

# Principles of Robot Autonomy I

Course overview, mobile robot kinematics



# Team

## Instructor



Prof. Jeannette Bohg

## Collaborators

Daniel Watzenig

## Labs



Center for Automotive  
Research at Stanford



IPRL

9/26/22

## Course Assistants



Zhengguan(Gary) Dai



Hao Li



Brian Dobkowski



Stephanie Newdick



Mason Murray-Cooper



Alvin Sun

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# From automation...



# ...to autonomy

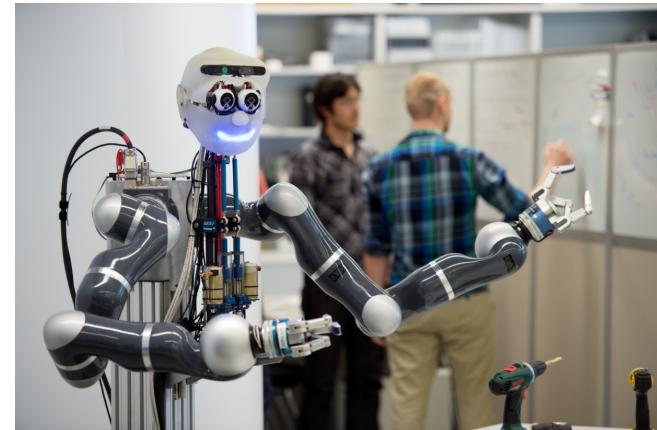
Waymo Self-Driving Car



Intuitive DaVinci Surgical Robot



Apollo Robot at MPI for Intelligent Systems



Boston Dynamics – Spot Mini

9/26/22



Astrobee - NASA



Zipline

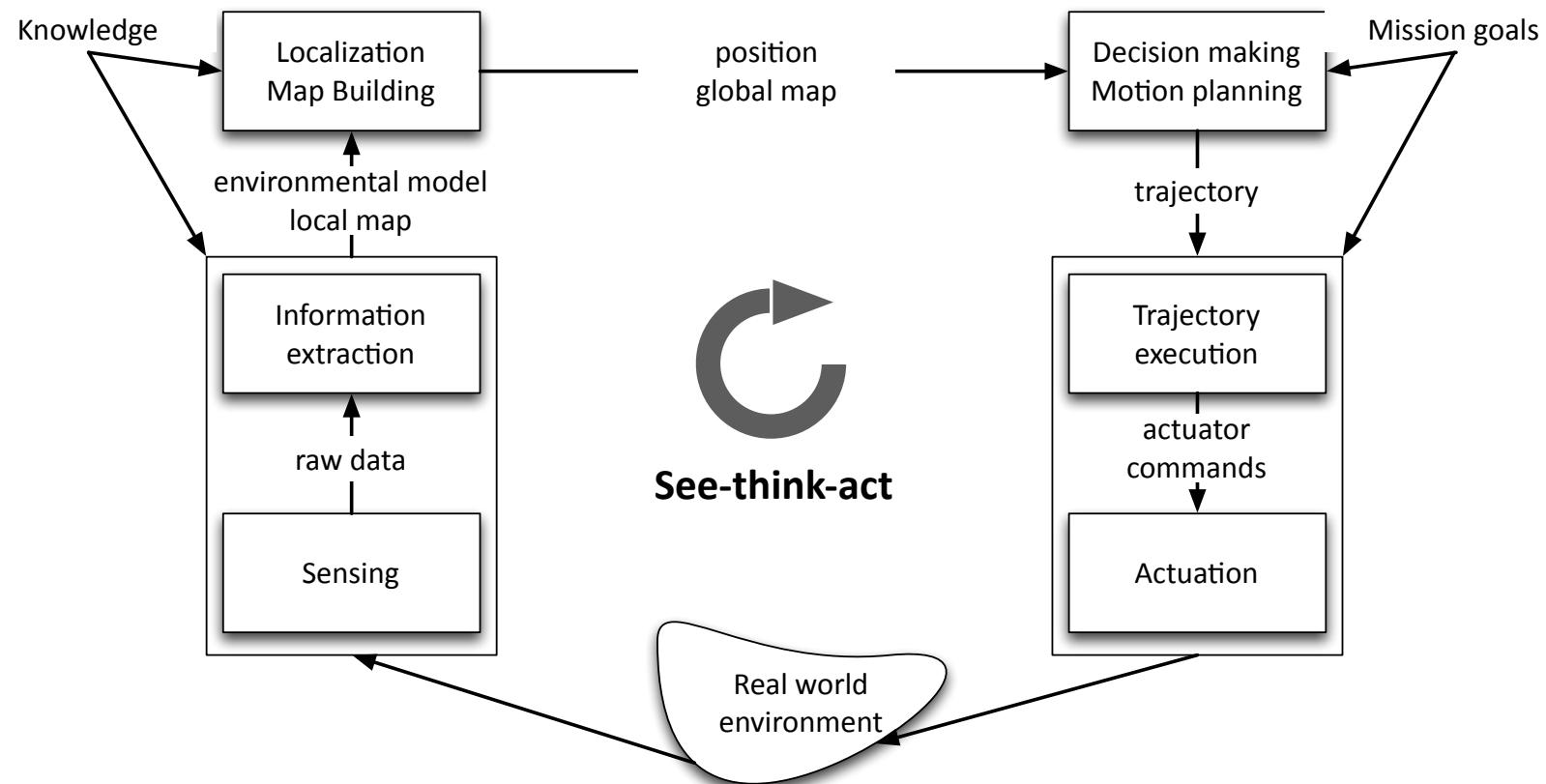
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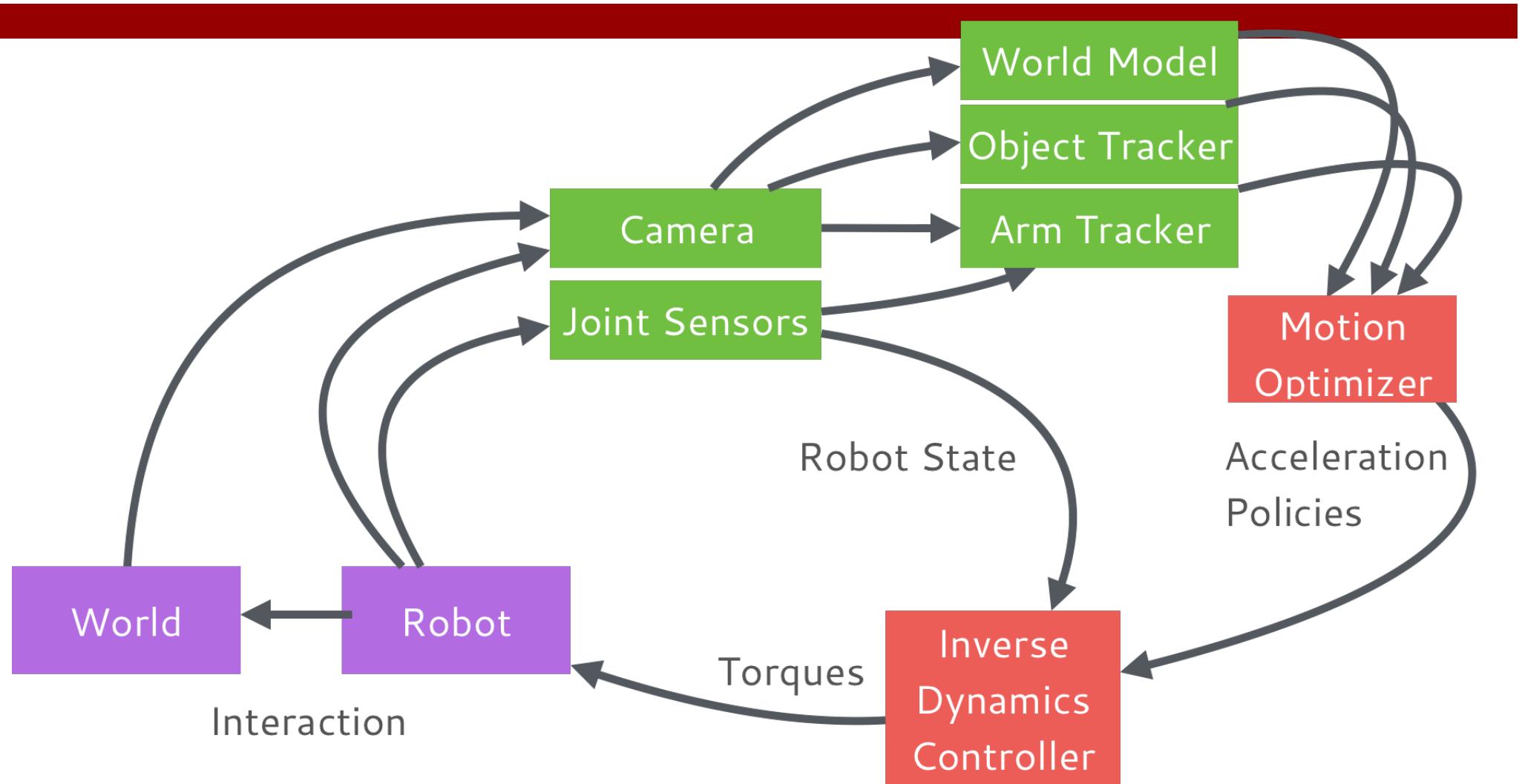
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# Course goals

- To learn the *theoretical, algorithmic, and implementation* aspects of main techniques for robot autonomy. Specifically, the student will
  1. Gain a fundamental knowledge of the “autonomy stack”
  2. Be able to apply such knowledge in applications / research by using ROS
  3. Devise novel methods and algorithms for robot autonomy

# The see-think-act cycle





# Course structure

- Four modules, roughly of equal length
  1. motion control and planning
  2. robotic perception
  3. localization and SLAM
  4. state machines and system architecture
- Extensive use of the Robot Operating System (ROS)
- Requirements
  - CS 106A or equivalent
  - CME 100 or equivalent (for calculus, linear algebra)
  - CME 106 or equivalent (for probability theory)
  - See also the [pre-knowledge quiz](#) on the course website

