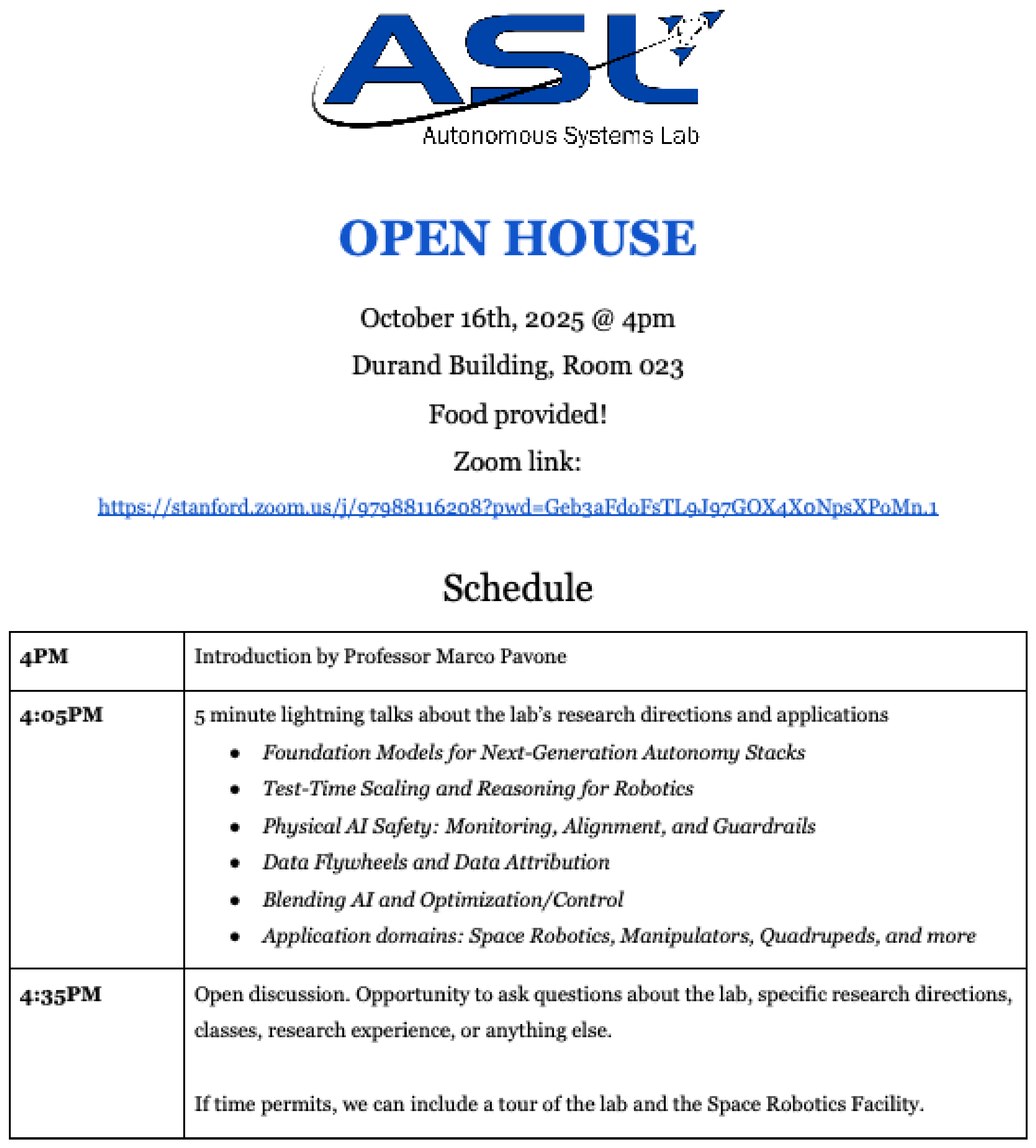
# Principles of Robot Autonomy I

Motion planning I: graph search algorithms

## Attendance Form



Autonomous Systems Lab

## OPEN HOUSE

October 16th, 2025 @ 4pm

Durand Building, Room 023

Food provided!

Zoom link:

https://stanford.zoom.us/j/97988116208?pwd=Geb3aFd0FsTL9J97GOX4X0NpsXP0Mn.1

### Schedule

| | |
|---|---|
| **4PM** | Introduction by Professor Marco Pavone |
| **4:05PM** | 5 minute lightning talks about the lab's research directions and applications<br>• *Foundation Models for Next-Generation Autonomy Stacks*<br>• *Test-Time Scaling and Reasoning for Robotics*<br>• *Physical AI Safety: Monitoring, Alignment, and Guardrails*<br>• *Data Flywheels and Data Attribution*<br>• *Blending AI and Optimization/Control*<br>• *Application domains: Space Robotics, Manipulators, Quadrupeds, and more* |
| **4:35PM** | Open discussion. Opportunity to ask questions about the lab, specific research directions, classes, research experience, or anything else.<br><br>If time permits, we can include a tour of the lab and the Space Robotics Facility. |

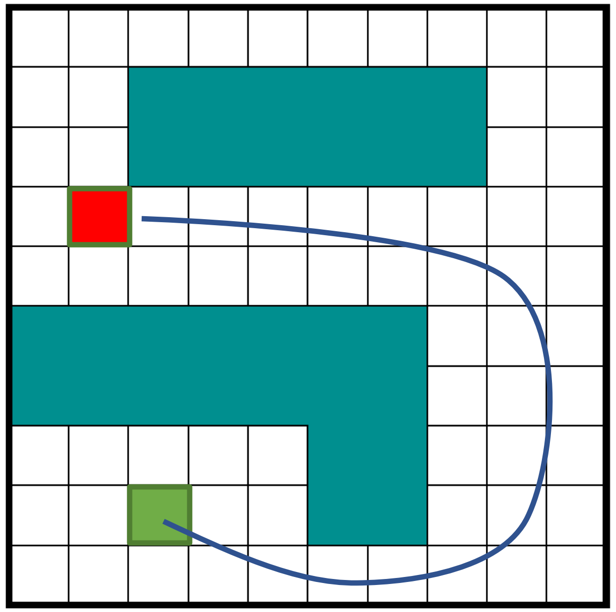
## Agenda

- Agenda
  - Introduction to motion planning
  - Search-based algorithms for motion planning
  - Configuration spaces and combinatorial motion planning

