10.1.4 Wing Load Distribution

The loads on the wing are made up of aerodynamic lift and drag forces, as well the concentrated or distributed weight of wing-mounted engines, stored fuel, weapons, structural elements, etc. This section will consider these as the first step in designing the internal structure for the wing.

Spanwise Lift Distribution. As a result of the finite aspect ratio of the wing, the lift distribution varies along the span, from a maximum lift at the root, to a minimum lift at the tip. The spanwise lift distribution should be proportional to the shape of the wing planform. It can readily be calculated using a vortex panel method. However, if the wing planform is elliptic in shape, with a local chord distribution, c(y) given as

$$c(y) = \frac{4\,S}{\pi b}\sqrt{1 - \left(\frac{2y}{b}\right)^2}\,, \qquad (10.21)$$

an analytic spanwise lift distribution exists. This is given as

$$L^E(y) = \frac{4\,S}{\pi b}\sqrt{1 - \left(\frac{2y}{b}\right)^2} \qquad (10.22)$$

where L^E is the total lift generated by the wing with an elliptic planform. In both these expressions, y is the spanwise coordinate of the wing, with y = 0 corresponding to the wing root, and y = ±b /2 corresponding to the wing tips. A schematic is shown in Figure 10.6.

The analysis of the elliptic planform wing shows that it results in an elliptic lift distribution in the spanwise direction. This is the basis for a semi-empirical method for estimating the spanwise lift distribution on untwisted wings with general trapezoidal plan-form shapes. The method is attributed to Schrenk (1940) and assumes that the spanwise lift distribution of a general untwisted wing has a shape that is the average between the actual planform chord distribution, c(y), and that of an elliptic wing. In this approach, the area under the spanwise lift distribution for the elliptic or general planform, must equal the total required lift.

For the trapezoidal wing, the local chord length, c(y), varies along the span as

$$c(y) = c_r\left[1 - \frac{2y}{b}\,(1 - \lambda)\right], \qquad (10.23)$$

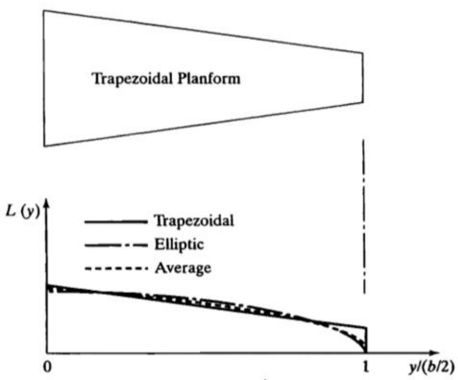

FIGURE 10.6: Schematic representation of two span wise lift distributions for an elliptic and trapezoidal planform shape, and the average of the two lift distributions using Schrenk' s (1940) approximation.

where again, Cr is the root chord length and)... is the taper ratio. Following the elliptic wing, we can take the spanwise lift distribution to vary like the spanwise chord variation. Therefore,

$$L^T(y) = L_r \left[1 - \frac{2y}{b}(1-\lambda)\right], \tag{10.24}$$

where Lr is the local lift value at the location of the wing root (y = 0).

Now the total lift must equal the value found by integrating the lift distribution in the spanwise direction. Therefore,

$$L = \int_{-b/2}^{b/2} L(y)dy = 2\,L_r \int_0^{b/2} \left[1 - \frac{2y}{b}(1-\lambda)\right] dy\,. \tag{10.25}$$

Evaluating the integral, we obtain

$$L = \frac{L_r\, b\,(1+\lambda)}{2}\,. \tag{10.26}$$

With this, we have an expression for Lr, which gives the necessary total lift for the trapezoidal lift distribution, namely,

$$L_r = \frac{2L}{b(1+\lambda)} \tag{10.27}$$

and, therefore,

$$L^T(y) = \frac{2L}{b(1+\lambda)}\left[1 - \frac{2y}{b}(1-\lambda)\right]\,. \tag{10.28}$$

As a check, for a planar wing (λ= 1), L^T (y) = L/b, which is the correct lift per span.

To use Schrenk's method, it is necessary to graph the spanwise lift distributions given in Eq. (10.19) for the elliptic planform and Eq. (10.28] for the trapezoidal planform. In each case, L is the required total lift. The approximated spanwise lift distribution is then the local average of the two distributions, namely,

$$\bar{L}(y) = \frac{1}{2}\left[L^T(y) + L^E(y)\right]. \tag{10.29}$$

An example of this corresponds to the dotted curve in Figure 10.6.

