Control Adaptation via Meta-Learning Dynamics

James Harrison*,1, Apoorva Sharma*,1, Xinrui Wang[1], Roberto Calandra[2], Marco Pavone[1]

[1] Stanford University, Stanford, CA,
{jharrison,apoorva,xinrui,pavone}@stanford.edu
[2] Facebook AI Research, Menlo Park, CA,
rcalandra@fb.com

Abstract. Deep neural networks are an increasingly popular approach to learning-based robotics planning and control. When these models are applied on systems that differ from those on which they were trained, model mismatch can lead to degraded performance, instability, and loss of control. These issues can be alleviated by performing online adaptation of the model as state transition data from the true system is observed during operation. However, standard neural network models are ill-suited to the data-efficient and computationally efficient adaption that is necessary for adaptive control. In this work, we present Control Adaptation via Meta-Learning Dynamics (CAMeLiD), an approach to nonlinear adaptive trajectory generation and control which leverages recent advances in meta-learning to enable the efficient adaptation of neural network models. We demonstrate that this framework, which extends the ALPaCA Bayesian meta-learning work for adaptive dynamics modeling and uses it within a model predictive controller, outperforms state-of-the-art in meta-learning-based adaptive control on two continuous control tasks. Moreover, we show the proposed framework outperforms state-of-the-art methods on prediction error and in terms of out-of-distribution meta-generalization.

1 Introduction

In recent years, deep neural networks are increasingly being used within model-based robotic control and planning, serving as data-driven approximations of system dynamics [1–4]. While hyper-parametric neural network models are attractive as universal approximators that can fit arbitrary nonlinear dynamics, these models typically require large amounts of data to train, which can be expensive or tedious to obtain on a single robot of interest. Approaches to alleviating this problem include transferring from simulation [5], or large-scale data collection across multiple robots [6], but both involving training a model on data from a source that is different from the target application and thus are prone to issues arising from model mismatch.

*These authors contributed equally. Order determined randomly.

This problem of model mismatch can be addressed through an adaptive control approach, wherein the state transitions observed during operation on the target system are used to adjust the model to better fit the true system dynamics. For this purpose, we not only desire a regression model that is *high-capacity*, in that it is able to capture nonlinear dynamics; but also one that is *data-efficient*, so that it can extract the maximal information about the true dynamics from the observed data. Further, in many robotic systems, it is critical to adapt quickly, during operation, so the adaptation should be *computationally efficient*. Typical neural network models fall short on these latter desiderata, as they require large amounts of data and several epochs of gradient descent steps to fit new data.

Recently, *meta-learning* has emerged as a promising approach to addressing these issues and has seen success in few-shot image classification and reinforcement learning, where deep networks are adapted from limited amounts of data at test-time [7]. In this work, we present a novel approach to nonlinear adaptive control and planning using a Bayesian meta-learned dynamics model. Our approach, Control Adaptation via Meta-Learning Dynamics (CAMeLiD), combines model-predictive control scheme with the ALPaCA meta-learning model proposed in [8], in which offline, the network weights are optimized such that it can adapt at test time through analytic, Bayesian recursive least-squares updates to the last-layer weights of the network.

Contributions: The contributions of this work are as follows: First, we extend the ALPaCA model with a multi-head architecture and demonstrate that this design allows better modeling relative to baseline ALPaCA. Next, we present CAMeLiD as a system-level framework for meta-learning for adaptive control and demonstrate that this outperforms state-of-the-art for meta-learning adaptive control. Finally, we conclude with a discussion of the general utility of the Bayesian modeling approach presented herein for the robotic control and planning context.

2 Related Work

Model learning is a core component of many robotic systems [9], and can be performed using a variety of strategies – system identification typically implies learning the model before acting [10], adaptive control implies learning and acting simultaneously over the course of one episode [11], and model-based reinforcement learning often implies interleaving learning and acting in an episodic fashion [12]. In this work, we consider a combination of these perspectives, in which prior experience allows a robotic system to learn strong priors, which are then adapted online over the course of one episode.

Adaptive control via Gaussian process modeling: Similarly to this paper, a body of work starting in the 1990s and the early 2000s has investigated adaptive control using Gaussian process models and radial basis function (RBF) networks. Murray-Smith et al. [13, 14] investigated adaptive control using non-parametric GP models in both the greedy (one-step ahead) and multi-step setting. They took a "cautious" or risk-sensitive approach, in which the optimization objective was the mean squared deviation from a reference trajectory, plus a variance penalty.

Their multi-step optimization approach relied on gradient descent on the cost function, which is expensive with kernel GPs. Indeed, it is unlikely that this approach could be run online for anything but low-dimensional systems, or with more than a very small amount of training data. In contrast, our approach relies on a parametric GP and associated derivatives, for which evaluation is simply evaluating a small neural network. In comparing non-parametric models to parametric, [14] claim that the relative advantage of non-parametric models is a representation of the model that depends on local data density. However, our model learns a structured representation of uncertainty via meta-learning. These learned correlations are capable of expressing uncertainty that is based on local data density if they are warranted for the class of systems under investigation, but are also able to learn richer correlations.

The model predictive control-based approach was extended to include constraints in [15], but computational efficiency was not addressed and the online optimization problem is constrained and nonconvex. More recently, Chowdhary et al. [16] leveraged GPs for model-reference adaptive control (MRAC) and achieved good online performance. However, this approach relies on selectively discarding data to reduce the computational expense of inference. In contrast, our parametric approach is capable of incorporating arbitrarily large amounts of data via recursive least squares, which results in constant complexity for inference and conditioning. As such, our proposed method has substantial computational efficiency advantages over kernel GPs and is better suited to online adaptive control. Note that leveraging GPs in adaptive control is a large subfield within the adaptive control literature, and there are many approaches that we have not surveyed here.

In addition to work on non-parametric GPs in adaptive control, there is a substantial body of work utilizing neural networks and parametric GP models. Early work on adaptive control with neural network models was presented in [17], in which a RBF network was used for direct adaptive control in the continuous time setting. Indeed, use of nonlinear basis functions (such as RBF networks) with linear adaptation has been one of the dominant approaches to both direct and indirect adaptive control in recent years [18–20]. Particularly relevant to our work, [21] use neural network dynamics models and perform Bayesian recursive least squares on the last layer to perform adaptive control. We apply this idea within the meta-learning setting, and thus empirically learn a prior that is optimized over the offline training data. Moreover, whereas their control law is a simple heuristic, we perform adaptive planning within the model predictive control loop.

Meta-learning for adaptive control: Closely related to our work is the literature framing model-based reinforcement learning as a meta-learning task [22, 23]. In [22], the authors investigated use of recurrent and gradient-based meta-learning approaches for the purpose of intra-episode dynamics adaptation. While this work is similar to the approach presented here, they use sampling-based MPC as opposed to a gradient-based control strategy. In addition, their approach is based on performing gradient descent on their dynamics neural network (trained using MAML [7]). A key advantage of MAML is that online adaptation is optimizing the full network via gradient descent, so in the limit of infinite data it can adapt to fit any arbitrary system at test time. However, to keep the

evaluation of the model efficient at runtime, [22] propose only keeping a sliding window of past observations. They then condition their original prior on only these data by taking a few gradient steps, so the system never enters the regime where this generality of MAML is useful. In contrast, our approach restricts online adaptation to the last layer of network. While this limits the flexibility of our online adaption, it enables efficient recursive update rules that directly solve the optimization problem associated with posterior computation, summarizing all the observed data without needing to store it and process it when making predictions. We compare to [22] directly in our experiments and show that our approach is more effective in several domains.

In [23], the authors instead use a Gaussian process dynamics model with a latent variable governing context. However, their approach is limited by the high computational expense associated with non-parametric Gaussian process regression, whereas we take a parametric approach that is more efficient to use online. Additionally, their approach is focused on adaptation over the course of multiple episodes, whereas our approach is focused on adaptive control over the course of one episode.

Overall, our approach leverages a meta-learned dynamics model that can be viewed as a parametric Gaussian process at test-time. While this limits the degree to which we can adapt to *arbitrary* systems at runtime as compared to non-parametric Gaussian Process or meta-learning approaches that optimize the entire network at test time, in return we enjoy improved computational complexity and online optimization dynamics through the analytic recursive least-squares updates. In our experiments, we demonstrate that this can enable better system-level performance of the nonlinear adaptive control framework.

