AA 274
Principles of Robotic Autonomy

Information extraction and intro to

machine learning for robot autonomy

Stanford University

ASL

Autonomous Systems Lab

Techniques for information extraction

- Aim
  - Learn how to extract information from sensor measurements

- Readings
  - SNS: 4.1.3, 4.6.1 - 4.6.5, 4.7.1 - 4.7.4

Information extraction

- Next step is to extract *information* from images, such as
  - Geometric primitives (e.g., lines and circles): useful, for example, for robot localization and mapping
  - Object recognition and scene understanding: useful, for example, for localization within a topological map and for high-level reasoning

Geometric feature extraction

- Geometric feature extraction: extract geometric primitives from sensor data (e.g., range data)
- Examples: line, circles, corners, planes, etc.
- We focus on *line extraction* from range data (a quite common task); other geometric feature extraction tasks are conceptually analogous
- The two main problems of line extraction from range data
  1. Which points belong to which line? -> *segmentation*
  2. Given an association of points to a line, how to estimate line parameters? -> *fitting*

Step #2: line fitting

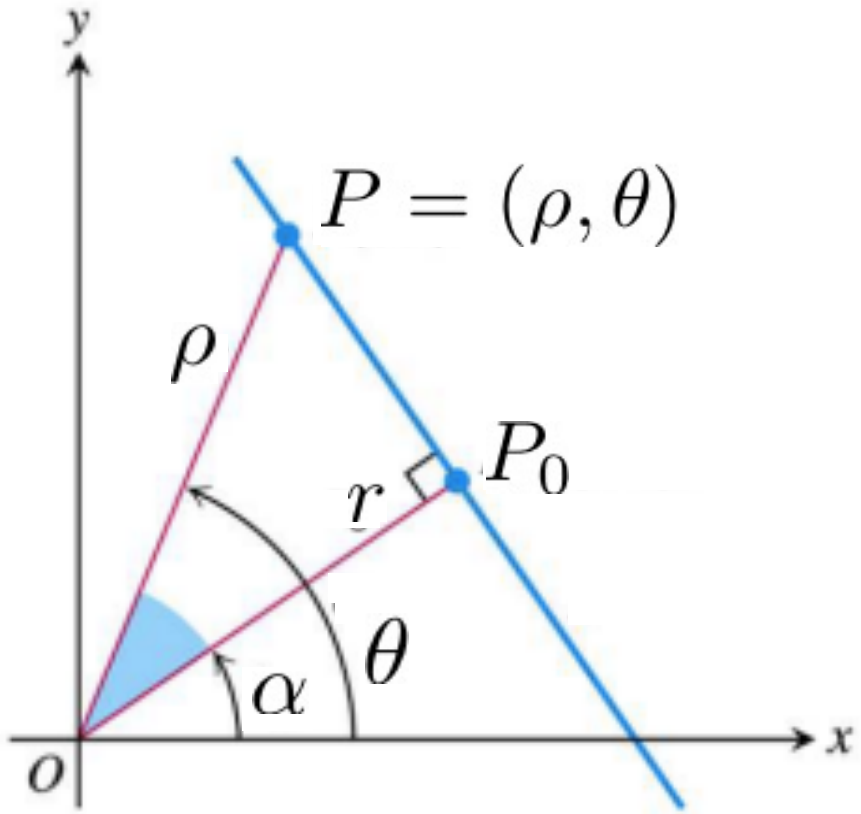
- Goal: fit a line to a set of sensor measurements
- It is useful to work in polar coordinates:

$$x = \rho \cos\theta, \quad y = \rho \sin\theta$$

- Equation of a line in polar coordinates
  - Let $P = (\rho, \theta)$ be an arbitrary point on the line
  - Since P, P_0, O determine a right triangle

$$\rho \cos(\theta - \alpha) = r \quad \text{or} \quad x \cos\alpha + y \sin\alpha = r$$

- (r, α) are the parameters of the line



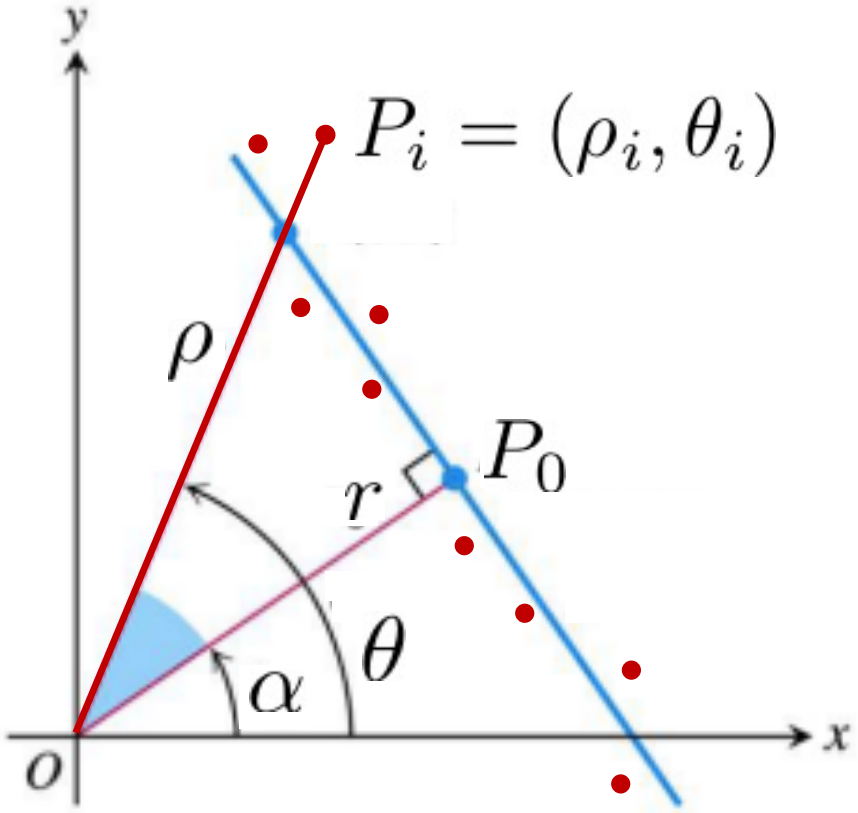
Step #2: line fitting

- Since there is measurement error, the equation of the line is only *approximately* satisfied

$$\rho_i \cos(\theta_i - \alpha) = r + d_i$$

Error

- Assume *n* ranging measurement points represented in polar coordinates as (ρ_i, θ_i)
- We want to find a line that best "fits" all the measurement points



Step #2: line fitting

- Consider, first, that all measurements are equally uncertain
- Find line parameters (r, α) that minimize squared error

$$S(r, \alpha) := \sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} (\rho_i \cos(\theta_i - \alpha) - r)^2$$

- Unweighted least squares

Step #2: line fitting

- Consider, now, the case where each measurement has its own, unique uncertainty
- For example, assume that the variance for each range measurement ρ_i is σ_i
- Associate with each measurement a weight, e.g., $w_i = 1/\sigma_i^2$
- Then, one minimizes

$$S(r, \alpha) := \sum_{i=1}^{n} w_i \, d_i^2 = \sum_{i=1}^{n} w_i \, (\rho_i \cos(\theta_i - \alpha) - r)^2$$

- Weighted least squares

Step #2: line fitting solution

- Assume that the *n* ranging measurements are independent
- Solution:

$$r^* = \frac{\sum_i w_i \rho_i \cos(\theta_i - \alpha)}{\sum_i w_i}$$

$$\alpha^* = \frac{1}{2}\text{atan}\left(\frac{\sum_i w_i \rho_i^2 \sin 2\theta_i - \frac{2}{\sum_i w_i}\sum_i \sum_j w_i w_j \rho_i \rho_j \cos\theta_i \sin\theta_j}{\sum_i w_i \rho_i^2 \cos 2\theta_i - \frac{1}{\sum_i w_i}\sum_i \sum_j w_i w_j \rho_i \rho_j \cos(\theta_i + \theta_j)}\right) + \frac{\pi}{2}$$

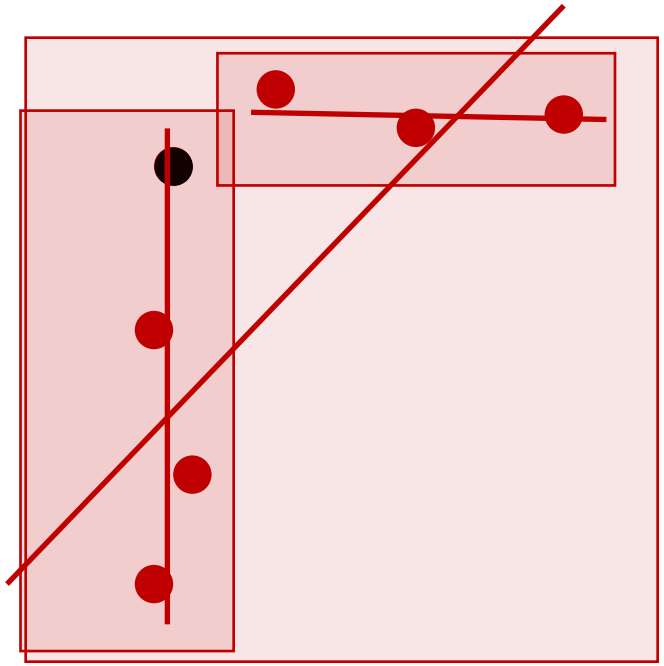
Step #1: line segmentation

- Several algorithms are available
- We will consider three popular algorithms
  1. Split-and-merge
  2. RANSAC
  3. Hough-Transform

