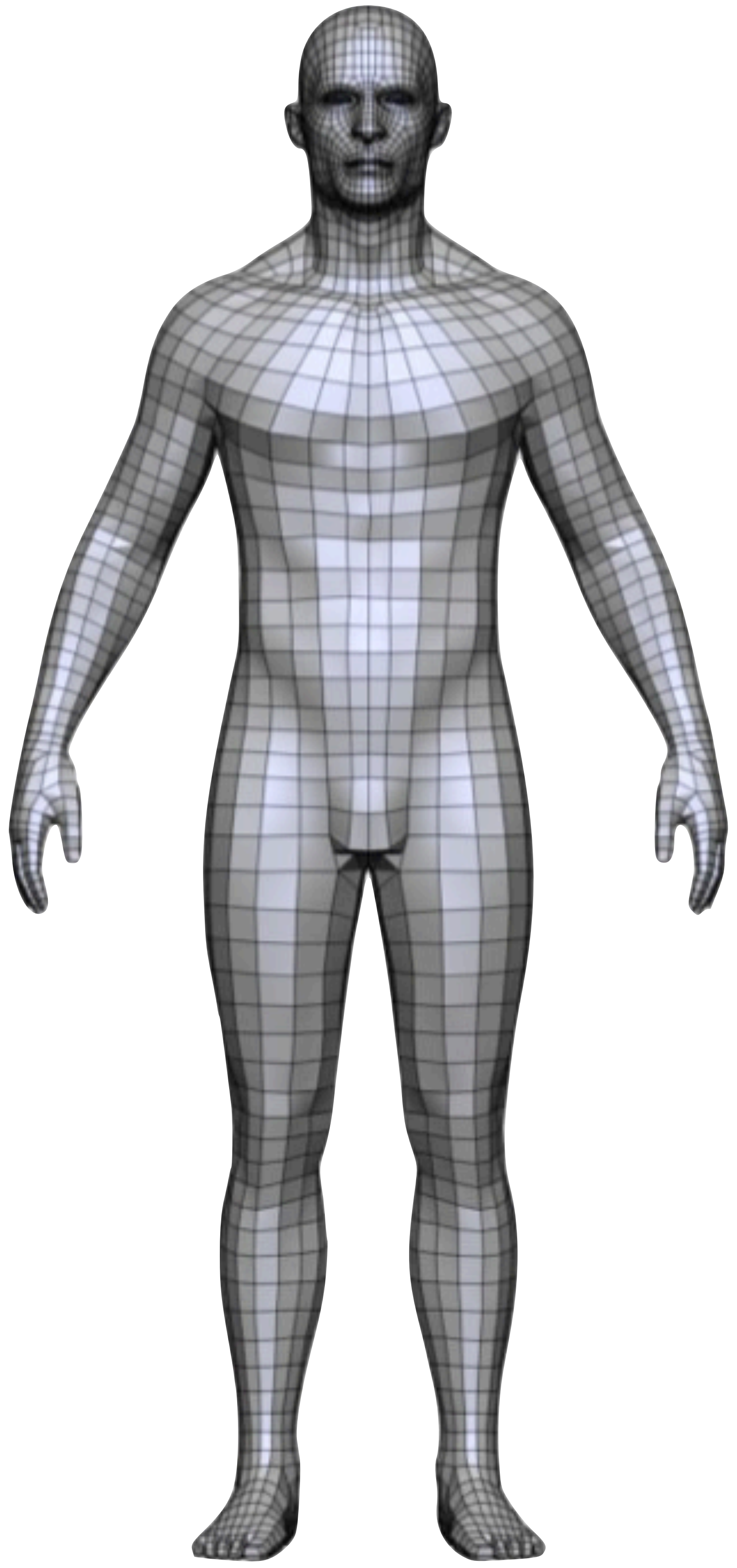
Skinning approach



Linear blend skinning







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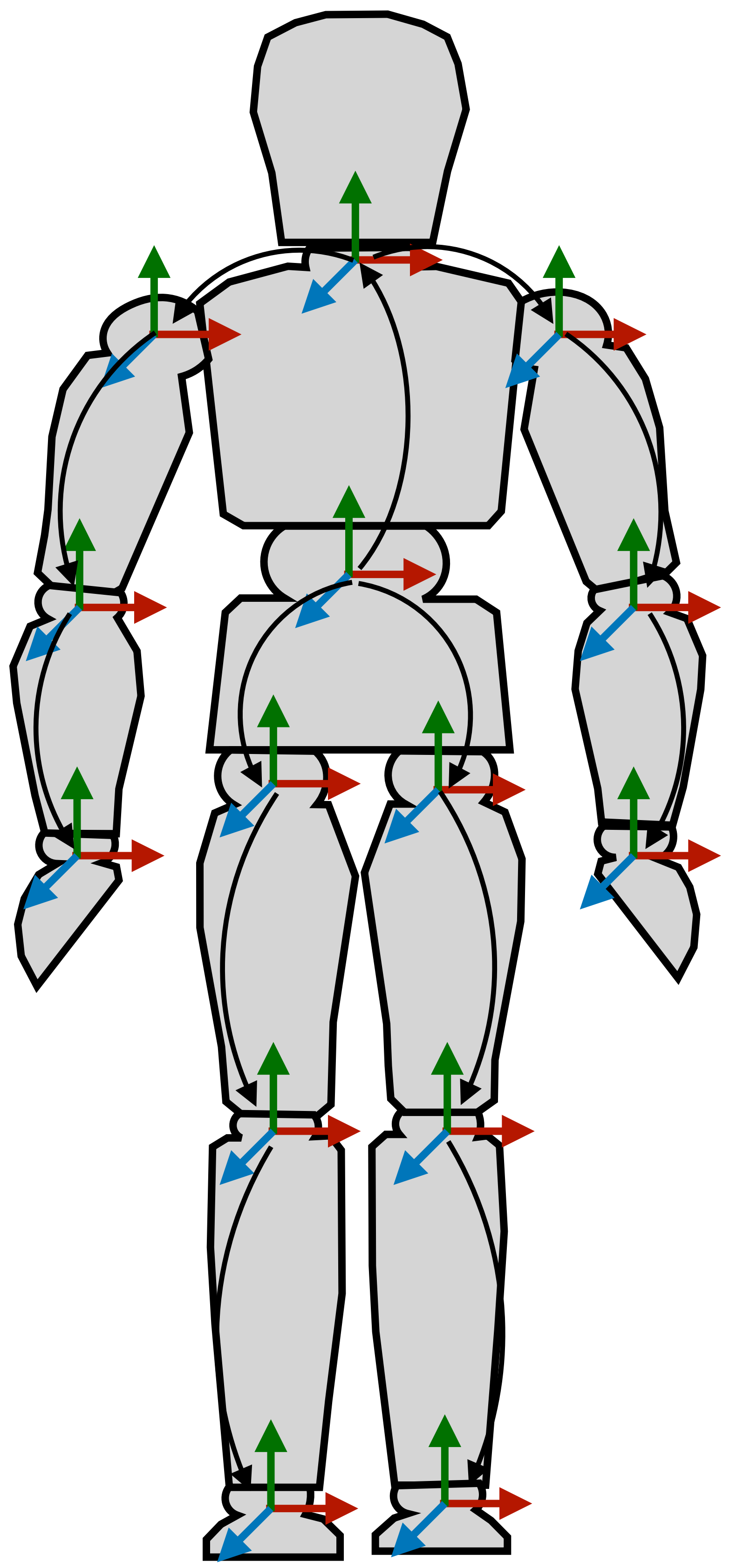

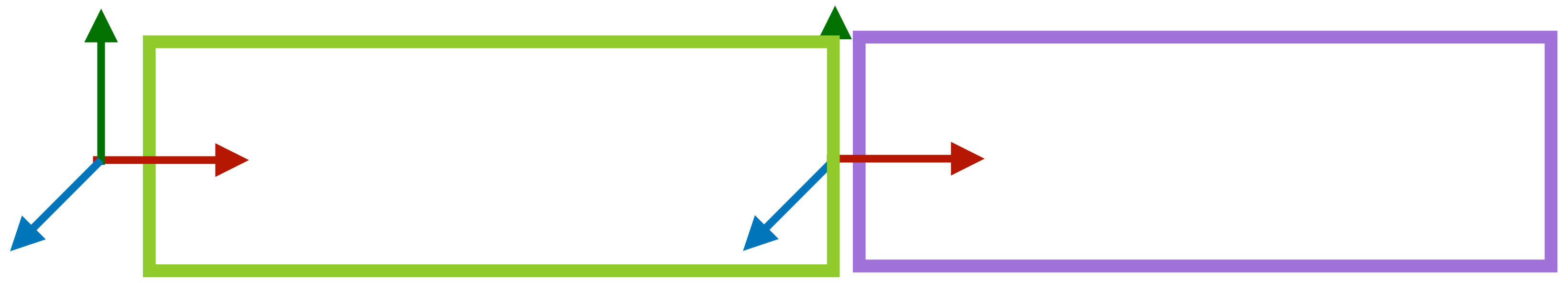

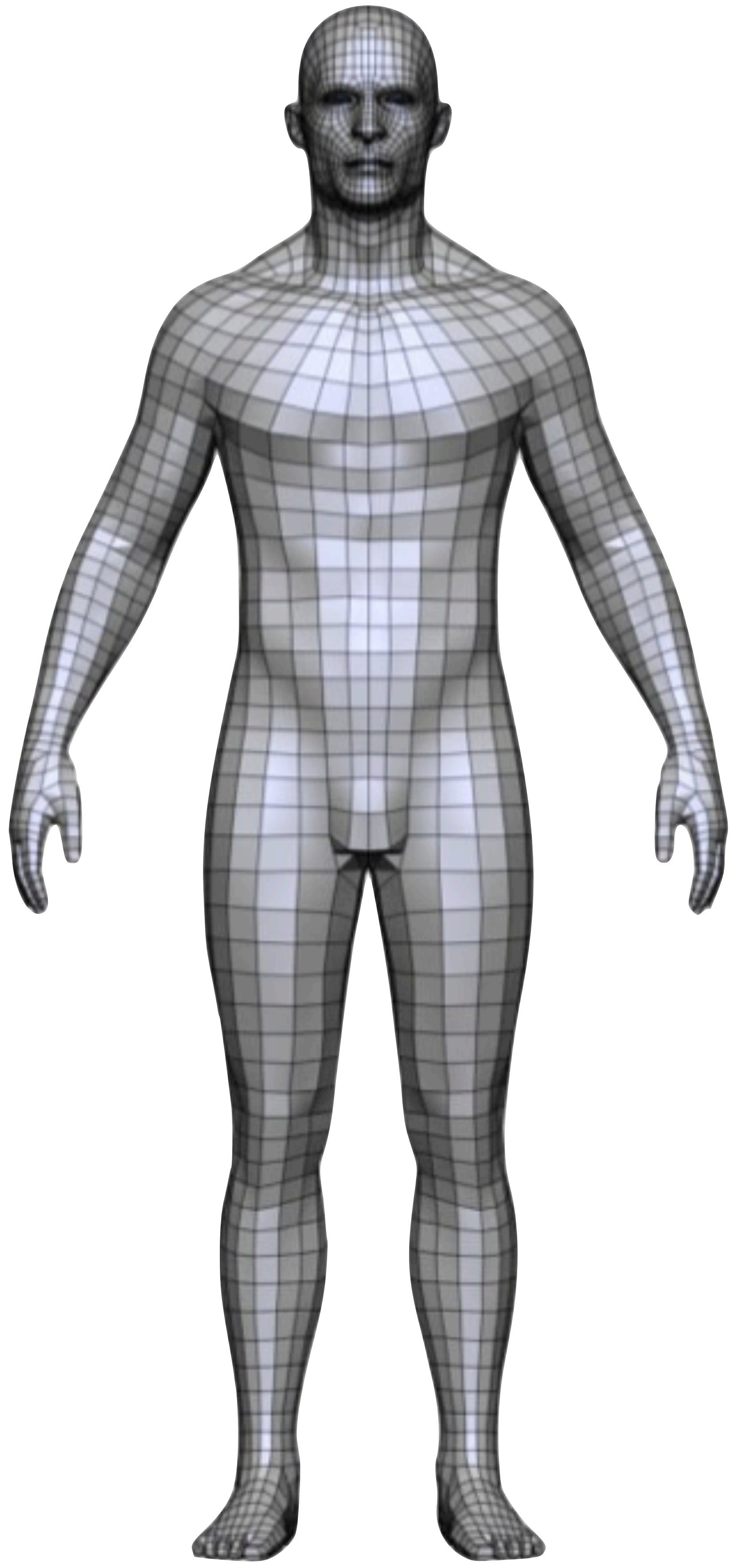

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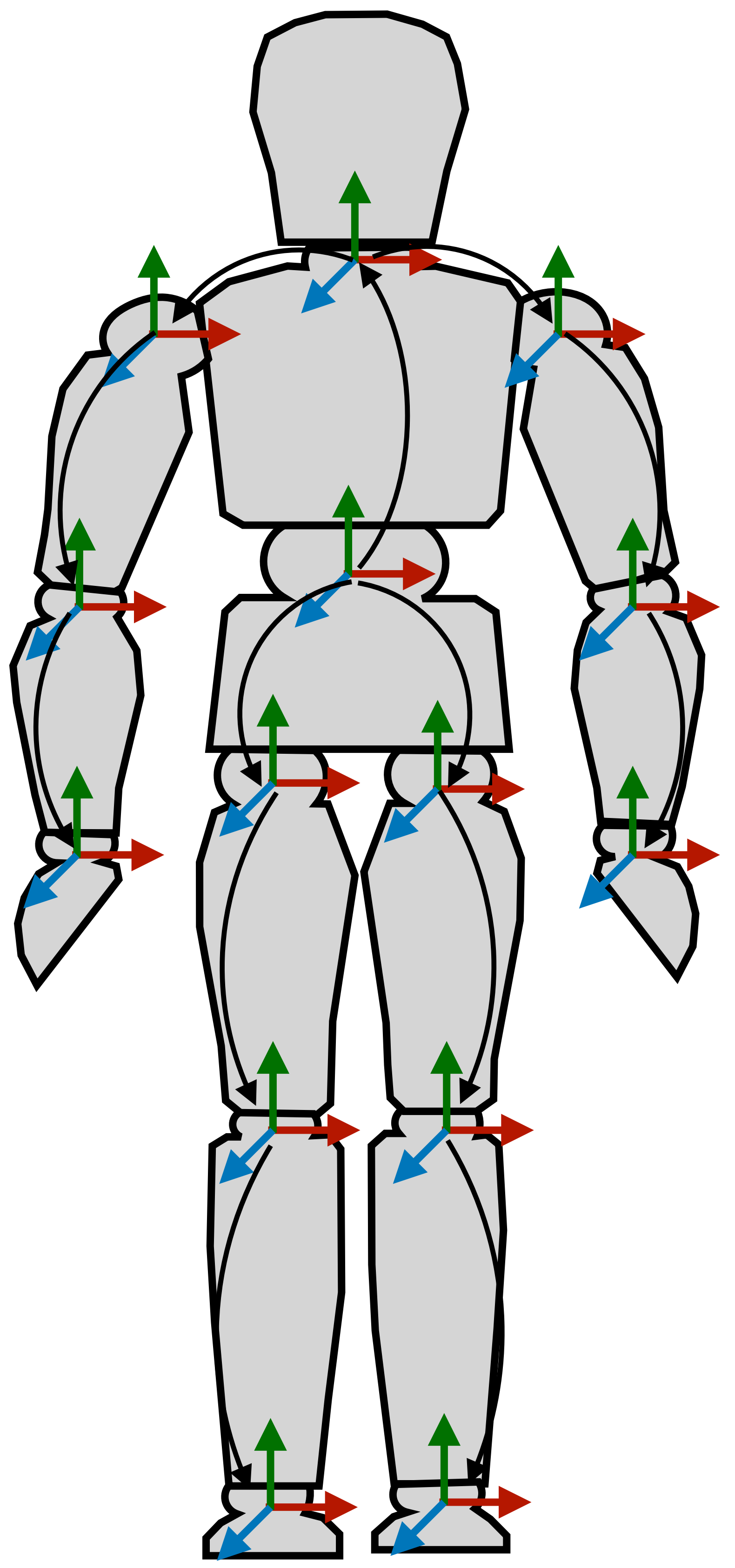