3 Problem Statement

We aim to control an unknown system, from which we have observed trajectory data with varying (unobservable) latent parameters. For example, if we have a delivery quadrotor, we may observe flight data for the vehicle under different loading conditions. By using efficient adaptive models that are capable of incorporating prior information, we wish to adaptively plan for and control this system, even without having knowledge about the exact payload being transported. Formally, we consider a discrete-time dynamical system of the form

$$\boldsymbol{x}_{t+1} = \boldsymbol{f}(\boldsymbol{x}_t, \boldsymbol{u}_t; \boldsymbol{\theta}) + \boldsymbol{\epsilon}\,, \quad \boldsymbol{\epsilon} \sim \mathcal{N}(0, \Sigma_\epsilon)\,, \tag{1}$$

where $\boldsymbol{f}(\cdot, \cdot; \cdot)$ is a continuous nonlinear function, $\boldsymbol{x} \in \mathcal{X}$ denotes the state, and $\boldsymbol{u} \in \mathcal{U}$ is the action. The dynamics are parameterized by latent parameters $\boldsymbol{\theta} \sim p(\boldsymbol{\theta})$. We assume the cost function, $\text{Cost}(\boldsymbol{x}, \boldsymbol{u})$ is known. We assume interaction with the system is episodic, and $\boldsymbol{\theta}$ is sampled randomly at the beginning of each episode and is fixed over the episode. Critically, however, we do not assume direct knowledge of $\boldsymbol{f}$, $\boldsymbol{\theta}$, or $p(\boldsymbol{\theta})$. Instead, we additionally assume access to trajectory data $\mathcal{D} = \{D_\tau(\boldsymbol{\theta}_j)\}_{j=1}^J$, where $D_\tau(\boldsymbol{\theta}_j) = \{(\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{x}_{t+1})\}_{t=0}^\tau$. From this data, we wish to learn an approximate model for both the dynamics function $\boldsymbol{f}$, as well as an approximate prior over $\boldsymbol{\theta}$. This setting is similar to the standard system identification setting, although $\boldsymbol{\theta}$ is not explicitly known to the system designer

as it may be in system identification. Our approach only requires samples from the system dynamics $\boldsymbol{f}$; it may be used with systems for which analytical models are not available.

Given this dataset $\mathcal{D}$, a episode horizon T, an initial state distribution $p(\boldsymbol{x}_0)$, the system-level control problem can be formulated as

$$\min_{\boldsymbol{\pi}_{0:T} \in \Pi} \mathbb{E}_{\boldsymbol{\theta} \sim p(\boldsymbol{\theta}), \boldsymbol{x}_0 \sim p(\boldsymbol{x}_0)} \left[\sum_{t=0}^{T} \text{Cost}\left(\boldsymbol{x}_t, \boldsymbol{\pi}_t(\boldsymbol{x}_t)\right) \right], \tag{2}$$

and subject to the dynamics of Eq. 1, where we write $\boldsymbol{\pi}_t$ to denote a time-dependent policy, and Π to denote the set of possible policies. While this notation is more commonly used in the model-free literature, we take a model-based approach in this paper. The problem we have presented can also be seen as a generalization of the standard adaptive control problem [11] to the "multi-task" case [24]. Alternatively, it may be interpreted as the meta-reinforcement learning problem [25–27] in which the cost function is known. This problem setting is a Bayes-adaptive Markov Decision Process (BAMDP), a specific form of POMDP in which the unobserved variables are not time-varying [28].

4 Meta-Learning an Adaptive Dynamics Model

In this work we build on ALPaCA, a Bayesian approach to meta-learning. In the following subsections, we first present the model as proposed in [8] as background and to better contextualize our contributions. Next, we highlight a limitation of the original model and present our modifications.

4.1 ALPaCA

ALPaCA [8] is an efficient Bayesian approach to function regression in a meta-learning/multi-task learning setting. In this formulation, function regression is performed by using observed data to update a posterior belief over a family of possible functions. As a Bayesian approach, ALPaCA maintains a belief over functions, and updates this belief via Bayes' Rule as samples of the function are observed. ALPaCA approximates this true distribution over functions by maintaining a distribution over the parameters of a neural network model $K\boldsymbol{\phi}(\boldsymbol{x}, \boldsymbol{u}; \boldsymbol{w})$, where $\boldsymbol{\phi}(\boldsymbol{x}, \boldsymbol{u}; \boldsymbol{w})$ represents a feed-forward neural network with weights $\boldsymbol{w}$ and output dimension n_ϕ, and K is a $n_x \times n_\phi$ matrix which can be thought of as the linear last-layer weights of the network. ALPaCA only maintains a delta distribution over $\boldsymbol{w}$, but a full matrix-normal distribution on the last-layer weights $K \sim \mathcal{MN}(\bar{K}_0, \Sigma_\epsilon, \Lambda_0^{-1})$. This particular form reduces the task of online function regression to Bayesian Linear Regression in the feature space defined by $\boldsymbol{\phi}$.

Given a set of observed transition tuples $D_{\text{context}} = \{(\boldsymbol{x}_\tau, \boldsymbol{u}_\tau, \boldsymbol{x}_{\tau+1})\}_{\tau=0}^{t-1}$ from a single fixed dynamical system $\boldsymbol{f}(\cdot; \boldsymbol{\theta})$, we can compute a posterior over the last-layer weights K, given by $p(K \mid X_t, \Phi_{t-1}) = \mathcal{MN}(\bar{K}_t, \Sigma_\epsilon, \Lambda_t^{-1},)$, where

$$\Lambda_t = \Phi_{t-1}^T \Phi_{t-1} + \Lambda_0 \,, \tag{3}$$

$$\bar{K}_t^T = \Lambda_t^{-1}(\Phi_{t-1} X_t + \Lambda_0 \bar{K}_0^T) \,, \tag{4}$$

with $X_t^T = [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_t] \in \mathbb{R}^{n_x \times t}$ and $\Phi_{t-1}^T = [\boldsymbol{\phi}(\boldsymbol{x}_0, \boldsymbol{u}_0), \ldots, \boldsymbol{\phi}(\boldsymbol{x}_{t-1}, \boldsymbol{u}_{t-1})] \in \mathbb{R}^{n_\phi \times t}$. We assume a Gaussian error model, giving rise to a Gaussian posterior predictive density $\hat{\boldsymbol{x}}_{t+1} \sim \mathcal{N}(\boldsymbol{\mu}_{t+1}, \Sigma_{t+1})$ where

$$\boldsymbol{\mu}_{t+1} = \bar{K}_t \boldsymbol{\phi}(\boldsymbol{x}_t, \boldsymbol{u}_t; \boldsymbol{w}) \,, \tag{5}$$

$$\Sigma_{t+1} = (1 + \boldsymbol{\phi}^T(\boldsymbol{x}_t, \boldsymbol{u}_t; \boldsymbol{w}) \Lambda_t^{-1} \boldsymbol{\phi}(\boldsymbol{x}_t, \boldsymbol{u}_t; \boldsymbol{w})) \Sigma_\epsilon \,. \tag{6}$$

A natural measure of the quality of this model is how well its posterior predictive density models future transition tuples drawn from the same system. If we have a set of query points $D_{\text{query}} = \{(\boldsymbol{x}_\tau, \boldsymbol{u}_\tau, \boldsymbol{x}_{\tau+1})\}_{\tau=t}^T$ from the same system, we can evaluate the negative log likelihood of these points under the posterior predictive density (Eq. 5 and 6) to obtain a loss function. As the posterior predictive density parameters are computed via analytic and differentiable operations, we can optimize the weights $\boldsymbol{w}$ and the prior parameters $\bar{K}_0, \Lambda_0^{-1}$ via stochastic gradient descent. We train on a meta-dataset of datasets $\mathcal{D} = \{(D_{\text{context}}^{(i)}, D_{\text{query}}^{(i)})\}_{i=1}^M$, where the transitions in $D_{\text{context}}^{(i)}$ and $D_{\text{query}}^{(i)}$ are sampled from the dynamical system $\boldsymbol{f}(\cdot; \boldsymbol{\theta}_i)$, and $\boldsymbol{\theta}_i$ is assumed to be representative samples from the prior $p(\boldsymbol{\theta})$. In this way, the model is able to capture the structure of the dynamical system in the neural network features $\boldsymbol{\phi}$, and express the effects of the parameter uncertainty through the prior on K.