Split-and-merge algorithm

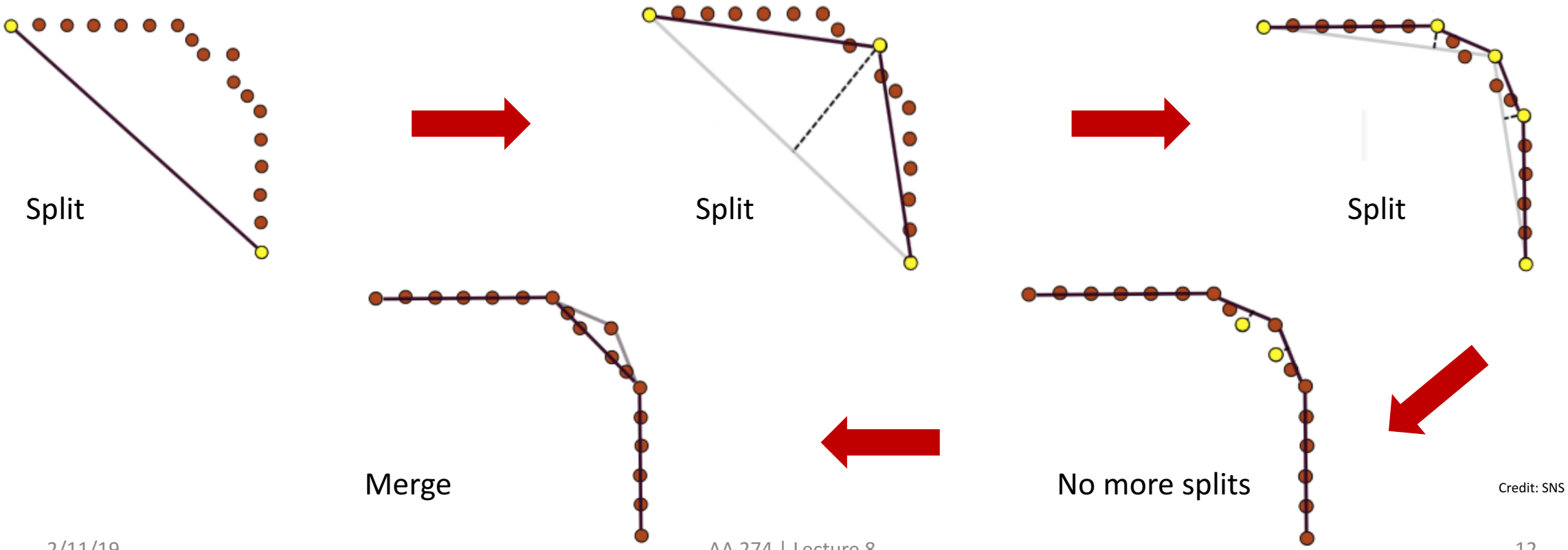
- Most popular line extraction algorithm

Data: Set S consisting of all N points, a distance threshold $d > 0$
Result: L, a list of sets of points each resembling a line
$L \leftarrow (S), i \leftarrow 1;$
while $i \leq len(L)$ **do**
  fit a line (r, α) to the set L_i;
  detect the point $P \in L_i$ with the maximum distance D to the line (r, α);
  if $D < d$ **then**
    $i \leftarrow i + 1$
  else
    split L_i at P into S_1 and S_2;
    $L_i \leftarrow S_1; L_{i+1} \leftarrow S_2;$
  end
end
Merge collinear sets in L;



Split-and-merge: iterative-end-point-fit variant

- Iterative-end-point-fit: split-and-merge where the line is constructed by simply connecting the first and last points



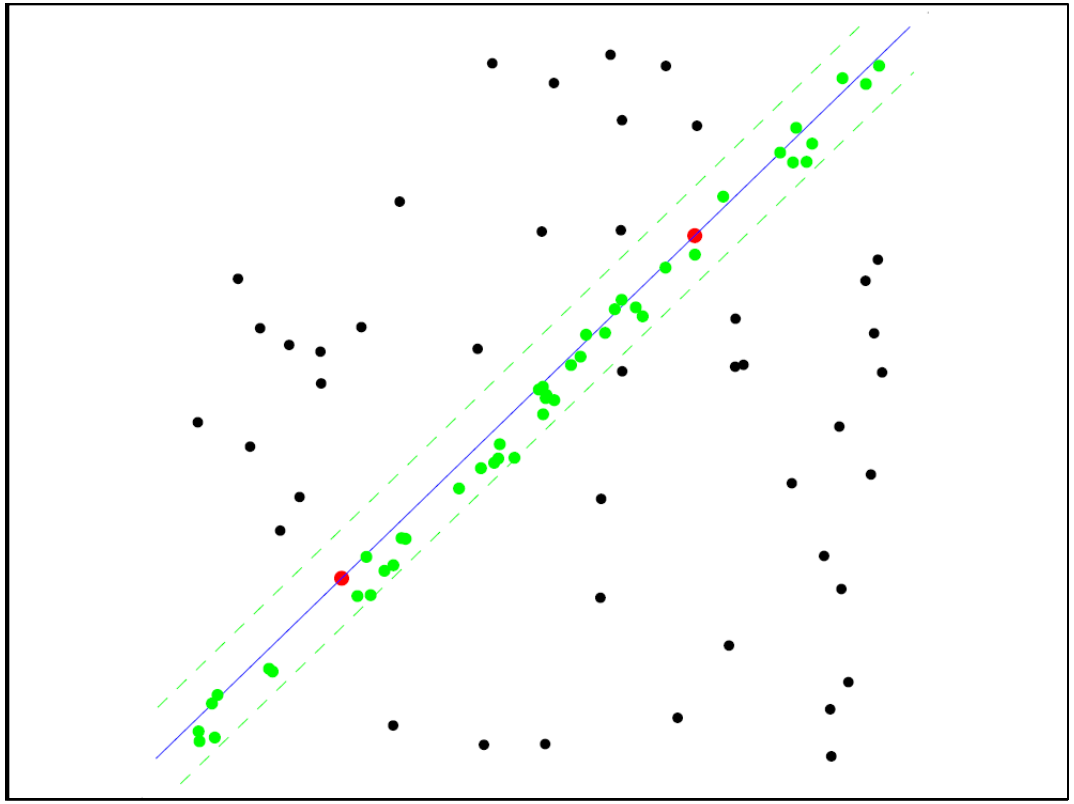
Credit: SNS

RANSAC

- RANSAC: **Ran**dom **Sa**mple **C**onsensus
- General method to estimate parameters of a model from a set of observed data in the presence of outliers, where outliers should have no influence on the estimates of the values
- Typical applications in robotics: line extraction from 2D range data, plane extraction from 3D point clouds, feature matching for structure from motion, etc.
- RANSAC is *iterative* and *non-deterministic*: the probability of finding a set free of outliers increases as more iterations are used

RANSAC

Data: Set S consisting of all N points
Result: Set with maximum number of inliers
(and corresponding fitting line)
while $i \leq k$ **do**
randomly select 2 points from S;
fit line l_i through the 2 points;
compute distance of all other points to line l_i ;
construct *inlier* set, i.e., count number of
points with distance to the line less than γ;
store line l_i and associated set of inliers;
$i \leftarrow i + 1$
end
Choose set with maximum number of inliers



RANSAC iterations

- In principle, one would need to check all possible combinations of 2 points in dataset
- If $|S| = N$, number of combinations is $N(N-1)/2$-> too many
- However, if we have a rough estimate of the percentage of inliers, we do not need to check all combinations…

RANSAC iterations: statistical characterization

- Let w be the percentage of inliers in the dataset, i.e.,

$$w = \frac{\text{number of inliers}}{N}$$

- Let p be the desired probability of finding a set of points free of outliers (typically, $p = 0.99$)
- Assumption: 2 points chosen for line estimation are selected independently
  - $P(\text{both points selected are inliers}) = w^2$
  - $P(\text{at least one of the selected points is an outlier}) = 1 - w^2$
  - $P(\text{RANSAC nevers selects two points that are both inliers}) = (1 - w^2)^k$

RANSAC iterations: statistical characterization

- Then minimum number of iterations $\bar{k}$ to find an outlier-free set with probability at least p is:

$$1 - p = (1 - w^2)^{\bar{k}} \Rightarrow \bar{k} = \frac{\log(1 - p)}{\log(1 - w^2)}$$

- Thus if we know w (at least approximately), after $\bar{k}$ iterations RANSAC will find a set free of outliers with probability p
- Note:
  - $\bar{k}$ depends only on w, not on N!
  - More advanced versions of RANSAC estimate w adaptively

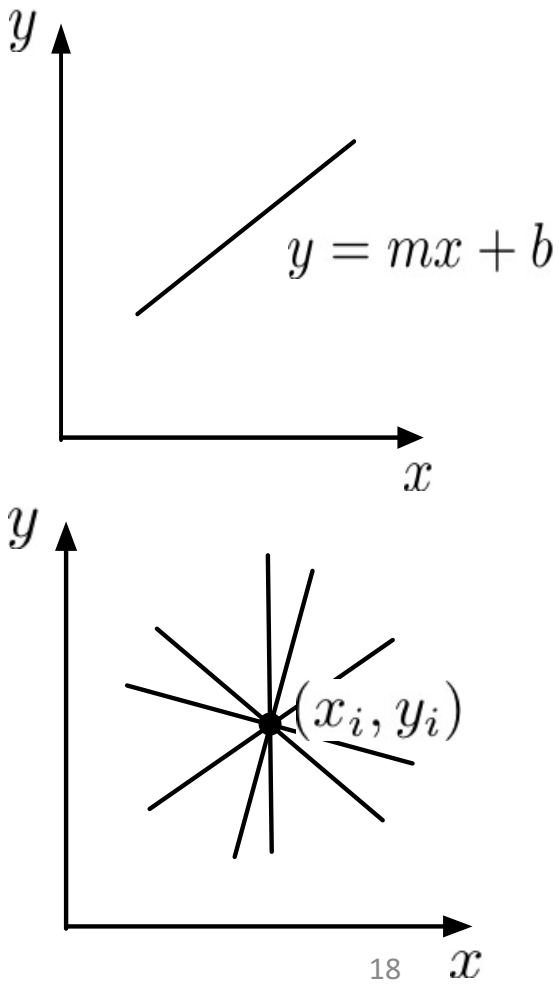
Hough transform

- Key idea: each point votes for a *set* of plausible line parameters

- A line has two parameters: (m, b)

