Causal Inference Using Potential Outcomes

Causal Inference Reading Group

3/20/2025

An example

- We are interested to know the relationship between taking **aspirin** (our treatment of interest T) and **headache severity** (our outcome of interest Y)

“Potential Outcomes Tradition”: Comparing two states of the world

ID	X_i (e.g. Age)	Potential Outcome Y (Severity of headache) STATE 1: if treated with aspirin Y_i^1	STATE 2: if NOT Treated with aspirin Y_i^0	Individual-level Causal effect of Aspirin $Y_i^1 - Y_i^0$		
1	34	100	100	0		
2	22	50	80	-30		
3	56	0	100	-100		
4	73	50	20	30		
5	86	0	0	0		

- Causal Effect of the aspirin is the difference in the severity of headache between the two states of the world

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Other Estimands of interest (i.e., the quantity that you want to estimate):

1. **Average Treatment Effect**: mean of the individual effects: E(**Y_i^1- Y_i^0**) **= -20**

"Potential Outcomes Tradition": Comparing two states of the world

ID	X (e.g. Age)	Potential Outcome Y (Severity of headache)		Individual-level Causal effect of Aspirin $Y_i^1 - Y_i^0$	Actual Aspirin Use	
		STATE 1: if treated with aspirin Y^1	STATE 2: if NOT Treated with aspirin Y^0			
1	34	100	100	0	1	
2	22	50	80	-30	1	
3	56	0	100	-100	0	
4	73	50	20	30	0	
5	86	0	0	0	0	

2. **Average Treatment Effect of the treated (ATT):** $E(Y_i^1 - Y_i^0 \mid \text{treated}) = -15$

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ID	X (e.g. Age)	Potential Outcome Y (Severity of headache)		Individual-level Causal effect of Aspirin $Y_i^1 - Y_i^0$	Actual Aspirin Use	
		STATE 1: if treated with aspirin Y^1	STATE 2: if NOT Treated with aspirin Y^0			
1	34	100	100	0	1	
2	22	50	80	-30	1	
3	56	0	100	-100	0	
4	73	50	20	30	0	
5	86	0	0	0	0	

3. **Average Treatment Effect of the untreated (ATU):** $E(Y_i^1 - Y_i^0 \mid \text{untreated}) = -23.3$

"The Fundamental Problem of Causal Inference" – we do not get to observe both states, one will always be missing

ID	X (e.g. Age)	Potential Outcome Y (Severity of headache)		Individual-level Causal effect of Aspirin $Y_i^1 - Y_i^0$	Actual Aspirin Use	Observed Outcome Y
		STATE 1: if treated with aspirin Y^1	STATE 2: if NOT Treated with aspirin Y^0			
1	34	100	100	0	1	100
2	22	50	80	-30	1	50
3	56	0	100	-100	0	100
4	73	50	20	30	0	20
5	86	0	0	0	0	0

- If we take the mean Y of the treated – mean Y of the untreated: E(Y|treated) - E(Y|untreated) = 35
- **≠ ATE (-20) because there is <u>selection bias</u> (e.g. older people were less likely to take aspirin), and/or <u>heterogeneous treatment effect bias</u>**

The assignment mechanisms

- Methods that assign treatments to individuals/units
- Can be seen as a special type of missing-data mechanism that creates missing potential outcomes
- Can be written as $P(T = 1 \mid X, Y^0, Y^1)$, where T indicates treatment assignment and X represents observed covariates.

Randomization

- Using a random mechanism to assign treatment
- Satisfies the “**Independence Assumption**” – treatment assignment is independent of the individuals’ potential outcomes.
- Randomization eliminates both the **selection bias** and **heterogeneous treatment effect bias**
- **E(Y|treated) - E(Y|untreated) = ATE!**

Non-random assignment and causal assumptions:

1. Exchangeability / unconfoundedness/ ignorability:

- The potential outcomes for individuals in different treatment groups are comparable, and the distribution of potential outcomes is the same across groups.

- After controlling for **observed covariates**, the treatment assignment is independent of the potential outcomes

    - *Example of violation:*
        - Those who had a positive experience with aspirin chose to take aspirin, and we do not have information on the individual’s aspirin preferences or history

Non-random assignment and causal assumptions:

2. Positivity

- Every individual has a non-zero probability of receiving each level of treatment. (Requires that the propensity of treatment assignment is between 0 and 1)

  - *Example of violation:*
    - Those who are at high risk of bleeding will never choose to take an aspirin for their headache (structural positivity violation)
    - Our sample size was small. All younger people took aspirin. (Stochastic violations)

More Causal Assumptions:
SUTVA Stable Unit Treatment Value Assumption

1. Consistency

- The **potential outcome** for an individual under a specific treatment condition is the same as the **observed outcome** when that individual actually receives that treatment

- **Allows us to see one of the potential outcomes** for each individual

- If a patient received aspirin, the outcome measured (severity of headache) should be the same as their potential outcome $\mathbf{Y^1}$

- *Example of violation:*
  - There are multiple versions of aspirin (e.g. different strength and delivery mechanisms), but we lumped them all as “aspirin”. The difference between a 80mb “baby aspirin” and an extra strength 500 mg aspirin may be significant. (A case of **poorly-defined exposure**)

SUTVA (cont'd)

2. No spillovers (No Externalities/ No interference)

- The potential outcome for an individual responds only to their OWN treatment status. Potential outcomes are **invariant to treatment assignment of others.**

- *Example of violation:*
  - In a vaccination study, if one person receives a vaccine, it may affect the probability of another person contracting an infectious disease
  - Seeing a friend's headache goes away after taking an aspirin makes your headache better, too.

SUTVA (cont'd)

3. No general equilibrium effects (Cunningham)

- The impact of a treatment on one individual should not be influenced by how many others are receiving the same treatment or a different treatment.

- *Example of violation:*
  - If a health educational program's effectiveness changes based on the number of students enrolled in the program, then the assumption is violated
  - The self-reported headache pain level changes when 1000 people are taking aspirin as the same time in the same room vs. when there was only 1 person.

Randomization Inference

- Inference based on different possible randomizations of treatment
- **Allows us to make probability calculations revealing whether the data are likely a draw from a truly random distribution or not**
- **Fisher's sharp null:** generate exact p-values for tests of a sharp null hypothesis
- Sharp null hypothesis: the treatment has no effect for any individual
  - H0: $Y_i^1 - Y_i^0 = 0$ for all *i*
  - *If sharp null is true:* $Y_i^1 = Y_i^0 = Y_i$
- Involves shuffling the treatment assignments among the participants to create a distribution of test statistics.
- Compare test statistic between the observed data and for each permutation of the treatment assignments.