4.2 Multi-head ALPaCA

While ALPaCA gives a heteroskedastic posterior predictive model, the posterior predictive covariance is restricted to be a scalar multiple of the error covariance Σ_ϵ. However, in many dynamical systems, the shape of the uncertainty in the transition function is often different across different dimensions. For example, systems where both position and velocities are observed, the position at the next timestep is only a function of the current velocity, while the next velocity may depend on the uncertain latent parameters, and thus have a different shape of uncertainty, as can be seen in Fig. 1a. To correct this, we propose making the modeling assumption that the uncertainty of each output dimension of f is independent. This allows us to use a separate, single-variable Bayesian linear regression model for each output dimension. These can be throught of as distinct "heads" of the model, hence we refer to it as a multi-head ALPaCA model. Formally, we can write the model and prior as

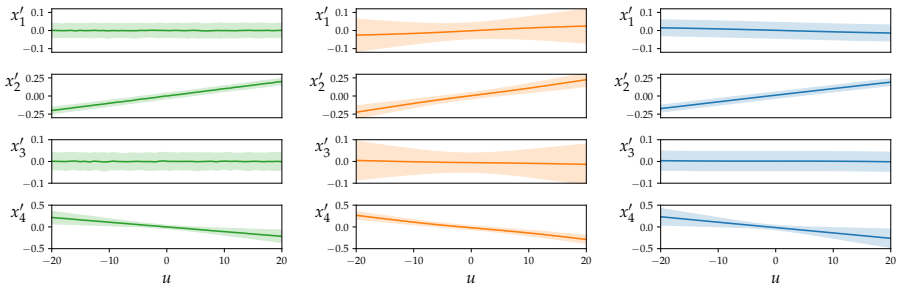
$$\boldsymbol{x}_{t+1} = K \boldsymbol{\phi}(\boldsymbol{x}_t, \boldsymbol{u}_t) + \boldsymbol{\epsilon} \,, \tag{7}$$

$$\boldsymbol{k}_i \sim \mathcal{N}(\bar{\boldsymbol{k}}_{i,0}, \sigma_i^2 \Lambda_{i,0}^{-1}) \,, \qquad \forall i = 1, \ldots, n_x \,, \tag{8}$$

$$\epsilon_i \sim \mathcal{N}(0, \sigma_i^2) \,, \qquad \forall i = 1, \ldots, n_x \,, \tag{9}$$

where each row $\boldsymbol{k}_i$ of the weight matrix K is drawn from an independent Gaussian distribution, with mean $\bar{\boldsymbol{k}}_{i,0}$ and covariance $\sigma_i^2 \Lambda_{i,0}^{-1}$, and each output dimension is corrupted with independent additive Gaussian noise.

These independence assumptions allow computing the posterior distribution on each $\boldsymbol{k}_i$ separately. Concretely, given D_{context}, the posterior distributions for



(a) True System (b) Single Prior [8] (c) Multi-Head (Ours)

Fig. 1: Mean and 95% confidence intervals of, from left to right, the true transition function $\boldsymbol{x}' = \boldsymbol{f}(\boldsymbol{0}, \boldsymbol{u}; \boldsymbol{\theta})$ with $\boldsymbol{\theta} \sim p(\boldsymbol{\theta})$ for the Cart-Pole system, the approximation of the ALPaCA model with a single prior on the last layer, and our variation using approximation from the multi-head ALPaCA model. Multi-head ALPaCA more accurately models the underlying function. Therefore, in the experiments of Section 6 we will use our variant of ALPaCA.

each output dimension i are given by $\mathcal{N}(\bar{\boldsymbol{k}}_{i,t}, \sigma_i^2 \Lambda_{i,t}^{-1})$, where

$$\Lambda_{i,t} = \boldsymbol{\phi}_{t-1}^T \Phi_{t-1} + \Lambda_{i,0}, \tag{10}$$

$$\bar{\boldsymbol{k}}_{i,t} = \Lambda_{i,t}^{-1}(\Phi_{t-1} X_{i,t} + \Lambda_{i,0} \bar{\boldsymbol{k}}_{i,0}), \tag{11}$$

$\Phi_t \in \mathbb{R}^{n_\phi \times t}$ is unchanged from before, and $X_{i,t} \in \mathbb{R}^t$ the vector containing dimension i of the next state for each of the t transition tuples in D_{context}. The posterior predictive for this model remains of the same form as in Eq. 5 and 6, where now the covariance is given by a diagonal matrix Σ_t with elements

$$(\Sigma_{t+1})_{ii} = \left(1 + \boldsymbol{\phi}^T(\boldsymbol{x}_t, \boldsymbol{u}_t; \boldsymbol{w}) \Lambda_{i,t}^{-1} \boldsymbol{\phi}(\boldsymbol{x}_t, \boldsymbol{u}_t; \boldsymbol{w})\right) \sigma_i^2 \,. \tag{12}$$

In the adaptive control context where context data is observed sequentially as the agent interacts with the true system, it is useful to leverage recursive updates of the posterior, as detailed in [8]. In our improved model, parameters of the posterior at time t can be computed given observed transition $(\boldsymbol{x}_{t-1}, \boldsymbol{u}_{t-1}, \boldsymbol{x}_t)$ and the values of those parameters at the previous timestep. For each state dimension i, this update rule is

$$\begin{aligned} \Lambda_{i,t}^{-1} &= \Lambda_{i,t-1}^{-1} - \frac{1}{1 + \boldsymbol{\phi}_{t-1}^T \Lambda_{i,t-1}^{-1} \boldsymbol{\phi}_{t-1}} \Lambda_{i,t-1}^{-1} \boldsymbol{\phi}_{t-1} \boldsymbol{\phi}_{t-1}^T \Lambda_{i,t-1}^{-1} \,, \\ \boldsymbol{q}_{i,t} &= (\boldsymbol{x}_t)_i \boldsymbol{\phi}_{t-1} + \boldsymbol{q}_{i,t-1} \,, \\ \bar{\boldsymbol{k}}_{i,t} &= \Lambda_{i,t}^{-1} \boldsymbol{q}_{i,t} \,, \end{aligned} \tag{13}$$

with each $\boldsymbol{q}_i \in \mathbb{R}^{n_\phi}$ is initialized to be $\boldsymbol{q}_i = \Lambda_0 \bar{\boldsymbol{k}}_{i,0}$, and $\boldsymbol{\phi}_{t-1}$ is shorthand for $\boldsymbol{\phi}(\boldsymbol{x}_{t-1}, \boldsymbol{u}_{t-1})$.

This choice of variance model corresponds to a structural assumptions on the covariance of $\text{vec}(K)$. The approach taken in [8] was to model K with a matrix-normal distribution, which corresponds to $\text{vec}(K) \sim \mathcal{N}(\text{vec}(\bar{K}), \Sigma_\epsilon \otimes \Lambda^{-1})$.

Note that the full (unstructured) covariance of $\text{vec}(K)$ contains $n_\phi^2 n_x^2$ parameters. For a moderately large system (say, $n_x = 10$) with 100 basis functions, this would require learning one million matrix entries. In contrast, the matrix normal factorization requires learning only n_ϕ^2 entries, which in this case corresponds to 1% of the parameters. However, this relies on Λ^{-1} being shared for each output dimension, which is typically overly restrictive. The approach we present here corresponds to a block-diagonal covariance, wherein $\text{vec}(K) \sim \mathcal{N}(\text{vec}(\bar{K}), \text{diag}([\Lambda_1^{-1}, \ldots, \Lambda_{n_x}^{-1}]))$. This factorization requires n_x RLS updates, and maintains $n_x n_\phi^2$ parameters. In comparison to a fully dense covariance matrix, we only sacrifice the ability to handle off-diagonal elements in the noise covariance, which are typically ignored in the modeling process anyway. Moreover, this choice of covariance factorization allows the RLS update to be performed via n_x independent updates with the precision matrices for each dimension, substantially reducing computational complexity. An extended discussion of this covariance factorization is presented in the appendix.