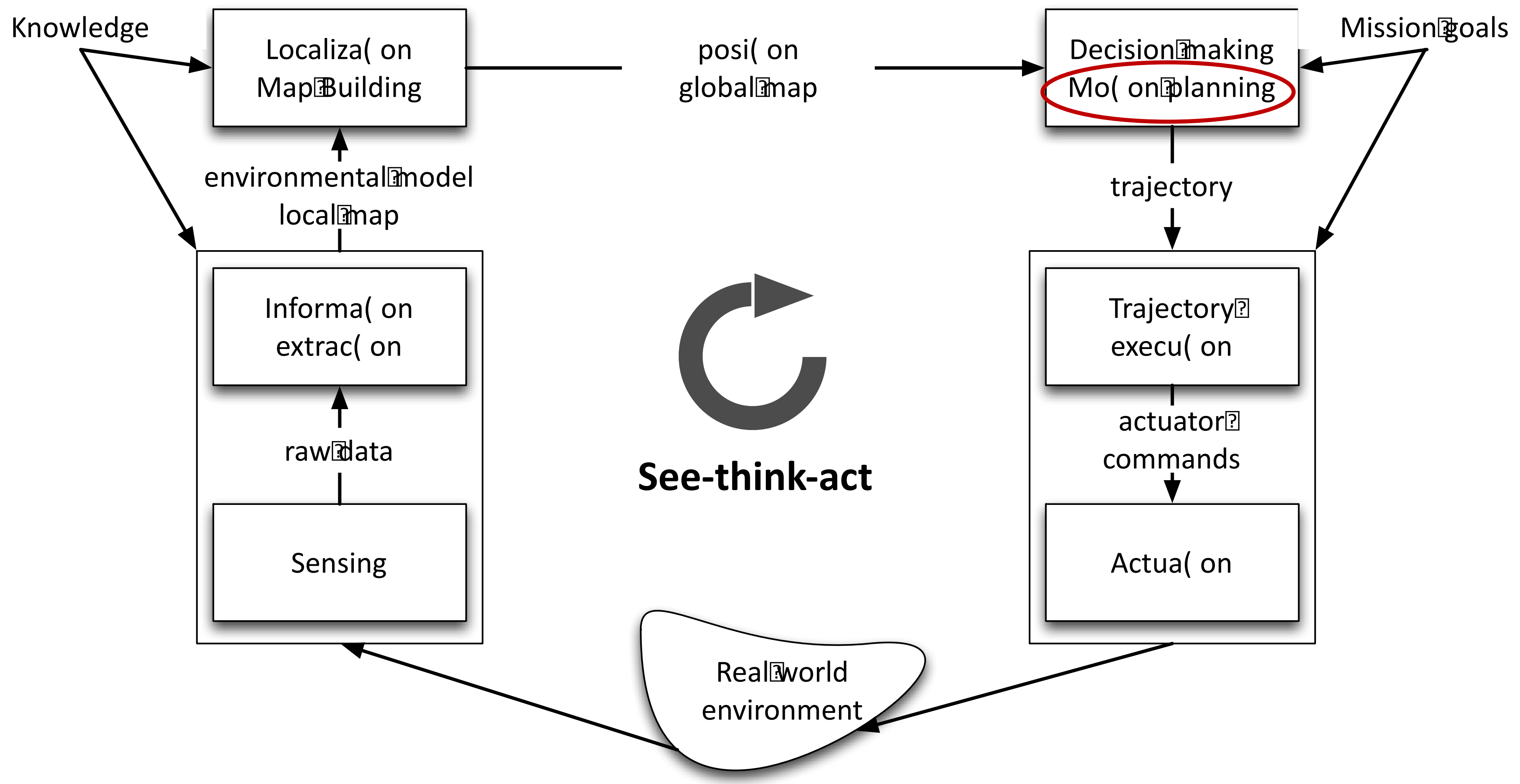
- Readings:
  - Chapter 4, sections 4.1 – 4.2 in D. Gammelli, J. Lorenzetti, K. Luo, G. Zardini, M. Pavone. *Principles of Robot Autonomy*. 2026.

