AA 274
Principles of Robotic Autonomy

SLAM I: non-parametric localization and EKF SLAM

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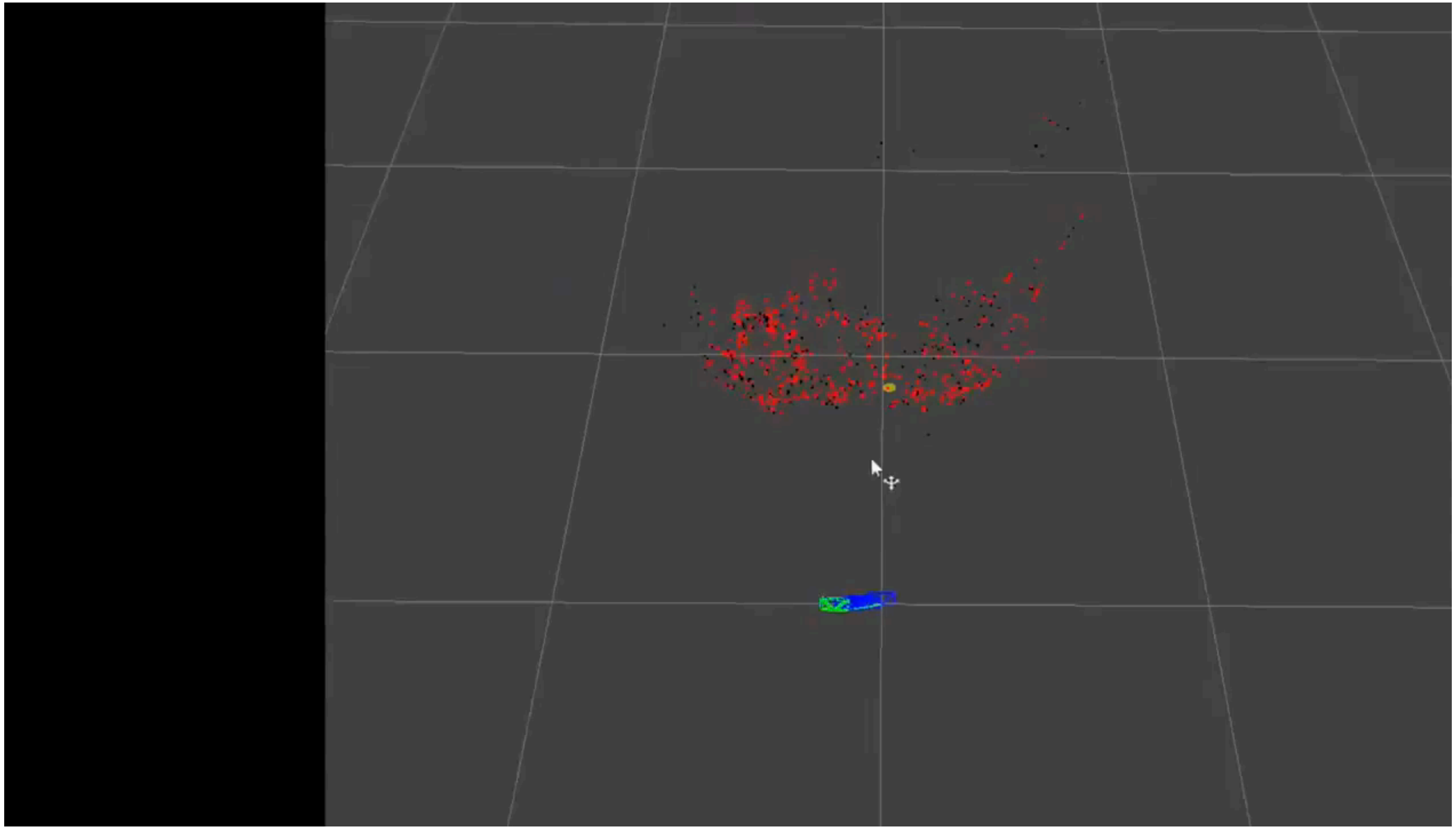
Today’s lecture

- Aim
  - Learn about the general SLAM problem
  - Learn about EKF SLAM

- Readings
  - S. Thrun, W. Burgard, and D. Fox. Probabilistic robotics. MIT press, 2005. Sections 8.1 – 8.3, 10.1 – 10.4

Simultaneous Localization and Mapping

The SLAM problem: given measurements $z_{1:t}$ and controls $u_{1:t}$, find the path (or pose) of the robot and acquire a map of the environment