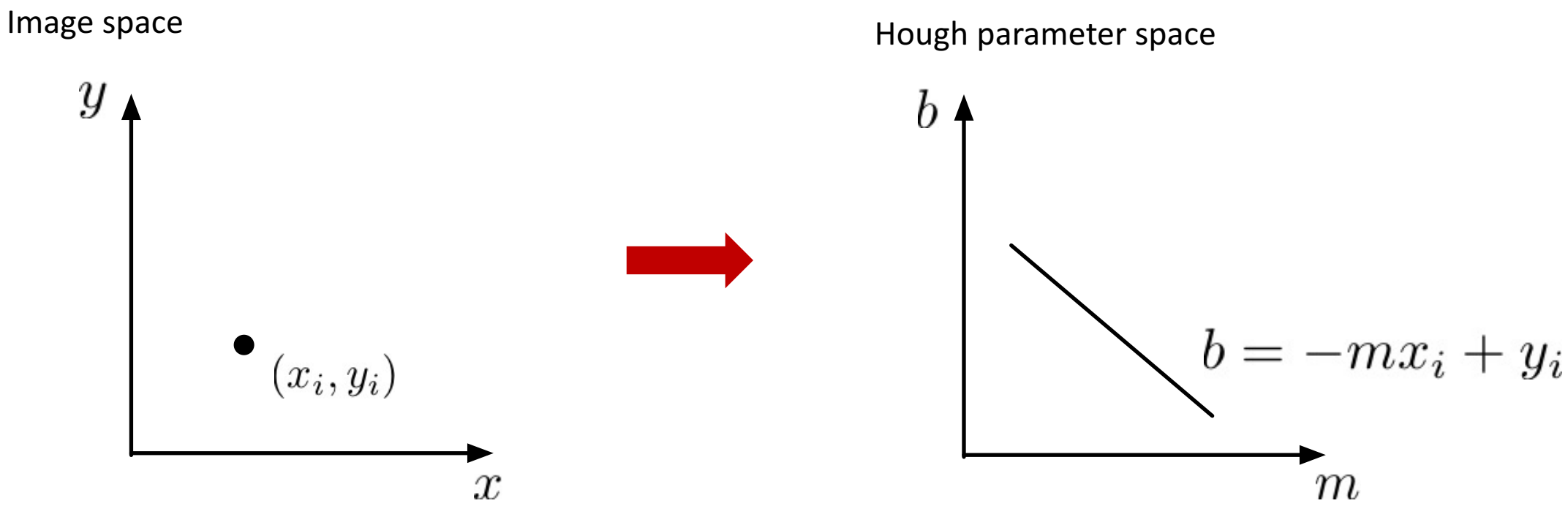
- Given a point (x_i, y_i), the lines that could pass through this point are all (m, b) satisfying

$$y_i = mx_i + b, \quad \text{or } b = -mx_i + y_i$$



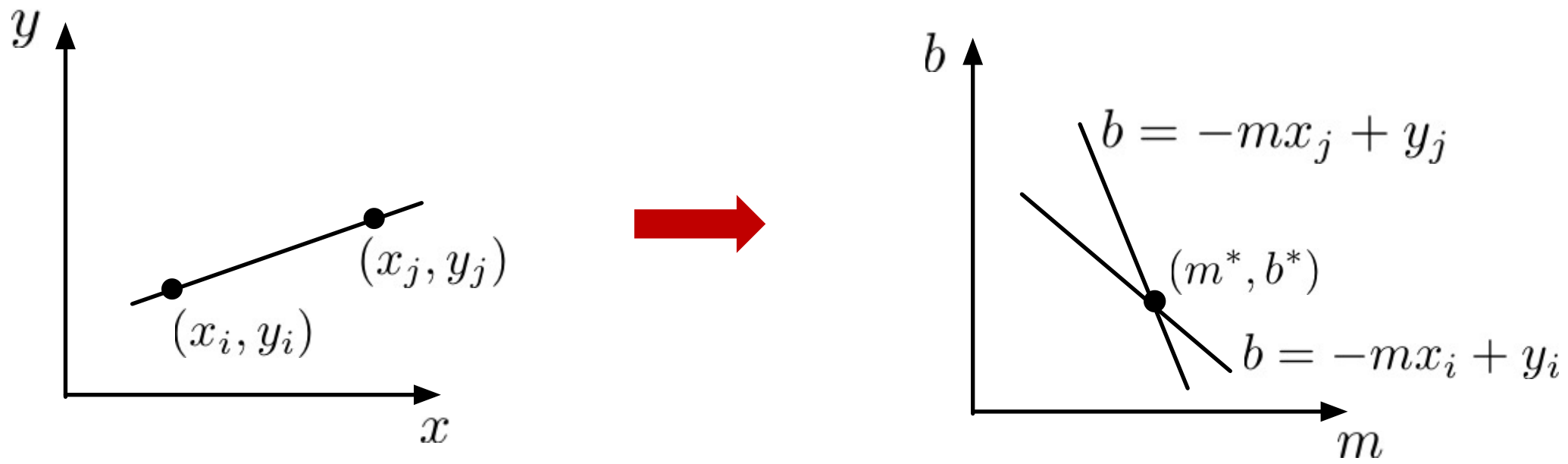
Hough transform

- A point in image space maps into a line in *Hough space*



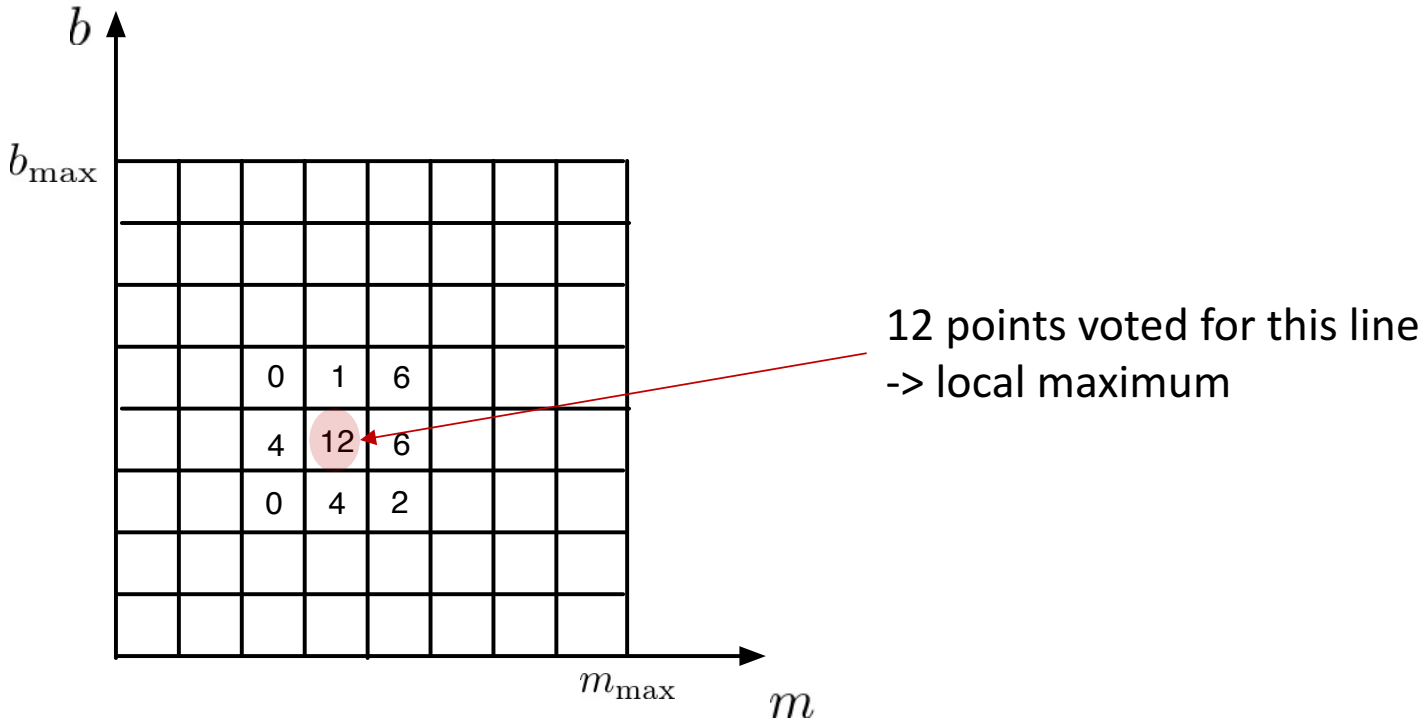
Hough transform

- Key fact: all points on a line in image space yield lines in parameter space which intersect at a *common point*, (m^*, b^*)



Hough transform algorithm

1. Initialize an accumulator array $H(m, b)$ to zero
2. For each point (x_i, y_i), increment all cells that satisfy $b = -x_i m + y_i$
3. Local maxima in array $H(m, b)$ corresponds to lines