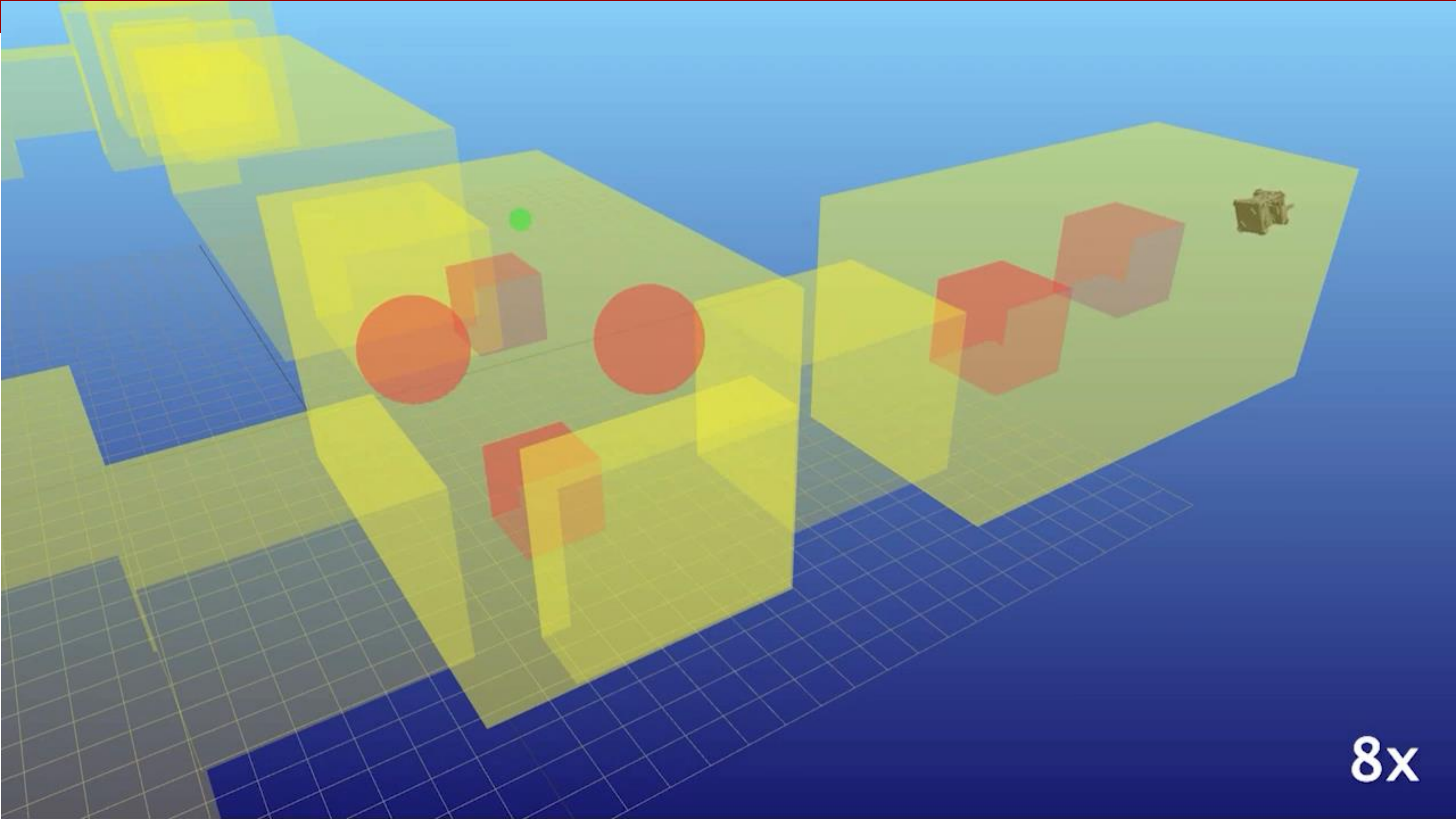
## Motion planning

Problem definition: Compute sequence of actions that drives a robot from an initial condition to a terminal condition while avoiding obstacles, respecting motion constraints, and *possibly* optimizing a cost function



# The see-think-act cycle

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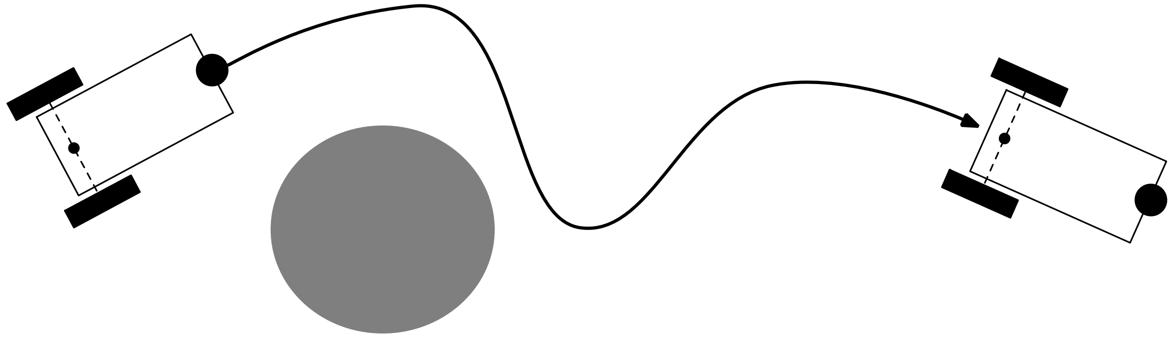
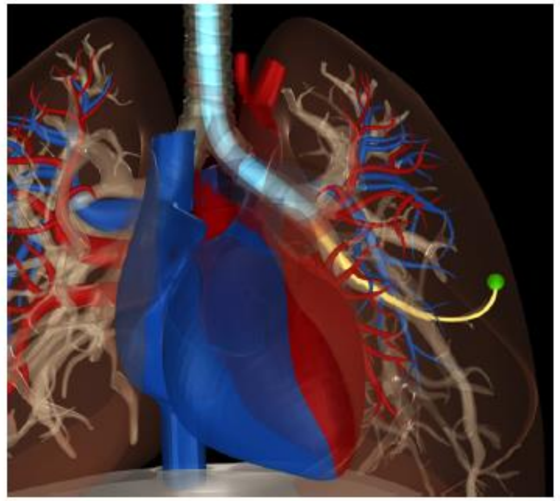
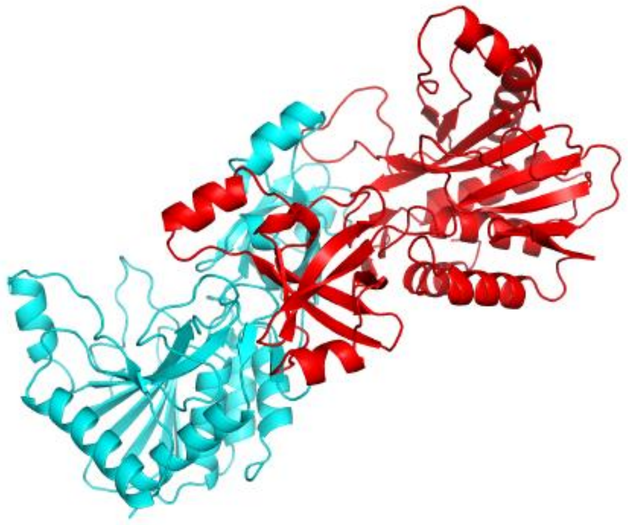
# More examples of motion planning

- Steering autonomous vehicles
- Controlling humanoid robot
- Surgery planning
- Protein folding
- …