Forms of SLAM

- Online SLAM problem: estimate the posterior over the momentary pose along with the map

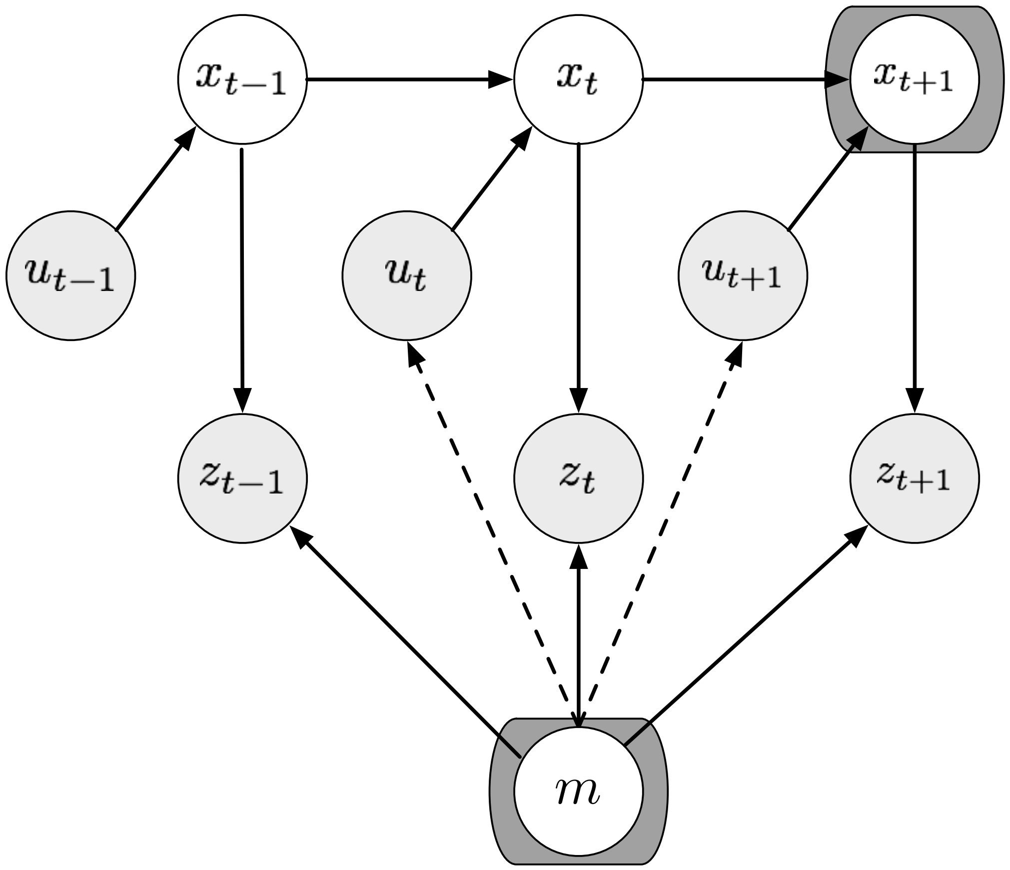
$$p(x_t, m \mid z_{1:t}, u_{1:t}) \quad \text{or} \quad p(x_t, m, c_t \mid z_{1:t}, u_{1:t})$$