We trained both the model from [8] as well as a model with the dimension specific priors on a cart-pole system with randomized parameters (details of the randomization are provided in the experiments section). Apart from the last layer structure, the models were identical. Fig. 1 visualizes each dimension of the next state when different control actions are applied to the system initially at $\boldsymbol{x}_0 = \mathbf{0}$. We see that the uncertainty in the parameters influences the transition in different ways for each dimension. It is evident that by having a separate prior for each output dimension, the model is better able to capture the true uncertainty structure in the dynamics. We observed that this added flexibility improved training stability, and led to improved quantitative performance as well: on the cart-pole system the multi-head ALPaCA obtains a NLL of -24.8, while the baseline ALPaCA only attains -22.67[3]. For this reason, we use the multi-head ALPaCA model for all of our experiments, and henceforth use ALPaCA to refer to this multi-head version.

5 Control Adaptation via Meta-Learning Dynamics

Our proposed framework for meta-learning enabled nonlinear adaptive control consists of two phases. First, *offline*, we use a meta-dataset of transitions drawn from a distribution of dynamical systems to optimize an adaptive dynamics model. Then, *online*, we use this adaptive model in a model predictive control framework, while adapting the model as data is observed. The high level operation of both of these phases is outlined in Alg. 1; in the following sections, we discuss each phase in more detail.

5.1 Offline Meta-learning of Dynamics

To efficiently adapt as transition data is observed online, it is necessary to leverage prior knowledge about the dynamical system expected to be seen in the online phase. In this work, we assume that this prior knowledge comes

[3] Additional quantitative results can be found in the appendix.

Algorithm 1 CAMeLiD

Require: training data $\mathcal{D}$, replanning frequency h, MPC horizon H

OFFLINE META-LEARNING:

Solve problem (14) to get $\bar{K}_0, \{\Lambda_{i,0}\}_{i=1}^{n_x}, \boldsymbol{w}$

ONLINE ADAPTIVE MPC:

Initialize $\boldsymbol{q}_{i,0} \leftarrow \Lambda_{i,0}\bar{\boldsymbol{k}}_{i,0}$ for all $i = 1, \ldots, n_x$

Get initial state x_0

for $t = 0$ to T **do**

  if $t \bmod h = 0$ **then**

    Compute rows of $\bar{K}_t$ (13)

    Compute policy $\pi_t, \ldots, \pi_{t+H}$ according to $\bar{K}_t$ by solving (18)

  end if

  Apply control $\boldsymbol{u}_t = \boldsymbol{\pi}_t(\boldsymbol{x}_t)$

  Observe next state $\boldsymbol{x}_{t+1}$

  Compute $\Lambda_{i,t+1}^{-1}, \boldsymbol{q}_{i,t+1}$ via recursive updates (13)

end for

in the form of a meta-dataset of datasets as discussed in the previous section $\mathcal{D} = \{(D_{\text{context}}^{(i)}, D_{\text{query}}^{(i)})\}_{i=1}^{M}$, drawn from the distribution of dynamical systems we expect to see online. This meta-dataset can be gathered in different ways: for example, it could be constructed from past interactions with dynamical systems of the same class, or generated by randomizing parameters of a simulator in physically realistic ways. In our experiments, we sample from the meta-dataset by first sampling the parameters $\boldsymbol{\theta}$ of the simulator from a prior distribution $p(\boldsymbol{\theta})$, and then sampling state transitions from the simulator.

Given this meta-dataset, the model is trained by minimizing the negative log likelihood of D_{query} under the posterior predictive density when conditioned on D_{context}, as in [5]:

$$\begin{aligned} \min_{\bar{K}_0, \Lambda_0, \boldsymbol{w}} \quad & \sum_{i=1}^{M} \sum_{(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{x}') \in D_{\text{query}}^{(i)}} \ell(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{x}', D_{\text{context}}^{(i)}; \bar{K}_0, \{\Lambda_{i,0}\}_{i=1}^{n_x}, \boldsymbol{w}) \,, \\ \text{subject to} \quad & \Lambda_{i,0} \succeq 0 \quad i = 1, \ldots, n_x \,, \end{aligned} \tag{14}$$

where

$$\ell(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{x}', D_{\text{context}}^{(i)}; \bar{K}_0, \Lambda_0, \boldsymbol{w}) = -\log \mathcal{N}(\boldsymbol{x}'; \boldsymbol{\mu}, \Sigma) \,, \tag{15}$$

$$\boldsymbol{\mu}, \Sigma = \text{PosteriorPredictive}(\boldsymbol{x}, \boldsymbol{u}, \bar{K}_t, \{\Lambda_{i,t}\}_{i=1}^{n_x}; \boldsymbol{w}) \,, \quad \text{(Eq. 5, 12)} \tag{16}$$

$$\bar{K}_t, \{\Lambda_{i,t}\}_{i=1}^{n_x} = \text{Posterior}(D_{\text{context}}; \bar{K}_0, \{\Lambda_{i,0}\}_{i=1}^{n_x}, \boldsymbol{w}) \,. \quad \text{(Eq. 10, 11)} \tag{17}$$

By parameterizing $\Lambda_{i,0} = Z_i Z_i^T$ for each i, we can implicitly enforce the positive semi-definite constraint, and thereby convert the problem into an unconstrained non-convex optimization problem, which we then approximately solve using mini-batch stochastic gradient descent.

5.2 Online Adaptive Model Predictive Control

At test time, the learned adaptive model can be applied within a model-predictive control framework to control an uncertain system. At each timestep, the agent

takes an action and uses the observed transition to update the posterior parameters through the recursive updates in Eq. 13. To compute which action to take, the agent uses the current model estimate to optimize a finite horizon control strategy. Formally, at time t, the agent optimizes for a horizon H using the current model estimate $\bar{K}_t$ such that

$$\begin{aligned} \min_{\pi \in \Pi} \quad & \sum_{\tau=0}^{H} \text{Cost}(\boldsymbol{x}_\tau, \boldsymbol{u}_\tau)\,, \\ \text{subject to} \quad & \boldsymbol{x}_{\tau+1} = \bar{K}_t \boldsymbol{\phi}(\boldsymbol{x}_\tau, \boldsymbol{u}_\tau; \boldsymbol{w})\,, \\ & \boldsymbol{u}_\tau = \boldsymbol{\pi}_\tau(\boldsymbol{x}_\tau)\,, \\ & \boldsymbol{u}_\tau \in \mathcal{U}\,. \end{aligned} \tag{18}$$

In this work, we solve this nonlinear optimal control problem using the iterative Linear Quadratic Regulator (iLQR) framework [29]. In contrast to sampling based optimization methods such as those used in [30], iLQR explicitly performs dynamic programming, and thus is better suited to optimizing over longer time horizons. The agent takes h steps from the resulting policy before recomputing a policy with the updated model. Since the ALPaCA dynamics model is pretrained to adapt quickly to fit the true observed transitions, the computed policies become increasingly effective on the true dynamical system.

6 Experimental Results

The goal of these experiments is to evaluate the performance of our meta-learned adaptive dynamics method, CAMeLiD, and compare it against prior literature. In particular, we aim to answer the questions: How fast can our approach adapt its dynamics model? How accurately can we control a system using the meta-learned dynamics? And finally, how well can our approach generalize to parameters outside of the meta-training distribution?

6.1 Experimental Settings

To answer these questions, we evaluate CAMeLiD on two benchmark systems for nonlinear systems, the 4 degree of freedom Cart-Pole system, and a 6 degree of freedom Planar Quadrotor model. We now detail these two systems and the baselines that we use as comparisons for our approach. More in-depth details of the cost function, dynamics, and randomization can be found in the appendix.

Cart-Pole: The Cart-Pole system is a classic benchmark nonlinear system for control in which a pole is attached as a pendulum onto a cart which can move horizontally. This system has one control input, the force applied to the cart. The key parameters of the nonlinear dynamics are the mass of the cart, the mass of the pole, and the length of the pole. In our experiments we consider distributions where each is varied independently, while the other remain fixed, as well as a scenario where all are varied simultaneously. The system starts with the pole upright, and the cart at position $x = -3$. The task is to move the cart to a position of $x = 0$ while keeping the pole upright, and this is encoded through a simple quadratic cost centered at the goal state.