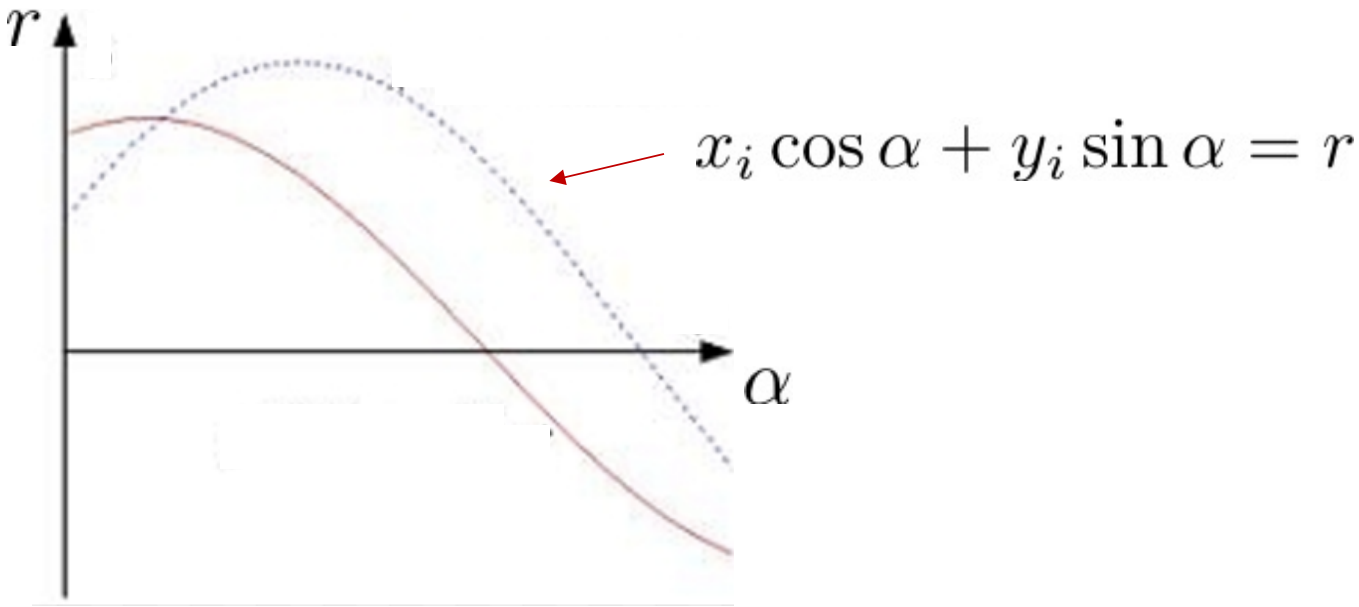


Hough transform algorithm: polar coordinate representation

- Equation of a line in polar coordinates

$$x \cos \alpha + y \sin \alpha = r$$

- The parameter space transform of a point is a sinusoidal curve

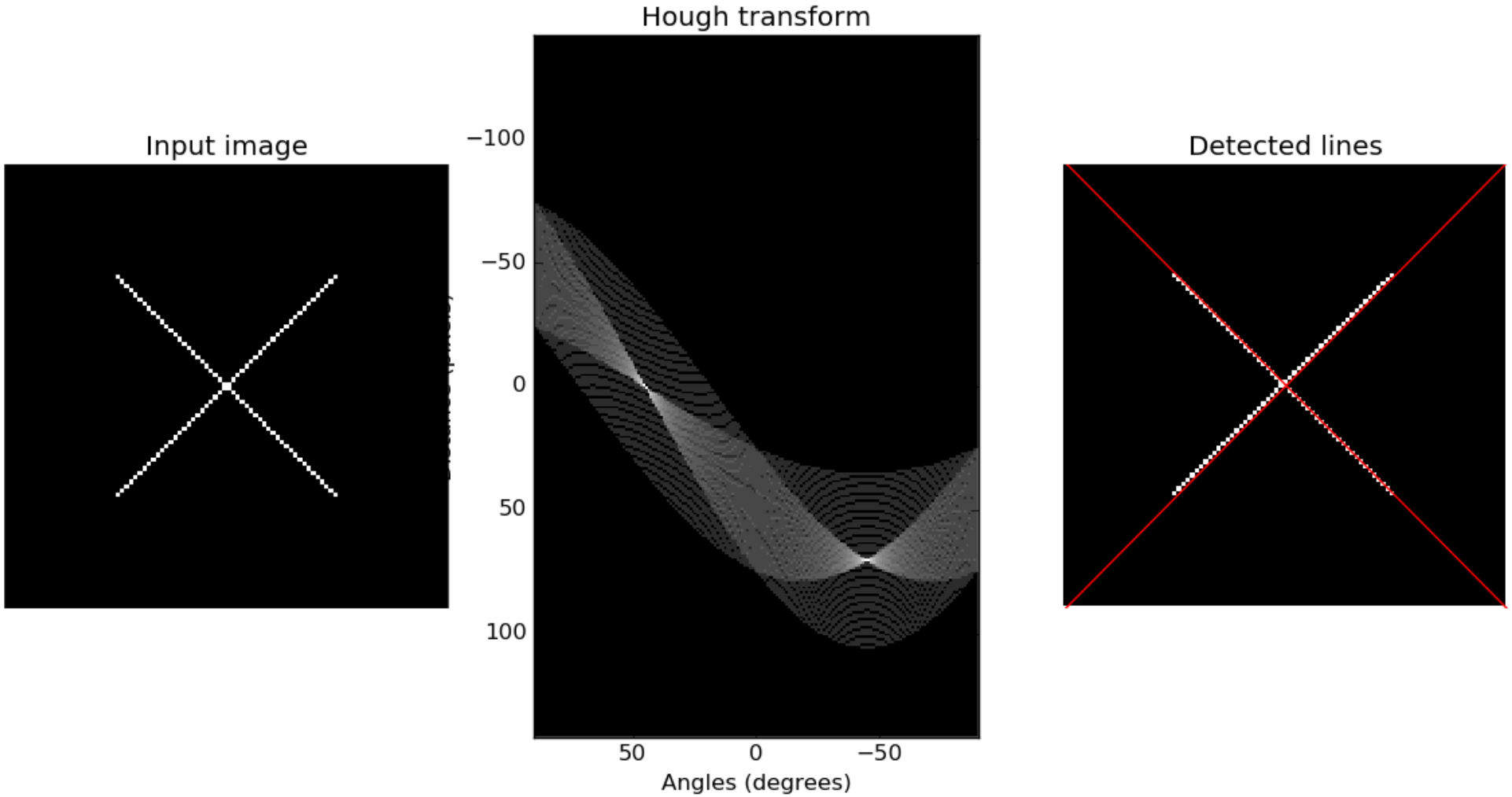


- Avoids infinite slope
- Constant resolution

Hough transform algorithm, revised

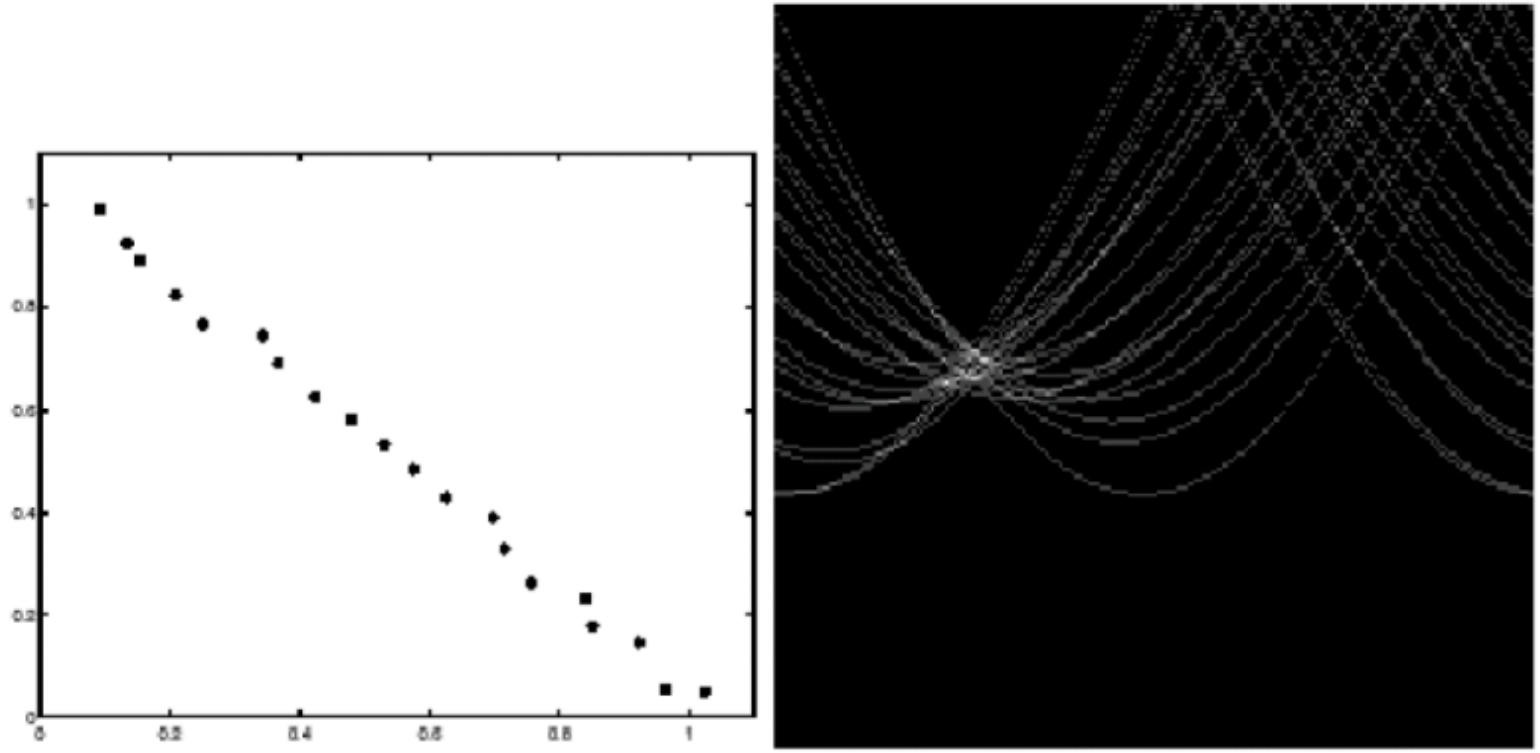
Data: Set S containing N points
Result: Line fitting the points in S
Initialize $n_\alpha \times n_r$ accumulator H with zeros;
foreach $(x_i, y_i) \in S$ **do**
  foreach $\alpha \in \{\alpha_1, \ldots, \alpha_{n_\alpha}\}$ **do**
    compute $r = x_i \cos\alpha + y_i \sin\alpha$;
    $H[\alpha, r] \leftarrow H[\alpha, r] + 1$;
  end
end
Choose (α^*, r^*) that corresponds to largest count in H;
Return line defined by (α^*, r^*)