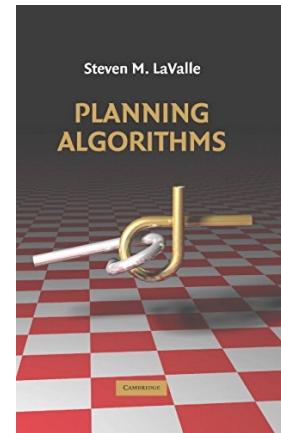
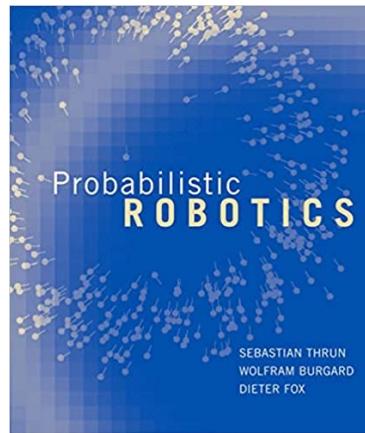
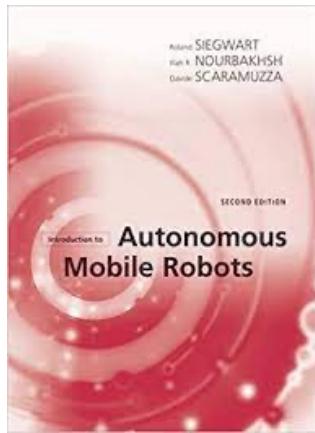
# Schedule

Week	Topic	
1	Course overview, mobile robot kinematics Introduction to the Robot Operating System (ROS) <i>Thursday: HW1 out</i>	
2	Trajectory optimization Trajectory tracking & closed loop control	
3	Motion planning I: graph search methods Motion planning II: sampling-based methods <i>Tuesday: HW1 due, HW2 out</i>	
4	Robotic sensors & introduction to computer vision Camera models & camera calibration	
5	Image processing, feature detection & description Information extraction & classic visual recognition <i>Tuesday: HW2 due, HW3 out</i>	
6	Intro to localization & filtering theory Parameteric filtering (KF, EKF, UKF)	
7	<i>Tuesday: No lecture (Democracy Day)</i> Nonparameteric filtering (PF) <i>Thursday: Final project released</i> <i>Tuesday: HW3 due, HW4 out</i>	
8	Object detection / tracking, EKF localization Simultaneous localization and mapping (SLAM)	
N/A	<i>Thanksgiving Break</i>	
9	Multi-sensor perception & sensor fusion I (by Daniel Watzenig) Multi-sensor perception & sensor fusion II (by Daniel Watzenig) <i>Tuesday: HW4 due</i>	
10	Stereo vision State machines <i>Tuesday: Final project check-in due</i>	
11	Final Project Presentation and Demo 12/15 3:30 - 6:30 PM	

In-Person attendance is not required!

# Logistics - Lectures

- Tuesdays and Thursdays, 10:30am – 11:50 (Gates B1)
- Recordings will be made available to all students on Canvas.
- Course Materials in addition to Course Notes:



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*Mobile Robot Kinematics*

*Mobile Robot Kinematics*

Motion planning and control are fundamental components of robotic autonomy<sup>1</sup>. For example, in order for an autonomous car to accomplish an objective (e.g. move from point A to B) it first needs to plan a trajectory and determine what control inputs (e.g. throttle and steering) will enable it to follow the trajectory. Both of these components require an understanding of the physical behavior of the robot in order to develop reasonable/actionable plans and controls. In the context of motion planning and control, a robot's physical behavior

<sup>1</sup> R. Siegwart, I. R. Nourbakhsh, and D. Scaramuzza. *Introduction to Autonomous Mobile Robots*. MIT Press, 2011

# Logistics – Homework Assignments

- 4 assignments
- First Assignment out on Thursday
- ~2 weeks to submit on Gradescope
- Assignments are due Tuesdays which is when a new assignment will be released
- Budget of 6 late days, max 3 days per assignment
- Cooperation and discussion is encouraged, but solutions must be prepared individually. Add names of classmates who you collaborated with. Copying from other students or other sources is considered a case of academic dishonesty.
- Need to be typeset in Latex!

# Logistics – Sections

- 2-hour, once-a-week sessions starting Week 2
- Hands-on exercises that complement the lecture material, build familiarity with ROS, develop skills necessary for the final project