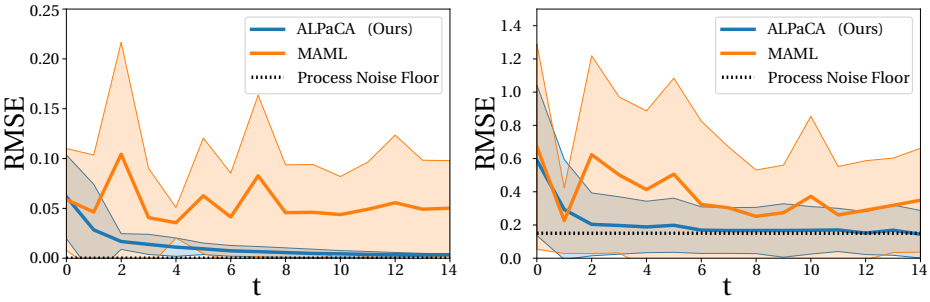
Quadrotor: To investigate the performance of the framework on a more complex system wherein the uncertain parameters enter into the dynamics nonlinearly, we consider a planar quad model, a standard benchmark problem in control and reinforcement learning [31–33]. We consider a quadrotor delivery scenario, in which a quadrotor has to carry an unknown mass attached at an unknown position on its frame. In addition to randomizing the amount and position of this mass, we also add a nonlinear ground effect model to the dynamics. The parameters in this model are typically fit from data, and thus are randomized as well. The model also has process noise on the velocities, simulating the effects of wind. We test on a simple translation task, by choosing a quadratic cost function centered around a goal position. If the quadrotor hits the floor ($y = 0$), we hold its state fixed for the remainder of the episode.

Baselines: We evaluate CAMeLiD, which combines the ALPaCA adaptive dynamics model with an iLQR controller, against a variety of alternative models. First, to investigate the efficacy of ALPaCA (with our modifications), we compare against the MAML dynamics model used in [30], as well as a baseline, non-adaptive, neural network model trained to minimize NLL on all the transition data in $\mathcal{D}$. All the neural network models use the same architecture, and are trained on the same data. For each dynamics model, we also compare the iLQR controller against MPPI [2], a derivative free Monte-Carlo based optimal control algorithm. The combination of the MAML dynamics model and the MPPI controller is the GrBAL framework proposed in [22].

We evaluate all six model-controller combinations on the two simulated control systems under different forms of randomization to investigate what forms of uncertainty this framework can address. For each system $\boldsymbol{f}(\cdot;\boldsymbol{\theta})$ with a particular distribution of parameters $p(\boldsymbol{\theta})$, we first train each of the dynamics models from data generated from this distribution. After training the model, we evaluate each model-controller combination by simulating rollouts on 100 systems sampled from the same distribution $p(\boldsymbol{\theta})$. From these simulated rollouts, we report the average realized cost in Table 1. We also investigate the model adaptation in isolation: at every timestep of simulation, we log the RMSE between the predicted next state and the actual observed next state. Fig. 2 compares ALPaCA against MAML in terms of these prediction RMSE values for the both systems.

6.2 Meta-learning of the Dynamics

Fig. 2a shows the adaptive performance of the ALPaCA meta-learned dynamics model. The results indicate that the RMSE predictions made by ALPaCA quickly decrease and approach 0 – meaning that the model is able to almost perfectly predict the next state after just 10 transitions. In contrast, the MAML model does not significantly improve over time in terms of RMSE prediction of the next state. This gap in performance may be explained by the fact that online, the MAML dynamics model is only taking a gradient step on all the parameters of the network. As such, the performance of MAML can be highly sensitive to hyperparameters such as the step-size to use for this online gradient step. In contrast, ALPaCA requires no such hyperparameter as it exactly updates the last layer weights to the analytic optimal through the recursive least-squares updates. Similar results are obtained also on the quadrotor task shown in Fig. 2b,



(a) Cart-Pole (b) Quadrotor

Fig. 2: Mean and 95% confidence of the RMSE of the prediction of the next state at each timestep of an episode. For both the cart-pole and the quadrotor tasks our approach substantially outperforms the MAML baseline by providing more accurate predictions. Moreover, the ALPaCA model adapts extremely rapidly and the decrease in RMSE is nearly monotonic.

where we observe that ALPaCA adapts more efficiently and stably than MAML, with the prediction RMSE here converging to the lower bound of the process noise RMSE after observing under 10 transition samples.

These two experiments confirm that our novel extension of the ALPaCA model is capable of automatically and efficiently adapting to different dynamics over the course of a single episode of interaction. With this promising result, we now proceed to evaluate the performance of ALPaCA when used within CAMeLiD for control.

6.3 Adaptive Control Performance

Accurate predictions do not always translate into precise control. Hence, we now proceed with evaluating the control performance of CAMeLiD of 4 variants of

Model	Cart-pole				Quadrotor
	Pole Length	Pole Mass	Cart Mass	All	
CAMeLiD	**12.21** $\pm$ 0.11	**12.10** $\pm$ 0.04	**12.01** $\pm$ 0.22	**12.28** $\pm$ 0.19	**9.12** $\pm$ 0.99
MAML/iLQR	49.31 $\pm$ 5.48	14.40 $\pm$ 0.14	20.92 $\pm$ 3.16	35.48 $\pm$ 6.47	18.07 $\pm$ 2.39
Baseline/iLQR	33.58 $\pm$ 2.24	42.17 $\pm$ 0.61	21.91 $\pm$ 1.52	90.22 $\pm$ 15.05	11.90 $\pm$ 1.38
ALPaCA/MPPI	34.05 $\pm$ 8.30	18.68 $\pm$ 0.60	20.36 $\pm$ 2.74	20.36 $\pm$ 2.74	101.96 $\pm$ 41.35
GrBAL [22]	67.82 $\pm$ 12.07	21.09 $\pm$ 0.97	31.86 $\pm$ 3.77	31.86 $\pm$ 3.77	83.45 $\pm$ 23.45
Baseline/MPPI	31.07 $\pm$ 3.11	43.80 $\pm$ 1.19	31.81 $\pm$ 2.96	31.81 $\pm$ 2.96	59.87 $\pm$ 14.18

Table 1: Mean and 95% confidence of the cost attained on different systems with different modes of randomization. CAMeLiD, a combination of ALPaCA and the iLQR controller, consistently demonstrates superior system-level performance. For Cart-Pole, we tested distributions where a single parameter was varied, as well as multiple together. Details of the distributions over system parameters for both domains can be found in the appendix.

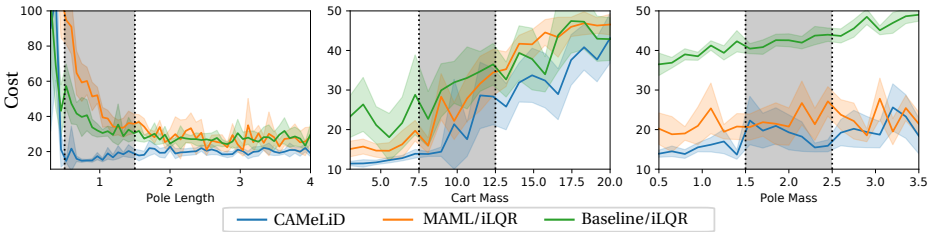


Fig. 3: Generalization capabilities of different approaches to out-of-distribution tasks. For each task, the greyed area indicates the interval of parameters used during the meta-training. The results show that the performance of CAMeLiD deteriorates the slowest compared to the baselines. This indicates that our approach is capable of better generalizing outside of the meta-training distribution.

the cart-pole task and on the quadrotor task. The results reported in Table 1 show that CAMeLiD outperforms the other models in terms of realized cost for all forms of randomization. Notably, we observe that the iLQR controller consistently outperforms MPPI in this setting. We suspect that this is because MPPI approximates the optimal sequence of actions via Monte-Carlo sampling. As such, the quality of the optimized trajectories depends on hyperparameters such as the number of action samples taken and the form of the action distribution, which may be hard to tune. Furthermore, as this task involves control around an unstable equilibrium of the system, sampling low-cost action sequences is low-probability.