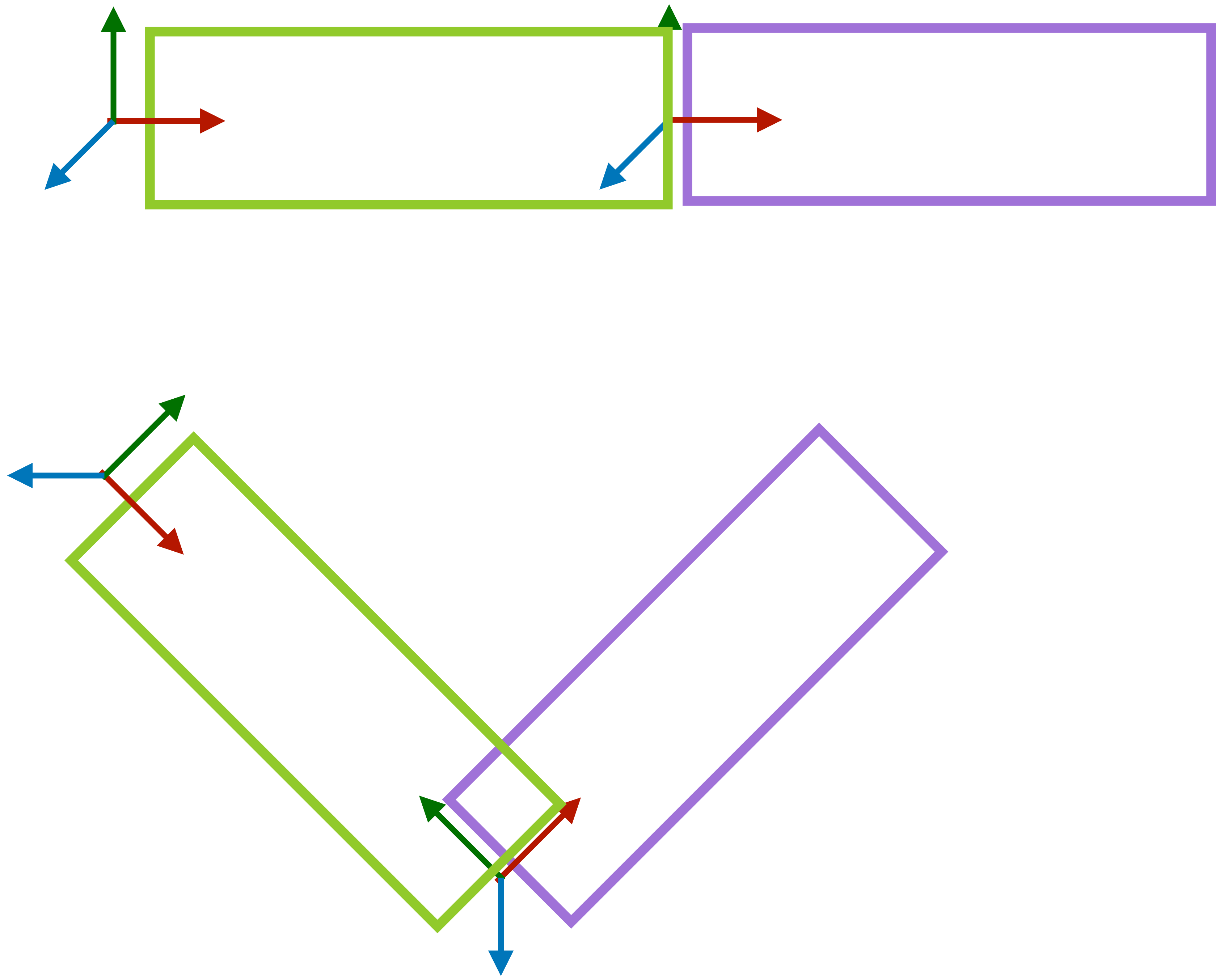

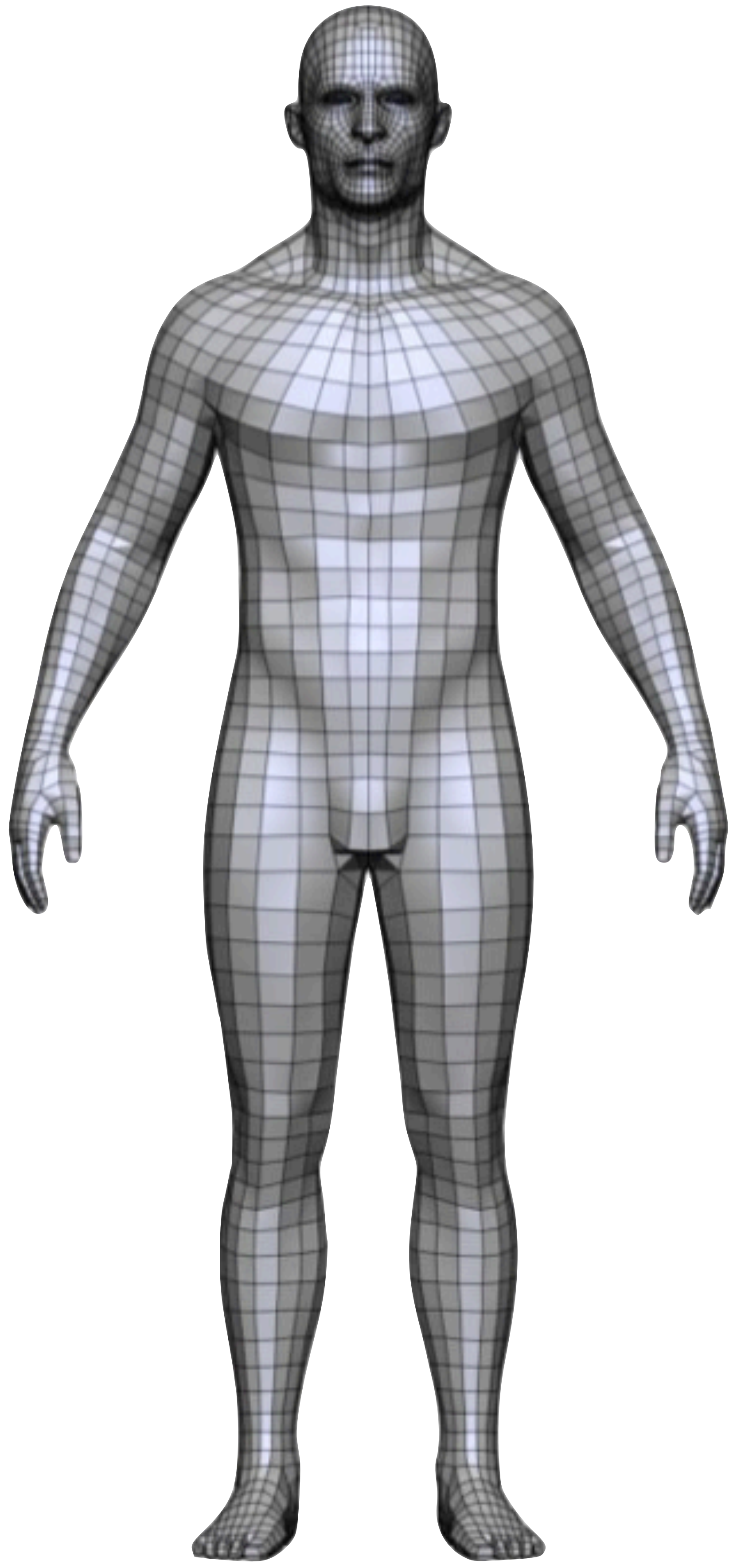

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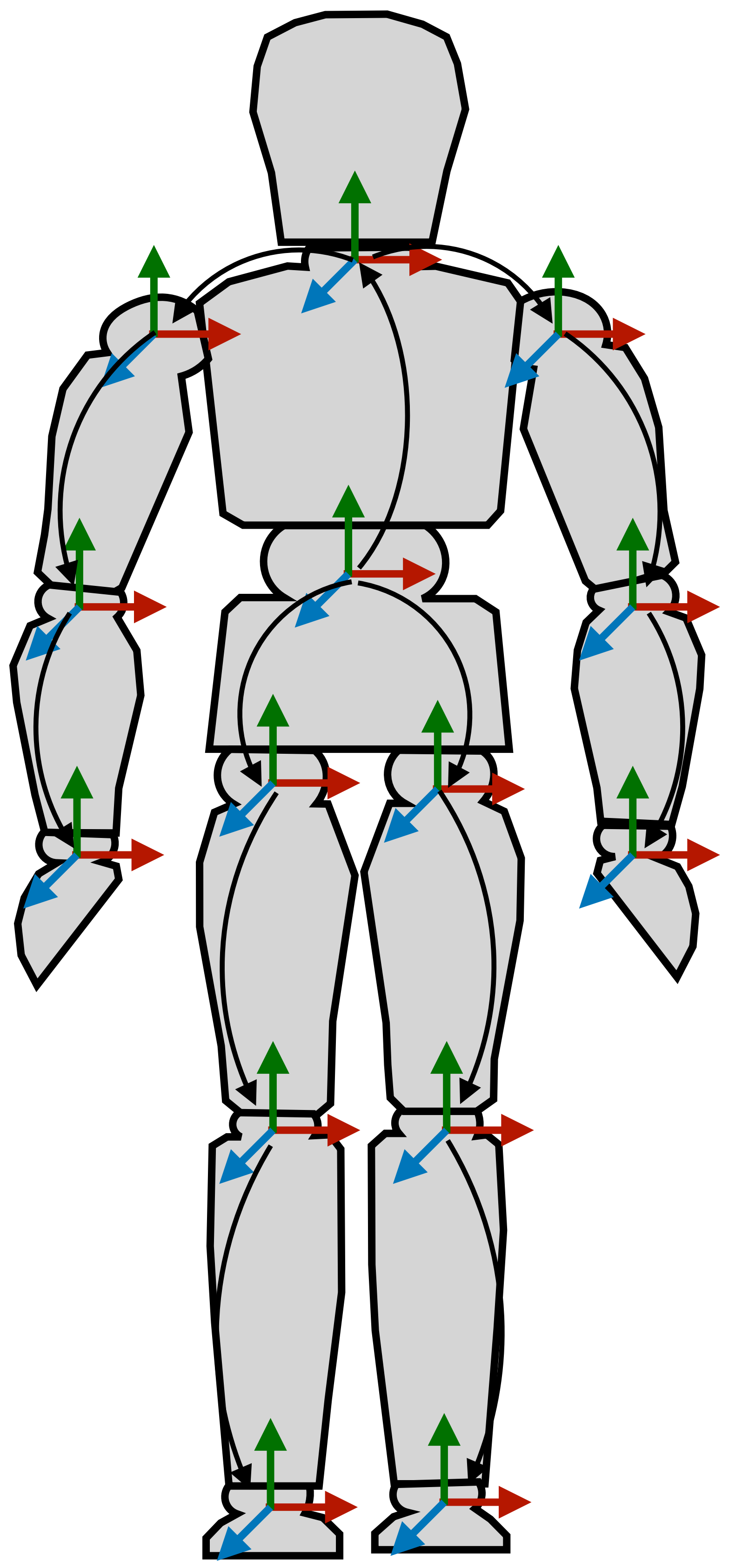

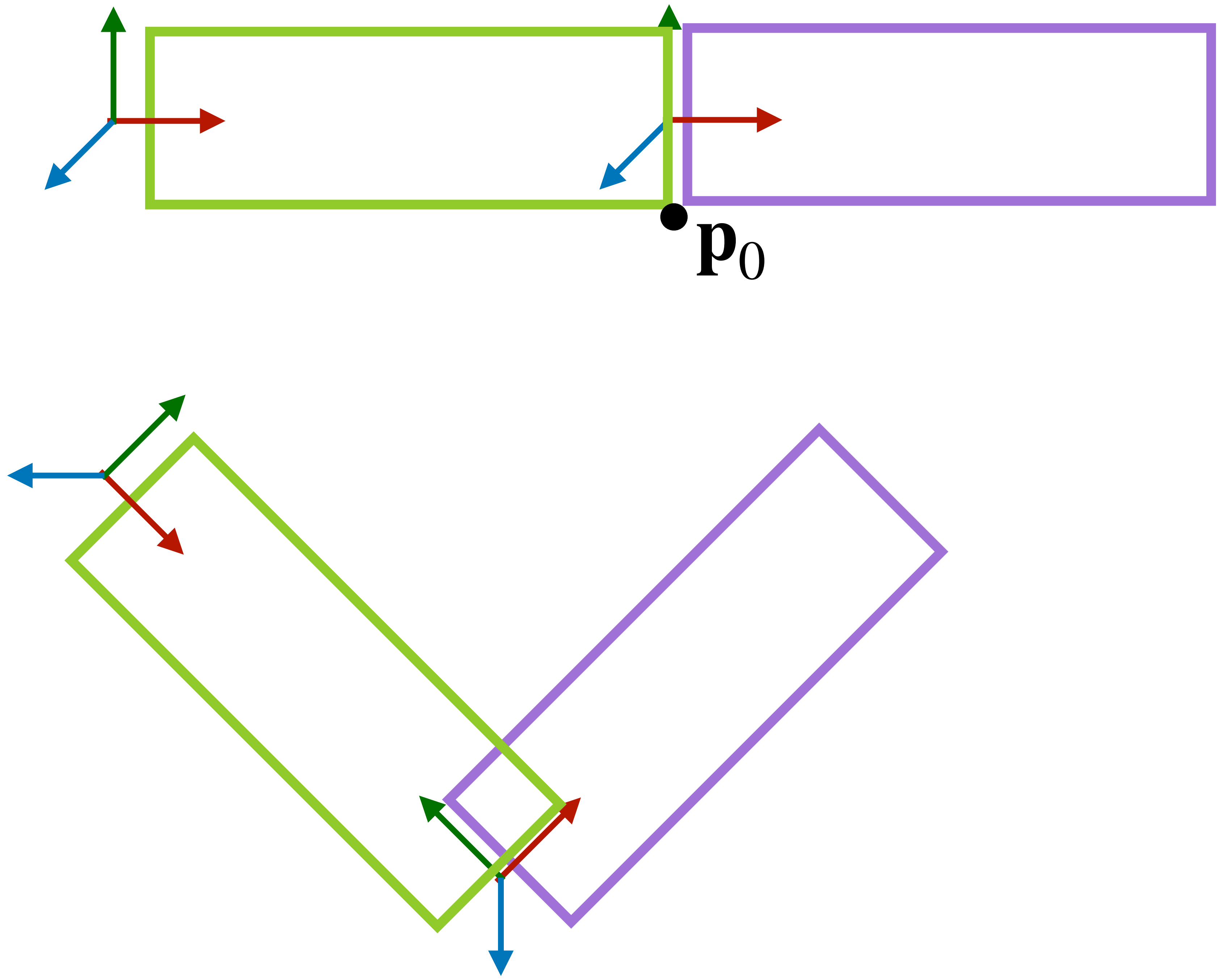

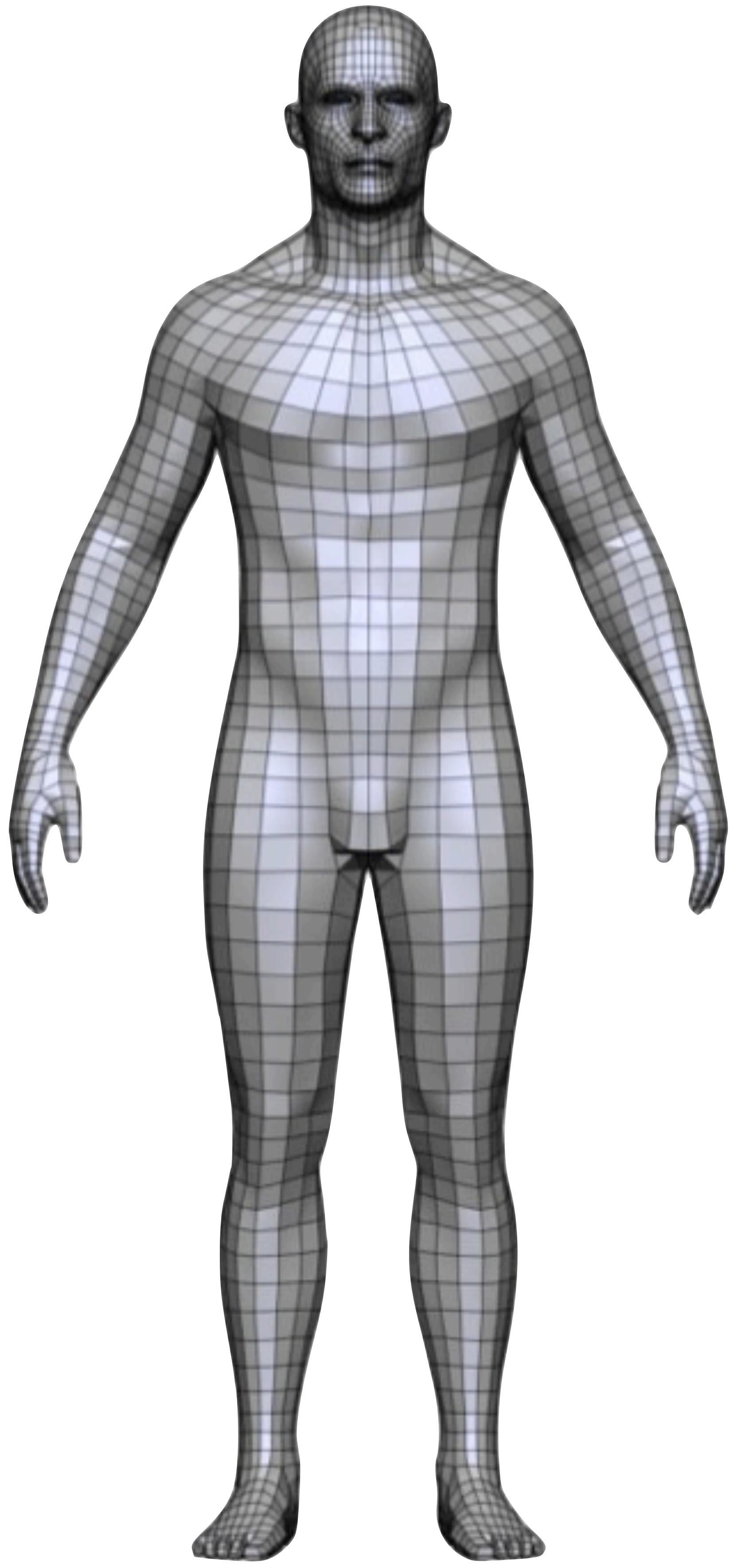

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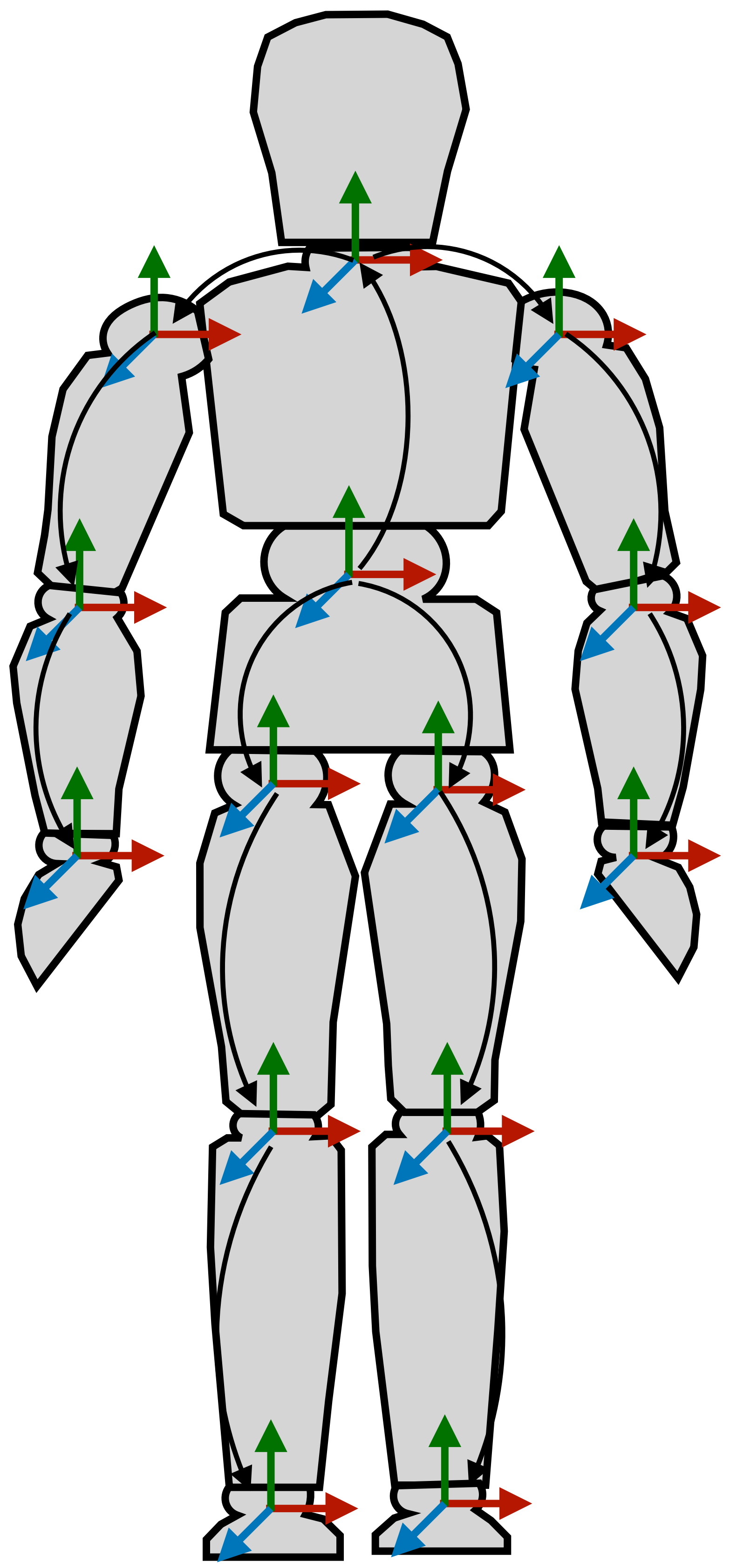

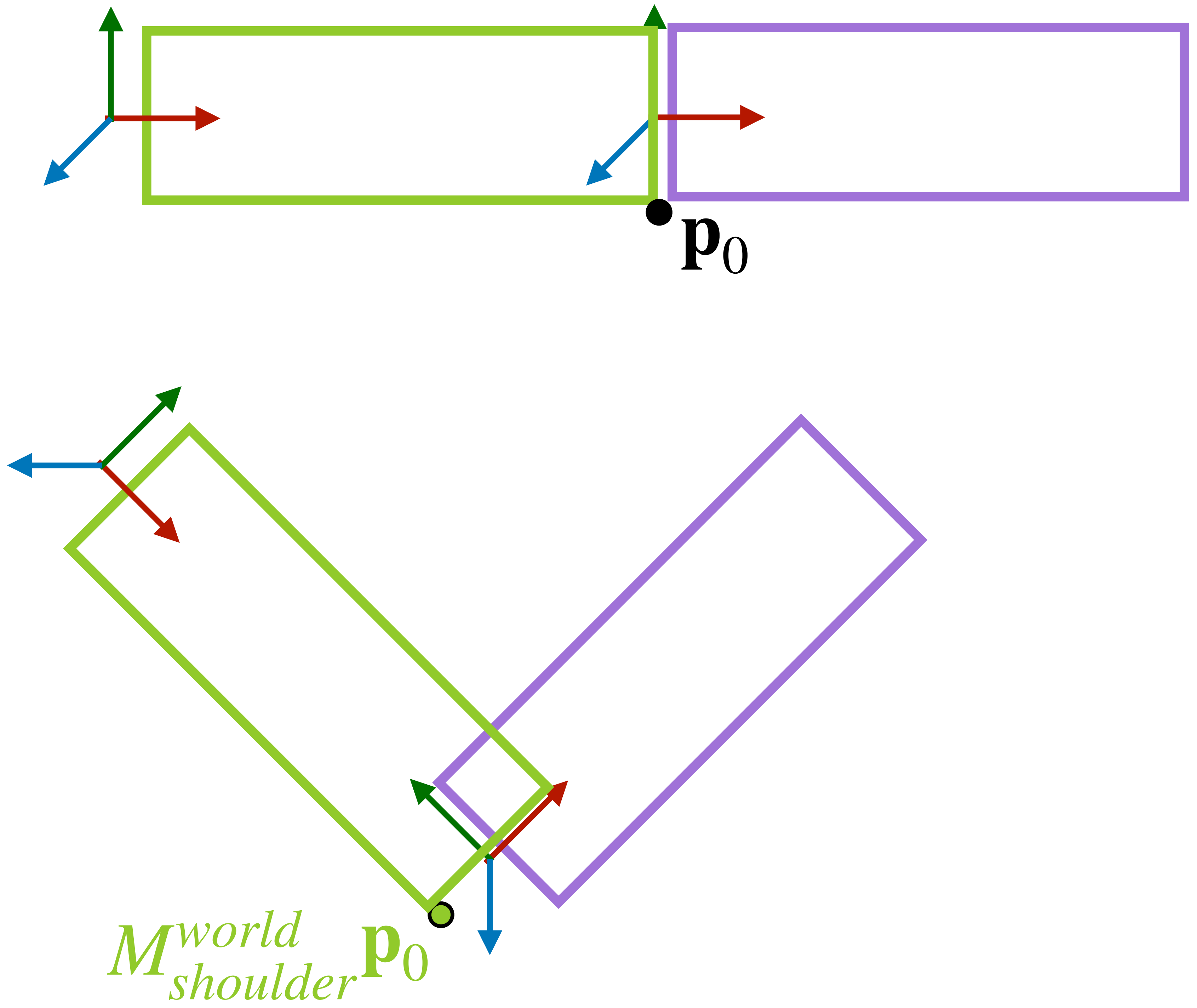

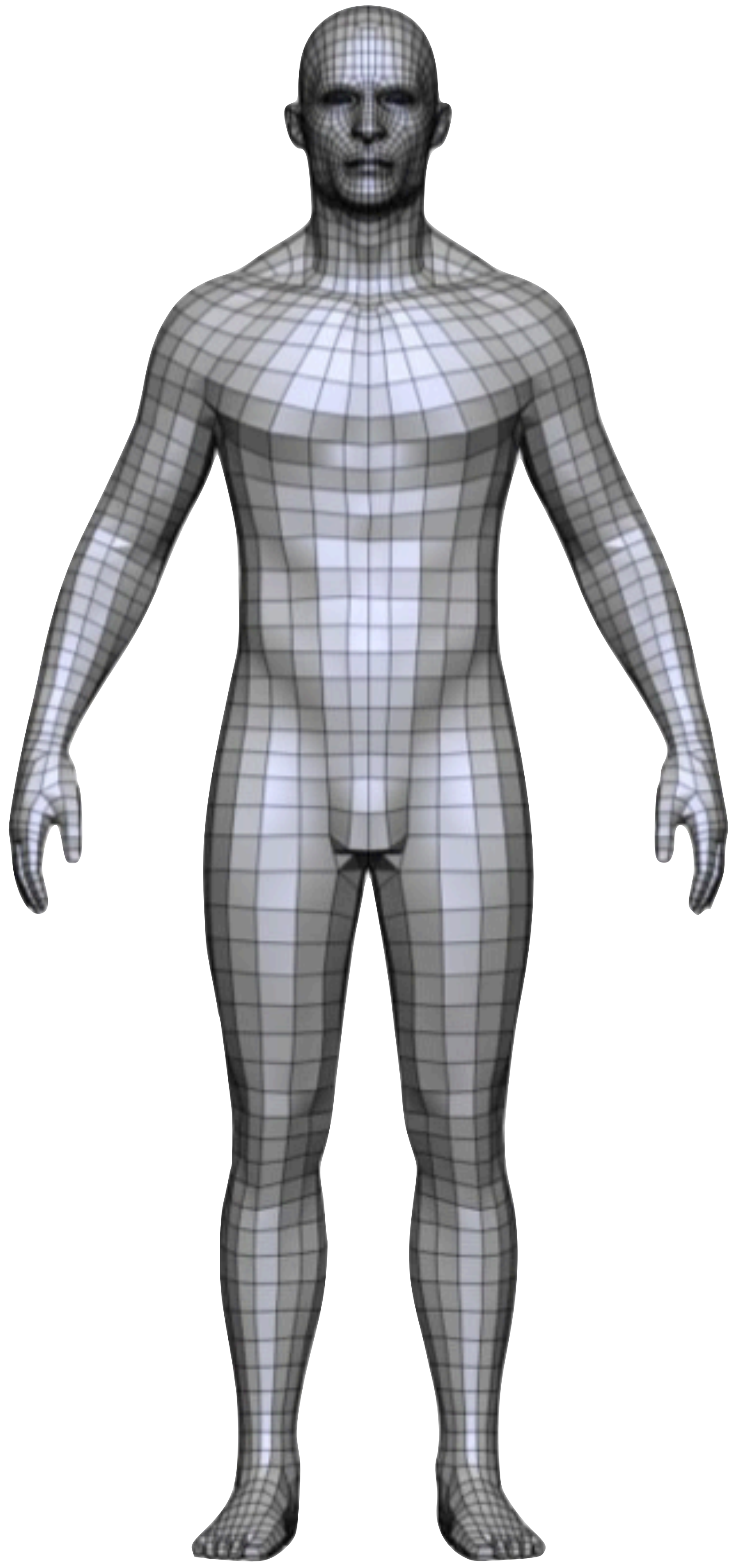

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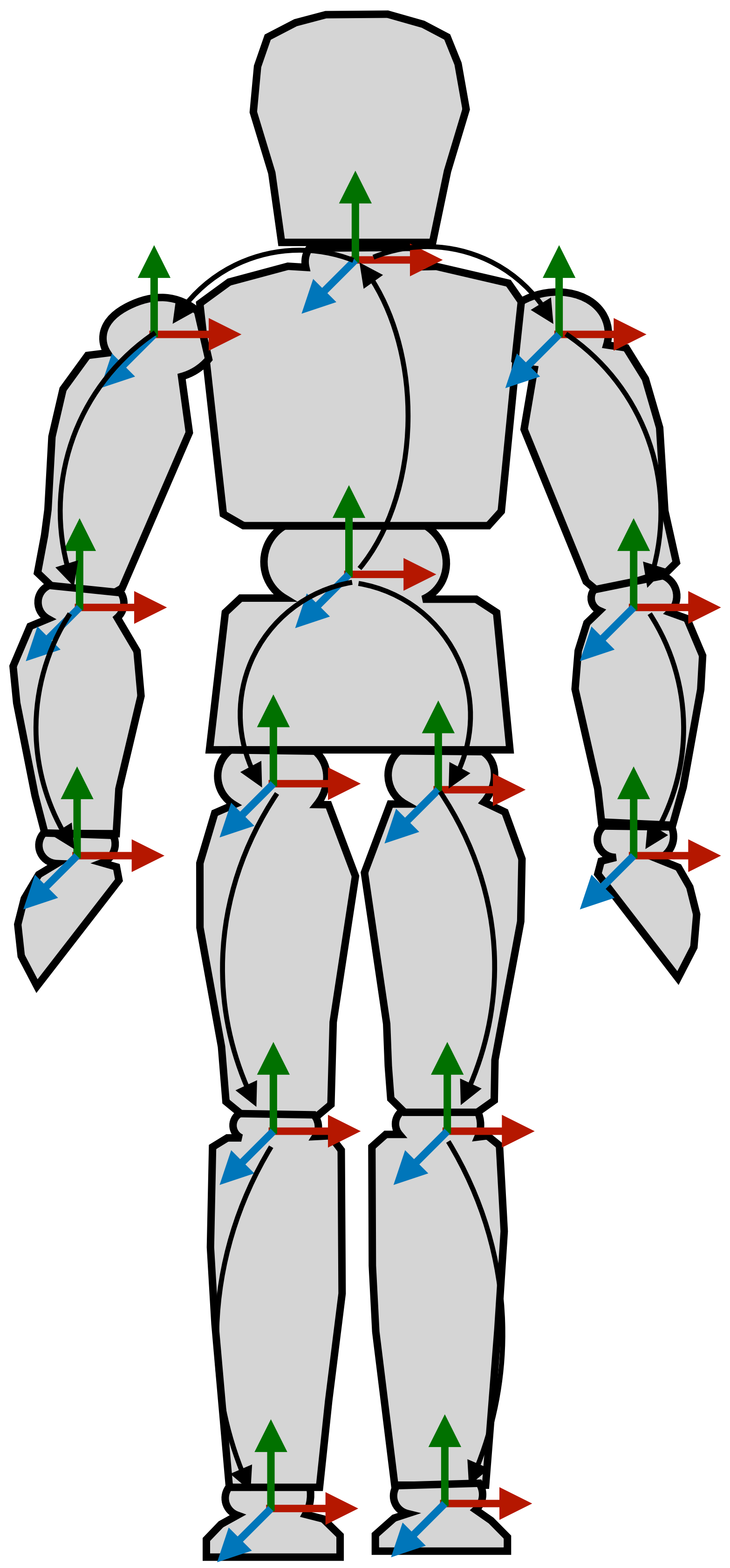