Monday: 5:30 – 7:30pm (virtual) `alvinsun`

Tuesday: 10:00am – 12:00pm (in-person) `li2053`

Tuesday: 4:30 – 6:30pm (in-person) `garydai`

Wednesday: 10:00am – 12:00pm (in-person) `masonmc`

Wednesday: 12:30 – 2:30pm (in-person) `snewdick`

Wednesday: 6:00 – 8:00pm (in-person) `bdobkows`

Thursday: 9:30 – 11:30am (virtual) `li2053`

Thursday: 12:00 – 2:00pm (in-person) `alvinsun`

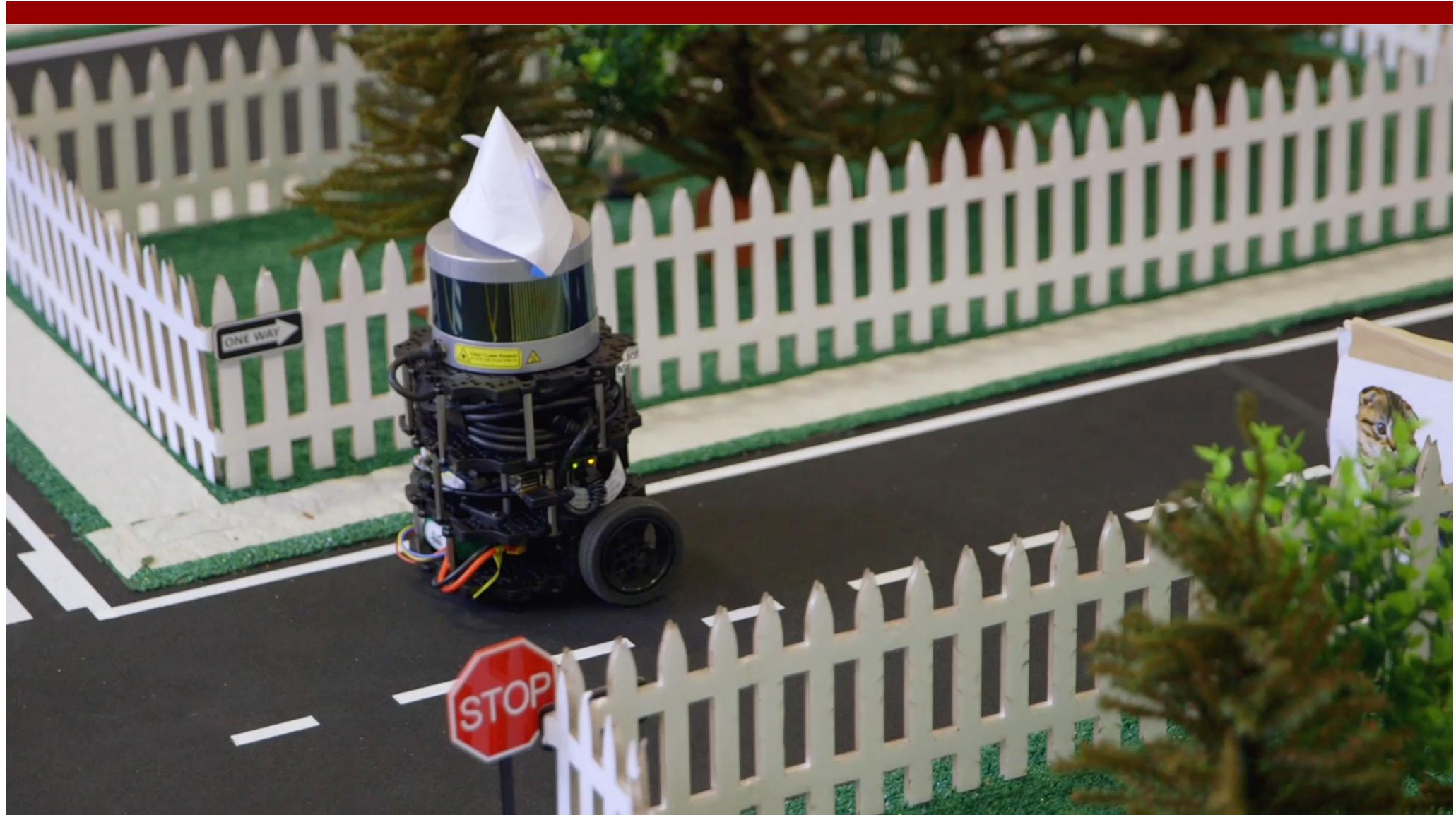
Thursday: 4:30 – 6:30pm (virtual) `garydai`

Friday: 9:30 – 11:30am (in-person) `snewdick`

Friday: 12:00 – 2:00pm (in-person) `bdobkows`

Friday: 2:30 – 4:30pm (in-person) `masonmc`

- Section sign-up sheet coming soon!





# Logistics - Grades

- (20%) final project.
- (60%) homework.
- (20%) sections.
- (extra 5%) participation on Ed Discussion

## 4 or 3 units?

- AA174A: 4 units
- AA 274A/CS 237A/EE 260A: 3 or 4. Taking this class for 4 units entails completing an additional homework problem per problem set and also writing a one-page review of a paper at the end of the quarter.