6.4 Generalization Capabilities

As a final experiment, we study the generalization performance of our approach when applied to tasks outside of the meta-training distribution. Ideally, we would like any adaptive controller to be robust to changes in the task (i.e., in the dynamics), and to perform consistently and optimally for all the possible tasks. This property is helpful both in terms of general-purpose robustness, but it also reduces the importance of correctly predicting the task distribution for an engineer designing the adaptive control system. To study this property we consider the previous three versions of the cart-pole task, in which the pole length, mass, and the cart mass are randomized independently. In this task, process noise was added to the cart and pole velocity. In Fig. 3, we report the results for these experiments. We see that our version of ALPaCA nearly always outperforms MAML and the baseline model. While the ability of a model to generalize is highly dependent on the particular parameter (and how it appears in the dynamics), the results for ALPaCA are highly promising.

7 Discussion and Extensions

While ALPaCA models are performing Bayesian regression, and thus maintaining a full posterior over the next state, we have not utilized this distribution in

our control scheme. Within adaptive control, our approach is *certainty equivalent*, as opposed to an approach which incorporates uncertainty, which is called *cautious*. Optimally, one would aim to incorporate both the uncertainty of predictions and the knowledge that information will be gained in the future. This approach, referred to as *dual control* [34], is typically computationally intractable. The relative merits of *certainty-equivalent* versus *cautious* adaptive control are still an open topic of research [35]. Cautious adaptive control tends to be overly conservative as it incorporates model-uncertainty without capturing information gain, especially over long horizons.

During this work we implemented two cautious adaptive control schemes: an iLQR-based approach that incorporates uncertainty, similar to [36], as well as probabilistic differential dynamic programming (PDDP) [37]. The former relies on re-sampling the model at each time, whereas the latter adds the uncertainty to the state vector and approximately propagates it forward in time. As such, they represent substantially different approaches to uncertainty-aware control. We repeatedly found that both methods performed worse than certainty-equivalent iLQR, which we believe represents an important negative result.

Beyond cautious adaptive control schemes, there are numerous ways in which the probabilistic predictions generated by our model may be leveraged, which we leave for future work. First, while incorporating the model uncertainty into the cost resulted in degraded performance, this model uncertainty may be used for probabilistic safety guarantees, via e.g. chance-constrained programming approaches [38]. Another possible application of the probabilistic modeling is Bayesian model testing/fault detection. Because we maintain a posterior distributions over outcomes, this may be compared to our prior model, and if the probability is large that there exists a mismatch between our posterior and the incoming data, we may e.g. reset to our prior. This approach could effectively perform adaptive control for systems with discontinuous changes in dynamics. A similar approach was taken in [30]. However, because they are using MAML [7] models, their probability evaluations were implicitly assuming isotropic unit variance (as they were using the 2-norm of the prediction error as a surrogate for likelihood) that did not concentrate as more data was observed online. Finally, in the reinforcement-learning setting in which episodes are performed sequentially, uncertainty may be used for exploration via e.g. Thompson sampling [39] or optimism [40].

8 Conclusions

A critical capability for truly autonomous systems is to be able to quickly adapt to new environments and conditions. In this paper, we frame this adaptation from a model-based control perspective, as adapting on-the-fly a forward dynamics model which is simultaneously used for control. To do so we propose CAMeLiD – a novel approach which leverage advances in meta-learning to efficiently and robustly adapt a neural network-based dynamics model within an adaptive control framework. Experimental results demonstrate that CAMeLiD outperform existing methods both in prediction accuracy, and in control performance. Moreover, our approach is less sensitive to tasks outside of the distribution used during meta-training, thus providing improved generalization capabilities.

Acknowledgments

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A Output Layer Factorization

In this section we will explicitly show that the block-diagonal covariance structure results in the RLS updates presented in Section 4.2. We will first show that the RLS updates of the block-diagonal system preserve block-diagonality. Then we will show the mean and covariance RLS updates are equivalent to those previously presented. We will rewrite Eq. 7 as

$$\boldsymbol{x}_{t+1} = \hat{\Phi}\hat{\boldsymbol{k}} + \boldsymbol{\epsilon} \tag{19}$$

where $\hat{\boldsymbol{k}} = \text{vec}(K) = [\boldsymbol{k}_1, \ldots, \boldsymbol{k}_{n_x}]^T$ where $\boldsymbol{k}_i$ is the i'th row of K, and

$$\hat{\Phi} = \begin{bmatrix} \boldsymbol{\phi}(\boldsymbol{x}_t, \boldsymbol{u}_t)^T & \boldsymbol{0}^T & & \\ \boldsymbol{0}^T & \boldsymbol{\phi}(\boldsymbol{x}_t, \boldsymbol{u}_t)^T & & \\ & & \ddots & \\ & & & \boldsymbol{\phi}(\boldsymbol{x}_t, \boldsymbol{u}_t)^T \end{bmatrix}. \tag{20}$$

One can verify that this is equivalent to Eq. 7. We will structure our prior as $\hat{\boldsymbol{k}}_0 = \mathcal{N}(\bar{\hat{\boldsymbol{k}}}_0, (\Sigma_\epsilon \otimes I)\hat{\Lambda}_0^{-1})$, with

$$\hat{\Lambda}_0^{-1} = \begin{bmatrix} \Lambda_{1,0}^{-1} & & \\ & \ddots & \\ & & \Lambda_{n_x,0}^{-1} \end{bmatrix}, \tag{21}$$

where the off-diagonal blocks are zeros. For matrix input, the RLS covariance update is

$$\hat{\Lambda}_t^{-1} = \hat{\Lambda}_{t-1}^{-1} - \hat{\Lambda}_{t-1}^{-1}\hat{\Phi}_{t-1}^T(I + \hat{\Phi}_{t-1}\hat{\Lambda}_{t-1}\hat{\Phi}_{t-1}^T)^{-1}\hat{\Phi}_{t-1}\hat{\Lambda}_{t-1}^{-1}. \tag{22}$$

Note that

$$(I+\hat{\Phi}_{t-1}\hat{\Lambda}_{t-1}\hat{\Phi}_{t-1}^T)^{-1} = \begin{bmatrix} (1 + \boldsymbol{\phi}_{t-1}^T\Lambda_{1,t-1}^{-1}\boldsymbol{\phi}_{t-1})^{-1} & & \\ & \ddots & \\ & & (1 + \boldsymbol{\phi}_{t-1}^T\Lambda_{n_x,t-1}^{-1}\boldsymbol{\phi}_{t-1})^{-1} \end{bmatrix} \tag{23}$$

with zero off-diagonals, and

$$\hat{\Lambda}_{t-1}^{-1}\hat{\Phi}_{t-1}^T = \begin{bmatrix} \Lambda_{1,t-1}^{-1}\boldsymbol{\phi}_{t-1} & & \\ & \ddots & \\ & & \Lambda_{n_x,t-1}^{-1}\boldsymbol{\phi}_{t-1} \end{bmatrix}, \tag{24}$$

again with zero off-diagonals. One may see, then, that all terms in Eq. 22 are block-diagonal, and so we recover the n_x independent RLS updates presented previously.

The mean update may be written as

$$\hat{\boldsymbol{q}}_t = \hat{\boldsymbol{q}}_{t-1} + \hat{\Phi}_{t-1}^T\boldsymbol{x}_t \tag{25}$$

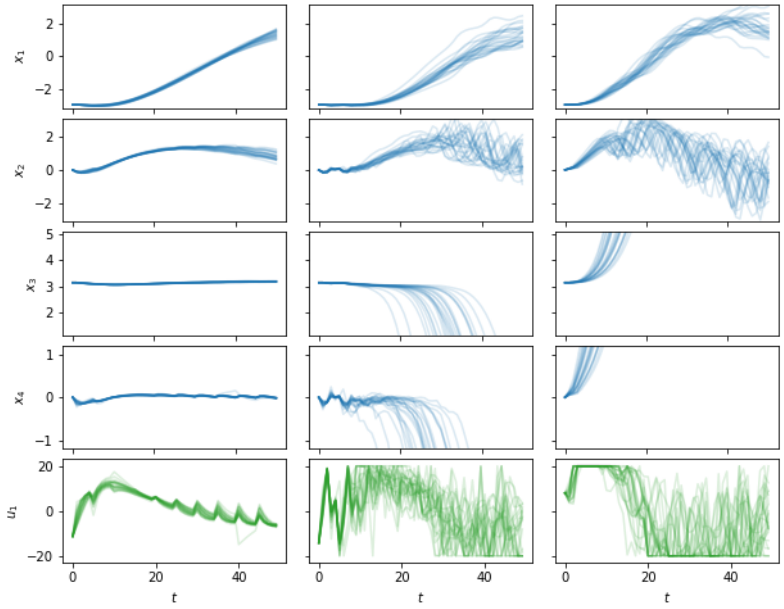

Fig. 4: State (top four rows) and action (bottom row) traces for the iLQR controller using the ALPaCA model (left), MAML (center), and baseline (right). Each figure shows 25 trials. Note that in these simulations there was no process noise, and so the performance of the ALPaCA model in particular is highly repeatable.

where $\hat{\boldsymbol{q}}$ is again stacked $\boldsymbol{q}_i$. The final RLS update is then

$$\bar{\hat{\boldsymbol{k}}}_t = \hat{\Lambda}_t^{-1} \hat{\boldsymbol{q}}_t. \tag{26}$$

Again, these updates are equivalent to those presented earlier. However, if one were to naïvely implement the matrix equations presented here, considerable computation would be wasted on the zero off-diagonal terms. The n_x separate RLS updates presented previously avoid this unnecessary computation.