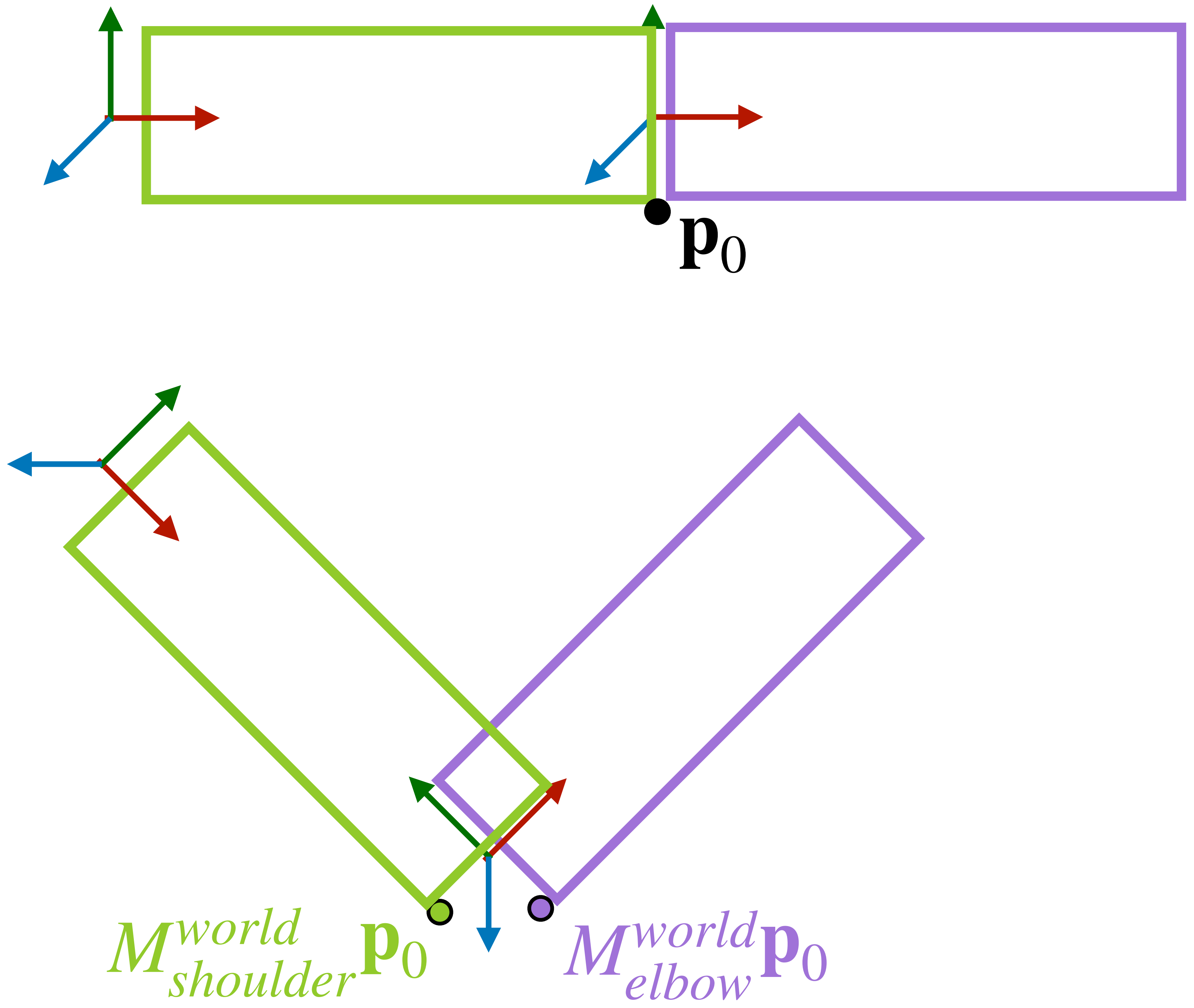

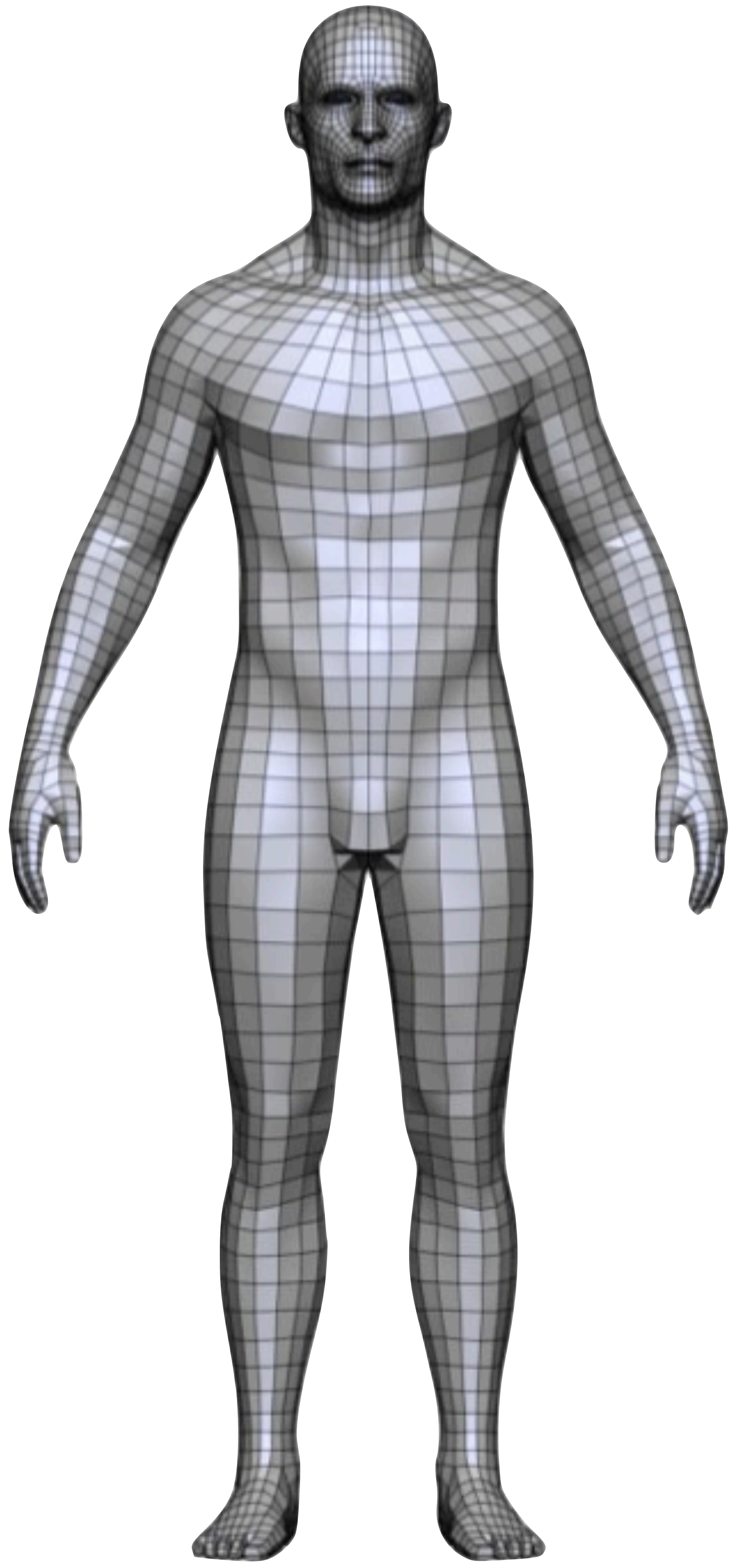

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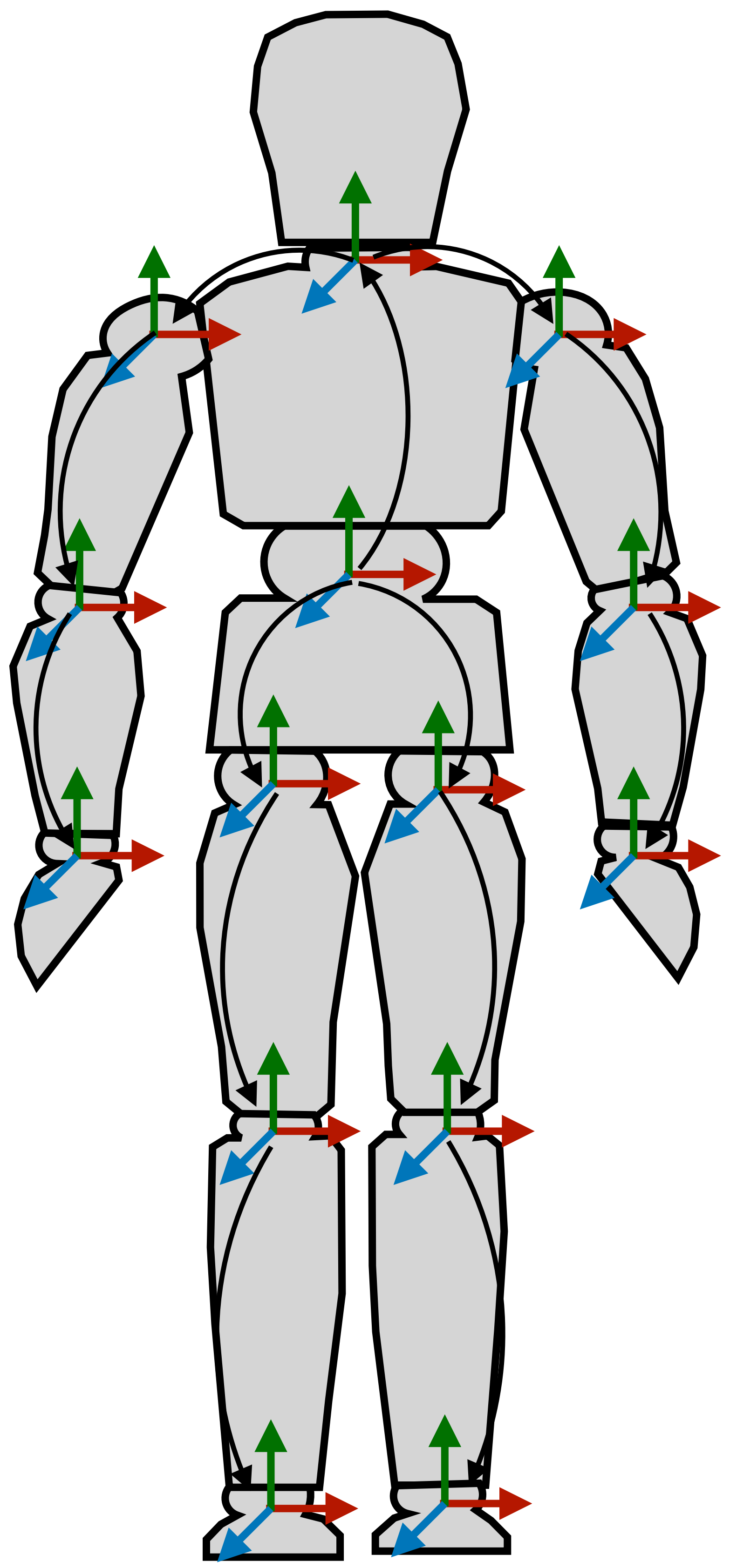