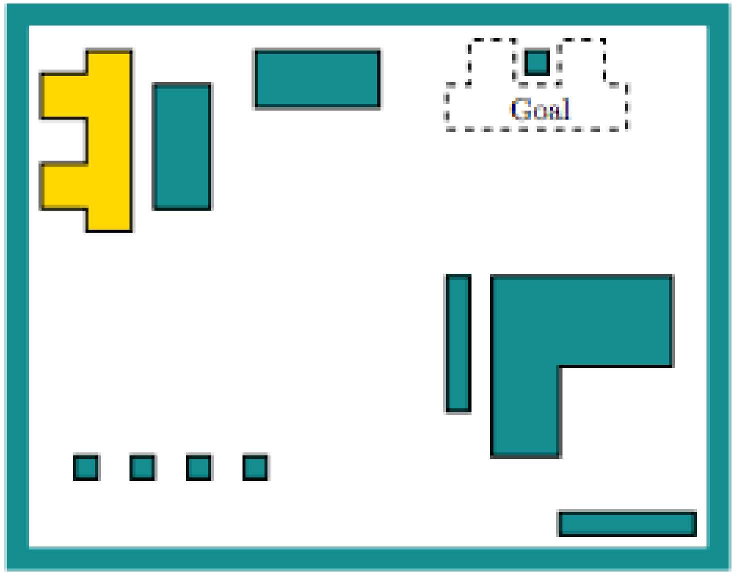


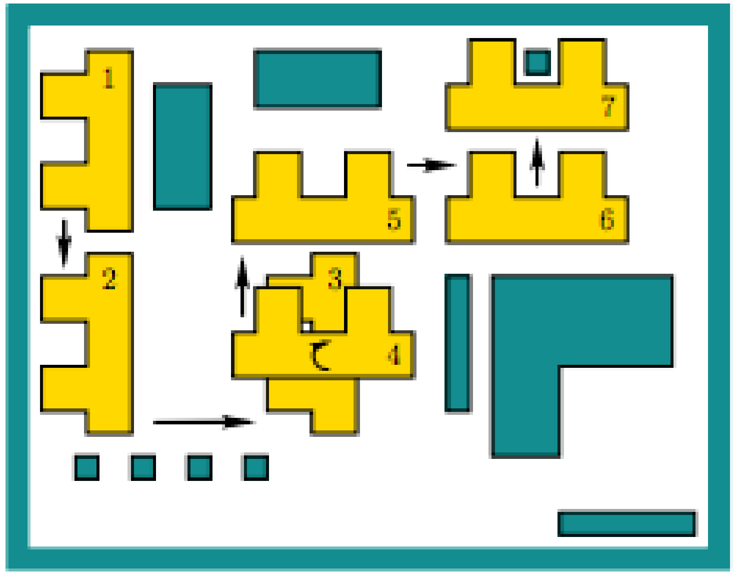
## Some history

- Formally defined in the 1970s
- Development of exact, combinatorial solutions in the 1980s
- Development of sampling-based methods in the 1990s
- Deployment on real-time systems in the 2000s
- Current research: inclusion of differential and logical constraints, planning under uncertainty, parallel implementation, and more

# Simplest setup

- Assume 2D workspace: $\mathcal{W} \subseteq \mathbb{R}^2$
- $\mathcal{O} \subset \mathcal{W}$ is the obstacle region with polygonal boundary
- Robot is a rigid polygon
- Problem: given initial placement of robot, compute how to gradually move it into a desired goal placement so that it never touches the obstacle region





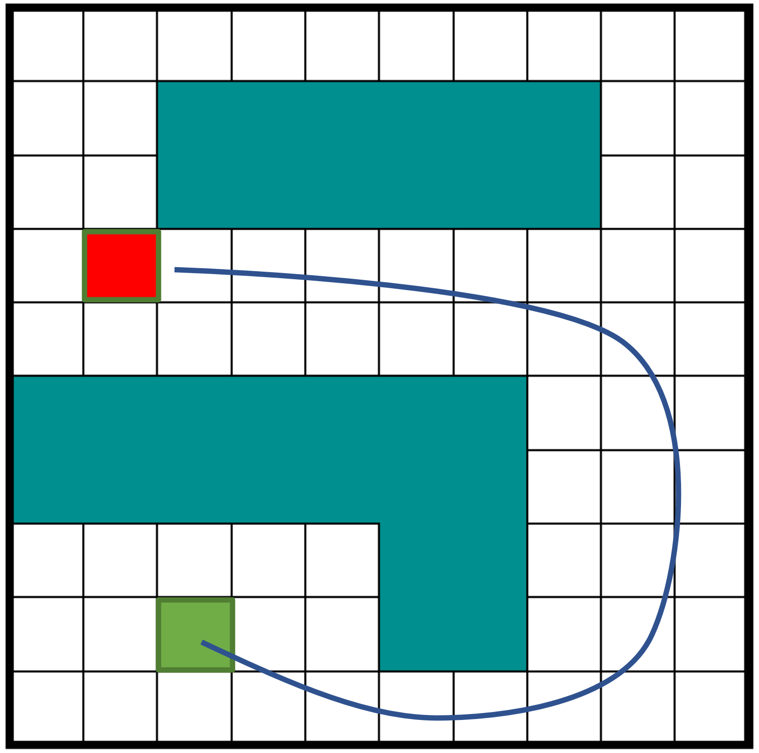
# Popular approaches

- *Potential fields* [Rimon, Koditschek, '92]: create forces on the robot that pull it toward the goal and push it away from obstacles
- *Grid-based planning* [Stentz, '94]: discretizes problem into grid and runs a graph-search algorithm (Dijkstra, A*, ...)
- *Combinatorial planning* [LaValle, '06]: constructs structures in the configuration (C-) space that completely capture all information needed for planning
- *Sampling-based planning* [Kavraki et al, '96; LaValle, Kuffner, '06, etc.]: uses collision detection algorithms to probe and incrementally search the C-space for a solution, rather than completely characterizing all of the $C_{\text{free}}$ structure

# Grid-based approaches

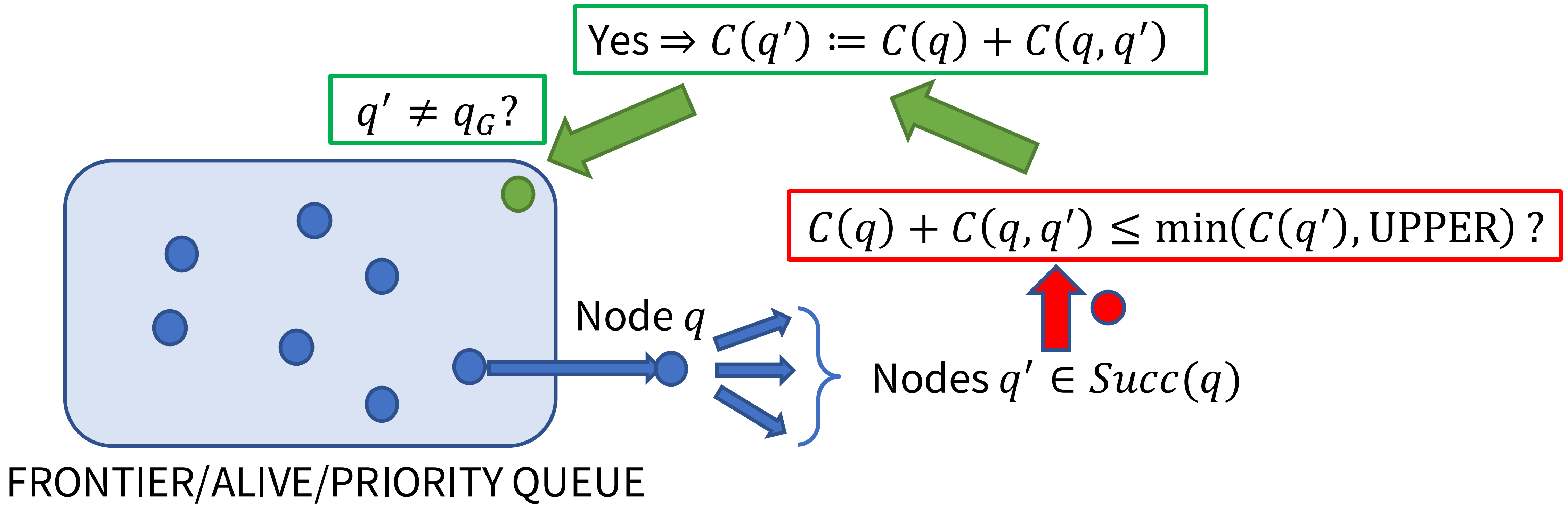
- Discretize the continuous world into a grid
    - Each grid cell is either free or forbidden
    - Robot moves between adjacent free cells
    - **Goal:** find sequence of free cells from start to goal