Finally, while we have discussed choice of a structured $\hat{\Lambda}^{-1}$, one may note that choice of a diagonal Σ_ϵ also results in the predictive covariance being equivalent. Thus, the structure we have proposed falls out naturally as a consequence of this diagonal process noise assumption, which is very common in controls engineering.

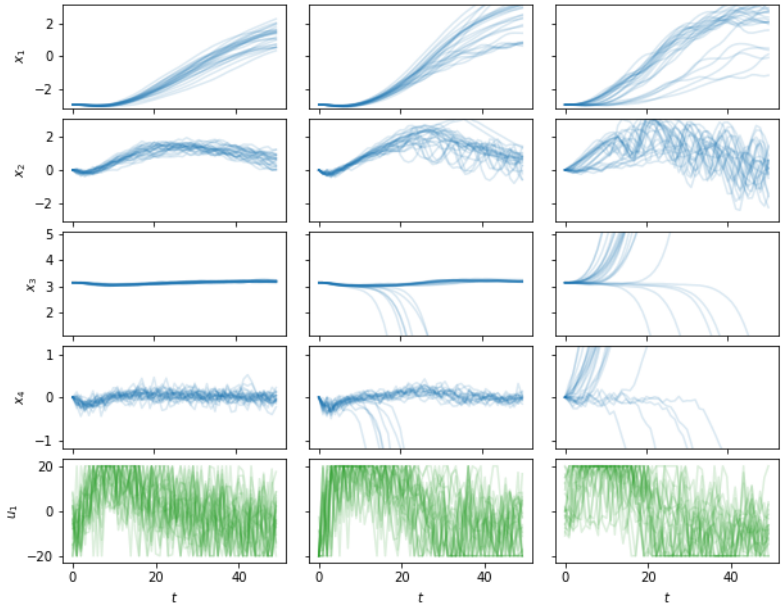

Fig. 5: State (top four rows) and action (bottom row) traces for the MPPI controller using the ALPaCA model (left), MAML (center), and baseline (right). Each figure shows 25 trials. The actions chosen for all models display considerably higher variance than for the iLQR controller.

B Experimental Details

B.1 Model Training

For all models and all systems, the neural network approximation of the dynamics is a 3-hidden layer network with 32 units in each hidden layer and tanh activations. For the ALPaCA models, this means the dimension of $\boldsymbol{\phi}(\boldsymbol{x}, \boldsymbol{u}; \boldsymbol{w})$, $n_\phi = 32$, and the last linear layer corresponds to the matrix K.

Since we have access to a simulator for our experiments, we draw new samples from the meta-dataset $\mathcal{D}$ for each minibatch SGD step. In effect, this makes the size of the meta-dataset infinite, and avoids concerns of meta-overfitting that arise from running SGD on a presampled, finite-size dataset of transitions. When training on finite-sized datasets, performance on a distinct validation meta-datset should be monitored to inform regularization or early termination of the SGD algorithm to avoid overfitting. Moreover, we emphasize that CAMeLiD is operating in an off-policy sense—the generating distribution of the data is invariant to our control actions. An important direction of future work is characterizing performing in the on-policy setting.

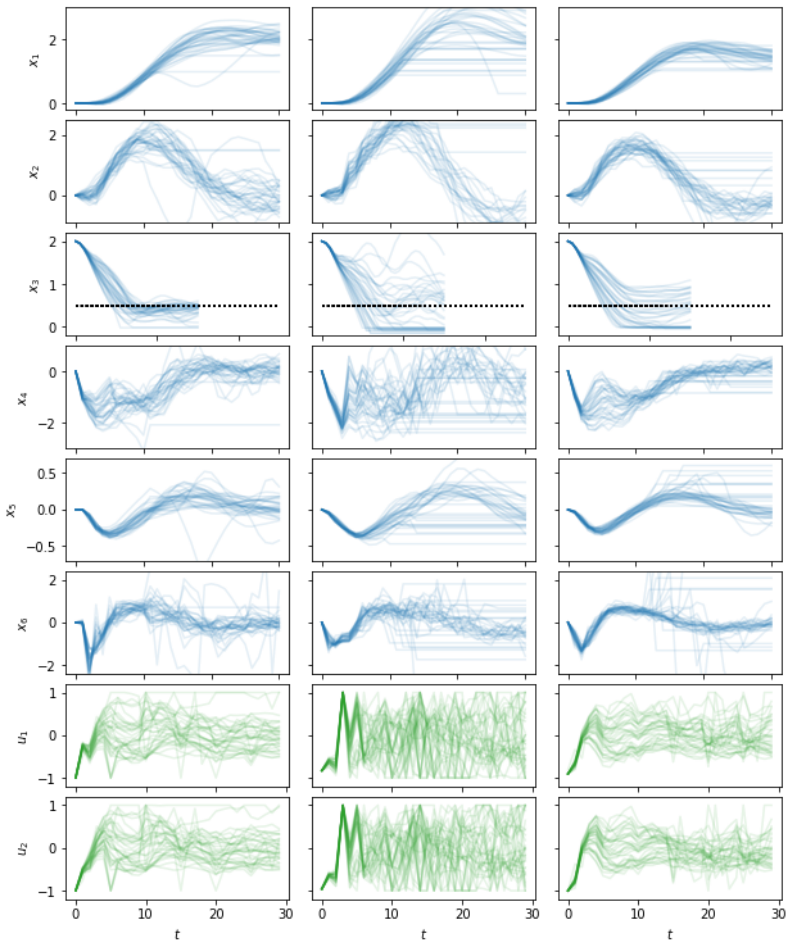

Fig. 6: State (top six rows) and action (bottom two rows) traces for the iLQR controller using the ALPaCA model (left), MAML (center), and baseline (right). Each figure shows 25 trials. When collision occurred with the ground, the state was held fixed for the remainder of the episode. Thus, traces in which collisions occur appear as a straight line in all state dimensions from the time of collision. The dotted grey line denotes the goal hover height.