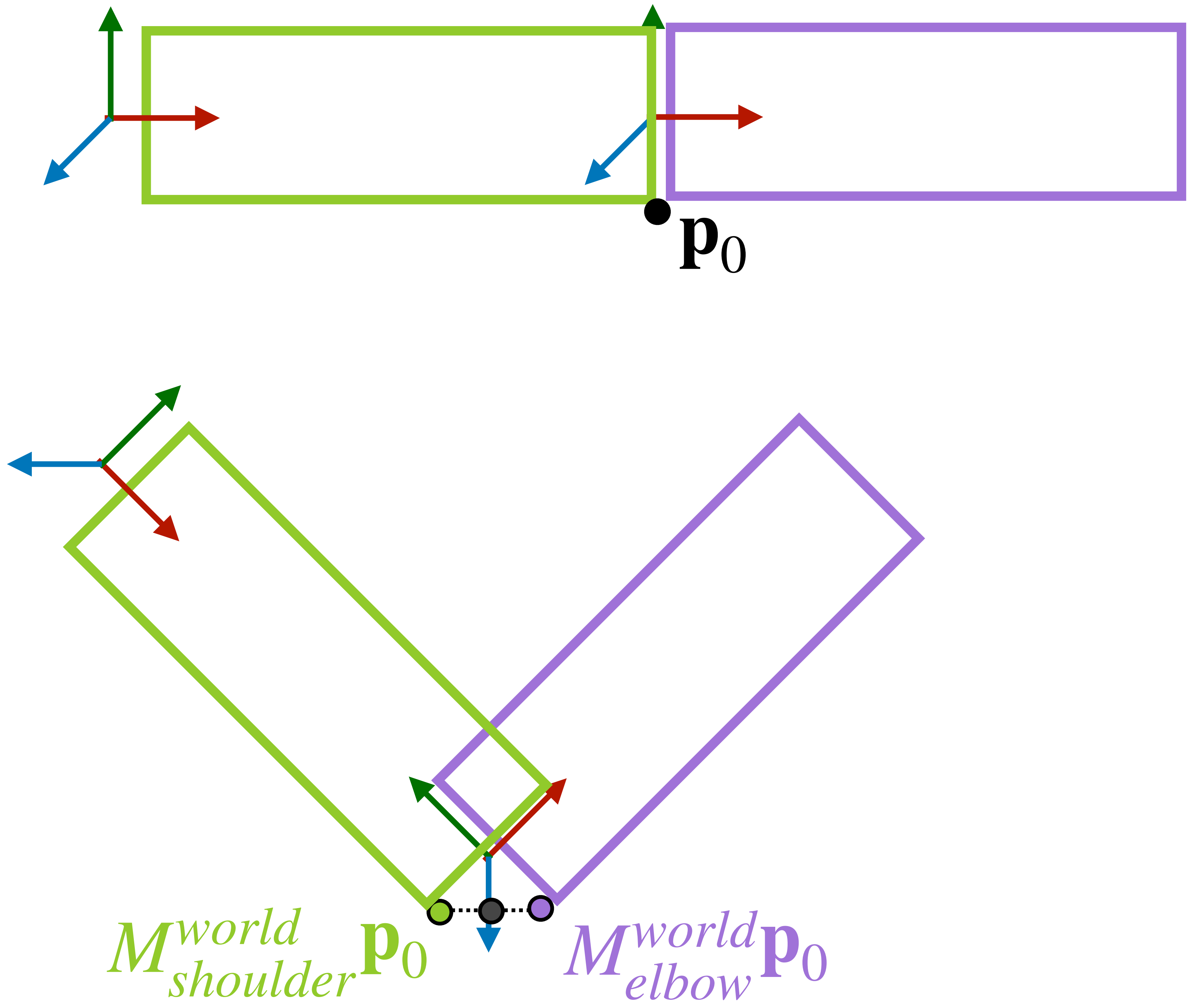

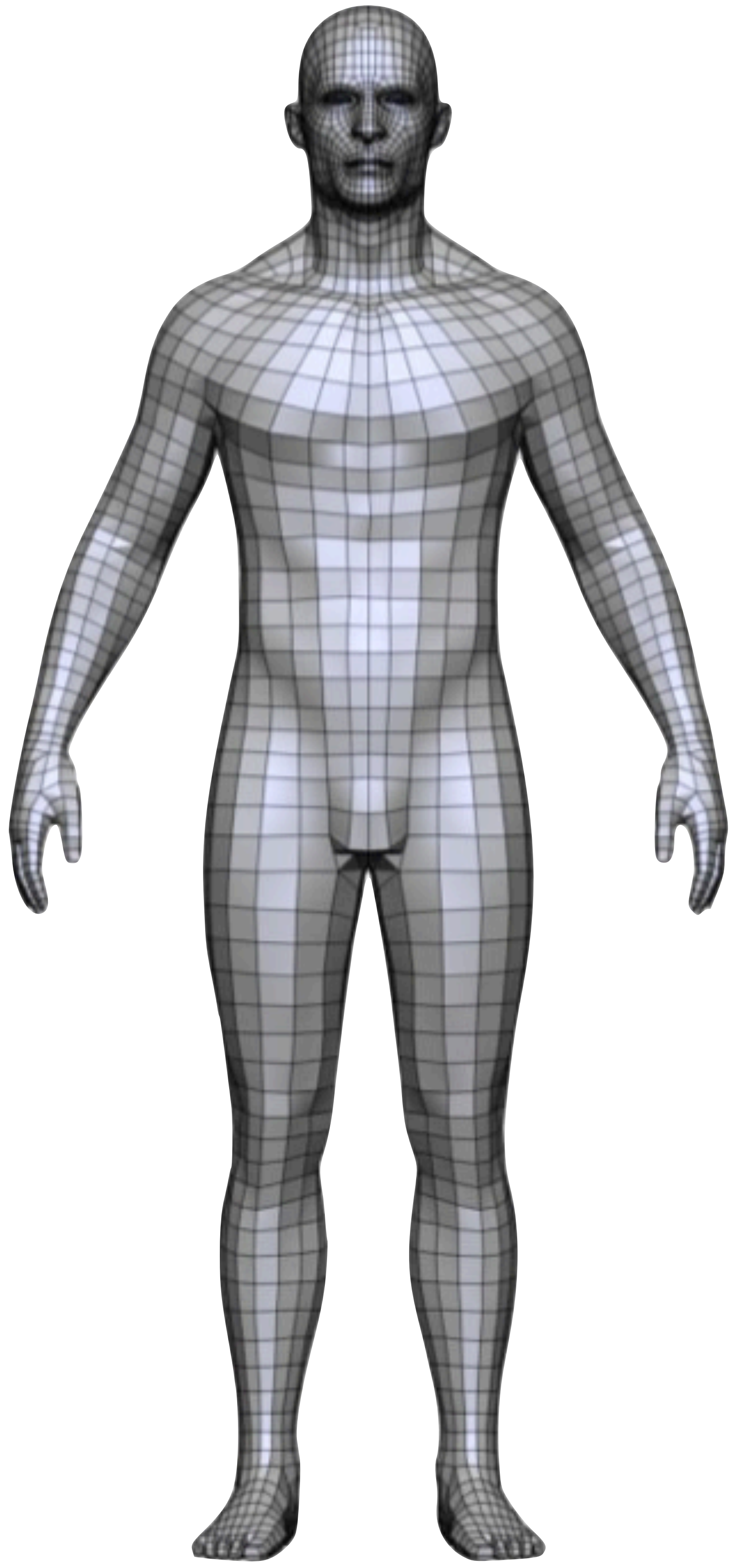

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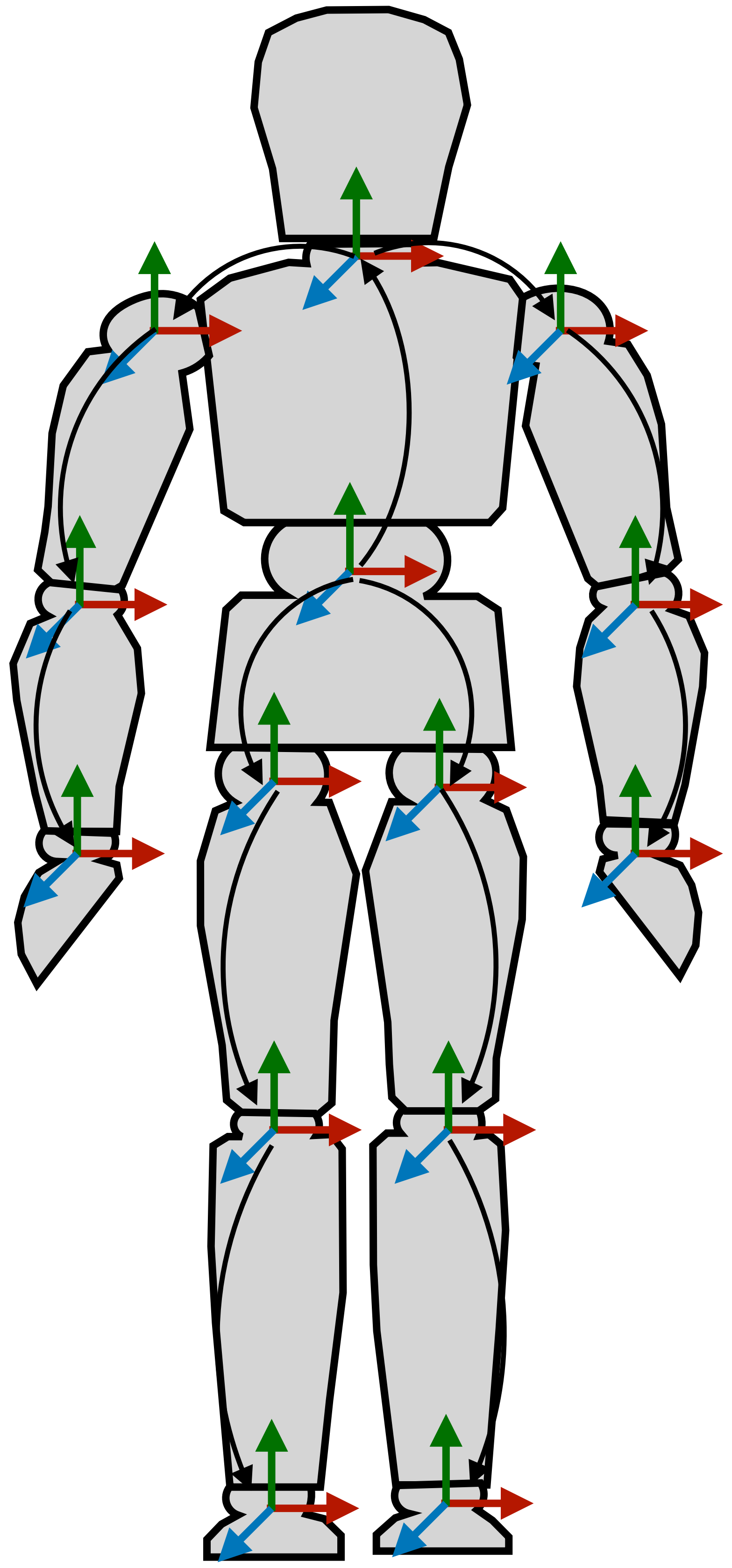
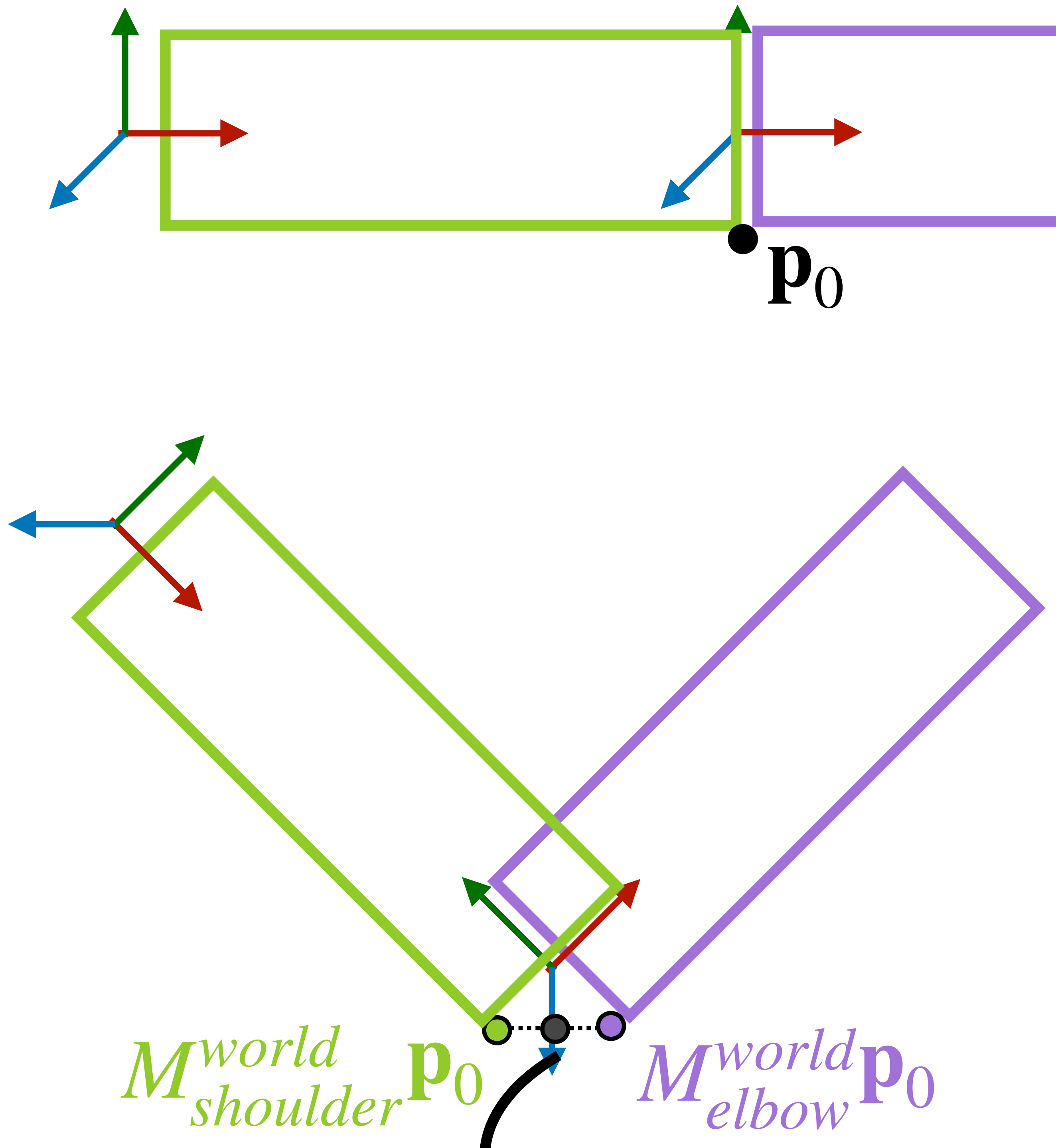
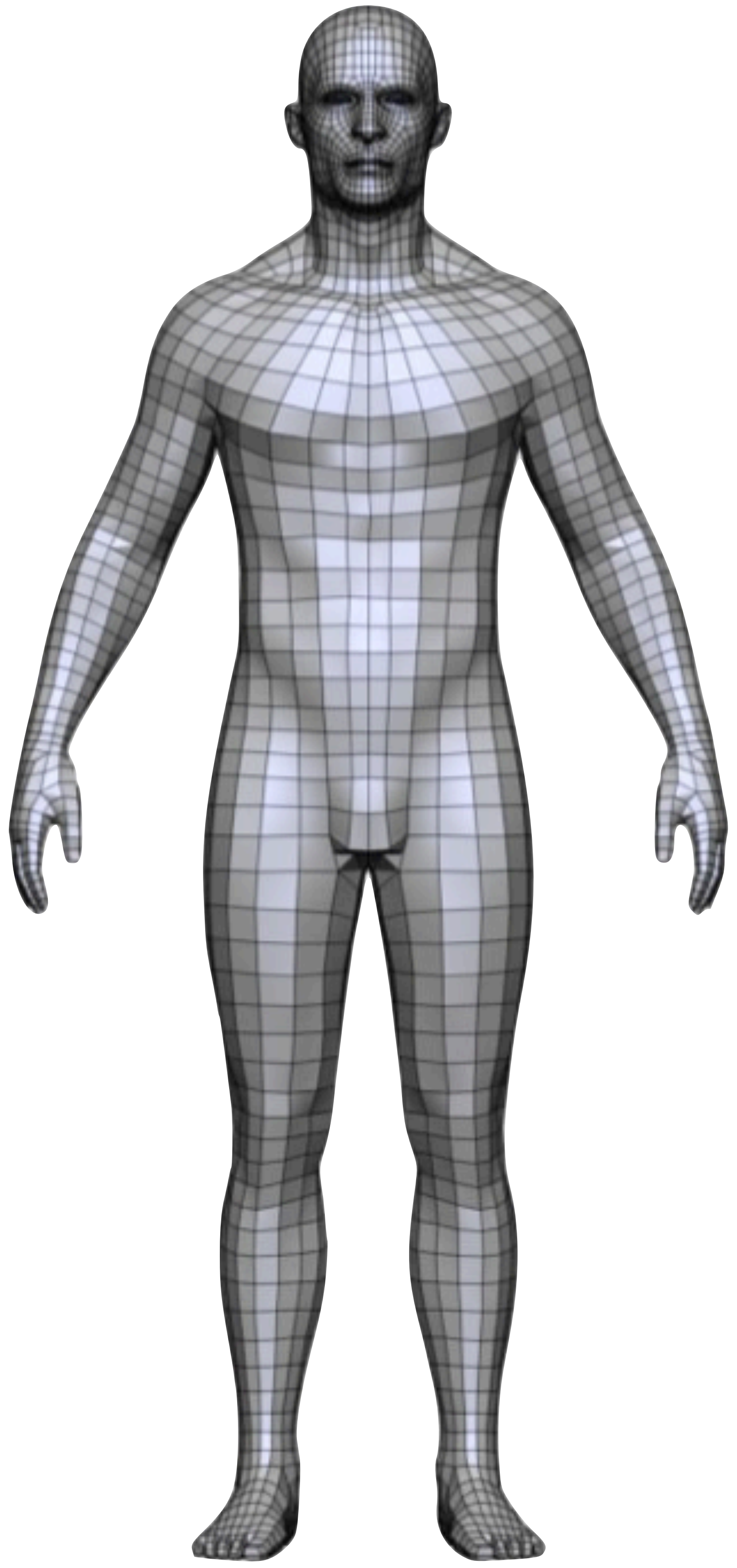




$$\mathbf{p} = 0.5 M^{world}_{shoulder}\mathbf{p}_0 + 0.5 M^{world}_{elbow}\mathbf{p}_0$$



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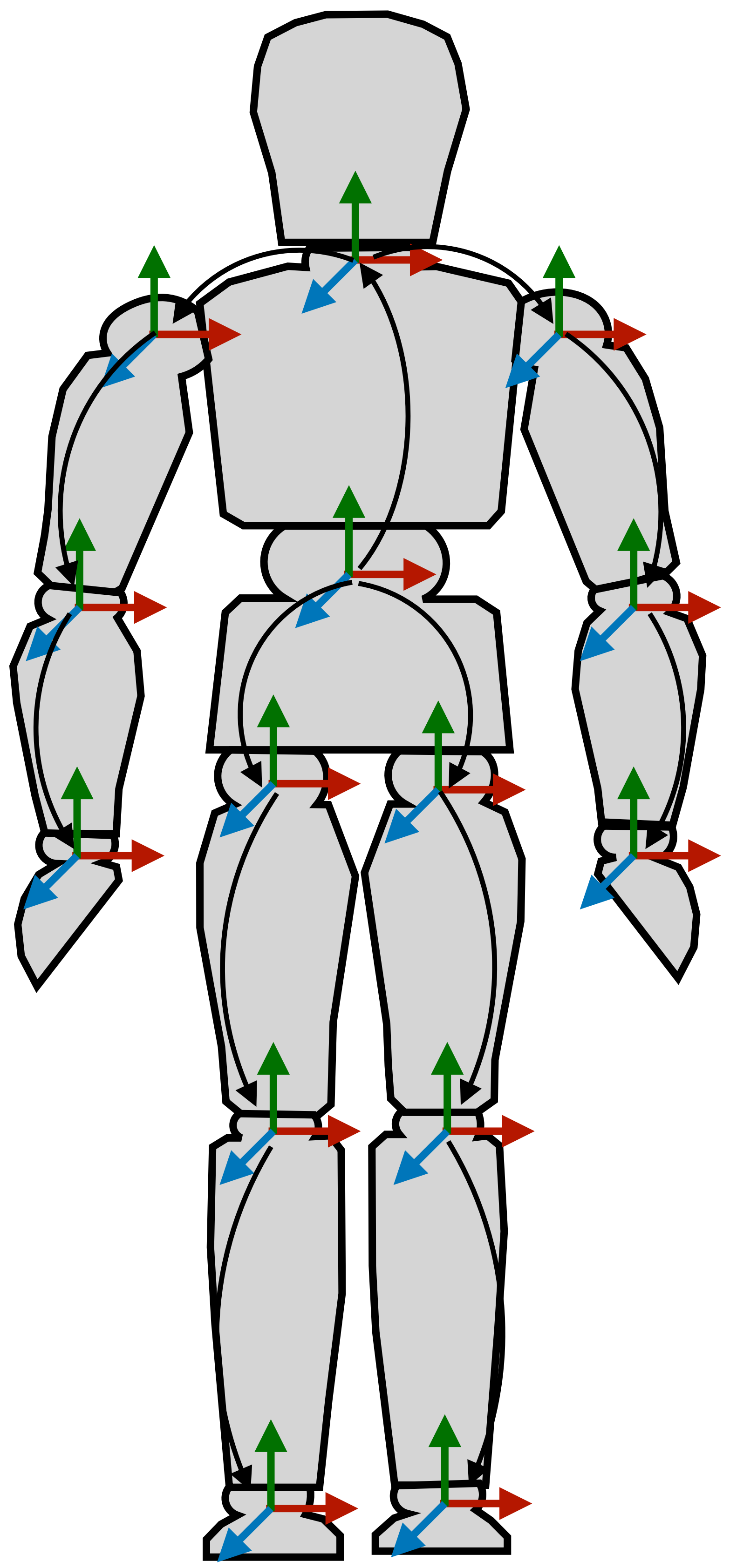