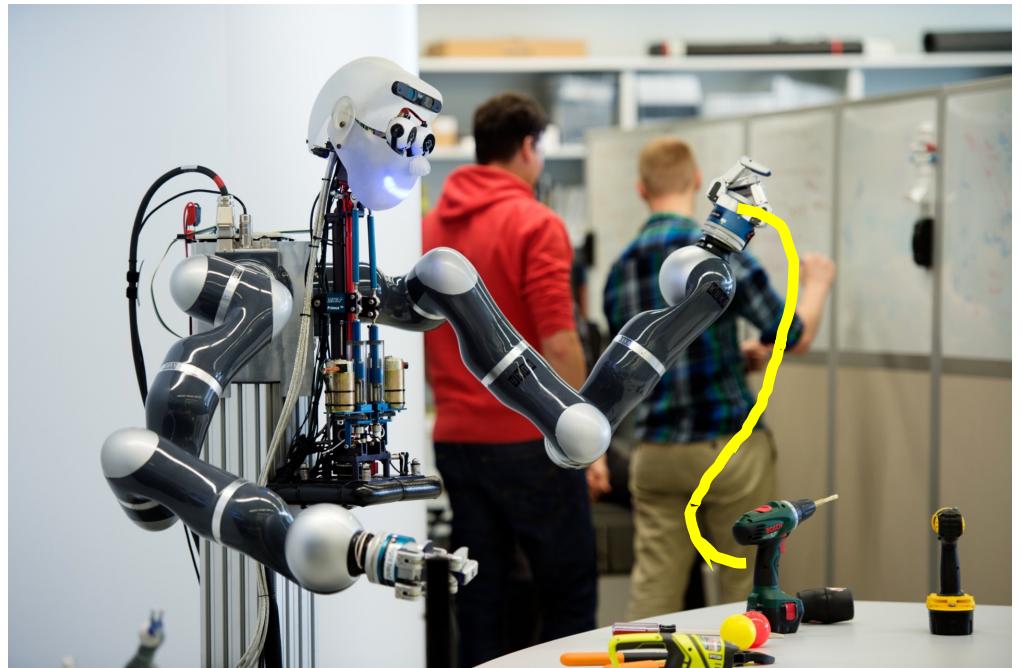
# Logistics

- Office hours:
  - Prof. Jeannette Bohg: Friday, 1-2pm (Gates 244 and Zoom)
  - CAs: Mondays 1 – 3pm (in-person), `garydai`, ...,  
Tuesdays 2pm – 4pm (virtual) `masonmc`, `snewdick`,  
Thursdays 6pm – 8pm (virtual) `alvinsun`, `bdkows`.  
Friday 10am – 12pm (virtual) `alvinsun`, `li2053`.
- Course websites:
  - For course content: <http://asl.stanford.edu/aa274a/>
  - For course-related questions: <https://edstem.org/us/courses/28635>
  - For homework submissions: <https://www.gradescope.com/courses/439779>
  - For announcements and lecture videos: <https://canvas.stanford.edu/courses/159179>
  - To contact the AA274 staff, use the email: [cs237a-aut2223-staff@lists.stanford.edu](mailto:cs237a-aut2223-staff@lists.stanford.edu)
- Syllabus has all the info!

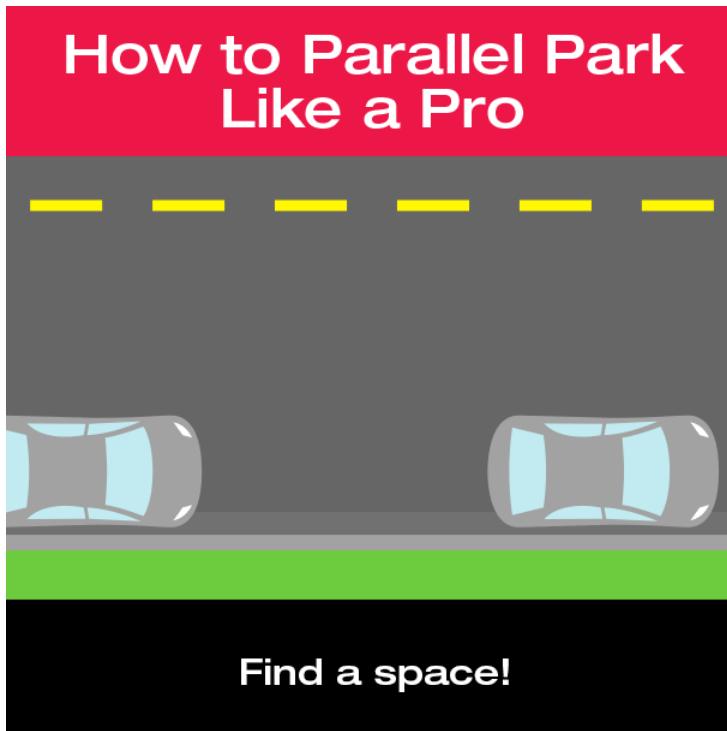
# Mobile robot kinematics

- Aim
  - Understand motion constraints
  - Learn about basic motion models for wheeled vehicles
  - Gain insights for motion control
- Readings
  - R. Siegwart, I. R. Nourbakhsh, D. Scaramuzza. *Introduction to Autonomous Mobile Robots*. MIT Press, 2nd Edition, 2011. Sections 3.1-3.3.
  - B. Siciliano, L. Sciavicco, L. Villani, G. Oriolo. *Robotics: Modelling, Planning, and Control*. Springer, 2008 (chapter 11).

# Motion Planning and Control

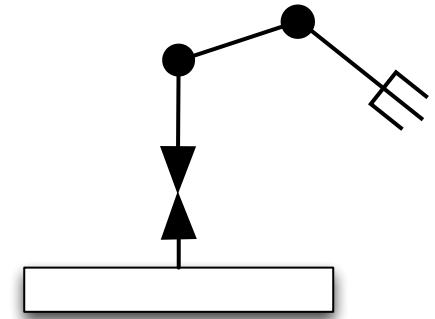
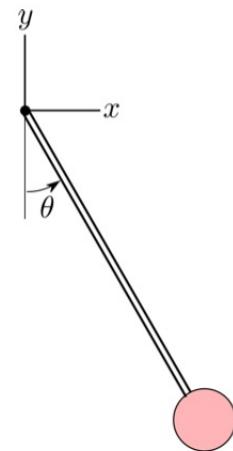
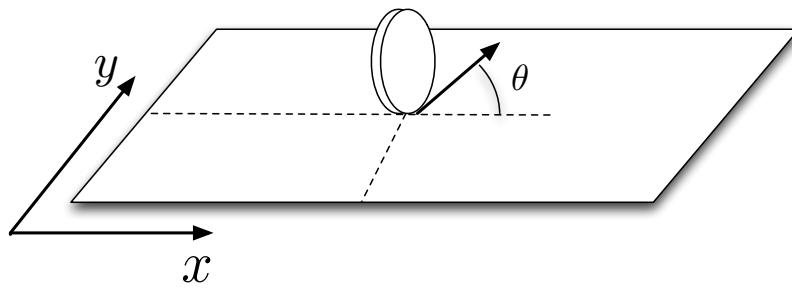