Hough transform: example



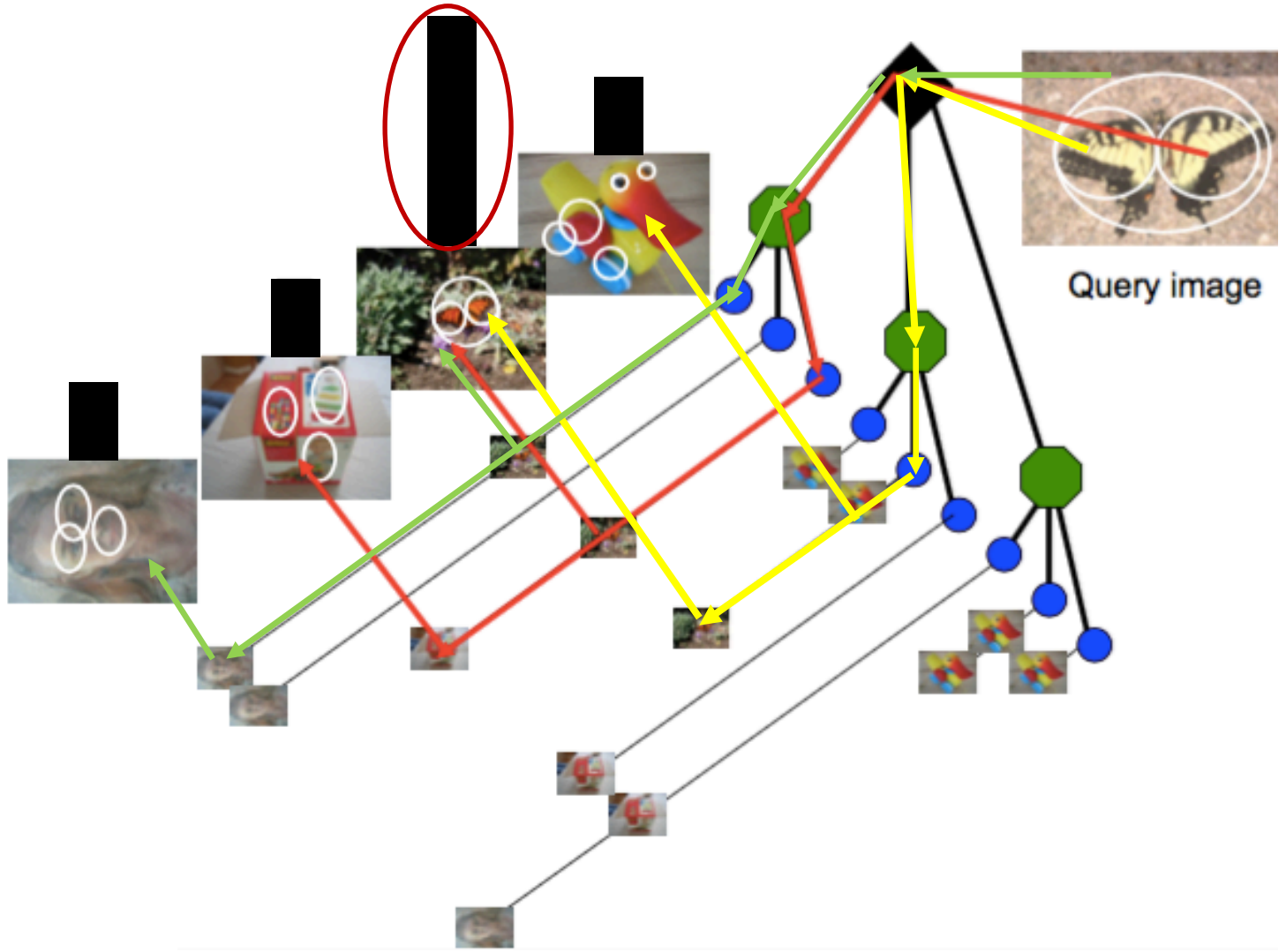
Hough transform: example

- With noise, peaks may be hard to detect



Object recognition and scene understanding

- Object recognition: capability of naming discrete objects in the world
- Popular approach: *bag of visual words*
- In this class, we will focus on neural network methods



Intro to ML

- Aim
  - Present and motivate modern ML techniques for vision applications

- Readings
  - CS231n (CNNs for Visual Recognition) class notes: http://cs231n.github.io/
    [vast majority of figures/slides borrowed from these notes]

Let’s zoom out

- What is a convolutional neural network (CNN)?
- What is a “deep” neural network?
- What is a neural network?
- What is machine learning?

Itinerary

- Stats/ML review
- Neural network basics
- Convolutional neural networks
- Robotic applications

Itinerary

- Stats/ML review
- Neural network basics
- Convolutional neural networks
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Machine learning

- Supervised learning (classification, regression)
  - Given $(x^1, y^1), \ldots, (x^n, y^n)$ choose a function $f(x) = y$

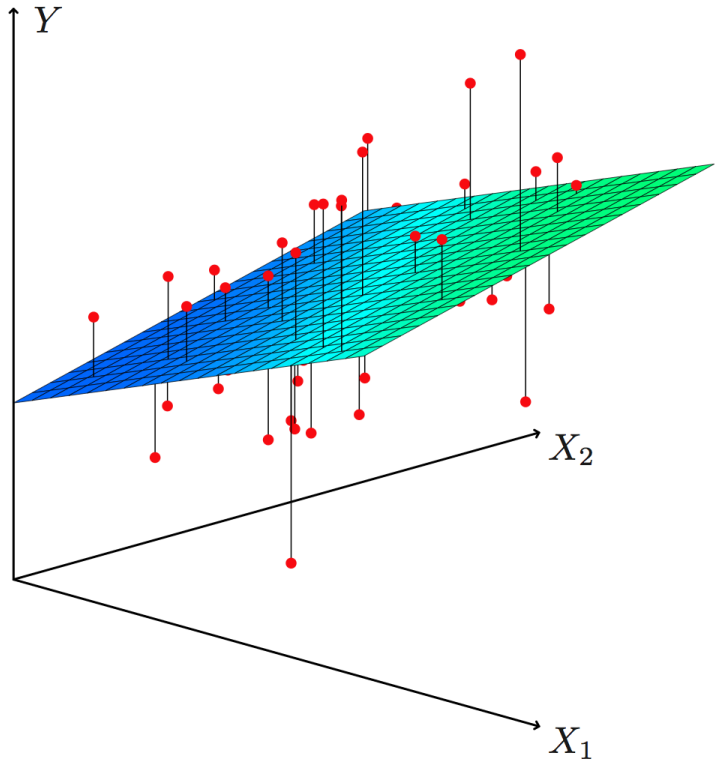
    $x_i = \text{data point}$

    $y_i = \text{class/value}$

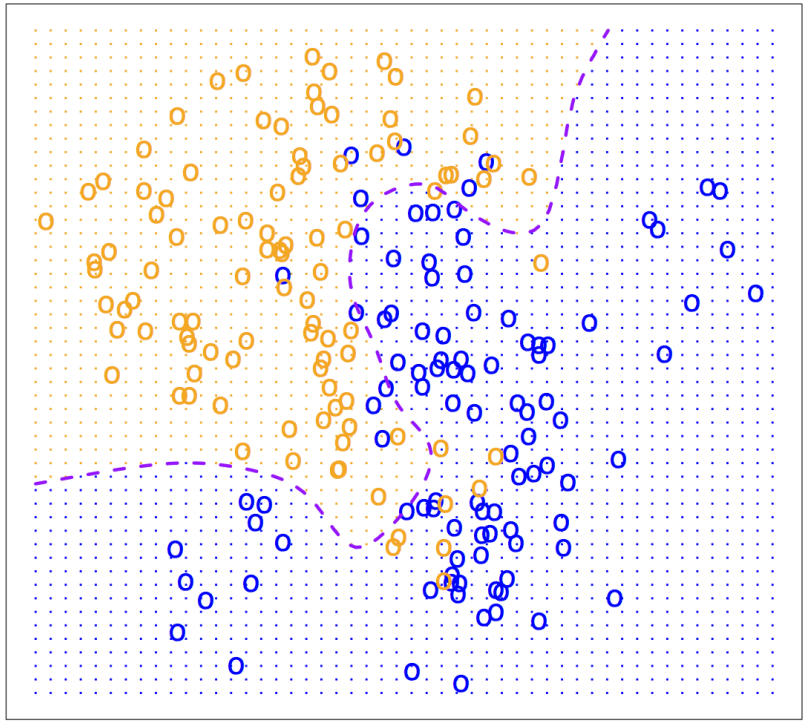
- Unsupervised learning (clustering, dimensionality reduction)
  - Given $(x^1, x^2, \ldots, x^n)$ find patterns in the data

Supervised learning

- Regression



- Classification



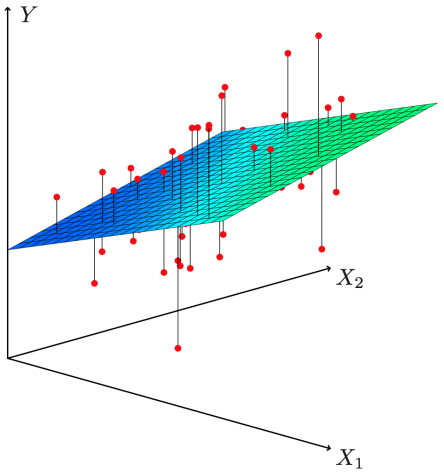
Loss functions

In selecting $f(x) \approx y$ we need a quality metric, i.e., a loss function to minimize