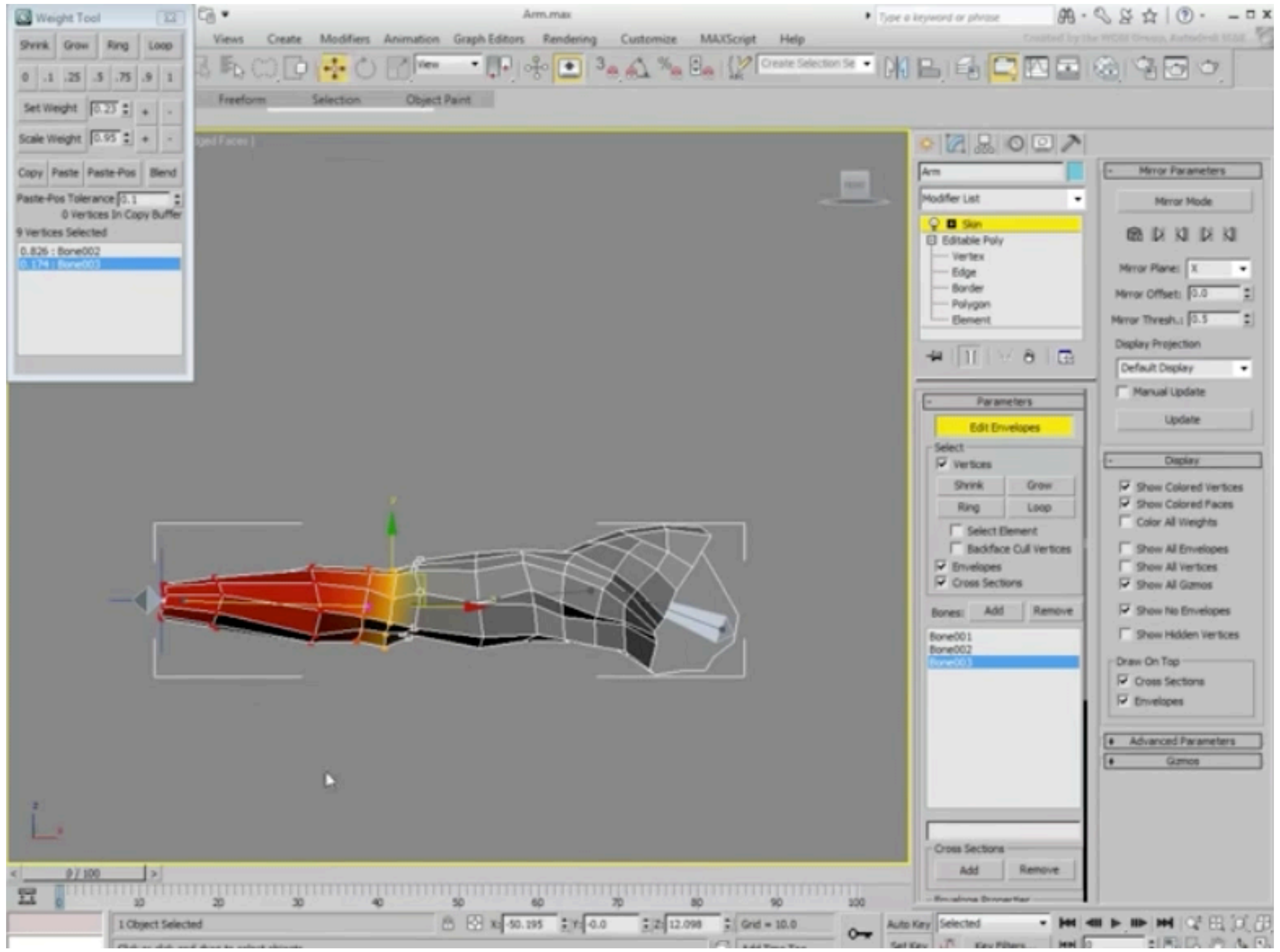

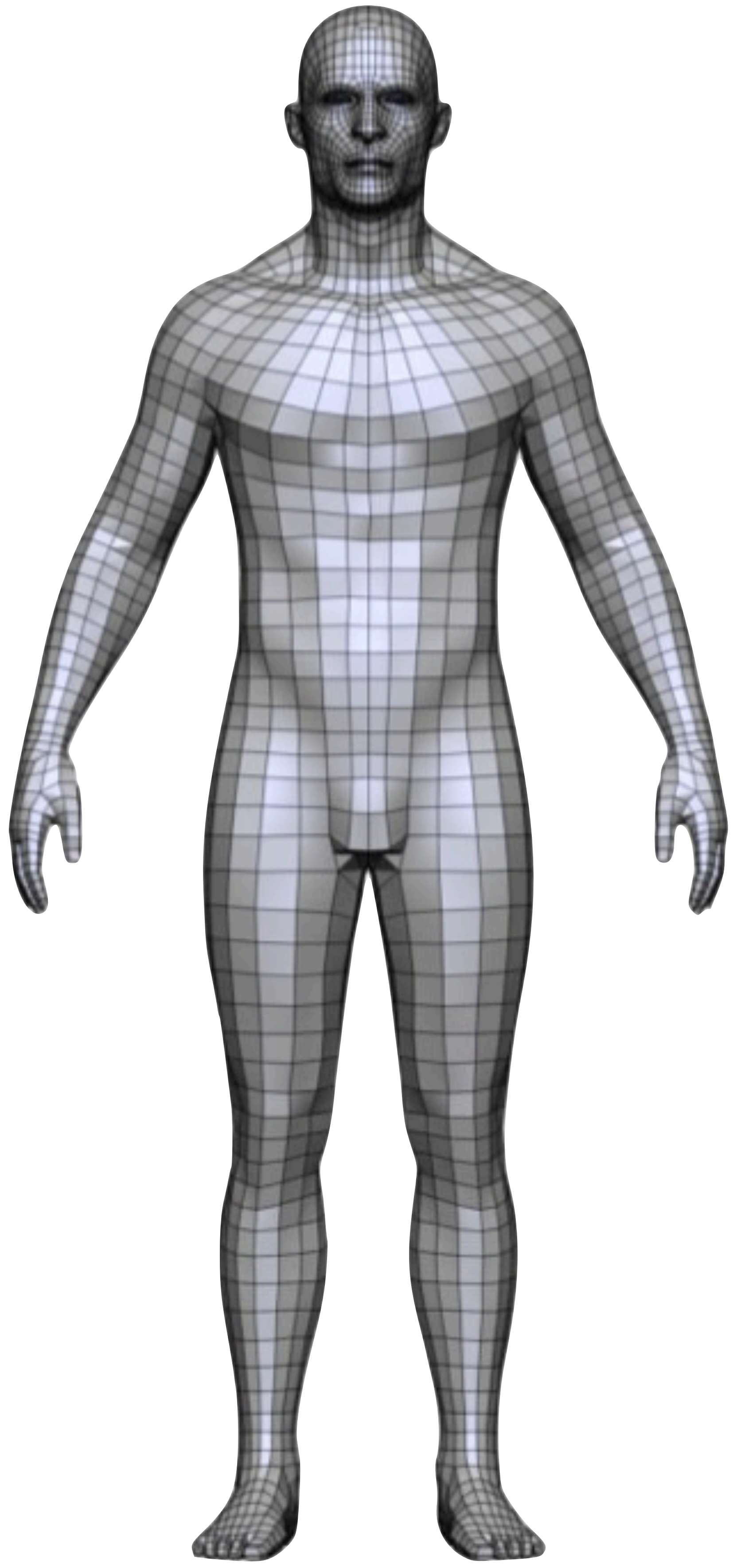

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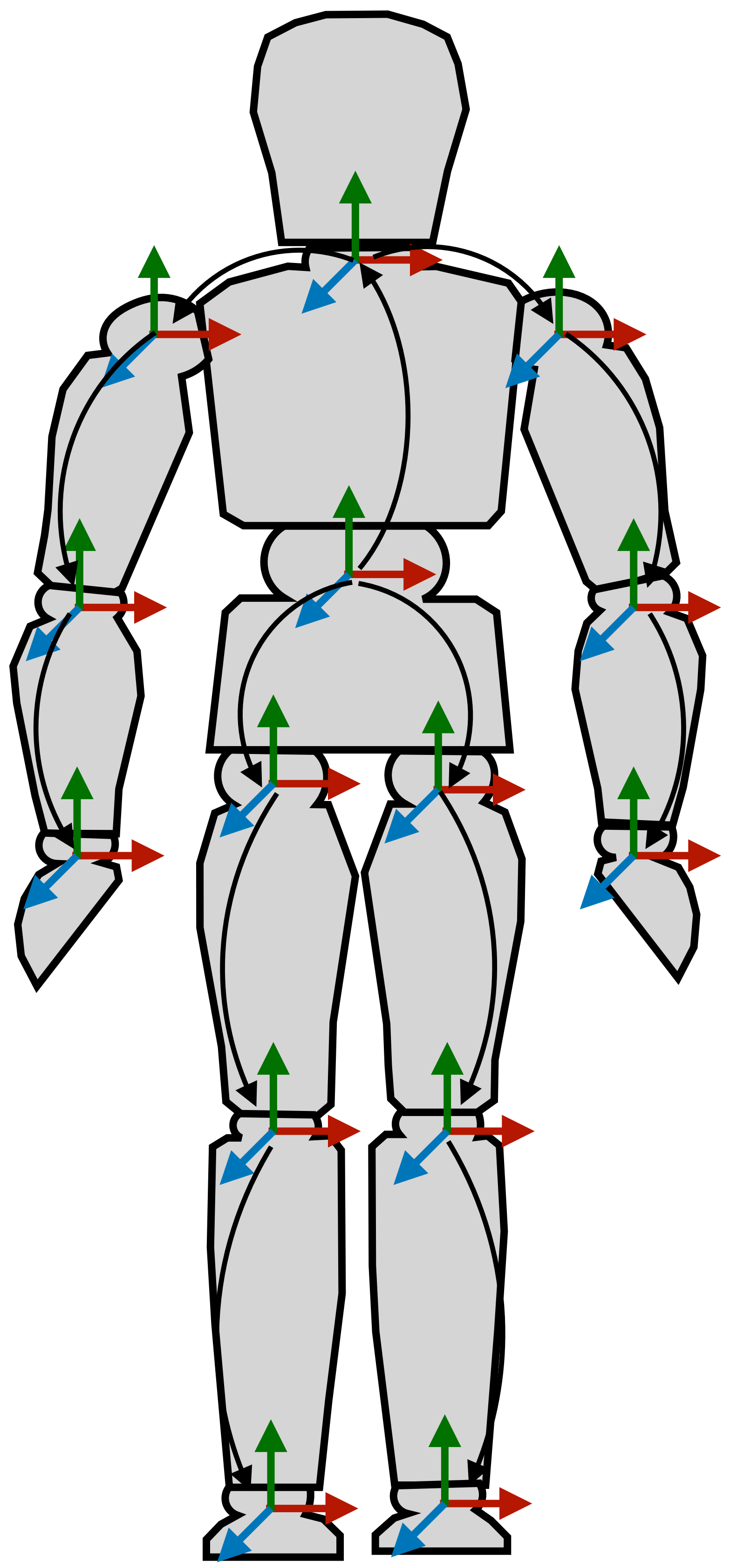
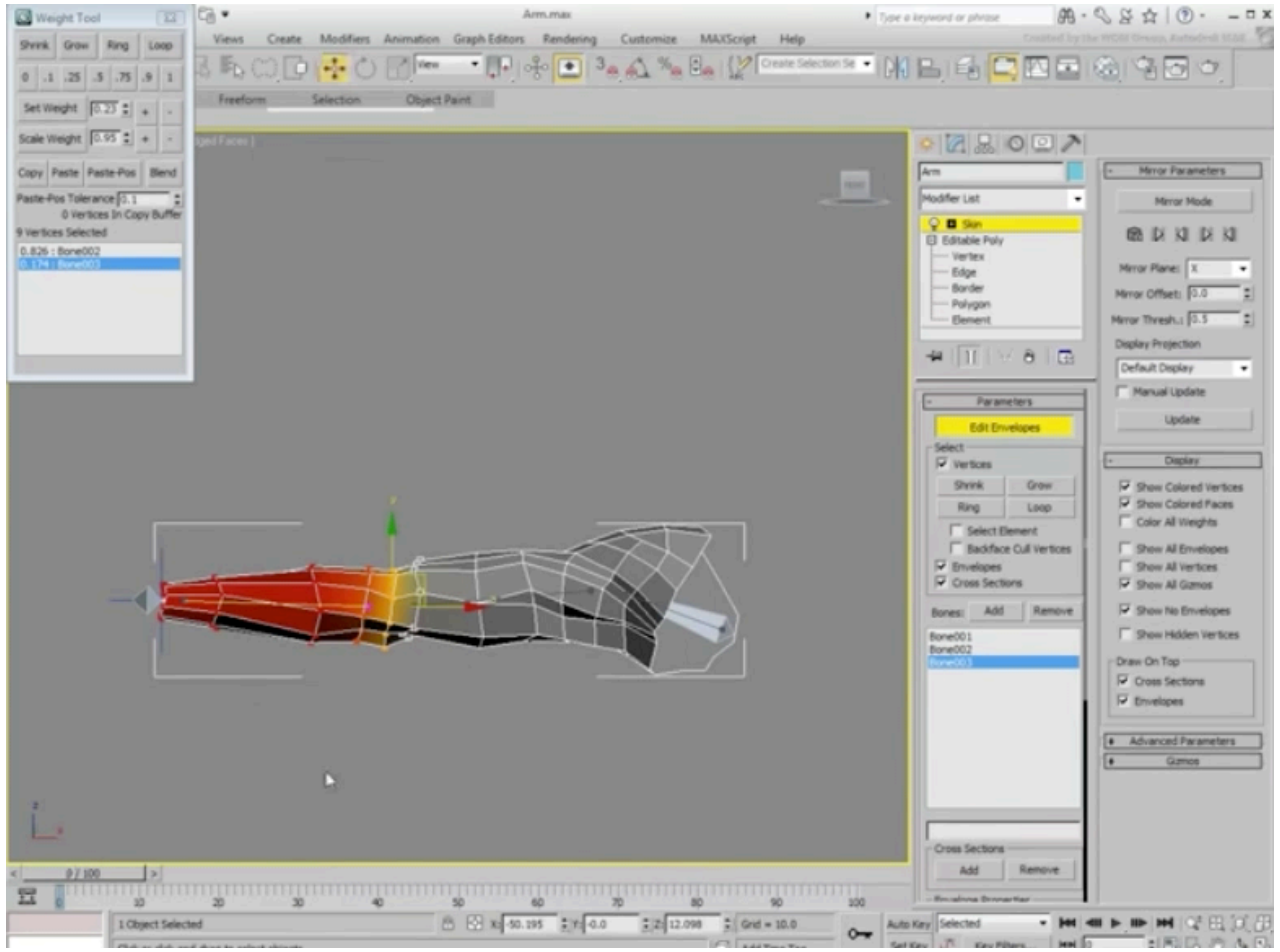
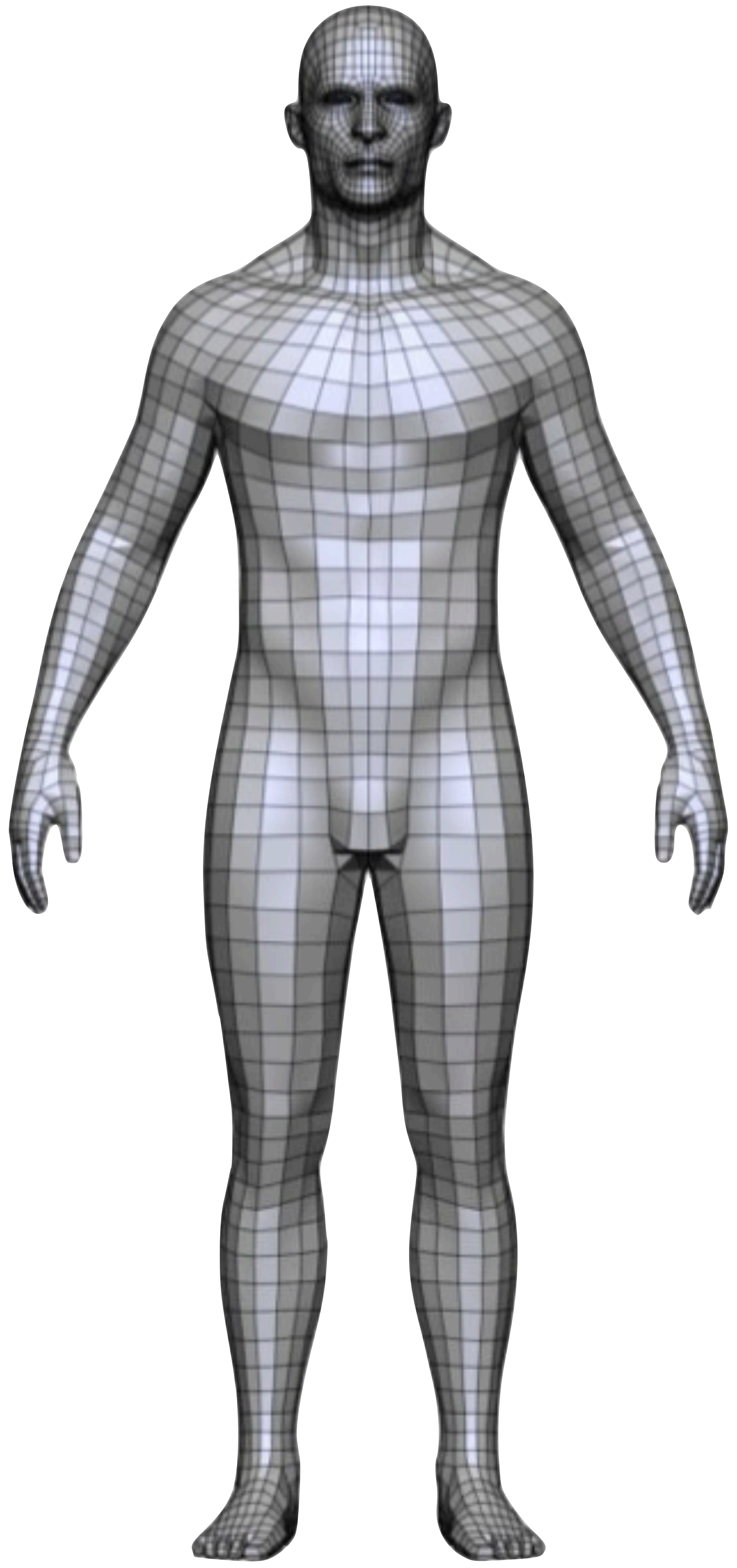
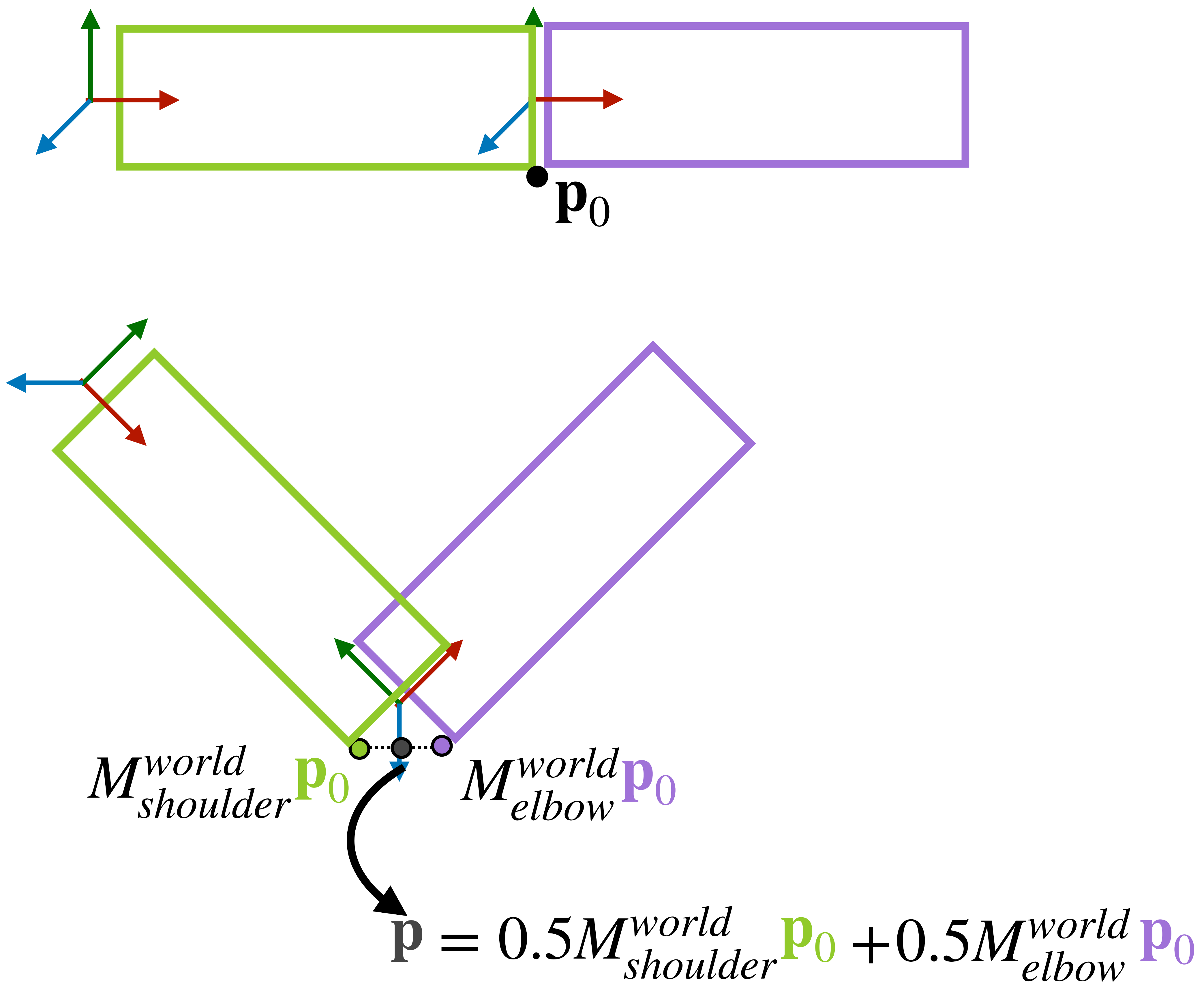
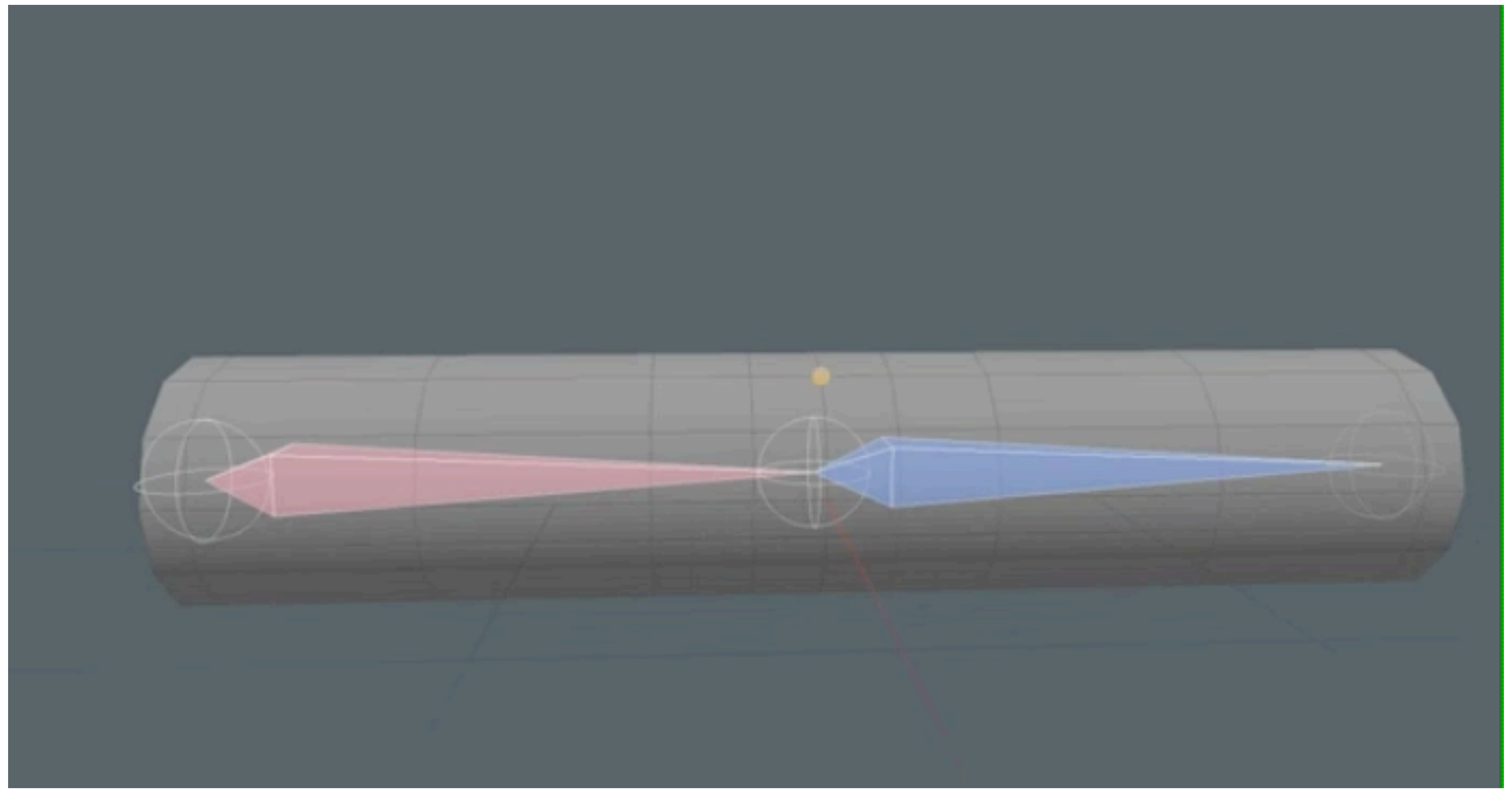






What can go wrong?