- Full SLAM problem: estimate posterior over the entire path along with the map

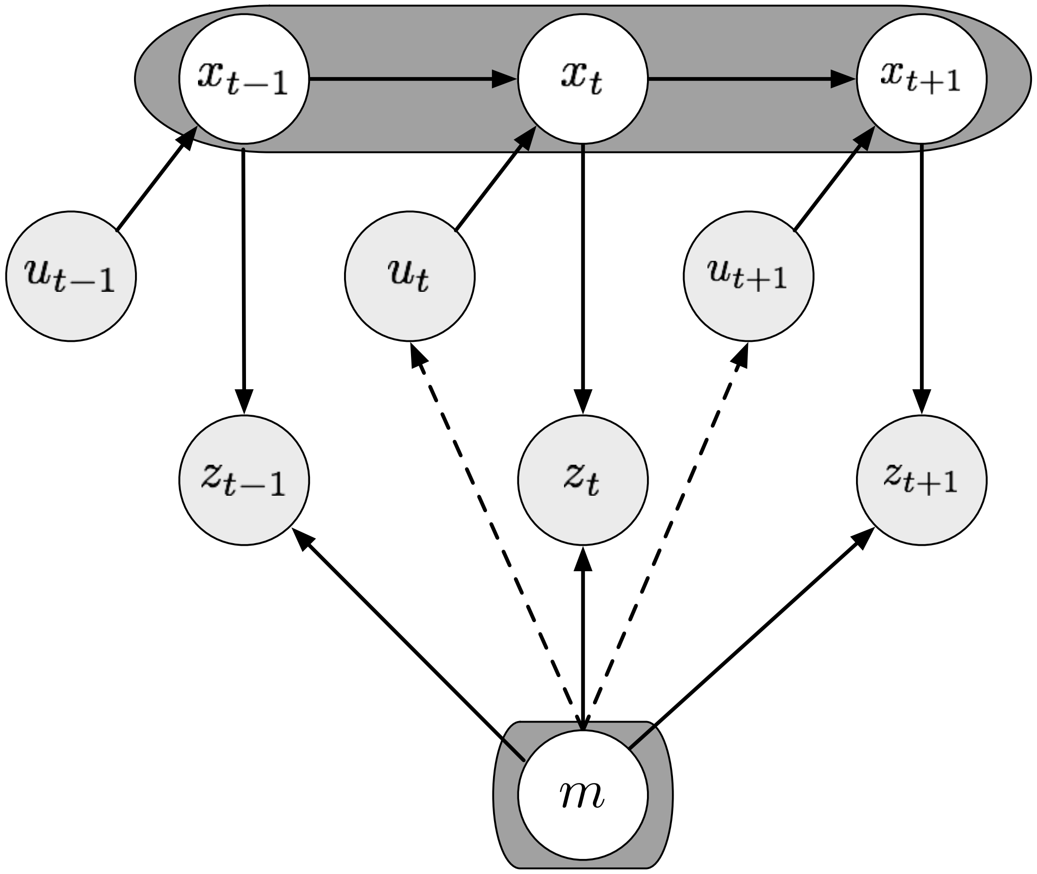
$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) \quad \text{or} \quad p(x_{1:t}, m, c_t \mid z_{1:t}, u_{1:t})$$