- Regression

$$\ell^2 \text{ loss}: \quad \sum_i |f(x^i) - y^i|^2$$

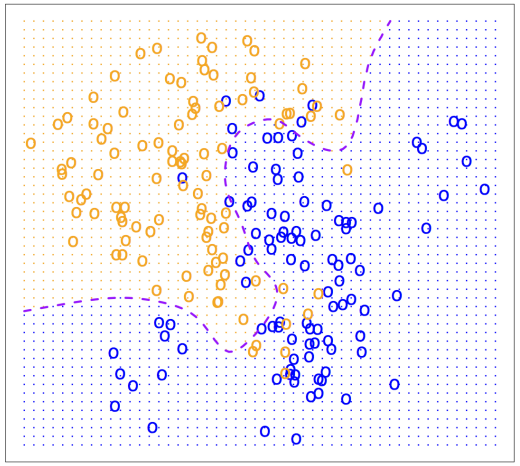
$$\ell^1 \text{ loss}: \quad \sum_i |f(x^i) - y^i|$$



- Classification

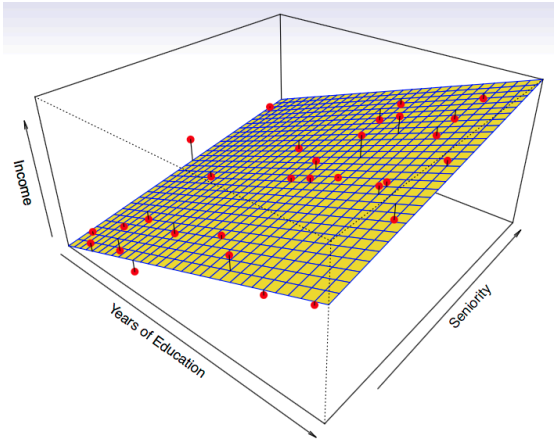
$$0-1 \text{ loss}: \quad \sum_i \mathbf{1}\{f(x^i) \neq y^i\}$$

$$\text{Cross entropy loss}: \quad -\sum_i (y^i)^T \log(f(x^i))$$

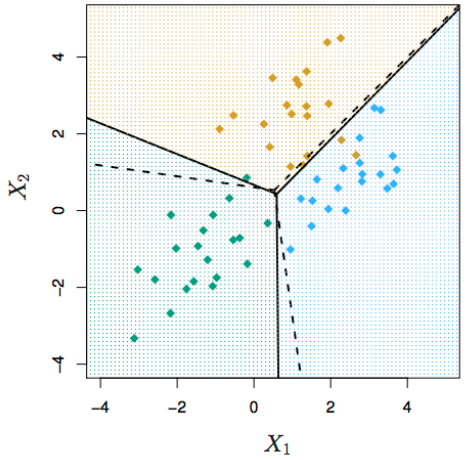


Learning models

Parametric models

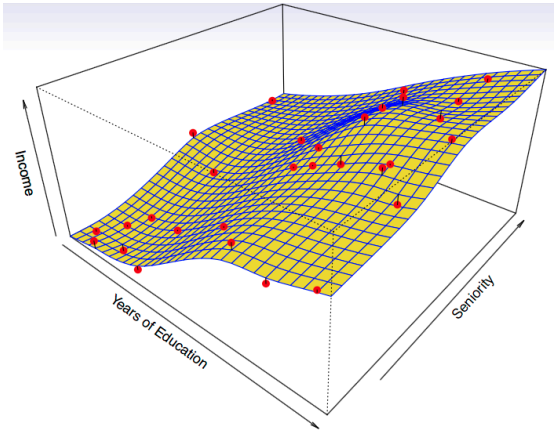


Linear regression

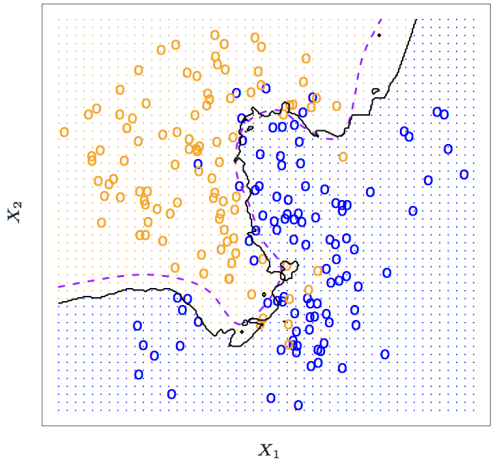


Linear classifier

Non-parametric models



Spline fitting



k-Nearest Neighbors

Machine learning as optimization

We'll focus on parametric models today – how can we choose the best (loss minimizing) parameters to fit our training data?*

Analytical solution

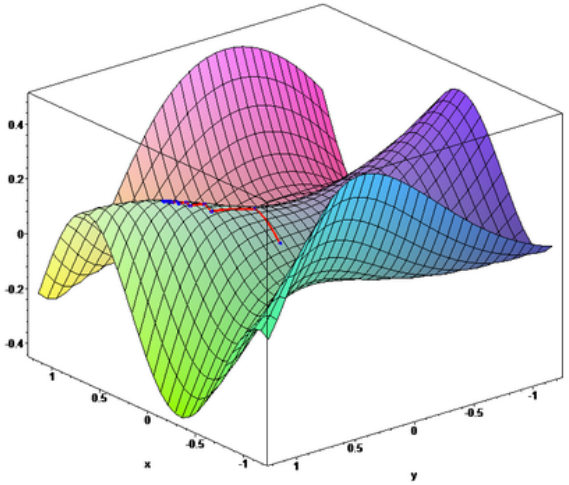
$$\begin{bmatrix} y_1^1 & y_2^1 \\ y_1^2 & y_2^2 \\ \vdots & \\ y_1^n & y_2^n \end{bmatrix} \approx \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_k^1 \\ x_1^2 & x_2^2 & \cdots & x_k^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^n & x_2^n & \cdots & x_k^n \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{11} & a_{12} \\ \vdots & \vdots \\ a_{k1} & a_{k2} \end{bmatrix}$$

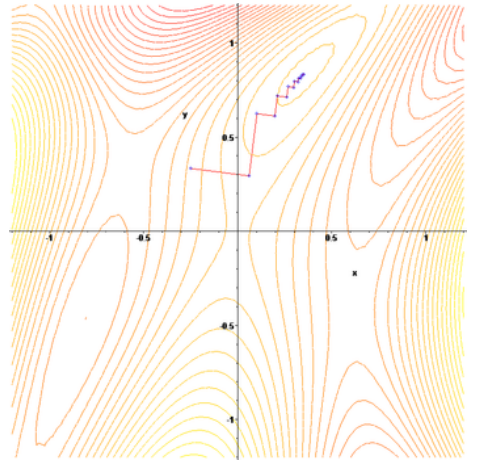
$$f_A(x) = xA, \quad \ell^2 \text{ loss}$$

$$\hat{A} = (X^T X)^{-1} X^T Y$$

(example: linear least squares)

Numerical optimization





(example: gradient descent)

* we'll come back to worrying about test data

Stochastic optimization

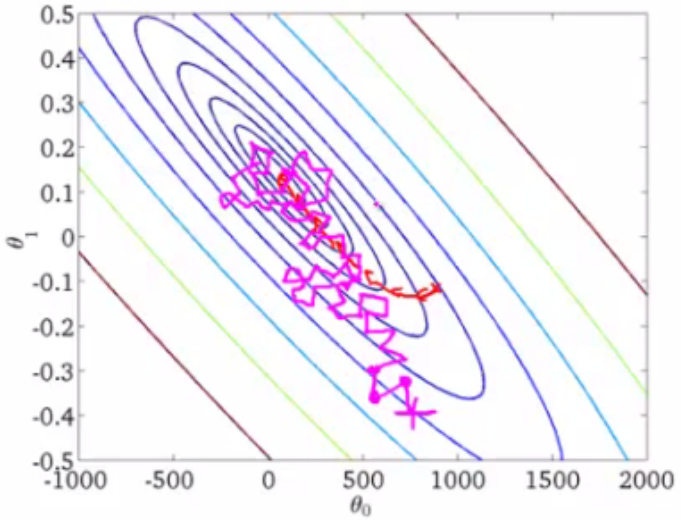
Our loss function is defined over the entire training dataset:

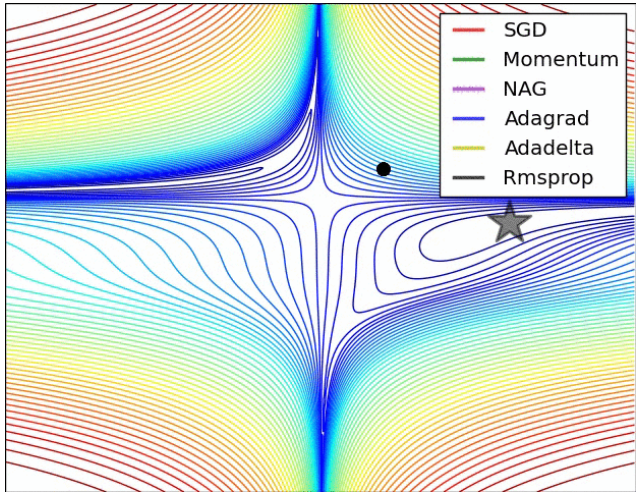
$$L = \frac{1}{n}\sum_{i=1}^{n} \left| f(x^i) - y^i \right|^2 = \frac{1}{n}\sum_{i=1}^{n} L_i$$

Computing ∇L could be very computationally intensive. We approximate:

$$\nabla L \approx \frac{1}{|S|} \sum_{i \in S \subset \{1,\dots,n\}} \nabla L_i$$

Stochastic gradient descent





Other variants

Regularization

To avoid overfitting on the training data, we may add additional terms to the loss function to penalize "model complexity."