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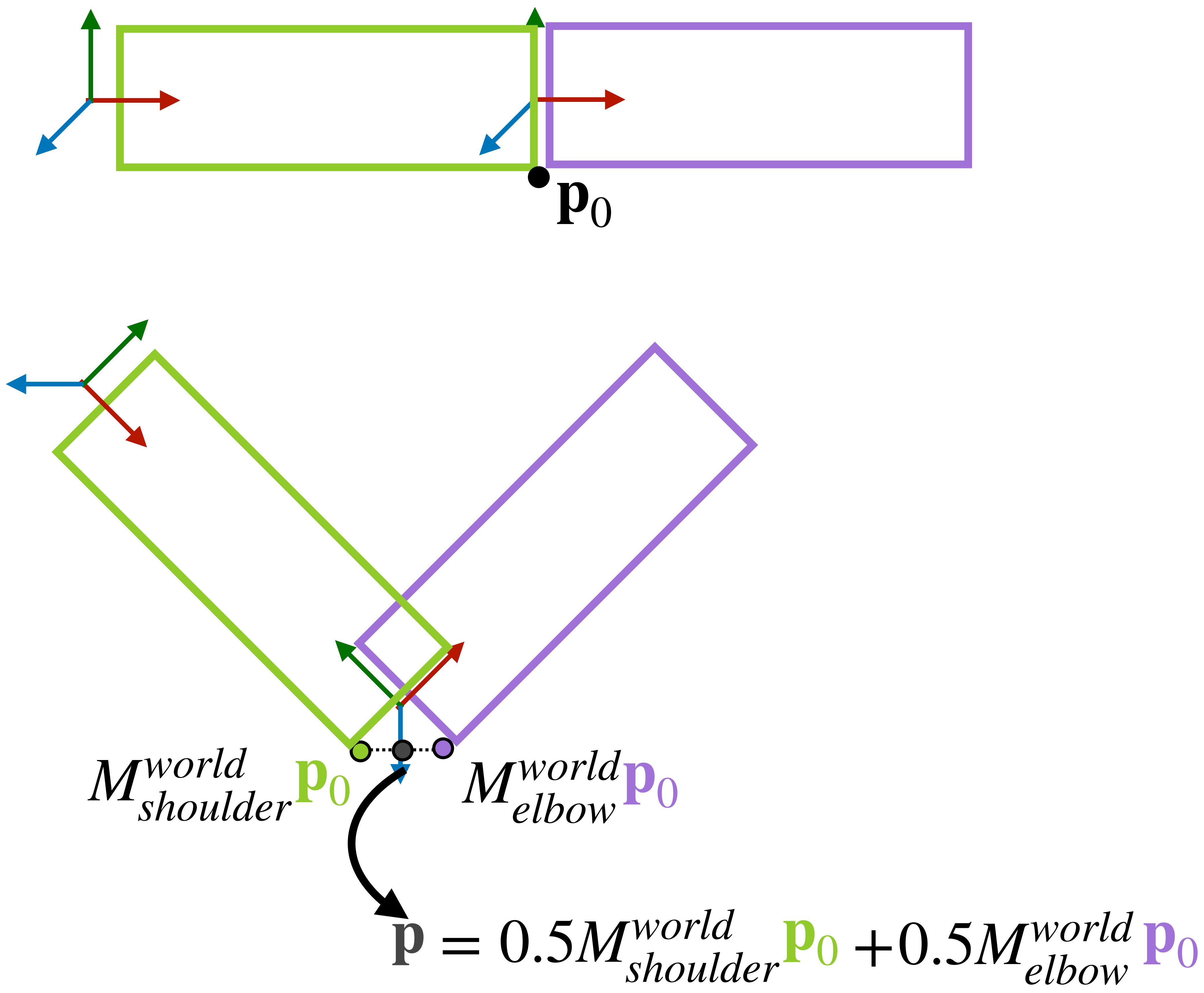
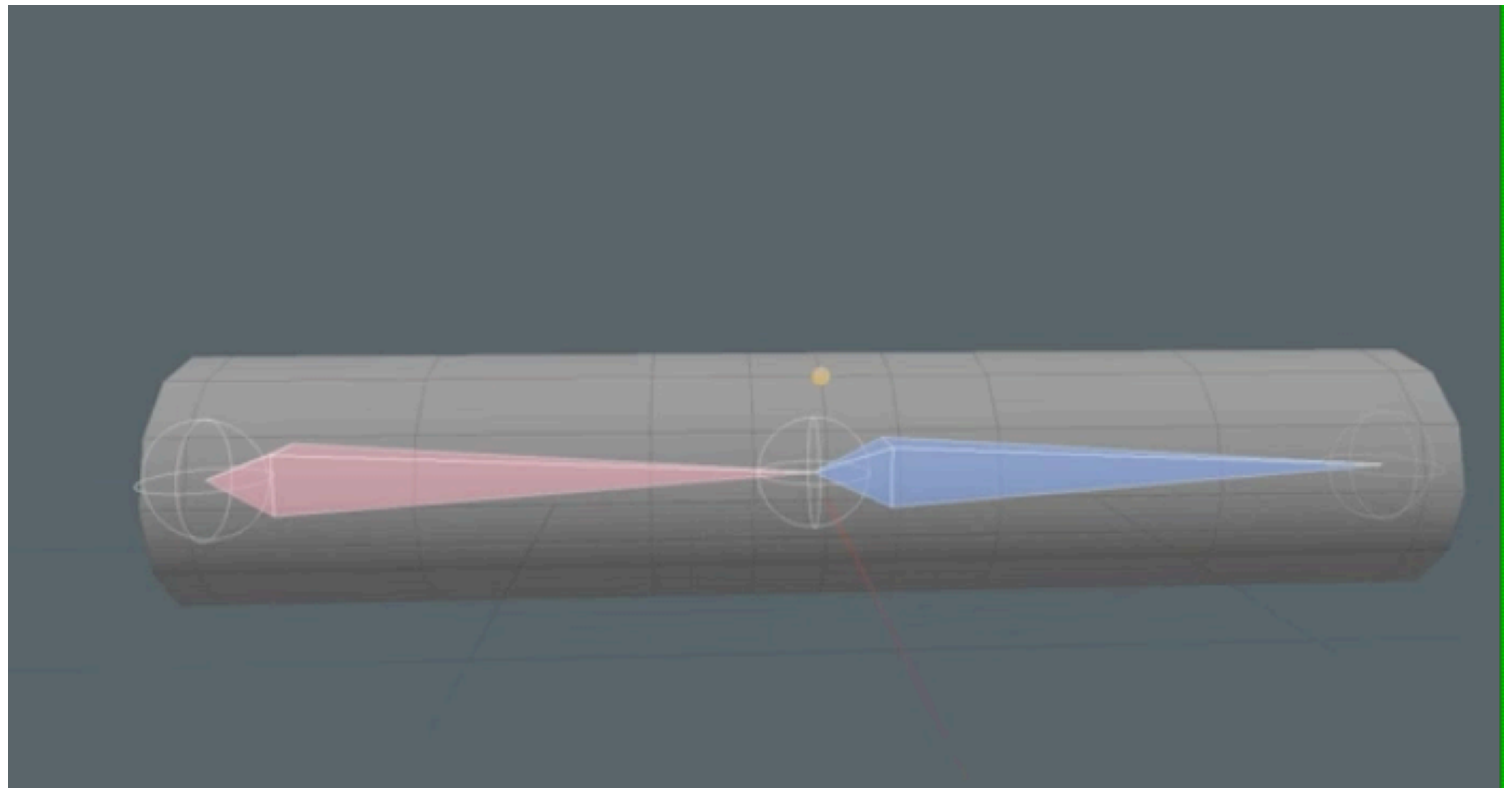
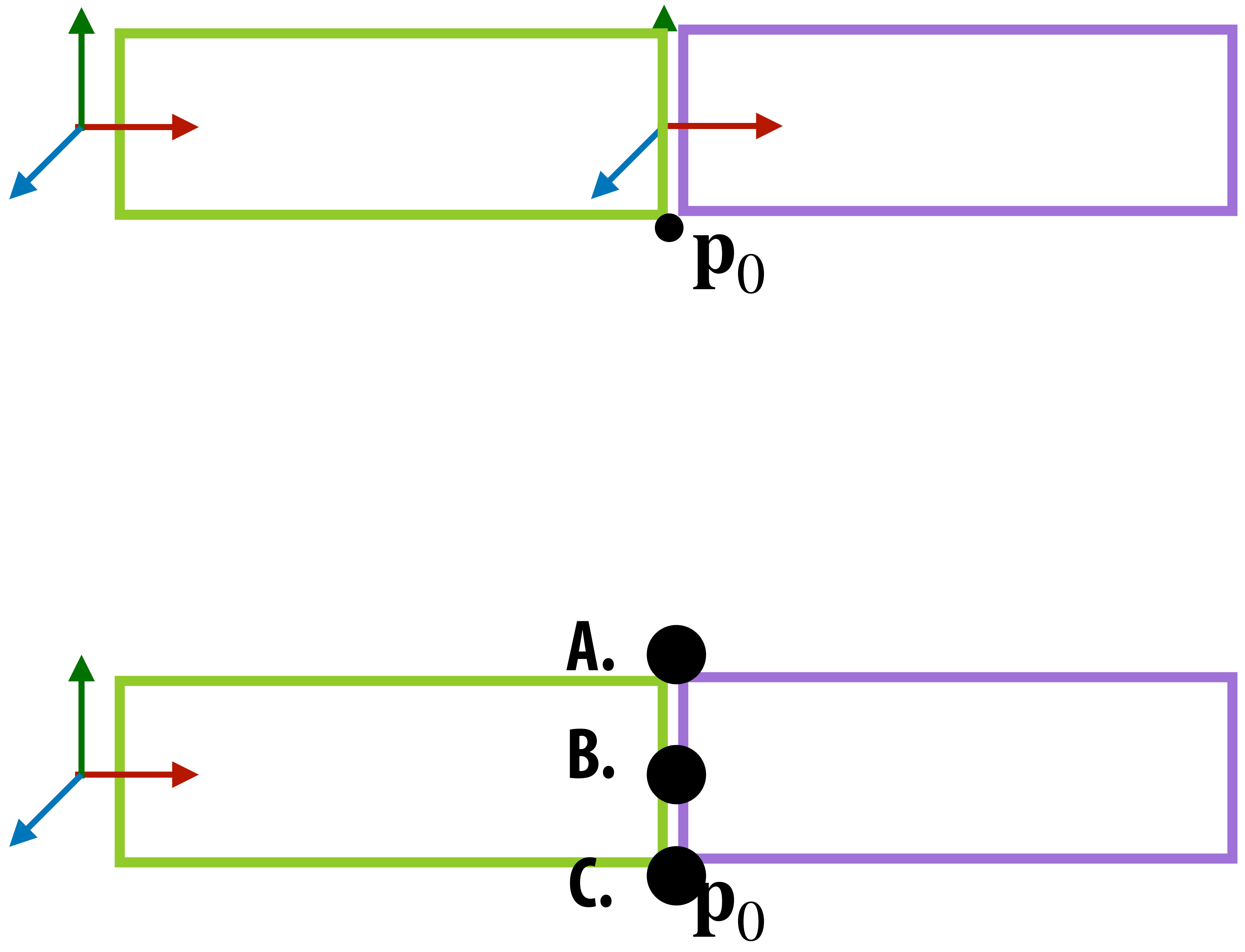
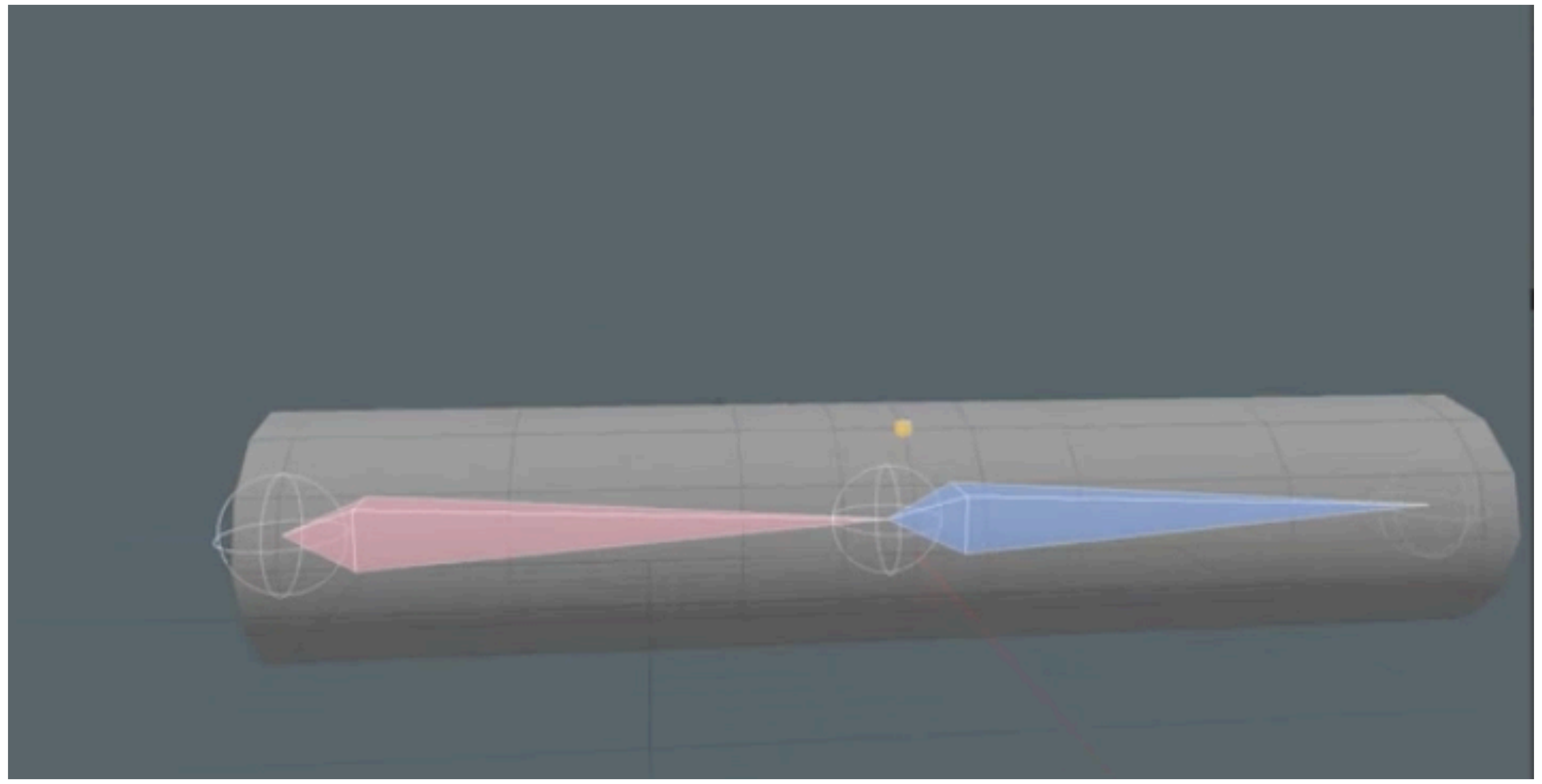




Quiz

- If p0 is assigned to both green and purple bones with 50/50 weighting, where would p0 be if the elbow twists for 180-degree?





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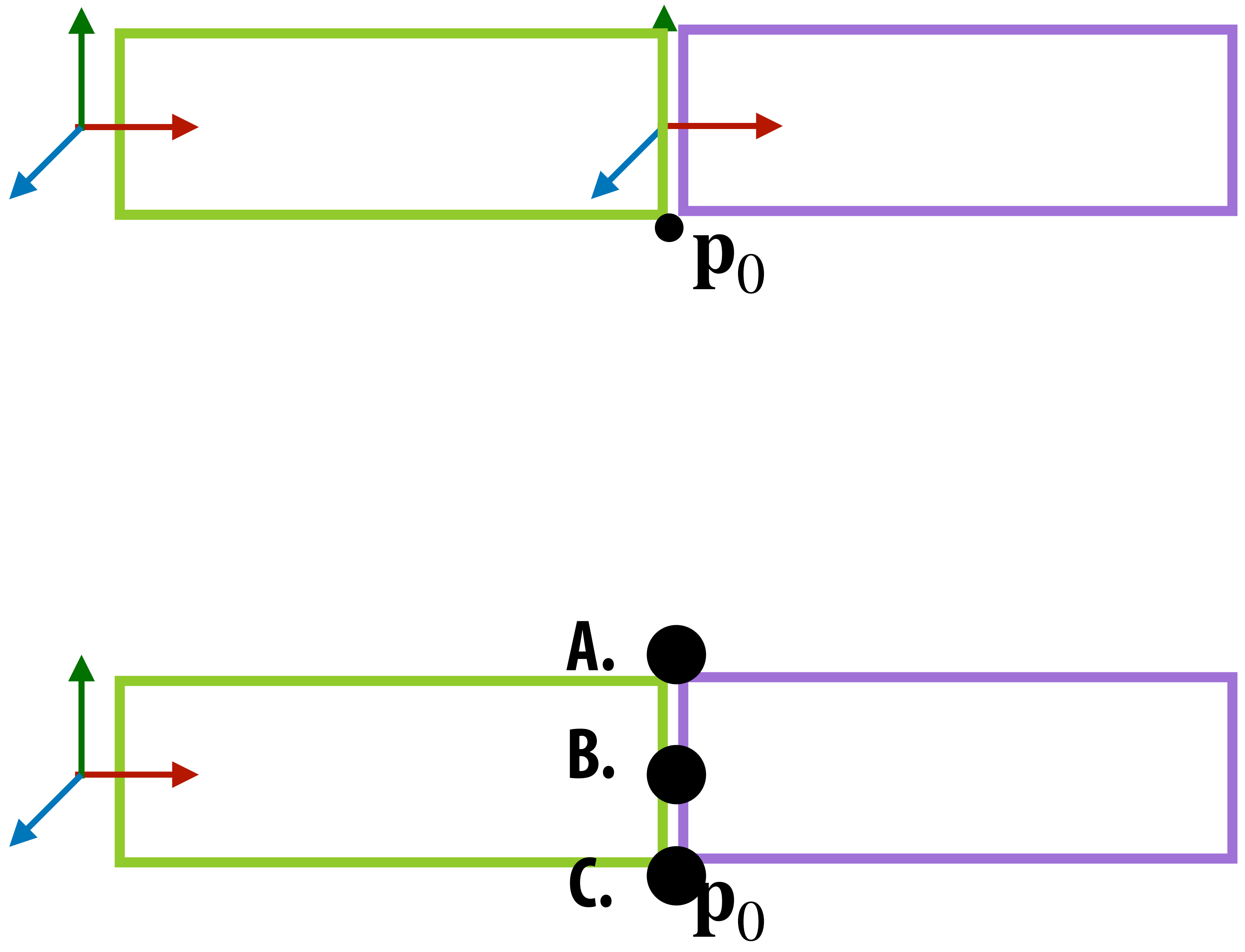

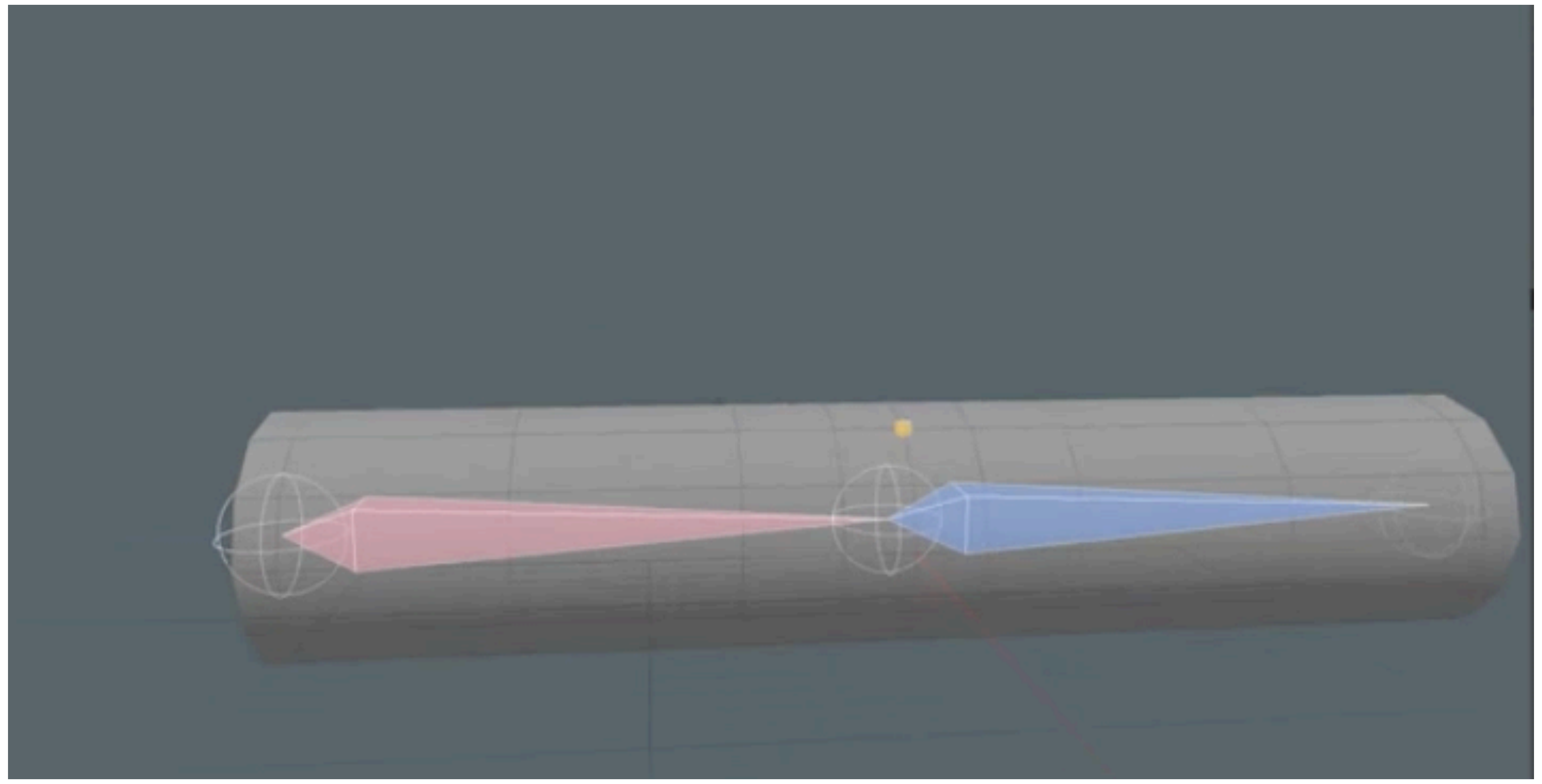

Solution: Dual quaternion

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- **The problem is that we linearly interpolate rigid transformations**

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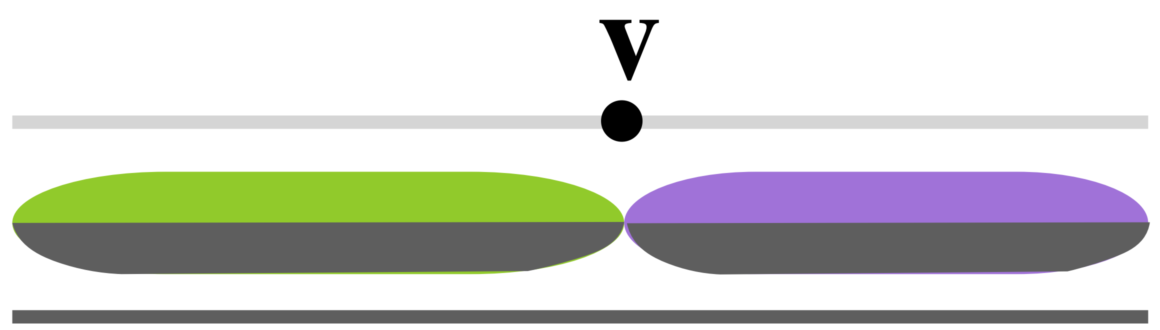

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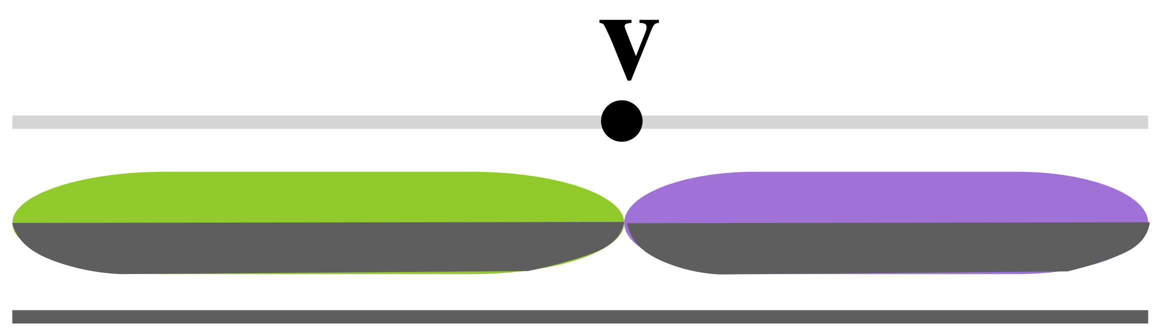

$$\mathbf{v} = \mathbf{W}_1\mathbf{p}_1$$

Solution: Dual quaternion

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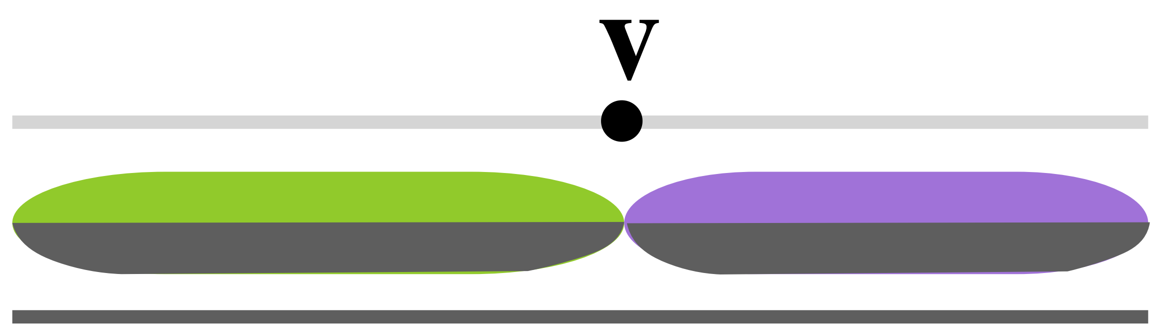

$\mathbf{v} = \mathbf{W}_1 \mathbf{p}_1$

transformation from world to the "green frame"

Solution: Dual quaternion

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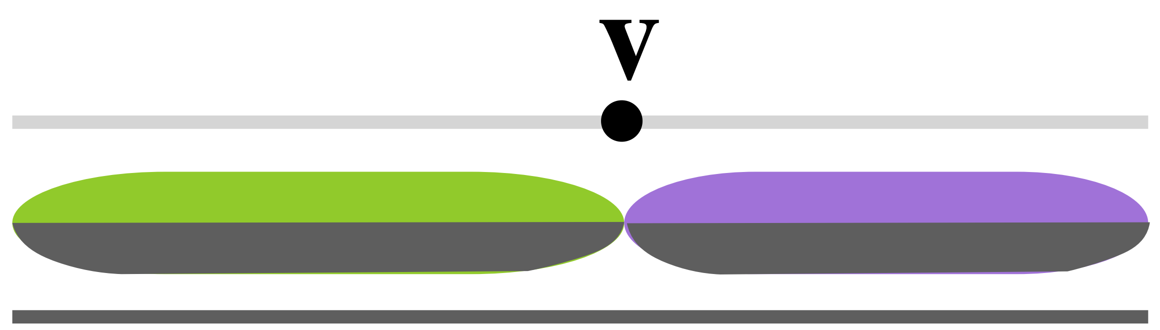

$$\mathbf{v} = \mathbf{W}_1 \mathbf{p}_1$$

transformation from world to the "green frame"

local coord of v in the "green frame"

