Sample average approximation for stochastic programming with equality constraints: trajectory optimization for a manipulator

July 14, 2024

We provide an additional example motivating the use of the sample average approximation (SAA) formulation proposed in [1], compared to using a classical SAA formulation. We consider the problem of computing a trajectory for a robotic manipulator with three joints. Starting from a configuration $q^0 \in \mathbb{R}^3$, the objective is to reach an uncertain target end-effector position in expectation, while accounting for uncertainty in the lengths of the robot's links. This problem corresponds to a robotic application where the robot interacts with the environment using noisy sensor measurements and attempts to reach an object located at an uncertain position. Manufacturing errors are captured by the uncertain link lengths of the robotic manipulator. The optimal control problem ($\mathbf{P}$) can be expressed as

$$\inf_{(u_0,\dots,u_{T-1})\in\mathbb{R}^{3T}} \quad \sum_{t=0}^{T-1} \|u_t\|^2 \tag{1a}$$

$$\text{s.t.} \quad q_{t+1} = q_t + u_t, \qquad t = 0,\dots,T-1, \tag{1b}$$

$$q_0 = q^0, \tag{1c}$$

$$\|u_t\|_\infty \leq 10, \qquad t = 0,\dots,T-1, \tag{1d}$$

$$\mathbb{E}[p_{\mathrm{e}}(q_T,\omega_\ell) - (\bar{p}_{\mathrm{e}} + \omega_{\mathrm{e}})] = 0, \tag{1e}$$

where $p_{\mathrm{e}} : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}^2, (q,\omega_\ell) \mapsto \sum_{i=1}^3 (\ell_i + \omega_{\ell_i})(\cos(q_i), \sin(q_i))$ gives the position of the end-effector given a configuration (i.e., joint angles) $q \in \mathbb{R}^3$ and uncertain link lengths $(\ell_i + \omega_{\ell_i})$, and $(\bar{p}_{\mathrm{e}} + \omega_{\mathrm{e}}) \in \mathbb{R}^2$ is the uncertain desired end-effector position. We use $T = 15$, $q^0 = \frac{\pi}{4}(1,2,3)$, $(\ell_1,\ell_2,\ell_3) = (0.4, 0.3, 0.3)$, and $\bar{p}_{\mathrm{e}} = p_{\mathrm{e}}(\frac{\pi}{8}(1,1,1), 0)$. Each entry of $(\omega_\ell, \omega_{\mathrm{e}})$ is Gaussian-distributed with mean 0 and standard deviation 0.01.

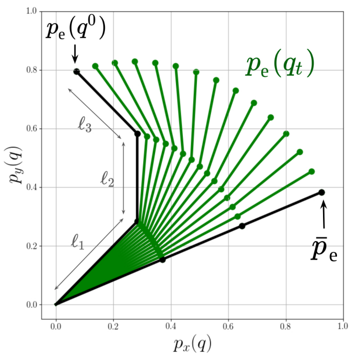

Figure 1: Robotic manipulator trajectory optimization problem.

We use the same numerical resolution procedure with IPOPT as in the benchmark study (Section 6.1 in [1]), solving 100 realizations of the sampled problems with $N = 10$ samples and $\delta_N = 0$ (corresponding to a standard SAA reformulation of $\mathbf{P}$) and $\delta_N = 10^{-2}$ (corresponding to $\mathbf{SP}_N(\bar{\omega})$). The number of successful numerical resolutions of the standard SAA reformulation of $\mathbf{P}$ ($\delta_N = 0$) is 49/100. In contrast, solving the reformulation $\mathbf{SP}_N(\bar{\omega})$ with $\delta_N = 10^{-2}$ results in 96/100 successful numerical resolutions. We conclude that relaxing the equality constraints using the proposed scheme leads to higher numerical resolution success rates. A trajectory solving the analytical reformulation of the problem is shown in Figure 1.

Code to reproduce experiments is available at `https://github.com/StanfordASL/stochastic-prog`.

References

[1] T. Lew, R. Bonalli, and M. Pavone, "Sample Average Approximation for Stochastic Programming with Equality Constraints," SIAM Journal on Optimization, 2024.