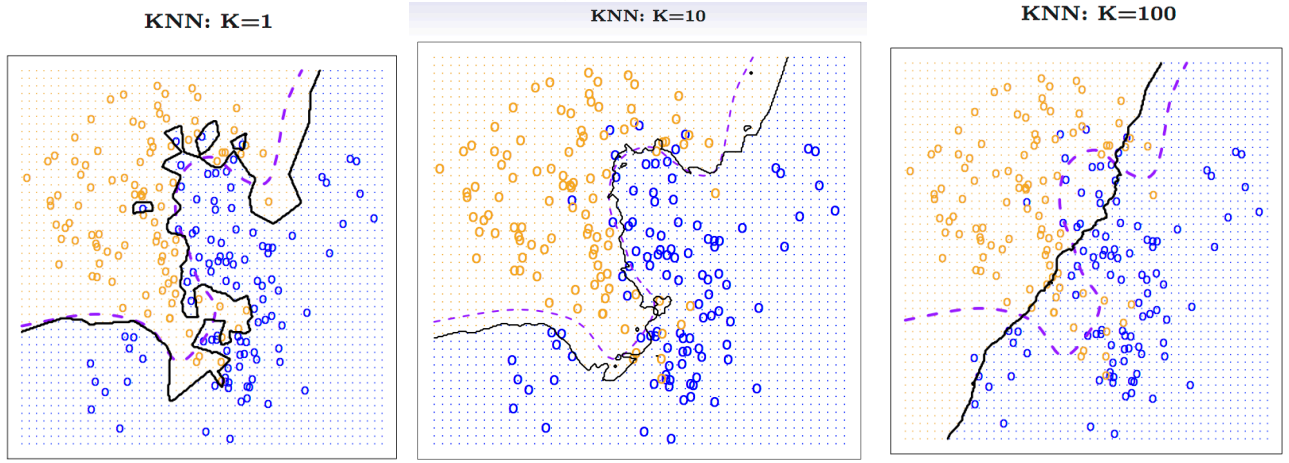
ℓ^2 regularization: $\|A\|_2$

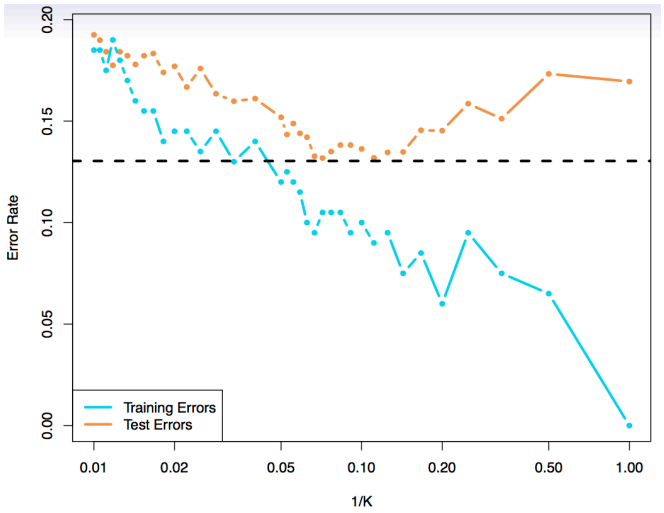
often corresponds to a Gaussian prior on parameters A

ℓ^1 regularization: $\|A\|_1$

often encourages sparsity in A (easier to interpret/explain)

Hyperparameter regularization:





Linear classifiers

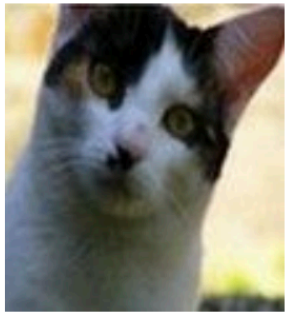
$$f(x, W) = Wx$$

3072x1

(+b) 10x1

10x1

10x3072



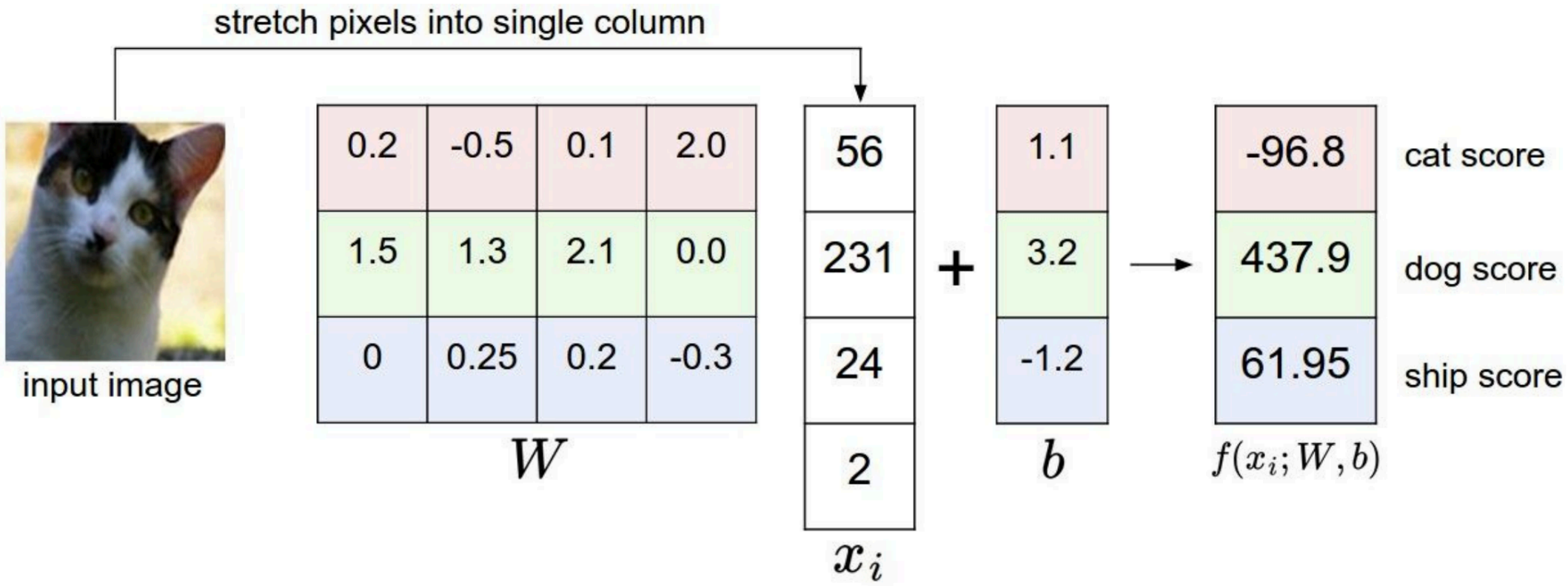
[32x32x3]
array of numbers 0...1

10 numbers, indicating class scores

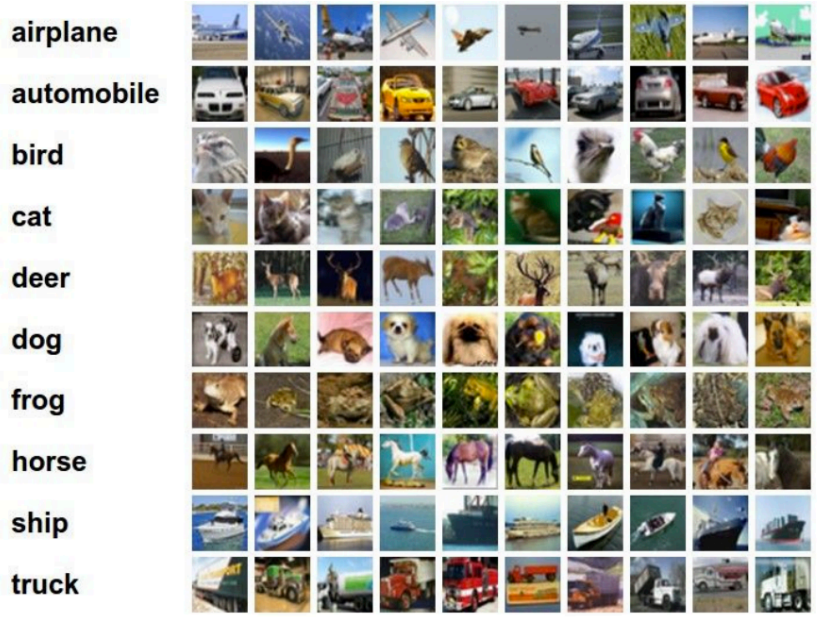
parameters, or “weights”

Linear classifiers

Example with an image with 4 pixels, and 3 classes (cat/dog/ship)



Linear classifiers – Interpretation



$$f(x_i, W, b) = Wx_i + b$$

Example trained weights of a linear classifier trained on CIFAR-10:

Each row of W can be thought of as a “template” for nearest neighbor classification

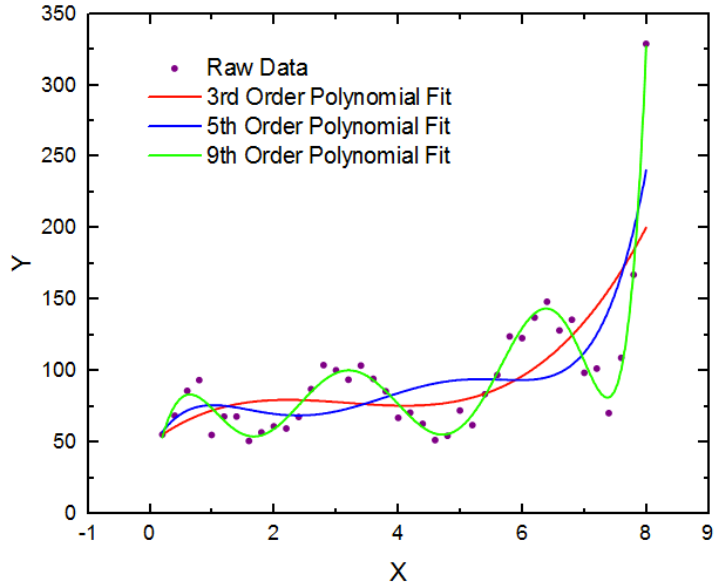
Softmax regression

Our class scores can be turned into a probability vector over classes using the softmax function:

$$\sigma(\mathbf{z}) = \begin{bmatrix} \frac{e^{z_1}}{\sum_k e^{z_k}} \\ \vdots \\ \frac{e^{z_m}}{\sum_k e^{z_k}} \end{bmatrix} \qquad p(y^i = j | x^i) = \frac{e^{x^i W_j + b_j}}{\sum_k e^{x^i W_k + b_k}}$$