Graphical models of SLAM

Online SLAM

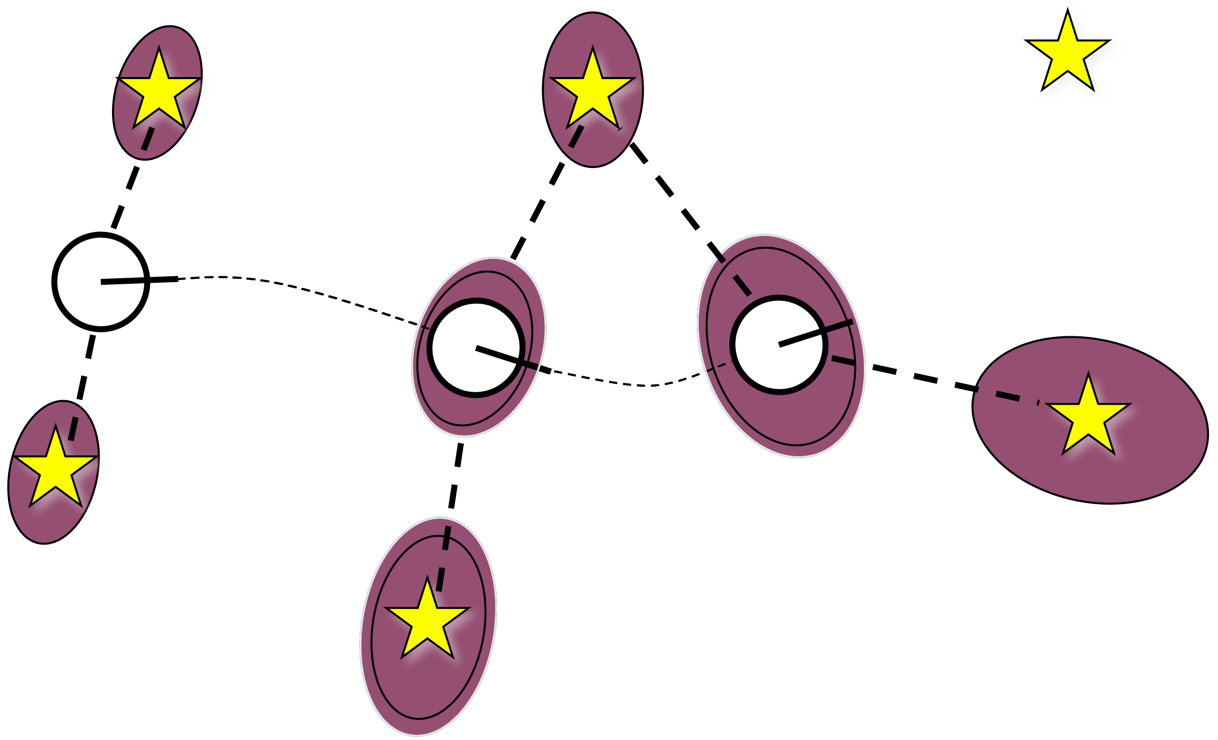


Full SLAM



The challenge of SLAM

- Robot path and map are both unknown



- Path error is correlated with map error

EKF SLAM

- Historically the earliest SLAM algorithm
- Key idea: apply EKF to online SLAM using maximum likelihood data association
- Assumptions:
  1. Gaussian assumption for motion and perception noise, and Gaussian approximation for belief (essential)
  2. Feature-based maps (essential)
- Two versions of the problem
  1. Correspondence variables are known
  2. Correspondence variables are not known (usual case)

EKF SLAM with known correspondences

- Similar to EKF localization algorithm with known correspondences
- Key difference: in addition to estimate the robot pose x_t, the EKF SLAM algorithm also estimates the coordinates of all landmarks
- Define combined state vector

$$y_t := \begin{pmatrix} x_t \\ m \end{pmatrix} = (x,\, y,\, \theta,\, m_{1,x},\, m_{1,y},\, m_{2,x},\, m_{2,y}\, \ldots\, m_{N,x},\, m_{N,y})^T$$

3 + 2N vector

- Goal: calculate the online posterior

$$p(y_t, m \,|\, z_{1:t}, u_{1:t})$$

Motion and sensing model

- (Following discussion is for illustration purposes; setup can be generalized to other motion and sensing models)
- Assume motion model with state $x_t = (x, y, \theta)$

$$y_t = g(u_t, y_{t-1}) + \epsilon_t, \qquad \epsilon_t \sim \mathcal{N}(0, R_t), \qquad G_t := J_g(u_t, \mu_{t-1})$$

where we assume that the landmarks are *static*, that is

1. $g(u_t, y_{t-1})$ *is a 3+2N vector, whose last 2N components are the same as those in* y_{t-1}
2. R_t has zero entries, except for the top left 3 x 3 block

Motion and sensing model

- Assume range and bearing measurement model

$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \operatorname{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \end{pmatrix}}_{:=h(y_t, j)} + \delta_t, \qquad \delta_t \sim \mathcal{N}(0, Q_t), \quad Q_t = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\phi^2 \end{pmatrix}$$

- Usual linear approximation for sensing model (with $j = c_t^i$)

$$h(y_t, j) \approx h(\overline{\mu}_t, j) + H_t^i (y_t - \overline{\mu}_t), \qquad \text{where } H_t^i := \frac{\partial h(\overline{\mu}_t, j)}{\partial y_t}$$

- Since h depends only on x_t and m_j, H_t^i can be factored as

$$H_t^i = h_t^i \, F_{x,j}$$

Motion and sensing model

- First term, a 2 x 5 matrix, is the Jacobian of $h(y_t, j)$ at $\bar{\mu}_t$ w.r.t. x_t and m_j:

$$h_t^i = \frac{\partial h(\overline{\mu}_t, j)}{\partial(x_t, m_j)} = \begin{pmatrix} \frac{\overline{\mu}_{t,x} - \overline{\mu}_{j,x}}{\sqrt{q_{t,j}}} & \frac{\overline{\mu}_{t,y} - \overline{\mu}_{j,y}}{\sqrt{q_{t,j}}} & 0 & \frac{\overline{\mu}_{j,x} - \overline{\mu}_{t,x}}{\sqrt{q_{t,j}}} & \frac{\overline{\mu}_{j,y} - \overline{\mu}_{t,y}}{\sqrt{q_{t,j}}} \\ \frac{\overline{\mu}_{j,y} - \overline{\mu}_{t,y}}{q_{t,j}} & \frac{\overline{\mu}_{t,x} - \overline{\mu}_{j,x}}{q_{t,j}} & -1 & \frac{\overline{\mu}_{t,y} - \overline{\mu}_{j,y}}{q_{t,j}} & \frac{\overline{\mu}_{j,x} - \overline{\mu}_{t,x}}{q_{t,j}} \end{pmatrix}$$

where $q_{t,j} := (\overline{\mu}_{j,x} - \overline{\mu}_{t,x})^2 + (\overline{\mu}_{j,y} - \overline{\mu}_{t,y})^2$