# Constraints in Motion Planning and Control



# Generalized Coordinates

- Let  $\xi = [\xi_1, \dots, \xi_n]^T$  denote the configuration of a robot (e.g.,  $\xi = [x, y, \theta]^T$  for a wheeled mobile robot)

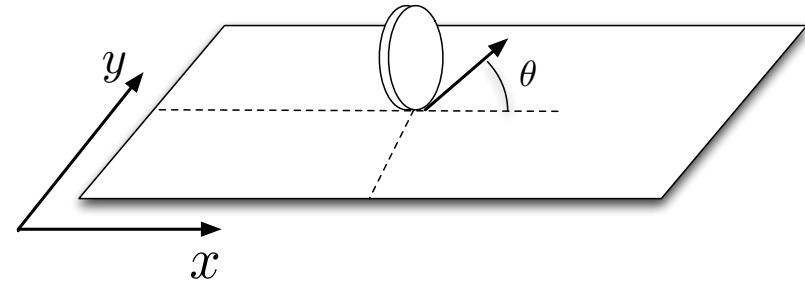
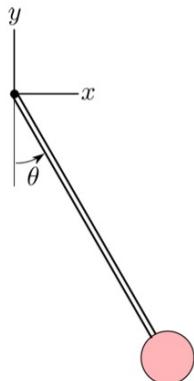


# Kinematic constraints

$$a_i(\xi, \dot{\xi}) = 0, \quad i = 1, \dots, k < n$$

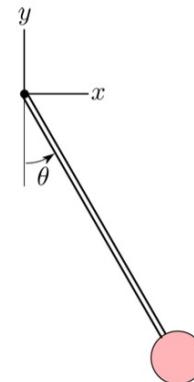
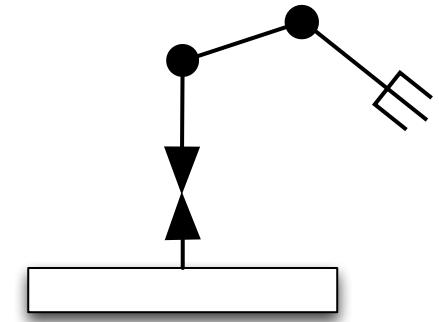
- constrain the instantaneous admissible motion of the mechanical system
- generally expressed in Pfaffian form, i.e., linear in the generalized velocities

$$a_i^T(\xi) \dot{\xi} = 0, \quad i = 1, \dots, k < n$$



# Holonomic constraints

- $h_i(\xi) = 0$ , for  $i = 1, \dots, k < n$
- Reduce space of accessible configurations to an  $n - k$  dimensional subset
- If all constraints are holonomic, the mechanical system is called holonomic
- Generally, the result of mechanical interconnections



# Examples of Holonomic constraints



Xiang, Qin, Mo et al., "SAPIEN: A SimulAted Part-based Interactive ENvironment", CVPR 2020

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# Kinematic constraints

$$a_i(\xi, \dot{\xi}) = 0, \quad i = 1, \dots, k < n$$

- constrain the instantaneous admissible motion of the mechanical system
- generally expressed in Pfaffian form, i.e., linear in the generalized velocities

$$a_i^T(\xi) \dot{\xi} = 0, \quad i = 1, \dots, k < n$$

- $k$  holonomic constraints imply the existence of an equal number of kinematic constraints

$$\frac{d h_i(\xi)}{dt} = \frac{\partial h_i(\xi)}{\partial \xi} \dot{\xi} = 0, \quad i = 1, \dots, k < n$$

- However, the converse is not true in general...

# Nonholonomic constraints

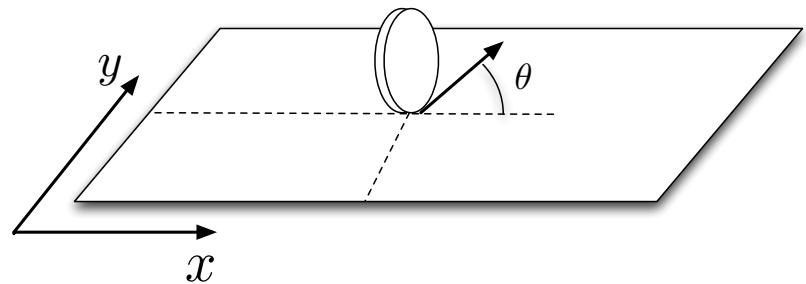
- If a kinematic constraint is not integrable in the form  $h_i(\xi) = 0$ , then it is said *nonholonomic* -> nonholonomic mechanical system
- Nonholonomic constraints reduce mobility in a completely different way. Consider a single Pfaffian constraint

$$a^T(\xi) \dot{\xi} = 0$$

- Holonomic
  - Can be integrated to  $h(\xi) = 0$
  - Loss of accessibility, motion constrained to a level surface of dimension  $n - 1$
- Nonholonomic
  - Velocities constrained to belong to a subspace of dimension  $n - 1$ , the null space of  $a^T(\xi)$
  - No loss of accessibility