Cross-entropy loss computed against “one-hot” class vector [0, …, 1, …, 0]

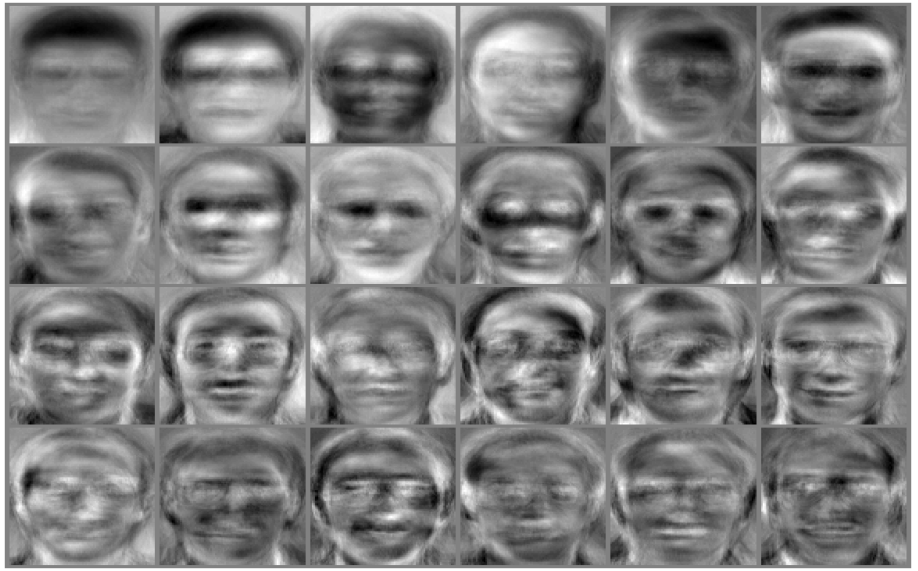
Generalizing linear models



$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix} \approx \begin{bmatrix} 1 & x_1 & x_1^2 & \dots & x_1^m \\ 1 & x_2 & x_2^2 & \dots & x_2^m \\ 1 & x_3 & x_3^2 & \dots & x_3^m \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & x_n^2 & \dots & x_n^m \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$$

Nonlinearity via basis functions

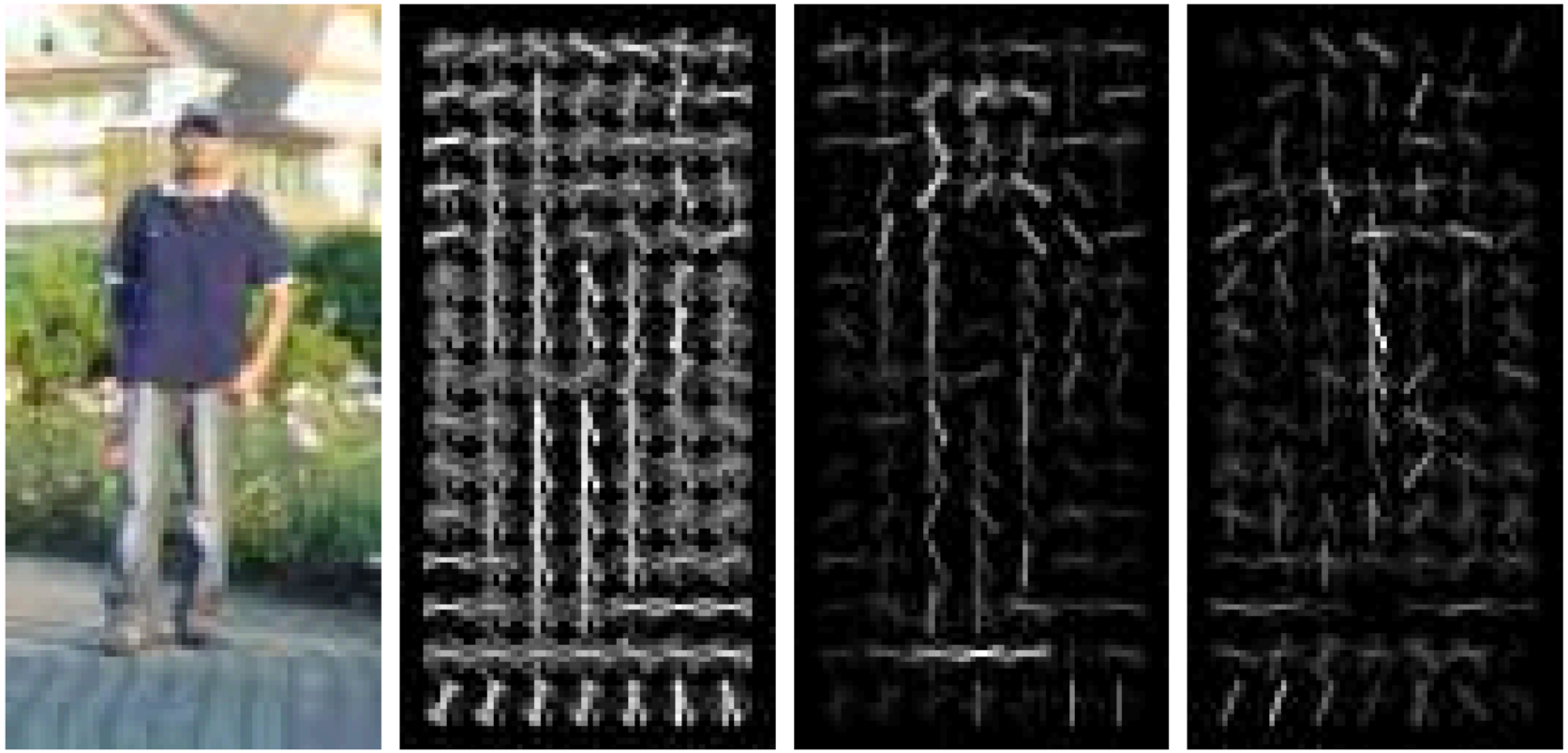
Linear regression/classification can be very powerful when empowered by the right features



Eigenfaces

Histogram of Oriented Gradients (HOG)

- Very successful feature set for shape classification/regression



Left to right:

- Test image
- Visualization of HOG descriptor
- HOG descriptor scaled by positive weights
- HOG descriptor scaled by negative weights

(Dalal and Triggs 2005)

Feature extraction

[32x32x3]

Human Ingenuity

vector describing various image statistics

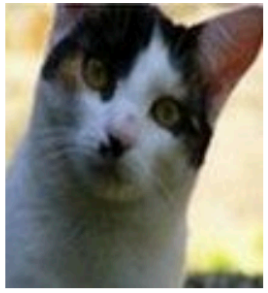
Feature Extraction

f

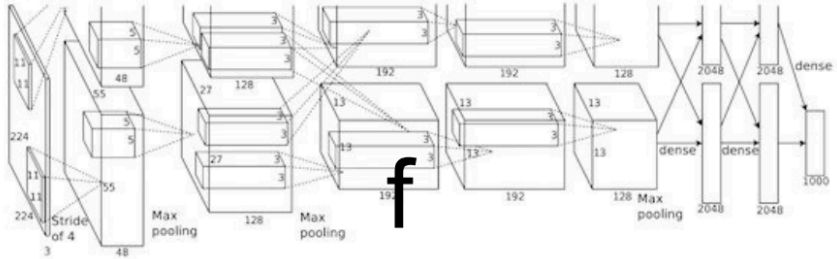
training

10 numbers, indicating class scores

Gradient Descent



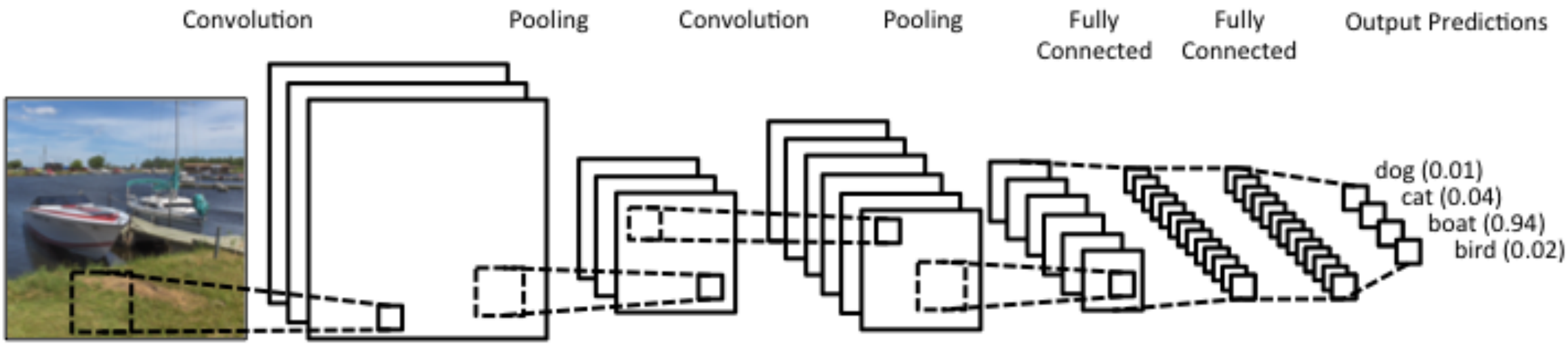
[32x32x3]



training

10 numbers, indicating class scores

Next time


Convolution
Pooling
Convolution
Pooling
Fully Connected
Fully Connected
Output Predictions
dog (0.01)
cat (0.04)
boat (0.94)
bird (0.02)