Spring 2020

Stanford
AA203: Optimal and Learning-based Control
Problem set 6, due on June 10

Problem 1: Find an extremal curve $x^*(t)$ for the functional

$$J = \int_0^1 \left[\frac{1}{2}\dot{x}^2(t) + 5x(t)\dot{x}(t) + x^2(t) + 5x(t)\right] dt$$

that passes through the points $x(0) = 1$ and $x(1) = 3$.

Problem 2: A ship must travel through a region of strong currents, which depend on position. The ship has a constant speed V, and its heading $\theta(t)$ can be controlled. The current is directed in the x direction with a speed

$$u = \frac{Vy(t)}{h}$$

for a given h. It is desired to find the ship's heading $\theta(t)$ required to move from a given initial position $(x(t_0), y(t_0))$ to the origin in minimum time. The equations of motion are

$$\begin{aligned}\dot{x}(t) &= V\ \cos\theta(t) + \frac{Vy(t)}{h}\\ \dot{y}(t) &= V\sin\theta(t)\end{aligned}$$

and the performance index is

$$J = \int_{t_0}^{T} 1\, dt.$$

(a) Show that the optimal control law takes the form of

$$\tan\theta(t) = \alpha + \frac{V(T-t)}{h},$$

where α is a constant. This law is referred to as linear tangent law.

(b) Compute the optimal transfer time, i.e., $T - t_0$, for the case where the current's speed is equal to a constant, i.e., $u = \beta > 0$.

Problem 3: Find the Hamiltonian and then solve the necessary conditions to compute the optimal control and state trajectory that minimize

$$J = \int_0^1 u^2(t)dt$$

for the system $\dot{x}(t) = -2x(t) + u(t)$ with initial state $x(0) = 2$ and terminal state $x(1) = 0$. Plot the optimal control and state response.

Learning goals for this problem set:

Problem 1: To familiarize with the process of solving calculus of variations problems.

Problems 2 & 3 To familiarize with the Hamiltonian equations for optimal control.