# Example of nonholonomic system

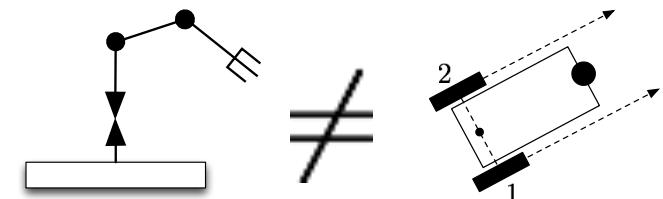
- System: disk that rolls without slipping
- $\xi = [x, y, \theta]^T$



- No side slip constraint

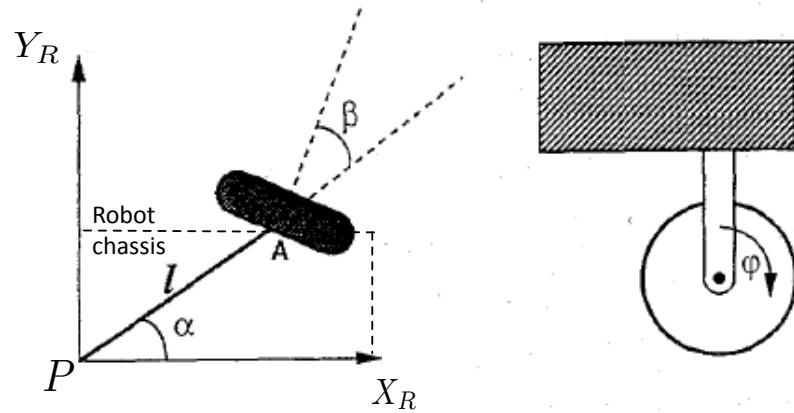
$$[\dot{x}, \dot{y}] \cdot \begin{bmatrix} \sin \theta \\ -\cos \theta \end{bmatrix} = \dot{x} \sin \theta - \dot{y} \cos \theta = [\sin \theta, -\cos \theta, 0] \dot{\xi} = 0$$

- Facts:
  - No loss of accessibility
  - Wheeled vehicles are generally nonholonomic

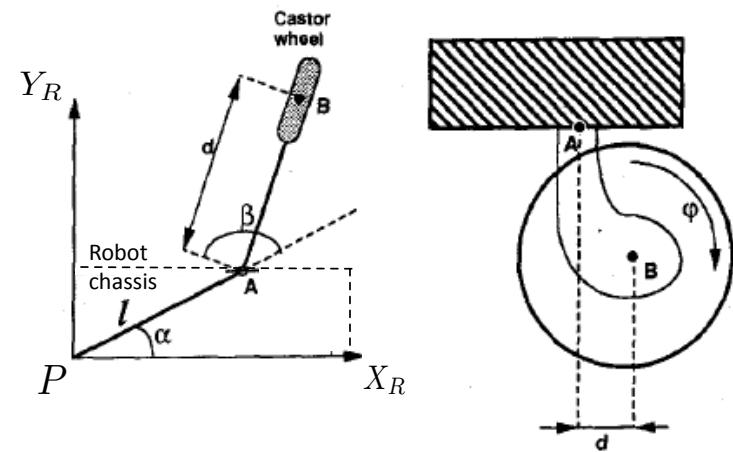


# Types of wheels

- Standard wheels (four types)



Standard wheel -- fixed or steerable



Standard, off-centered wheel (caster)  
-- passive or active

- Special wheels: achieve omnidirectional motion (e.g., Swedish or spherical wheels)

# Kinematic models

- Assume the motion of a system is subject to  $k$  Pfaffian constraints

$$\begin{bmatrix} a_1^T(\xi) \\ \vdots \\ a_k^T(\xi) \end{bmatrix} \dot{\xi} := A^T(\xi) \dot{\xi} = 0$$

- Then, the admissible velocities at each configuration  $\xi$  belong to the  $(n - k)$ -dimensional null space of matrix  $A^T(\xi)$
- Denoting by  $\{g_1(\xi), \dots, g_{n-k}(\xi)\}$  a basis of the null space of  $A^T(\xi)$ , admissible trajectories can be characterized as solutions to

$$\dot{\xi} = \sum_{j=1}^{n-k} g_j(\xi) u_j = G(\xi) u$$

Input vector

## Example: unicycle

- Consider pure rolling constraint for the wheel:

$$\dot{x} \sin \theta - \dot{y} \cos \theta = [\sin \theta, -\cos \theta, 0] \dot{\xi} = a^T(\xi) \dot{\xi} = 0$$

- Consider the matrix

$$G(\xi) = [g_1(\xi), g_2(\xi)] = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix}$$