Solution: Dual quaternion

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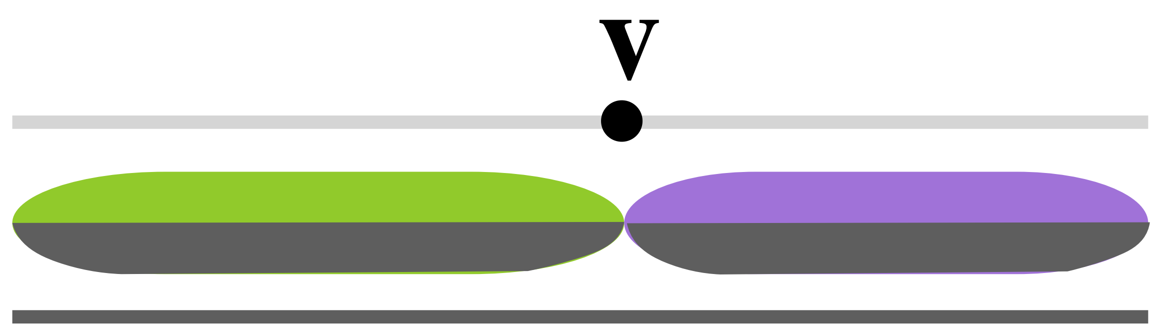

$$\mathbf{v} = \mathbf{W}_1\mathbf{p}_1 = \mathbf{I}\mathbf{p}_1$$

Solution: Dual quaternion

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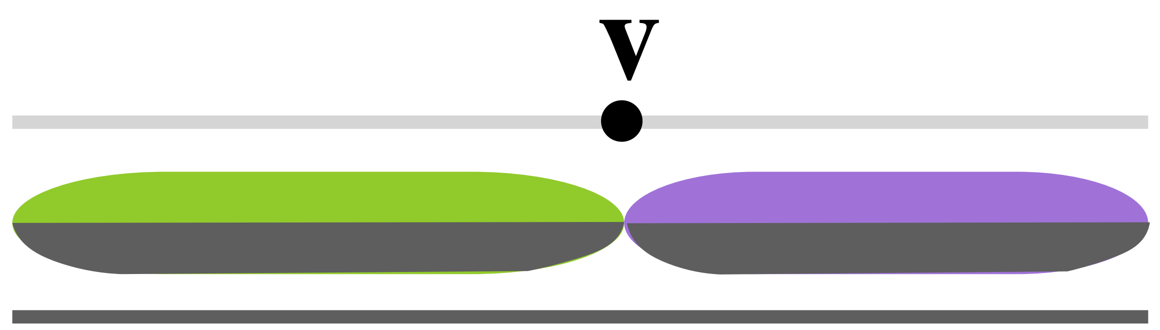

$$\mathbf{v} = \mathbf{W}_1\mathbf{p}_1 = \mathbf{I}\mathbf{p}_1$$

$$\mathbf{p}_1 = \mathbf{v}$$

Solution: Dual quaternion

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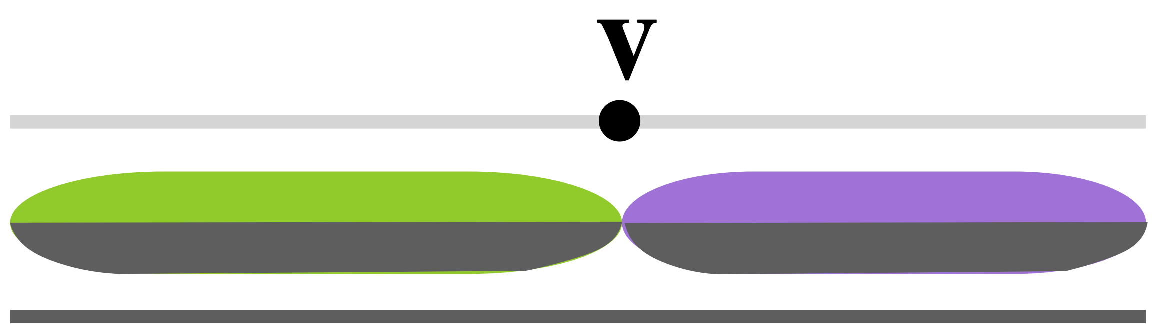

$$\mathbf{v} = \mathbf{W}_1\mathbf{p}_1 = \mathbf{I}\mathbf{p}_1$$

$$\mathbf{p}_1 = \mathbf{v}$$

$$\mathbf{v} = \mathbf{W}_2\mathbf{p}_2$$

Solution: Dual quaternion

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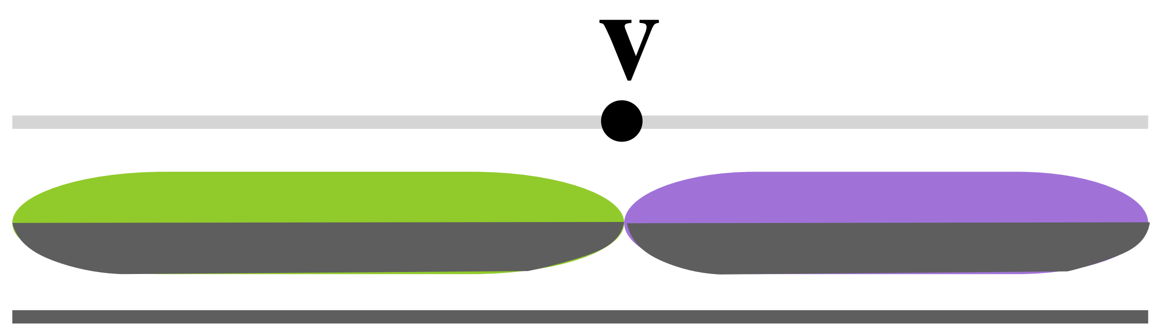

$$\mathbf{v} = \mathbf{W}_1\mathbf{p}_1 = \mathbf{I}\mathbf{p}_1$$

$$\mathbf{p}_1 = \mathbf{v}$$

$$\mathbf{v} = \mathbf{W}_2\mathbf{p}_2 = \mathbf{I}\mathbf{T}_{12}\mathbf{I}\mathbf{p}_2$$

Solution: Dual quaternion

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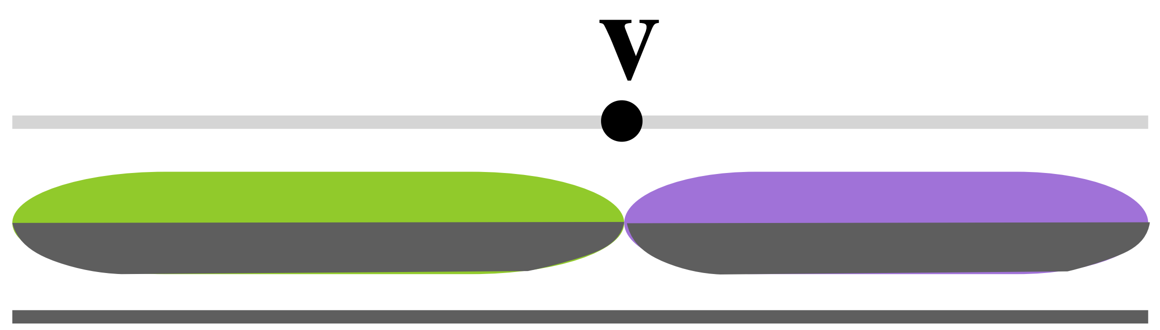


$$\mathbf{v} = \mathbf{W}_1\mathbf{p}_1 = \mathbf{I}\mathbf{p}_1$$

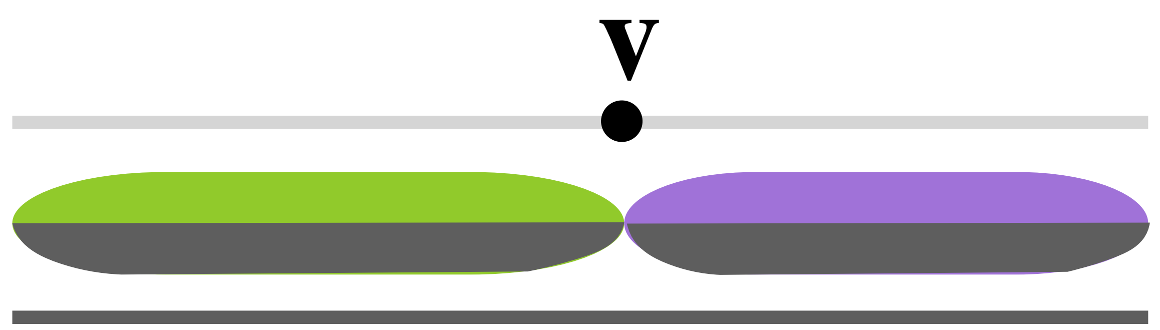
$$\mathbf{p}_1 = \mathbf{v}$$

$$\mathbf{v} = \mathbf{W}_2\mathbf{p}_2 = \mathbf{I}\mathbf{T}_{12}\mathbf{I}\mathbf{p}_2$$

$$\mathbf{p}_2 = \mathbf{T}_{12}^{-1}\mathbf{v}$$

Solution: Dual quaternion

- The problem is that we linearly interpolate rigid transformations



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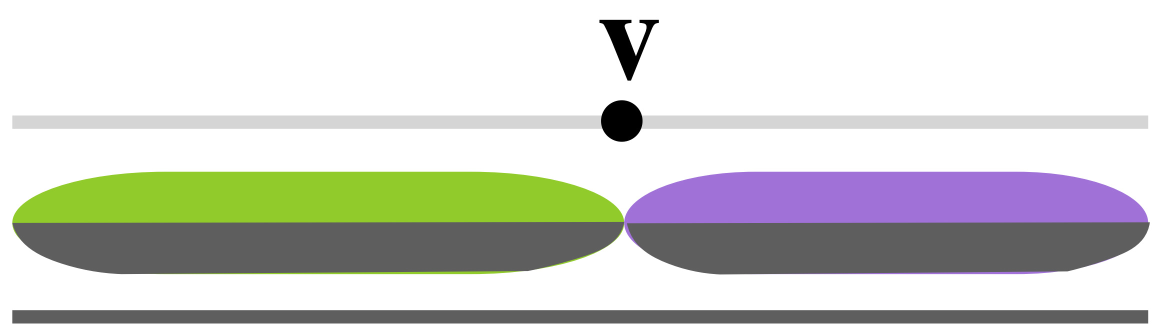

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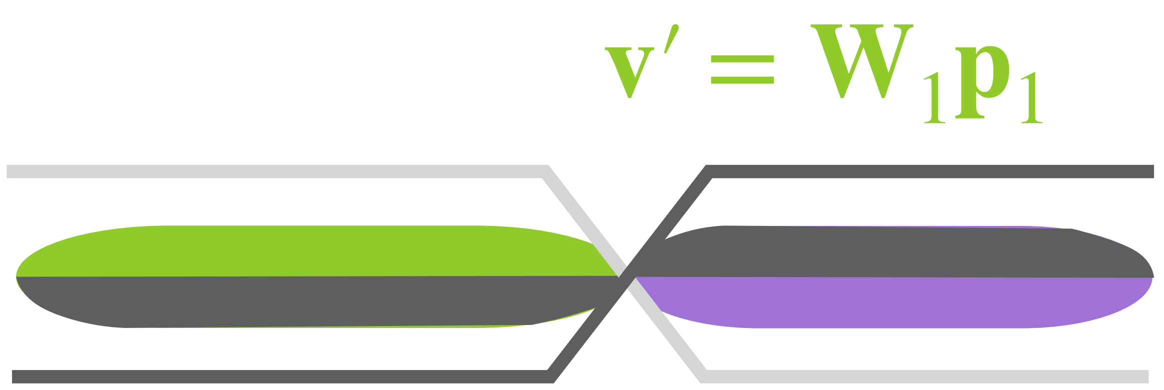

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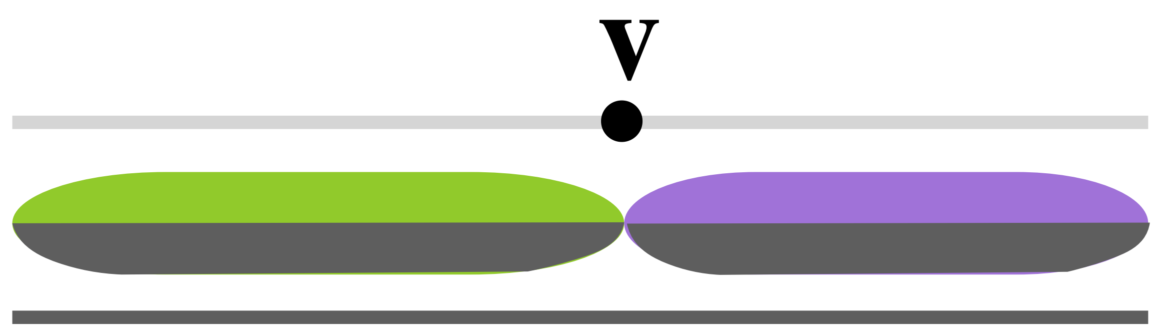

$$\mathbf{v} = \mathbf{W}_1\mathbf{p}_1 = \mathbf{I}\mathbf{p}_1$$

$$\mathbf{p}_1 = \mathbf{v}$$

$$\mathbf{v} = \mathbf{W}_2\mathbf{p}_2 = \mathbf{I}\mathbf{T}_{12}\mathbf{I}\mathbf{p}_2$$

$$\mathbf{p}_2 = \mathbf{T}_{12}^{-1}\mathbf{v}$$

$$\mathbf{v}' = \mathbf{W}_1\mathbf{p}_1 = \mathbf{I}\mathbf{p}_1 = \mathbf{v} = \mathbf{C}_1\mathbf{v}$$

Solution: Dual quaternion

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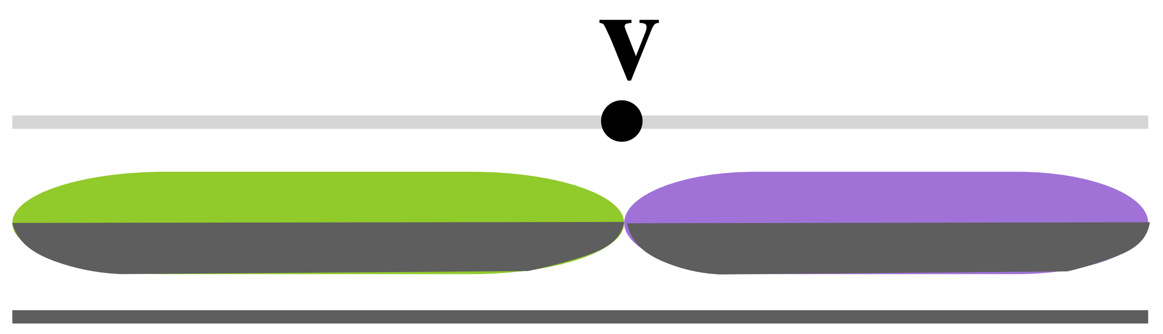


$$\mathbf{v}' = \mathbf{W}_1\mathbf{p}_1 = \mathbf{I}\mathbf{p}_1 = \mathbf{v} = \mathbf{C}_1\mathbf{v}$$

$$\mathbf{v} = \mathbf{W}_1\mathbf{p}_1 = \mathbf{I}\mathbf{p}_1$$

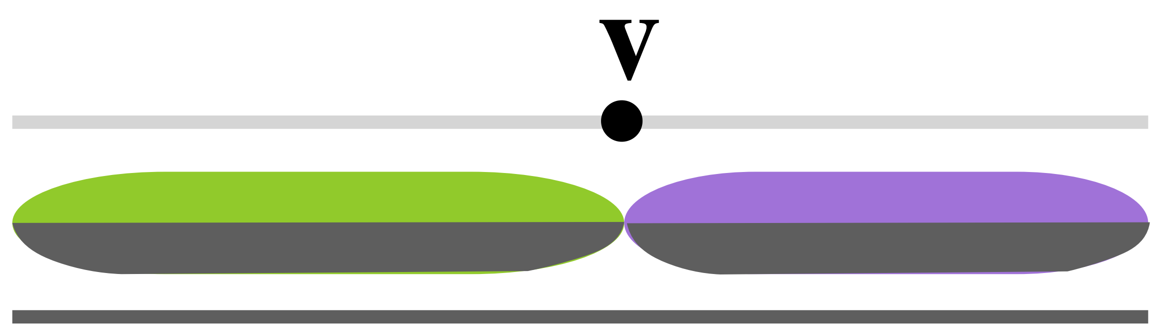
$$\mathbf{p}_1 = \mathbf{v}$$

$$\mathbf{v} = \mathbf{W}_2\mathbf{p}_2 = \mathbf{I}\mathbf{T}_{12}\mathbf{I}\mathbf{p}_2$$

$$\mathbf{p}_2 = \mathbf{T}_{12}^{-1}\mathbf{v}$$

Solution: Dual quaternion

- The problem is that we linearly interpolate rigid transformations



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$$\mathbf{p}_2 = \mathbf{T}_{12}^{-1}\mathbf{v}$$

$$\mathbf{v}' = \mathbf{W}_1\mathbf{p}_1 = \mathbf{I}\mathbf{p}_1 = \mathbf{v} = \mathbf{C}_1\mathbf{v}$$

$$\mathbf{v}' = \mathbf{W}_2\mathbf{p}_2$$

Solution: Dual quaternion

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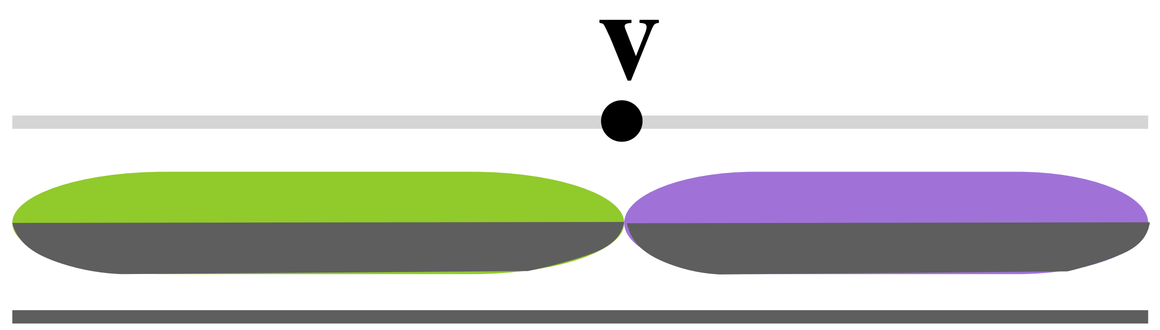

$$\mathbf{v}' = \mathbf{W}_1\mathbf{p}_1 = \mathbf{I}\mathbf{p}_1 = \mathbf{v} = \mathbf{C}_1\mathbf{v}$$

$$\mathbf{v}' = \mathbf{W}_2\mathbf{p}_2 = \mathbf{I}\mathbf{T}_{12}\mathbf{R}(\pi)\mathbf{p}_2$$

$$\mathbf{v} = \mathbf{W}_1\mathbf{p}_1 = \mathbf{I}\mathbf{p}_1$$

$$\mathbf{p}_1 = \mathbf{v}$$

$$\mathbf{v} = \mathbf{W}_2\mathbf{p}_2 = \mathbf{I}\mathbf{T}_{12}\mathbf{I}\mathbf{p}_2$$

$$\mathbf{p}_2 = \mathbf{T}_{12}^{-1}\mathbf{v}$$

Solution: Dual quaternion

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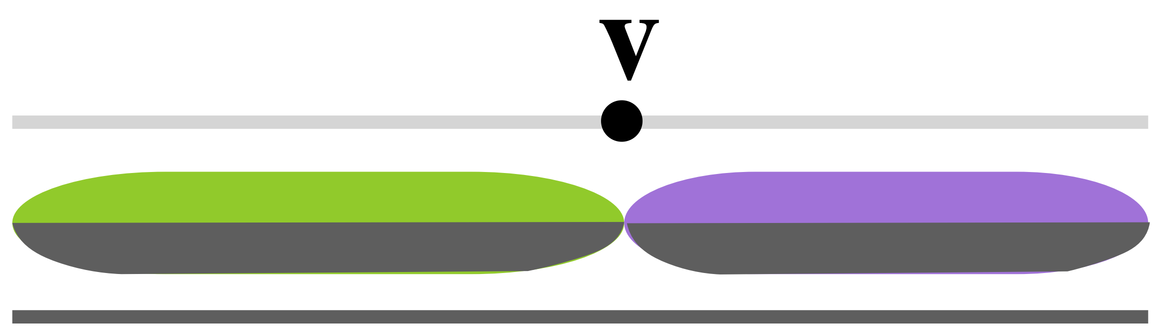

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$$\mathbf{p}_2 = \mathbf{T}_{12}^{-1}\mathbf{v}$$

$$\mathbf{v}' = \mathbf{W}_1\mathbf{p}_1 = \mathbf{I}\mathbf{p}_1 = \mathbf{v} = \mathbf{C}_1\mathbf{v}$$

$$\mathbf{v}' = \mathbf{W}_2\mathbf{p}_2 = \mathbf{I}\mathbf{T}_{12}\mathbf{R}(\pi)\mathbf{p}_2$$

$$= \mathbf{T}_{12}\mathbf{R}(\pi)\mathbf{T}_{12}^{-1}\mathbf{v} = \mathbf{C}_2\mathbf{v}$$

Solution: Dual quaternion

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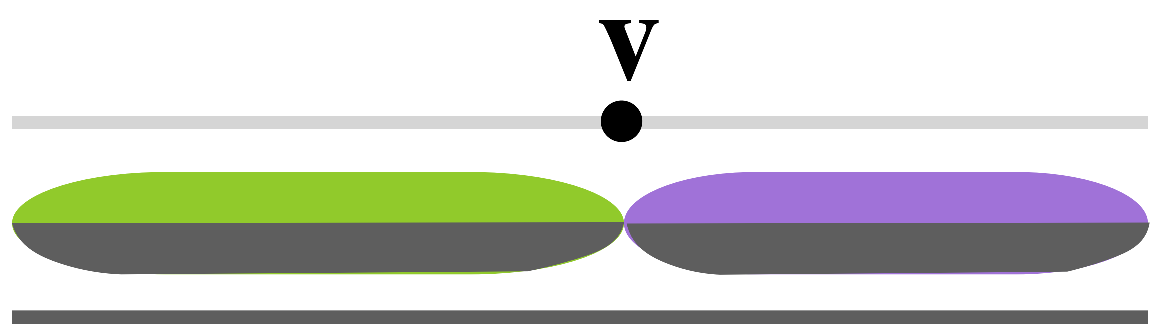


$$\mathbf{v} = \mathbf{W}_1\mathbf{p}_1 = \mathbf{I}\mathbf{p}_1$$

$$\mathbf{p}_1 = \mathbf{v}$$

$$\mathbf{v} = \mathbf{W}_2\mathbf{p}_2 = \mathbf{I}\mathbf{T}_{12}\mathbf{I}\mathbf{p}_2$$

$$\mathbf{p}_2 = \mathbf{T}_{12}^{-1}\mathbf{v}$$

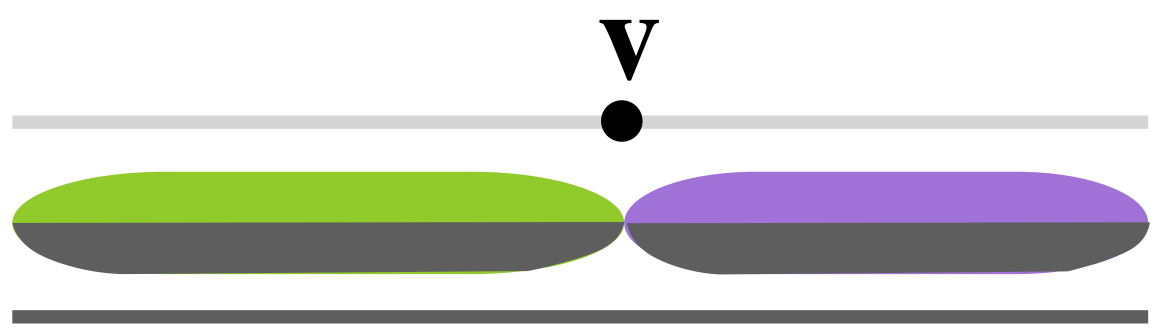
$$\mathbf{v}' = \mathbf{W}_1\mathbf{p}_1 = \mathbf{I}\mathbf{p}_1 = \mathbf{v} = \mathbf{C}_1\mathbf{v}$$

$$\mathbf{v}' = \mathbf{W}_2\mathbf{p}_2 = \mathbf{I}\mathbf{T}_{12}\mathbf{R}(\pi)\mathbf{p}_2$$

$$= \mathbf{T}_{12}\mathbf{R}(\pi)\mathbf{T}_{12}^{-1}\mathbf{v} = \mathbf{C}_2\mathbf{v}$$