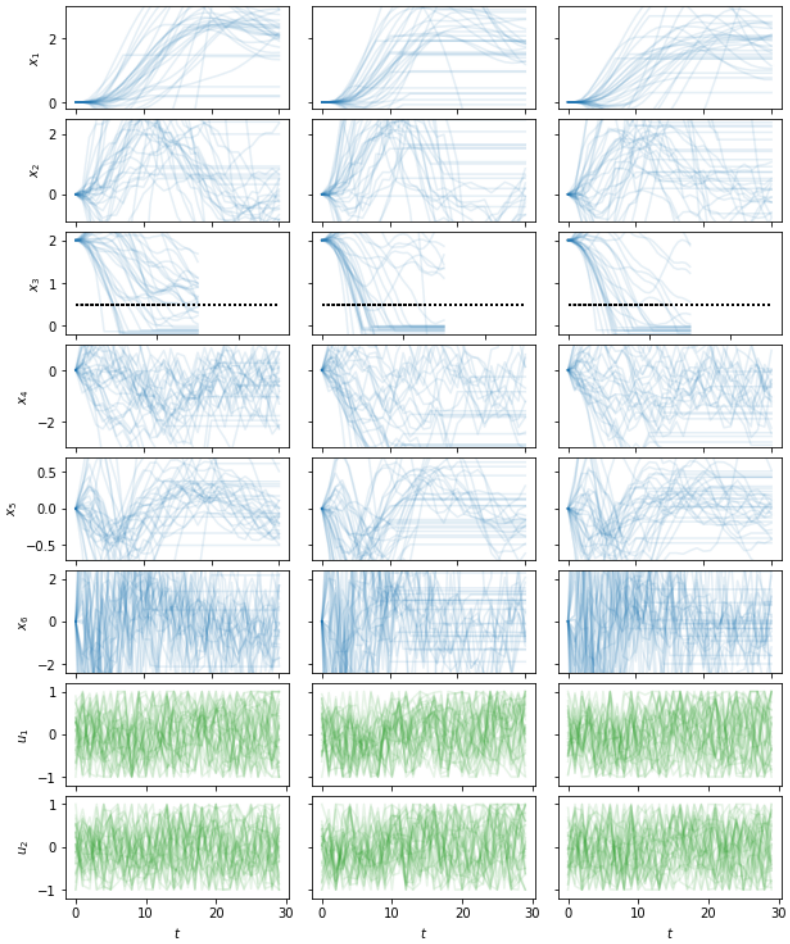

Fig. 7: State (top six rows) and action (bottom two rows) traces for the MPPI controller using the ALPaCA model (left), MAML (center), and baseline (right). Each figure shows 25 trials. When collision occurred with the ground, the state was held fixed for the remainder of the episode. The dotted grey line denotes the goal hover height.

B.2 Cart-Pole

Dynamics and Randomization: The state of the Cart-Pole system is $\boldsymbol{x} = [x, \dot{x}, \theta, \dot{\theta}]$, the position and velocity of the cart and the angular position and velocity of the pole. We use the Cart-Pole dynamics as defined in [41]. Our simulator forward integrates the dynamics simply via forward Euler integration, to improve performance. The uncertain parameters $\boldsymbol{\theta}$ are the cart mass m_c, the pole mass m_p, and the pole length ℓ_p. The distribution of parameters $p(\boldsymbol{\theta})$ we use are:

$$m_c \sim \text{Unif}(7.5, 12.5) \tag{27}$$
$$m_p \sim \text{Unif}(1.5, 2.5) \tag{28}$$
$$\ell_p \sim \text{Unif}(0.5, 1.5) \tag{29}$$

When only one parameter is randomized, the others are held fixed at the mean value.

Cost Function: To define the task of translating the cart to a goal position of $x = 0$ while keeping the pole upright $\theta = \pi$, we use the following quadratic cost function

$$\text{Cost}(\boldsymbol{x}, \boldsymbol{u}) = 0.05x^2 + 0.001\dot{x}^2 + 0.005(\theta - \pi)^2 + 0.01\dot{\theta}^2 + 0.001u^2 \tag{30}$$

We note that this places relatively high weight on $\dot{\theta}$. This has a substantial stabilizing effect on the controller, at the cost of leading to frequent overshoot of the goal horizontal position. We found that this relatively high cost term was necessary to achieve good performance with MPPI. With a lower cost on this element of the state, iLQR regulated to $x = 0$ more effectively.

Controller Hyperparameters: When using the iLQG controller, we use a replanning frequency $h = 5$. Since the MPPI controller only returns a sequence of actions rather than a feedback policy, we use $h = 1$ for the MPPI trials. For both controllers, we use a control horizon of $H = 25$, and simulate an episode of length $T = 50$. For MPPI, we use 1000 samples.

B.3 Quadrotor

Dynamics and Randomization: As in [32], the state of the quadrotor is $\boldsymbol{x} = [x, \dot{x}, y, \dot{y}, \phi, \dot{\phi}]$, and the control action maps to the commanded thrust for each of the two motors. The dynamics of the quadrotor, with a point mass m_a added at position $r_a\ell$ along its frame, are given by:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{x} \\ -\frac{1}{m+m_a} C_{d,v}\dot{x} - \frac{T_1}{m+m_a}\sin(\phi) - \frac{T_2}{m+m_a}\sin(\phi) \\ \dot{y} \\ -\frac{1}{m+m_a} C_{d,v}\dot{y} + \frac{T_1}{m+m_a}\cos(\phi) + \frac{T_2}{m+m_a}\cos(\phi) \\ \dot{\phi} \\ \frac{1}{I_{zz}+m(r_a\ell)^2}\left(-C_{d,\omega}\dot{\phi} - \ell T_1 + \ell T_2 - m_a g r_a \ell\cos(\phi)\right) \end{bmatrix} \tag{31}$$

Parameter	Interpretation	Value / Randomization Range
g	Gravity	9.81
m	Mass of Quad	1
ℓ	Length of Arm	0.25
$C_{d,v}$	Coefficient of Translational Drag	0.25
$C_{d,\omega}$	Coefficient of Rotational Drag	0.02255
I_{zz}	Moment of Inertia of Quad	$\text{Unif}(0.02, 0.04)$
$T_{\min}$	Minimum Thrust	0
$T_{\max}$	Maximum Thrust	$2mg$
m_a	Added mass	$\text{Unif}(0, 0.5)$
r_a	Position of addded mass	$\text{Unif}(-0.6, 0.6)$
β	Ground effect decay scale	$\text{Unif}(1, 4)$
c_1	Ground effect scale	$\text{Unif}(0.4, 0.6)$
c_2	Ground height offset	$\text{Unif}(0.05, 0.08)$
c_3	Ground height scale	$\text{Unif}(0.15, 0.4)$

Table 2: Quadrotor Parameters and Associated Randomizations. $\text{Unif}(a, b)$ denotes that the parameter is sampled uniformly within the range $[a, b)$.

The realized thrusts T_1 and T_2 are a function of the commanded thrusts $\hat{T}_1, \hat{T}_2$ that accounts for ground effect, based on the model proposed in [42]. This model was augmented with rotational effects, and randomized parameter scaling terms were added, which may capture a variety of dynamic effects. The realized thrusts become

$$T_1 = \gamma(y - \ell \sin(\phi))\hat{T}_1 \tag{32}$$

$$T_2 = \gamma(y + \ell \sin(\phi))\hat{T}_2 \tag{33}$$

where the relative increase in thrust is a function of the height z of the rotor from the ground, and is given by

$$\gamma(z) = 1 + \frac{1}{\cosh(\beta\phi)} \left(\max\left(1, \min\left(\frac{1}{1 - \frac{c_1 R^2}{16(c_3 z + c_2)^2}}, 2 \right)\right) - 1 \right) \tag{34}$$

The commanded actions $u_1, u_2 \in [-1, 1]$ are mapped linearly to commanded thrusts:

$$\hat{T}_j = T_{\min} + \frac{1}{2}(T_{\max} - T_{\min})(u_j + 1) \qquad j = 1, 2 \tag{35}$$

The interpretation and values of the parameters used in these dynamics are given in Table 2.

Cost Function: We use a quadratic cost function centered around the desired goal position, with zero velocities and zero angle:

$$\text{Cost}(\boldsymbol{x}, \boldsymbol{u}) = 0.1(x - x_g)^2 + 0.001\dot{x}^2 + 0.1 * (y - y_g)^2 + 0.001\dot{y}^2 + 0.1\phi^2 + 0.1\dot{\phi}^2 \tag{36}$$

For the comparisons against the baselines in Table 1, we use $x_g = 2$ and $y_g = 0.5$. For the RMSE prediction results in Figure 2, we use $y_g = 3$ to prevent avoid crashes into the floor, as the dynamics models are not trained on transitions from crashed states.

Controller Hyperparameters: For these experiments, we use $H = 30$, and simulate for an episode length of $T = 30$. We also use a terminal cost, in the model predictive control of

$$\text{Cost}_T(\boldsymbol{x}) = (x - x_g)^2 + \dot{x}^2 + (y - y_g)^2 + \dot{y}^2 + \phi^2 + \dot{\phi}^2 \tag{37}$$

As in the Cart-Pole experiments, we use a replan rate of $h = 5$ with the iLQR controller, and a replan rate of $h = 1$ for MPPI. For MPPI, we use 1000 action samples, sampled with mean 0 and standard deviation 0.2.