- Second term, a 5 x (3+2*N*) matrix, maps h_t^i into H_t^i:

$$F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{2j-2} & 0 & 1 & \underbrace{0 \cdots 0}_{2N-2j} \end{pmatrix}$$

Initialization

- Initial belief expressed as

$$\mu_0 = (0, 0, 0 \ldots 0)^T$$

Initialization for pose variables

(3+2*N*) x (3+2*N*)

$$\Sigma_0 = \begin{pmatrix} \boxed{\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}} & \begin{matrix} 0 & \ldots & 0 \\ 0 & \ldots & 0 \\ 0 & \ldots & 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{matrix} & \begin{matrix} \infty & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \infty \end{matrix} \end{pmatrix}$$

Initialization

- When a landmark is observed for the first time, the landmark estimate $(\bar{\mu}_{j,x}, \bar{\mu}_{j,y})^T$ is initialized with the expected position, that is

$$\begin{pmatrix} \overline{\mu}_{j,x} \\ \overline{\mu}_{j,y} \end{pmatrix} = \begin{pmatrix} \overline{\mu}_{t,x} \\ \overline{\mu}_{t,y} \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \overline{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \overline{\mu}_{t,\theta}) \end{pmatrix}$$

- Bearing only SLAM would require multiple sightings

EKF SLAM algorithm

- Similar to EKF localization; main differences:
  - Augmented state vector
  - Augmented dynamics (with trivial dynamics for the landmarks)
  - Initialization of unseen landmarks
  - Augmented measurement Jacobian

Data: $(\mu_{t-1}, \Sigma_{t-1}), u_t, z_t, c_t$
Result: (μ_t, Σ_t)
$\overline{\mu}_t = g(u_t, \mu_{t-1});$
$\overline{\Sigma}_t = G_t \Sigma_{t-1} {G_t}^T + R_t;$

foreach $z_t^i = (r_t^i, \phi_t^i)^T$ **do**

$j = c_t^i;$

if *landmark j never seen before* **then**

$$\begin{pmatrix} \overline{\mu}_{j,x} \\ \overline{\mu}_{j,y} \end{pmatrix} = \begin{pmatrix} \overline{\mu}_{t,x} \\ \overline{\mu}_{t,y} \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \overline{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \overline{\mu}_{t,\theta}) \end{pmatrix};$$

end

$$\hat{z}_t^i = \begin{pmatrix} \sqrt{(\overline{\mu}_{j,x} - \overline{\mu}_{t,x})^2 + (\overline{\mu}_{j,y} - \overline{\mu}_{t,y})^2} \\ \text{atan2}(\overline{\mu}_{j,y} - \overline{\mu}_{t,y}, \overline{\mu}_{j,x} - \overline{\mu}_{t,x}) - \overline{\mu}_{t,\theta} \end{pmatrix};$$