Solution: Dual quaternion

- **The problem is that we linearly interpolate rigid transformations**



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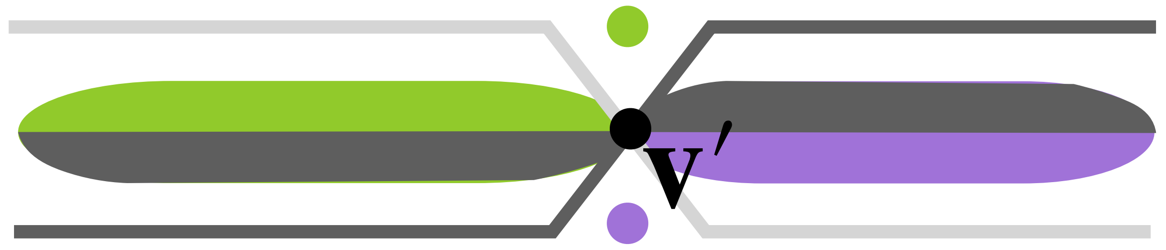

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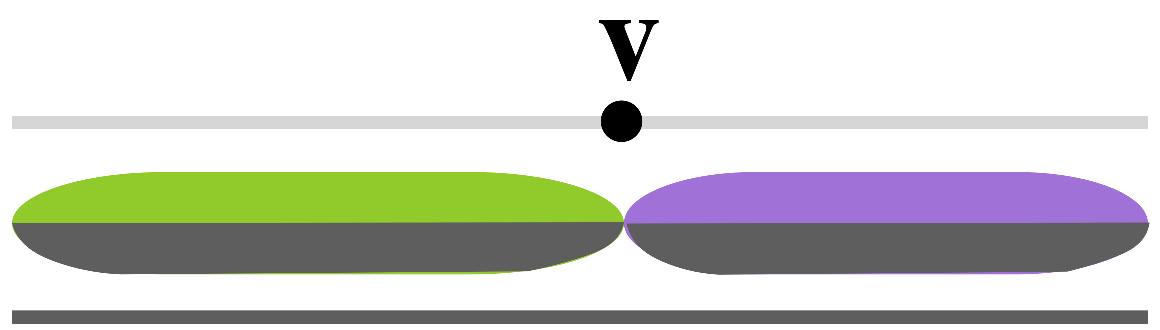

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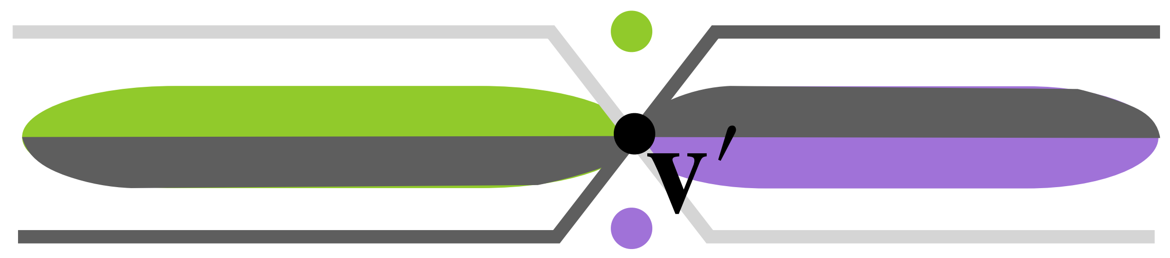


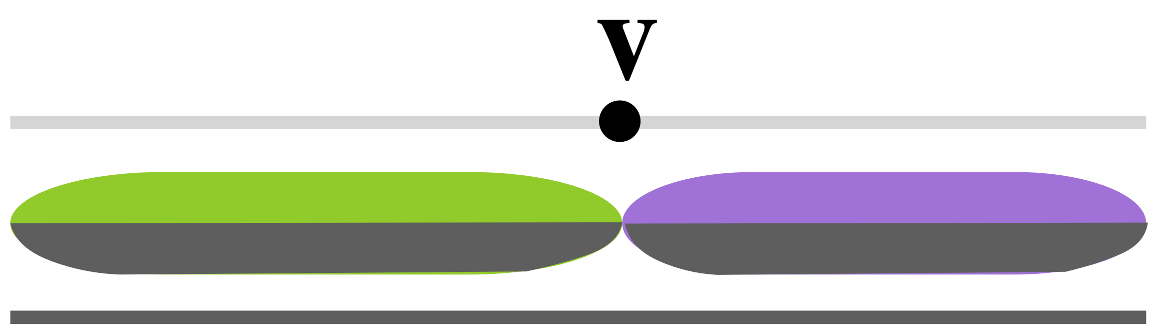
$$\mathbf{v}' = \mathbf{W}_2\mathbf{p}_2 = \mathbf{I}\mathbf{T}_{12}\mathbf{R}(\pi)\mathbf{p}_2$$

$$= \mathbf{T}_{12}\mathbf{R}(\pi)\mathbf{T}_{12}^{-1}\mathbf{v} = \mathbf{C}_2\mathbf{v}$$

$$\mathbf{v}' = 0.5\ \mathbf{C}_1\mathbf{v} + 0.5\ \mathbf{C}_2\mathbf{v}$$

Solution: Dual quaternion

- The problem is that we linearly interpolate rigid transformations



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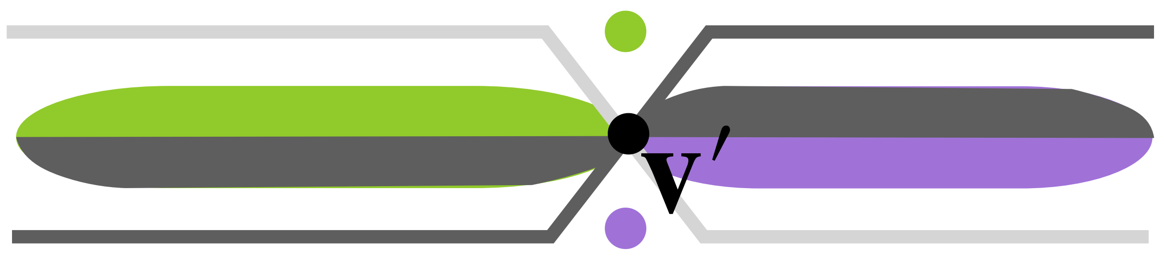

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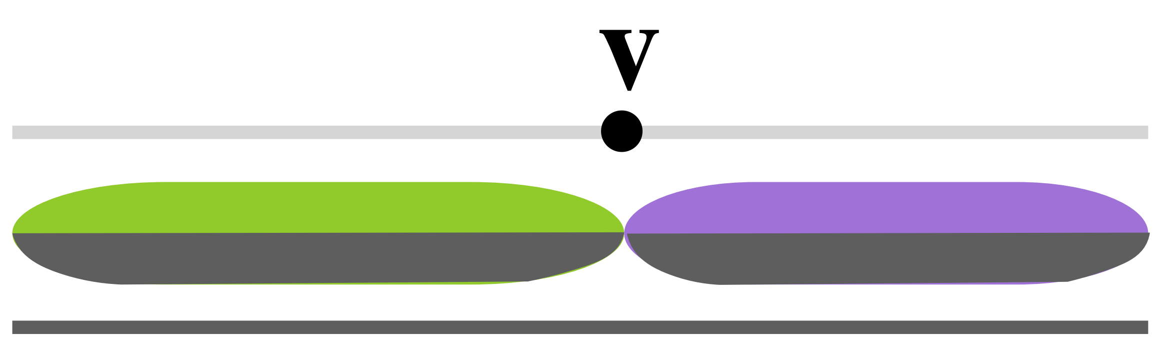
$$\mathbf{v}' = 0.5\ \mathbf{C}_1\mathbf{v} + 0.5\ \mathbf{C}_2\mathbf{v} = \left(0.5\ \mathbf{C}_1 + 0.5\ \mathbf{C}_2\right)\mathbf{v}$$

Bad idea: linear interpolating two matrices!

Solution: Dual quaternion

- The problem is that we linearly interpolate rigid transformations



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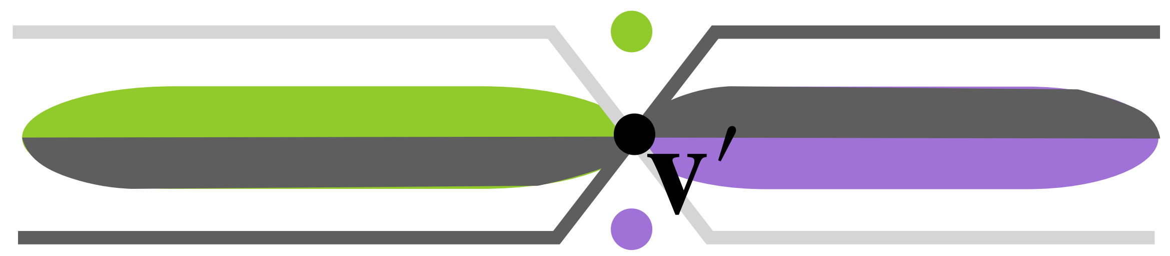

$$\mathbf{v}' = \mathbf{W}_2\mathbf{p}_2 = \mathbf{I}\mathbf{T}_{12}\mathbf{R}(\pi)\mathbf{p}_2$$

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$$\mathbf{v}' = 0.5\ \mathbf{C}_1\mathbf{v} + 0.5\ \mathbf{C}_2\mathbf{v} = \left(0.5\ \mathbf{C}_1 \underset{\uparrow}{+} 0.5\ \mathbf{C}_2\right)\mathbf{v}$$

Bad idea: linear interpolating two matrices!

- We need a better interpolation method that properly blends centers of rotation and the 3D rotations

Solution: Dual quaternion

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- **Dual quaternion definition:**

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$$\hat{\mathbf{q}} = \begin{bmatrix} \hat{w} \\ \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = \begin{bmatrix} w_0 + \epsilon w_\epsilon \\ x_0 + \epsilon x_\epsilon \\ y_0 + \epsilon y_\epsilon \\ z_0 + \epsilon z_\epsilon \end{bmatrix} = \mathbf{q}_0 + \epsilon \mathbf{q}_\epsilon$$

Solution: Dual quaternion

- **Dual quaternion definition:**

$$\hat{\mathbf{q}} = \begin{bmatrix} \hat{w} \\ \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = \begin{bmatrix} w_0 + \epsilon w_\epsilon \\ x_0 + \epsilon x_\epsilon \\ y_0 + \epsilon y_\epsilon \\ z_0 + \epsilon z_\epsilon \end{bmatrix} = \mathbf{q}_0 + \epsilon \mathbf{q}_\epsilon$$

- **Given a 3D rotation q_0 and a 3D translation (t_x, t_y, t_z), the rigid transformation can be expressed as a dual quaternion:**

Solution: Dual quaternion

- **Dual quaternion definition:**

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- **Given a 3D rotation q_0 and a 3D translation (t_x, t_y, t_z), the rigid transformation can be expressed as a dual quaternion:**

$$\hat{\mathbf{q}} = \mathbf{q}_0 + \begin{bmatrix} 1 \\ \frac{\epsilon}{2} t_x \\ \frac{\epsilon}{2} t_y \\ \frac{\epsilon}{2} t_z \end{bmatrix} \mathbf{q}_0$$

Solution: Dual quaternion

- **Dual quaternion definition:**

$$\hat{\mathbf{q}} = \begin{bmatrix} \hat{w} \\ \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = \begin{bmatrix} w_0 + \epsilon w_\epsilon \\ x_0 + \epsilon x_\epsilon \\ y_0 + \epsilon y_\epsilon \\ z_0 + \epsilon z_\epsilon \end{bmatrix} = \mathbf{q}_0 + \epsilon \mathbf{q}_\epsilon$$

- **Given a 3D rotation $\mathbf{q}_0$ and a 3D translation (t_x, t_y, t_z), the rigid transformation can be expressed as a dual quaternion:**

$$\hat{\mathbf{q}} = \mathbf{q}_0 + \begin{bmatrix} 1 \\ \frac{\epsilon}{2} t_x \\ \frac{\epsilon}{2} t_y \\ \frac{\epsilon}{2} t_z \end{bmatrix} \mathbf{q}_0$$

- **Using dual quaternion algebra, we can easily interpolate two rigid transformations:**

Solution: Dual quaternion

- **Dual quaternion definition:**

$$\hat{\mathbf{q}} = \begin{bmatrix} \hat{w} \\ \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = \begin{bmatrix} w_0 + \epsilon w_\epsilon \\ x_0 + \epsilon x_\epsilon \\ y_0 + \epsilon y_\epsilon \\ z_0 + \epsilon z_\epsilon \end{bmatrix} = \mathbf{q}_0 + \epsilon \mathbf{q}_\epsilon$$

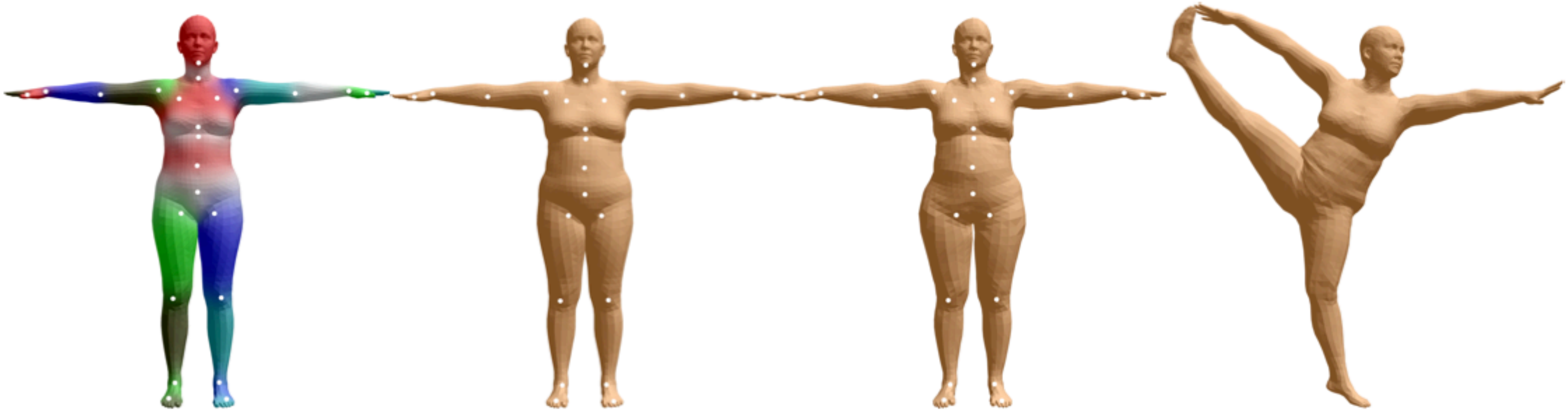
- **Given a 3D rotation $\mathbf{q}_0$ and a 3D translation (t_x, t_y, t_z), the rigid transformation can be expressed as a dual quaternion:**

$$\hat{\mathbf{q}} = \mathbf{q}_0 + \begin{bmatrix} 1 \\ \frac{\epsilon}{2} t_x \\ \frac{\epsilon}{2} t_y \\ \frac{\epsilon}{2} t_z \end{bmatrix} \mathbf{q}_0$$

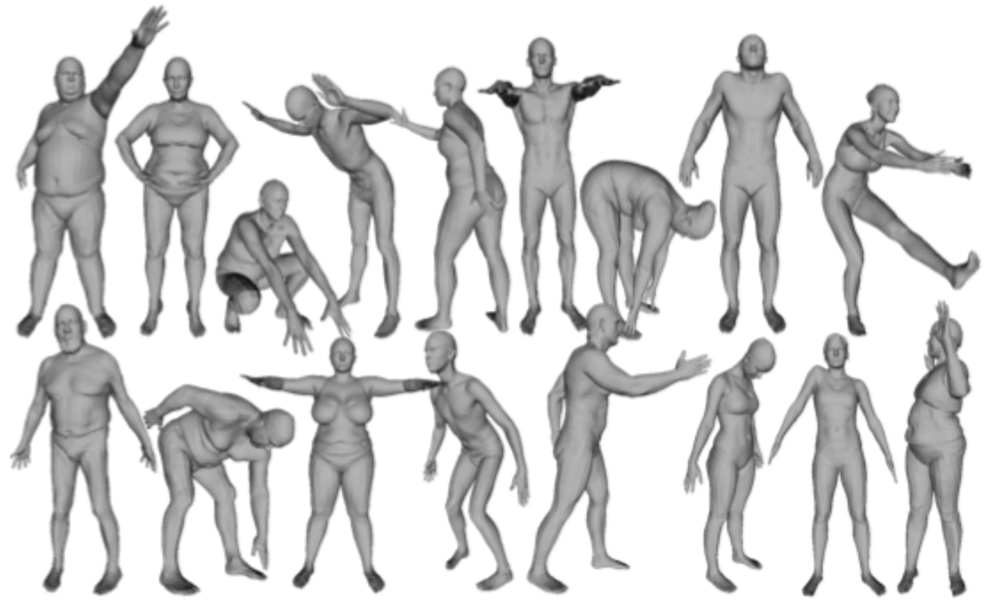
- **Using dual quaternion algebra, we can easily interpolate two rigid transformations:**

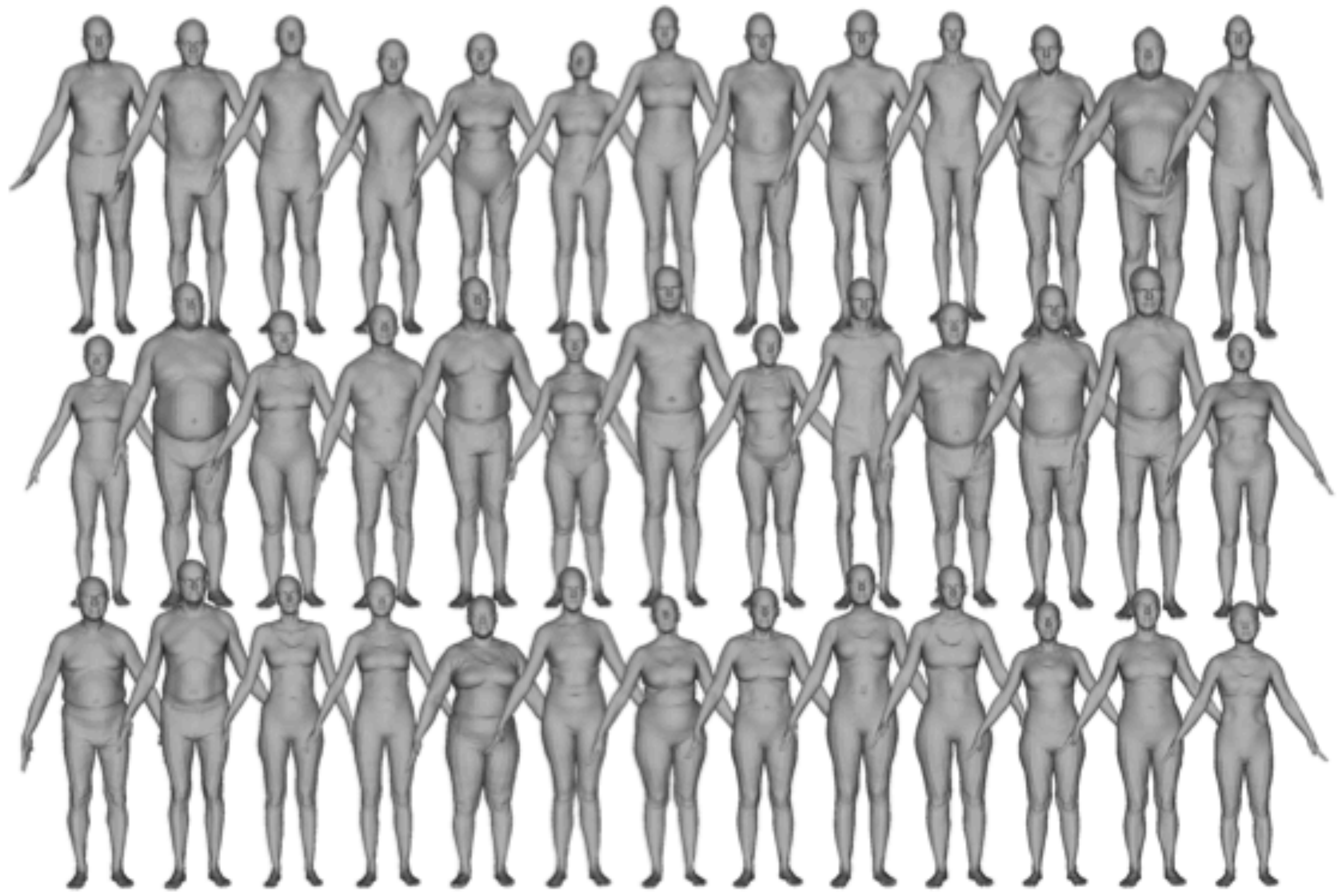
$$\hat{\mathbf{q}}(t) = \frac{(1-t)\hat{\mathbf{q}}_1 + t\hat{\mathbf{q}}_2}{\|(1-t)\hat{\mathbf{q}}_1 + t\hat{\mathbf{q}}_2\|}$$

Learning-based skinning: SMPL representation



(a) $\bar{\mathbf{T}}, \mathcal{W}$ **(b)** $\bar{\mathbf{T}} + B_S(\vec{\beta}), J(\vec{\beta})$ **(c)** $T_P(\vec{\beta}, \vec{\theta}) = \bar{\mathbf{T}} + B_S(\vec{\beta}) + B_P(\vec{\theta})$ **(d)** $W(T_P(\vec{\beta}, \vec{\theta}), J(\vec{\beta}), \vec{\theta}, \mathcal{W})$





$$M(\vec{\beta}, \vec{\theta}; \Phi) = W\left(T_P(\vec{\beta}, \vec{\theta}; \bar{\mathbf{T}}, \mathcal{S}, \mathcal{P}), J(\vec{\beta}; \mathcal{J}, \bar{\mathbf{T}}, \mathcal{S}), \vec{\theta}, \mathcal{W}\right)$$

References

- Georgios Pavlakos: https://ps.is.mpg.de/publications/smplex-2019
- Nancy Pollard: http://graphics.cs.cmu.edu/nsp/course/15-462/Spring04/slides/05-hierarchy.pdf
- Radiopaedia: https://radiopaedia.org/articles/joints-1?lang=us
- Physiopedia: https://www.youtube.com/watch?v=l7h2FJnSXyw&feature=emb_logo
- Randale Sechrest: https://www.youtube.com/watch?v=D3GVKjeY1FM
- Geometric Skinning with Approximate Dual Quaternion Blending, Kavan et al. ACM TOG 2008
- SMPL: A Skinned Multi-Person Linear Model, Loper et al. ACM TOG 2015