It should be pointed out that Schrenk's method does not provide a suitable estimate of the spanwise lift distribution for highly swept wings. In that instance, a panel method approach or other computational method is necessary.

Added Flap Loads. Leading-edge and trailing-edge flaps enhance the lift over the spanwise extent where they are placed. The lift force is assumed to be uniform in the region of the flaps and to add to the local spanwise lift distribution that is derived for the unflapped wing.

The determination of the added lift force produced by the flaps requires specifying a velocity. For this, the velocity is taken to be twice the stall value, $2V_s$, with flaps down.

Spanwise Drag Distribution. The drag force on the wing varies along the span, with a particular concentration occurring near the wing tips. An approximation that is suitable for the conceptual design is to assume that

1. the drag force is constant from the wing root to 80 percent of the wing span and equal to 95 percent of the total drag on the wing;

2. the drag on the outward 20 percent of the wing is constant and equal to 120 percent of the wing total drag.

In most cases, the wing structure is inherently strong (stiff) in the drag component direction because the relevant length for the bending moment of inertia is the wing chord, which is large compared to the wing thickness. Therefore, the principle bending of the wing occurs in the lift component direction. The design of the internal structure of the wing is then primarily driven by the need to counter the wing-thickness bending moments.

Concentrated and Distributed Wing Weights. Other loads on the wing, besides the aerodynamic loads, are due to concentrated weights, such as wing-mounted engines, weapons, fuel tanks, etc., and due to distributed loads such as the wing structure.

TABLE 10.4: Statistical weights of major components on various types of aircraft.

Component	Multiplier			Factor*
	Combat	Transport/ Bomber	General Aviation	
Main Wing	9.0	10.0	2.5	S_W
Horizontal Tail	4.0	5.5	2.0	S_W
Vertical Tail	5.3	5.5	2.0	S_W
Installed Engine	1.3	1.3	1.4	Uninstalled W_{engine}
Landing Gear	0.033 (Navy, 0.045)	0.043	0.057	W_{TO}
	(Navy, 0.045)			W_{TO}
Fuselage	4.8	5.0	1.4	$S_{\text{fuse-wetted}}$

*Note: Areas have units of f^2 and weights have units of pounds.

Since the structure is being designed at this step, it is difficult to know precisely what the final weight will be. Therefore, historic weight trends for different aircraft are used to make estimates at this stage of the design. A refined weight analysis will be done later as the initial step in determining the static stability coefficients for the aircraft. Table l0.4 gives historic weights for the major components of a range of different aircraft. These include the main wing, horizontal and vertical tails, fuselage, installed engine, and landing gear. The weights of these components are determined from the table as

$$W(lb_f) = \text{Multiplier x Factor}, \qquad (l0.30)$$

where the "multiplier" is a number that corresponds to a general type of aircraft and the "Factor" is a reference portion of the aircraft, such as the wing planform area, S_w, or the fuselage wetted area, $S_{\text{fuse-wetted}}$.

Engines and landing gear mounted on the wing can be treated as concentrated loads. The wing structure will be considered as a distributed load. It is reasonable to consider that the weight of a spanwise section of the wing would scale with the wing chord length, so that with a linear tapered wing, the distributed weight would decrease in proportion to the local chord from the root to the tip.

10.1.5 Shear and Bending Moment Analysis

The wing structure can be considered to be a cantilever beam, which is rigidly supported at the wing root. The critical loads that need to be determined are the shear forces and bending moments along the span of the wing. These take into account the loads on the wing produced by the aerodynamic forces and component weights, which were discussed in the previous section. A generic load arrangement is listed in Table l0.5 and illustrated in Figure 10.7.

To determine the shear force and bending moments along the span, it is useful to divide the wing into spanwise segments of width Δy. A schematic of such an element is shown in Figure l0.8.

TABLE 10.5: Load summary for generic wing which is illustrated in Figure 10.7.