- Mathematically, this corresponds to pathfinding in a discrete graph $G = (V, E)$
    - Each vertex $v \in V$ represents a free cell
    - Edges $(v, u) \in E$ connect adjacent grid cells



# Graph search algorithms

- Having determined decomposition, how to find “best” path?
- **Label-Correcting Algorithms**: $C(q)$: *cost-of-arrival* from $q_I$ to $q$



Yes $\Rightarrow C(q') := C(q) + C(q,q')$
$q' \neq q_G$?
$C(q) + C(q,q') \leq \min(C(q'), \text{UPPER})$ ?
Node $q$
Nodes $q' \in Succ(q)$
FRONTIER/ALIVE/PRIORITY QUEUE

# Label correcting algorithm

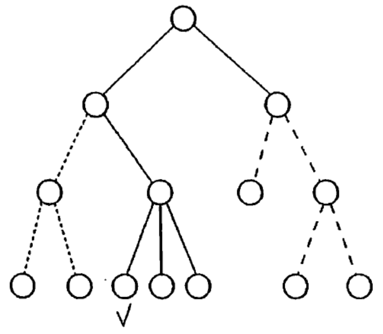
**Step 1.** Remove a node $q$ from frontier queue and for each child $q'$ of $q$, execute step 2

**Step 2.** If $C(q) + C(q, q') \leq \min(C(q'), \mathrm{UPPER})$, set $C(q') \coloneqq C(q) + C(q, q')$ and set $q$ to be the parent of $q'$. In addition, if $q' \neq q_G$, place $q'$ in the frontier queue if it is not already there, while if $q' = q_G$, set UPPER to the new value $C(q) + C(q, q_G)$

**Step 3.** If the frontier queue is empty, terminate, else go to step 1

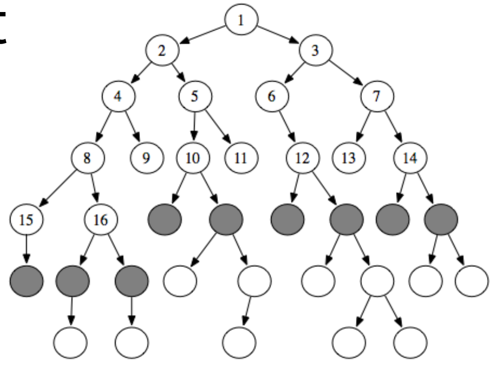
**Initialization**: set the labels of all nodes to ∞, except for the label of the origin node, which is set to 0

# GetNext() ?



Depth-First-Search (DFS): Maintain $Q$ as a **stack** – Last in/first out

- Lower memory requirement (only need to store part of graph)



Breadth-First-Search (BFS, Bellman-Ford): Maintain $Q$ as a **list** – First in/first first out

- Update cost for all edges up to current depth before proceeding to greater depth
- Can deal with negative edge (transition) costs

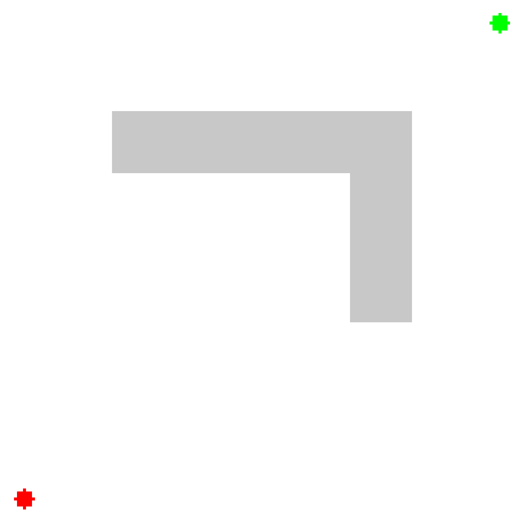
Best-First (BF, Dijkstra): Greedily select next $q$: $q = \operatorname{argmin}_{q \in Q} C(q)$

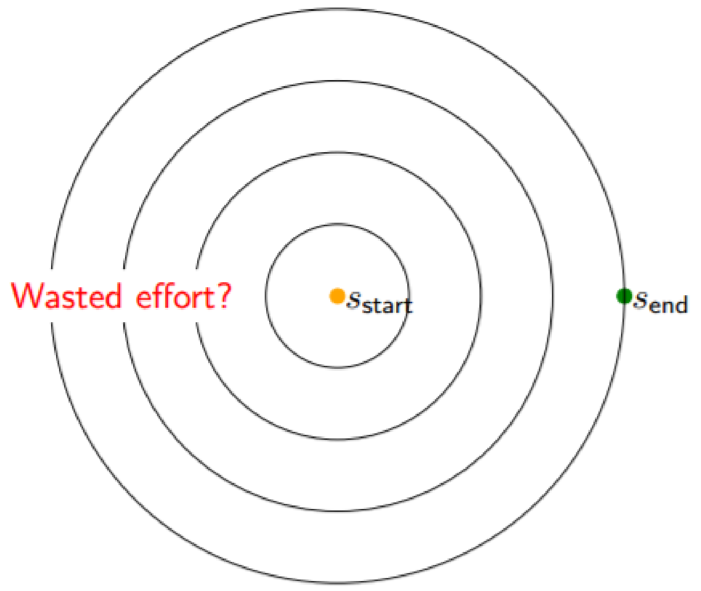
- Node will enter the frontier queue at most *once*
- Requires costs to be non-negative

# Correctness and improvements

**Theorem**

If a feasible path exists from $q_I$ to $q_G$, then algorithm terminates in finite time with $C(q_G)$ equal to the optimal cost of traversal, $C^*(q_G)$.