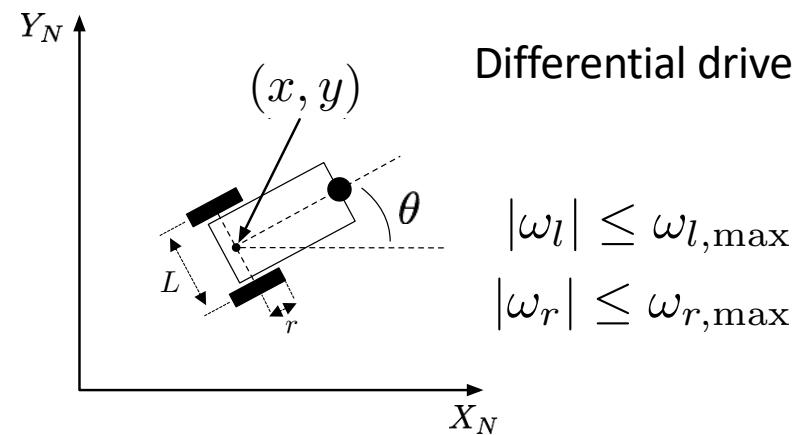
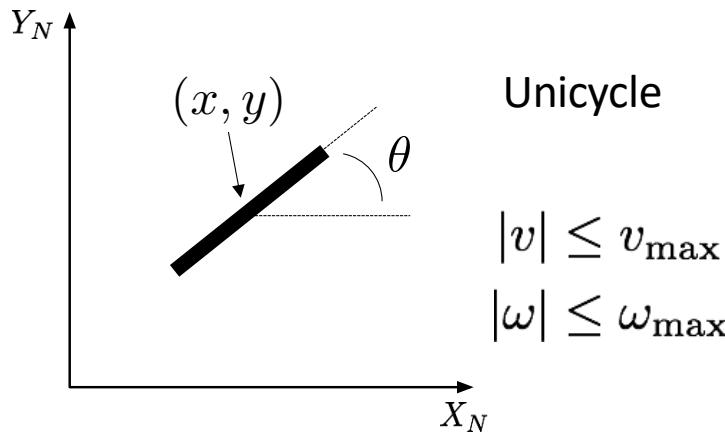
where  $[g_1(\xi), g_2(\xi)]$  is a basis of the null space of  $a^T(\xi)$

- All admissible velocities are therefore obtained as linear combination of  $g_1(\xi)$  and  $g_2(\xi)$

# Unicycle and differential drive models

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos \theta \\ \sin \theta \\ 0 \end{pmatrix} v + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \omega$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \frac{r}{2}(\omega_l + \omega_r) \cos \theta \\ \frac{r}{2}(\omega_l + \omega_r) \sin \theta \\ \frac{r}{L}(\omega_r - \omega_l) \end{pmatrix}$$



The kinematic model of the unicycle also applies to the differential drive vehicle, via the one-to-one input mappings:

$$v = \frac{r}{2}(\omega_r + \omega_l) \quad \omega = \frac{r}{L}(\omega_r - \omega_l)$$

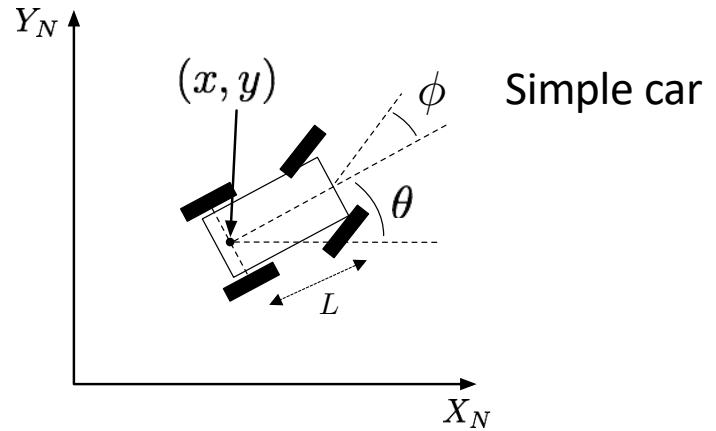
# Simplified car model

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} v \cos \theta \\ v \sin \theta \\ \frac{v}{L} \tan \phi \end{pmatrix}$$

$$|v| \leq v_{\max}, \ |\phi| \leq \phi_{\max} < \frac{\pi}{2}$$

$$v \in \{-v_{\max}, v_{\max}\}, \ |\phi| \leq \phi_{\max} < \frac{\pi}{2}$$

$$v = v_{\max}, \ |\phi| \leq \phi_{\max} < \frac{\pi}{2}$$



- Simple car model
- Reeds&Shepp's car
- Dubins' car

References: (1) J.-P. Laumond. Robot Motion Planning and Control. 1998. (2) S. LaValle. Planning algorithms, 2006.

# From kinematic to dynamic models

- A kinematic state space model should be interpreted only as a subsystem of a more general dynamical model
- Improvements to the previous kinematic models can be made by placing **integrators** in front of action variables
- For example, for the unicycle model, one can set the speed as the integration of an action  $a$  representing acceleration, that is

$$\dot{x} = v \cos \theta, \quad \dot{y} = v \sin \theta, \quad \dot{\theta} = \omega, \quad \dot{v} = a$$

Next time