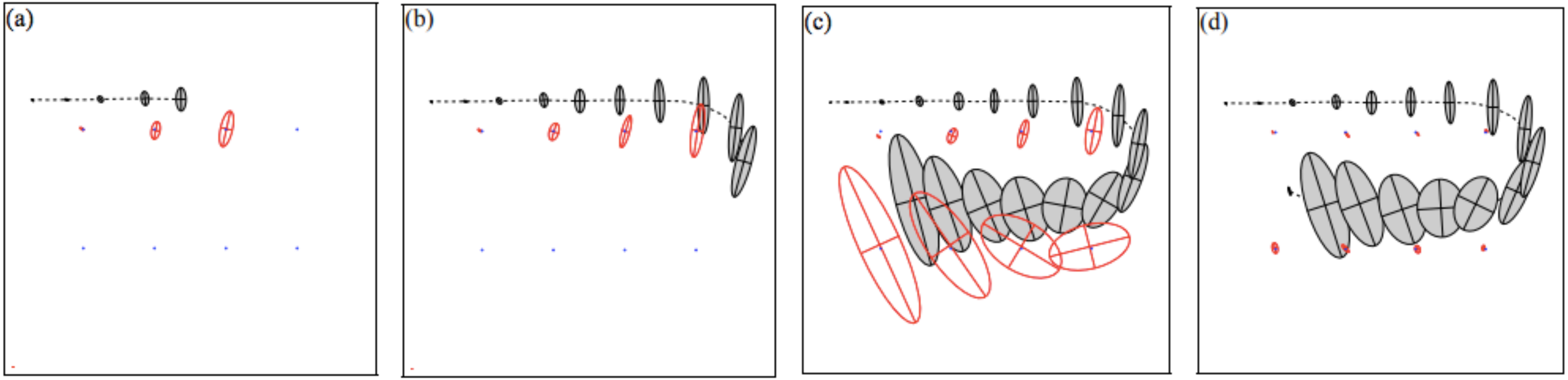
$H_t^i = h_t^i \, F_{x,j};$

$S_t^i = H_t^i \, \overline{\Sigma}_t \, [H_t^i]^T + Q_t;$
$K_t^i = \overline{\Sigma}_t \, [H_t^i]^T \, [S_t^i]^{-1};$
$\overline{\mu}_t = \overline{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i);$
$\overline{\Sigma}_t = (I - K_t^i H_t^i) \, \overline{\Sigma}_t;$

end

$\mu_t = \overline{\mu}_t$ and $\Sigma_t = \overline{\Sigma}_t;$
Return (μ_t, Σ_t)

Example



EKF SLAM with unknown correspondences

- Key idea: use an incremental maximum likelihood estimator to determine correspondences
- Similar to EKF localization with unknown correspondences, but now we also need to create hypotheses for new landmarks
- Caveat: maximum likelihood data association often makes the algorithm brittle, as it is not possible to revise past data associations

EKF SLAM with unknown correspondences

- In the measurement update loop, we first create the hypothesis of a new landmark
- A new landmark is created if the Mahalanobis distance to all existing landmarks exceeds the value α

Data: $(\mu_{t-1}, \Sigma_{t-1}), u_t, z_t, N_{t-1}$
Result: (μ_t, Σ_t)
$N_t = N_{t-1};$
$\bar{\mu}_t = g(u_t, \mu_{t-1});$
$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^{\,T} + R_t;$

foreach $z_t^i = (r_t^i, \phi_t^i)^T$ **do**

Hypothesis for new landmark

$$\begin{pmatrix} \bar{\mu}_{N_t+1,x} \\ \bar{\mu}_{N_t+1,y} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \end{pmatrix};$$

for $k = 1$ *to* $N_t + 1$ **do**

$$\hat{z}_t^k = \begin{pmatrix} \sqrt{(\bar{\mu}_{j,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{j,y} - \bar{\mu}_{t,y})^2} \\ \operatorname{atan2}(\bar{\mu}_{j,y} - \bar{\mu}_{t,y}, \bar{\mu}_{j,x} - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{pmatrix};$$

$H_t^k = h_t^k\, F_{x,k};$
$S_t^k = H_t^k\, \bar{\Sigma}_t\, [H_t^k]^T + Q_t;$
$\pi_k = (z_t^i - \hat{z}_t^k)^T\, [S_t^k]^{-1}\, (z_t^i - \hat{z}_t^k);$

Mahalanobis distance

end

$\pi_{N_t+1} = \alpha;$
$j(i) = \operatorname{argmin}_k\ \pi_k;$

Hypothesis test

$N_t = \max\{N_t,\ j(i)\};$
$K_t^i = \bar{\Sigma}_t\, [H_t^{j(i)}]^T\, [S_t^{j(i)}]^{-1};$
$\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^{j(i)});$
$\bar{\Sigma}_t = (I - K_t^i H_t^{j(i)})\, \bar{\Sigma}_t;$

end

$\mu_t = \bar{\mu}_t$ and $\Sigma_t = \bar{\Sigma}_t;$
Return (μ_t, Σ_t)

Making EKF SLAM robust

- A key issue is represented by the fact that fake landmarks might be created; furthermore, EKF can diverge if nonlinearities are large
- Several techniques exist to mitigate such issues
  1. Outlier rejection schemes, for example via provisional landmark lists
  2. Strategies to enhance the distinctiveness of landmarks
     - Spatial arrangement
     - Signatures
     - Enforcing geometric constraints

- Dilemma of EKF SLAM: accurate localization typically requires dense maps, but EKF requires sparse maps due to quadratic update complexity

Next time