* https://en.wikipedia.org/wiki/Dijkstra%27s_algorithm

## A*: Improving Dijkstra

- Dijkstra orders by optimal "*cost-to-arrival*"
- Faster results if order by "*cost-to-arrival*"+ *(approximate)* "*cost-to-go*"
- That is, strengthen test

$$C(q) + C(q, q') \leq \text{UPPER}$$

to

$$C(q) + C(q, q') + h(q') \leq \text{UPPER}$$

where $h(q)$ is *a* heuristic for optimal cost-to-go (specifically, a positive *underestimate*)

- In this way, fewer nodes will be placed in the frontier queue
- This modification still guarantees that the algorithm will terminate with a shortest path
- Many variations are possible… see (Problem 2 in pset 2)

# Grid-based approaches: summary

- Pros:
  - Simple and easy to use
  - Fast (for some problems)
- Cons:
  - Resolution dependent
    - Not guaranteed to find solution if grid resolution is not small enough
  - Limited to simple robots
    - Grid size is exponential in the number of DOFs

# Back to continuous motion planning

- Assume 2D workspace: $\mathcal{W} \subseteq \mathbb{R}^2$
- $\mathcal{O} \subset \mathcal{W}$ is the obstacle region with polygonal boundary
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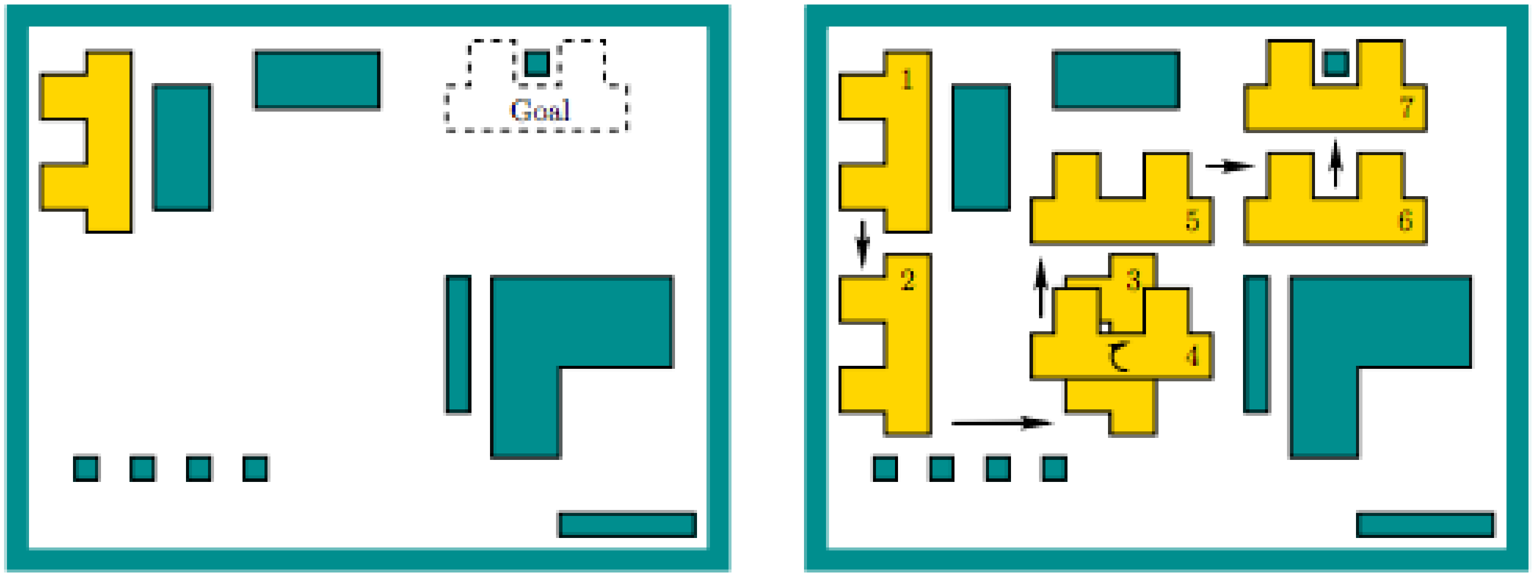
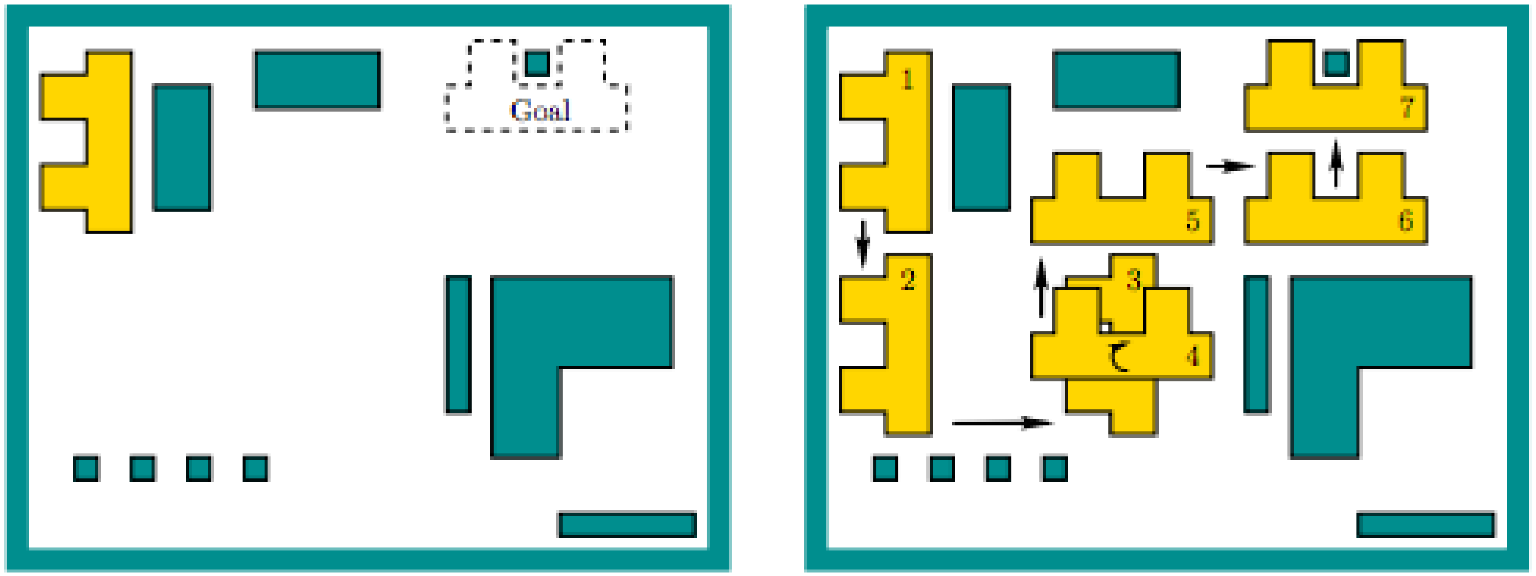
- Problem: Given initial placement of robot, compute how to gradually move it into a desired goal placement so that it never touches the obstacle region



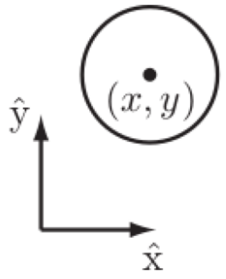
# Back to continuous motion planning

Key point: motion planning problem described in the real-world, but it really lives in another space -- the configuration (*C*-) space!

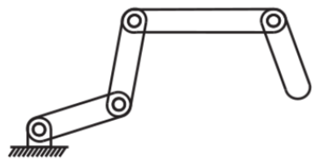


# Configuration space

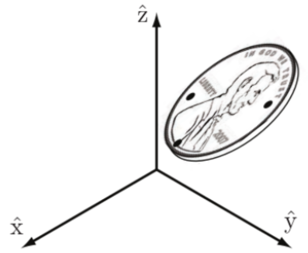
- *C*- space: captures all degrees of freedom (all rigid body transformations)
- More in detail, let $\mathcal{R} \subset \mathbb{R}^2$ be a polygonal robot (e.g., a triangle)
- The robot can rotate by angle $\theta$ or translate $(x_t, y_t) \subset \mathbb{R}^2$
- Every combination $q = (x_t, y_t, \theta)$ yields a *unique* robot placement: configuration
- So *C*- space is a subset of $\mathbb{R}^3$
- Note: $\theta \pm 2\pi$ yields equivalent rotations $\Rightarrow$ *C*- space is: $\mathbb{R}^2 \times \mathcal{S}^1$
- Concept of *C*- space extends naturally to higher dimensions (e.g., robot linkages)



$d = 2$



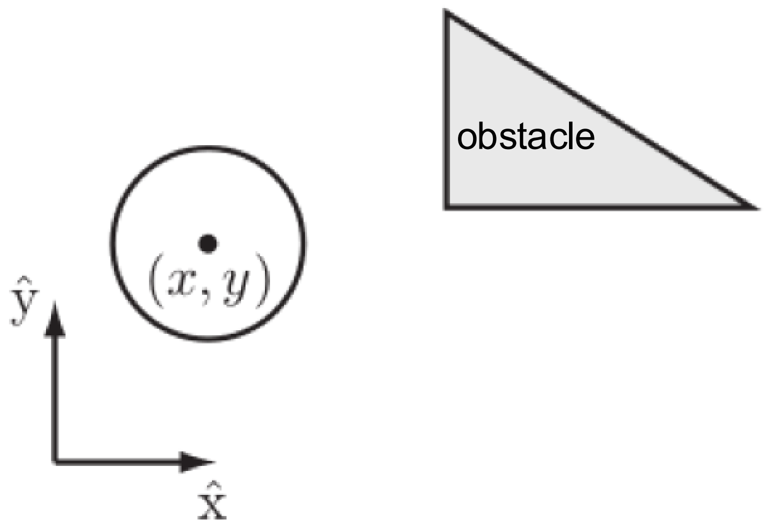
$d = 4$

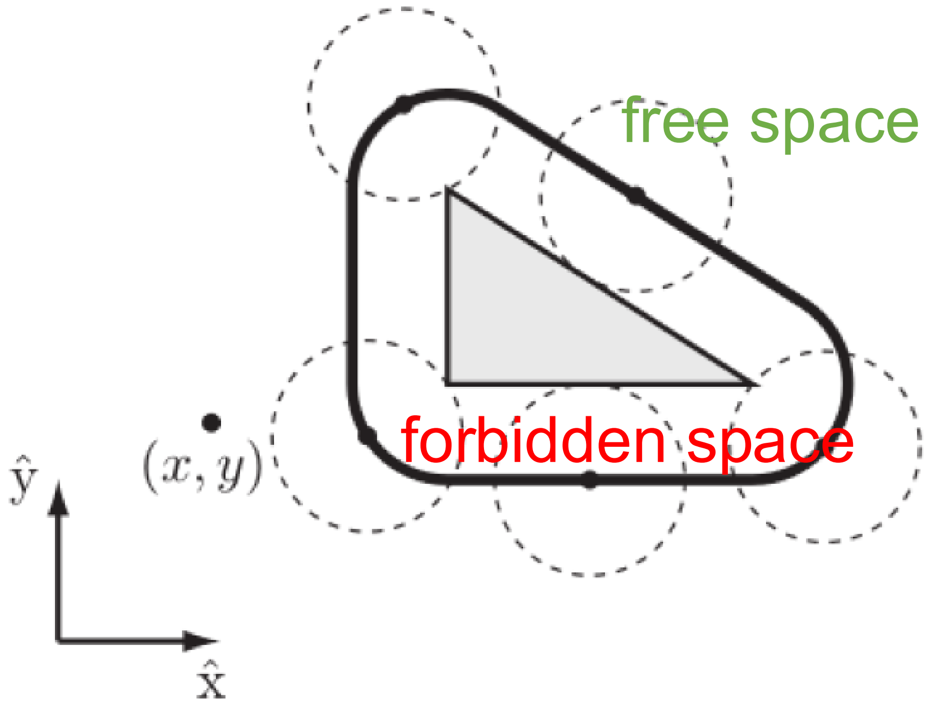


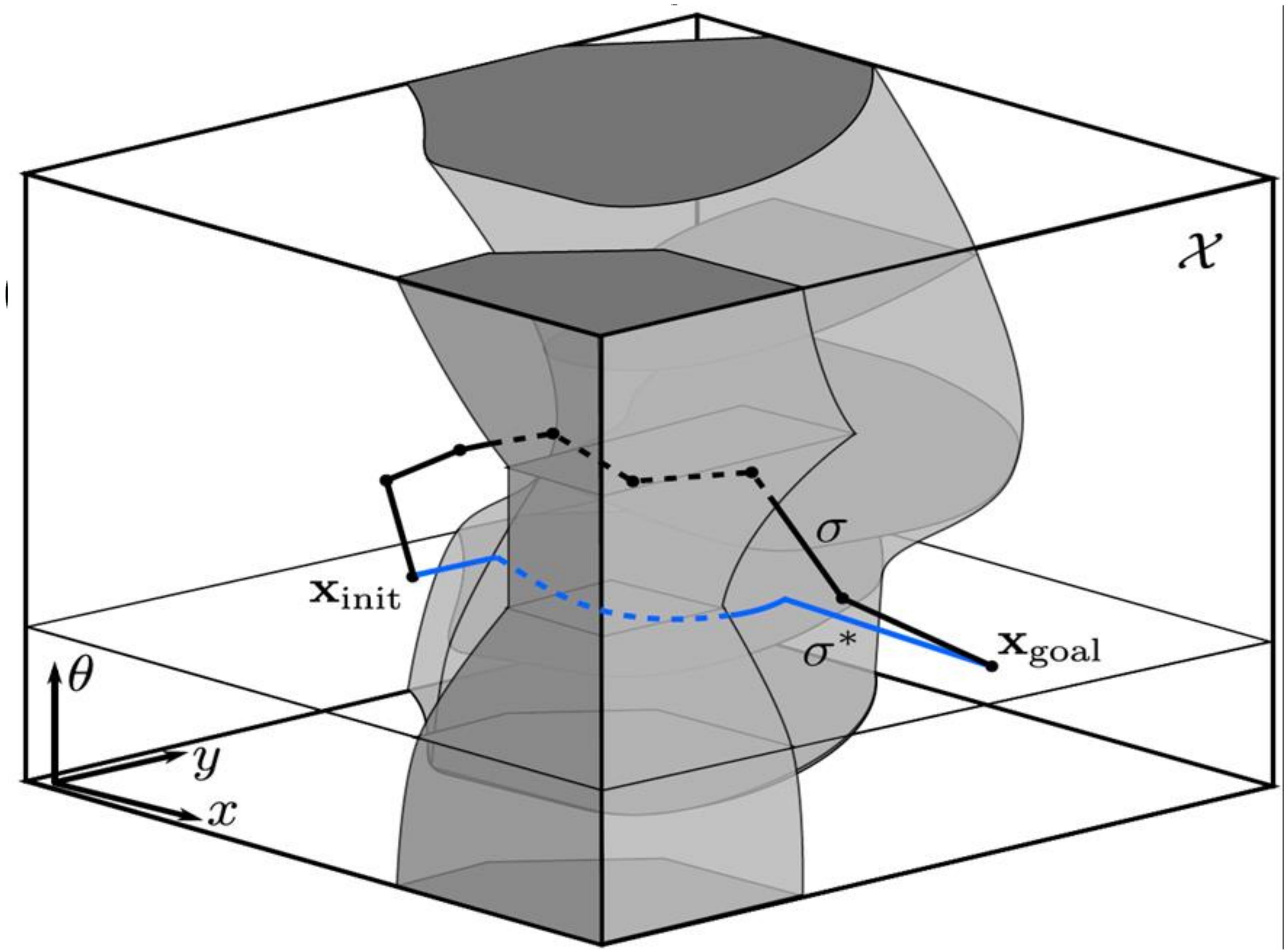
$d = 6$

# Configuration free space

- The subset $\mathcal{F} \subseteq \mathcal{C}$ of all collision free configurations is the **free space**



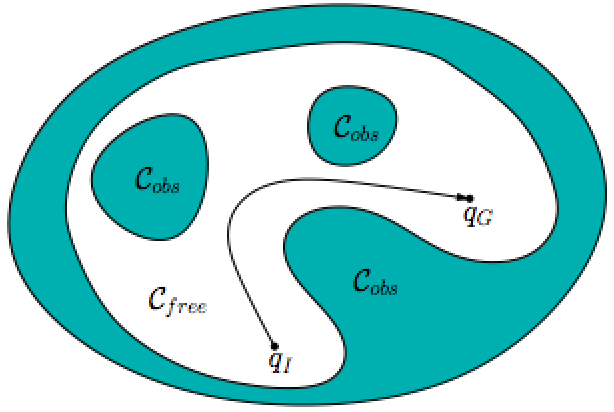



$\mathcal{X}$
$\sigma$
$\mathbf{x}_{\text{init}}$
$\sigma^*$
$\mathbf{x}_{\text{goal}}$
$\theta$
$y$
$x$

Bottom line: *explicitly* computing C free spaces in high-dimensional settings is hard!

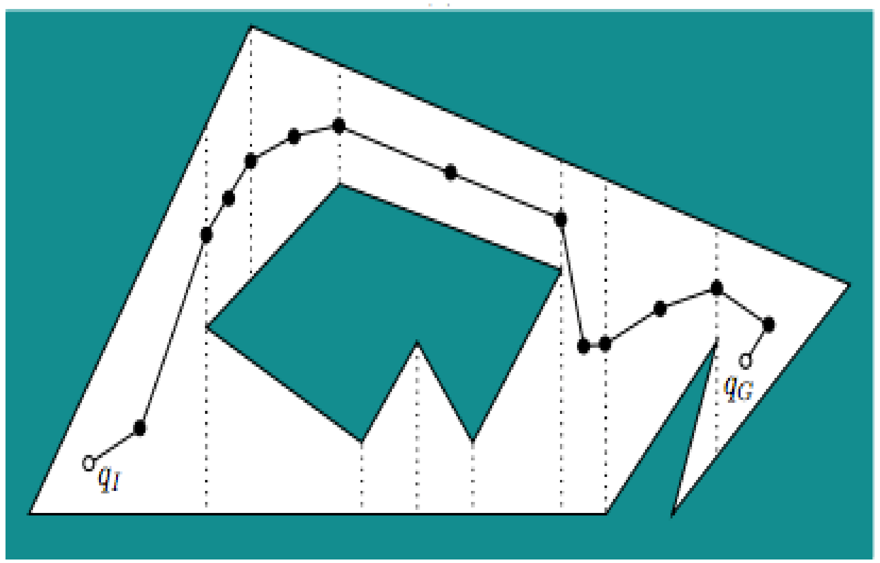
# Planning in $C$-space

- Let $R(q) \subset W$ denote set of points in the world occupied by robot when in configuration $q$
- Robot in collision $\Leftrightarrow R(q) \cap O \neq \emptyset$
- Accordingly, *free* space is defined as: $C_{\text{free}} = \{q \in C | R(q) \cap O = \emptyset\}$
- Path planning problem in $C$-space: compute a **continuous** path: $\tau: [0,1] \rightarrow C_{\text{free}}$, with $\tau(0) = q_I$ and $\tau(1) = q_G$



# Combinatorial planning

- Combinatorial approaches to motion planning find paths through continuous configuration space without resorting to approximations
- Key idea: compute a roadmap, which provides a discrete representation of continuous motion planning problem without losing any of the original connectivity information needed to solve it
- Such approaches are typically complete (i.e., guaranteed to find a solution), but are typically limited to small number of DOFs due to the challenge of *exactly* computing C free spaces



A roadmap is a graph in which each vertex is a configuration in $\mathcal{C}_{\text{free}}$ and each edge is a path through $\mathcal{C}_{\text{free}}$ that connects a pair of vertices

# Next time: sampling-based planning