Load Type	Magnitude (lb_f)	$y/(b/2)_{\text{start}}$	$y/(b/2)_{\text{end}}$
Lift (unflapped)	10,000	0	1
Lift (flapped)	5000	0	0.4
Fuel	3000	0	0.4
Engine	3000	0.3	0.3
Structure	4000	0	1

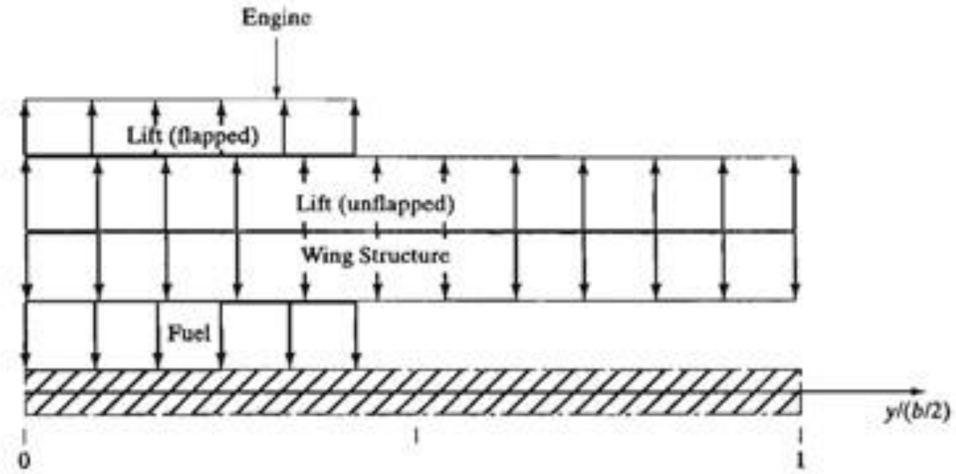

FIGURE 10.7: Schematic representation of spanwise load distribution on a generic trapezoidal wing (λ = 1).

As an example, the element shows a distributed load, W (y). The resultant load acting on the element is then W(y) Δy. In the limit as Δy goes to zero, Δy approaches the differential length, dy, and the resultant load is W(y)dy.

The element shear force, V, is related to the resultant load as

$$W = \frac{dV}{dx} . \tag{10.31}$$

The bending moment, M, acting on the element is related to the shear force by

$$V = \frac{dM}{dy} . \tag{10.32}$$

In integral form,

$$V = \int W \, dy \tag{10.33}$$

and

$$M = \int V \, dy \,. \tag{10.34}$$

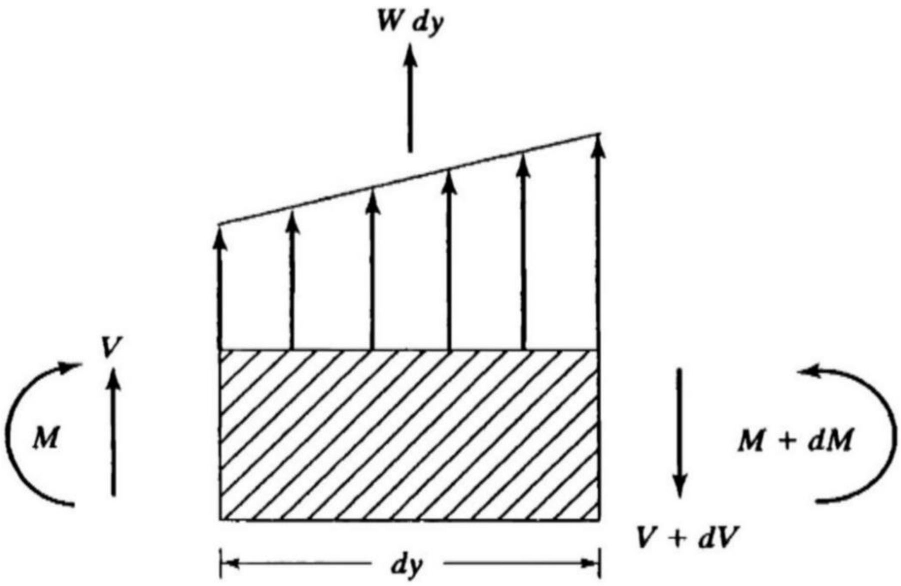

FIGURE 10.8: Schematic representation of shear loads and bending moments on a span wise element of the wing.

These integrals can be approximated by sums, namely,

$$V = \sum_{i}^{N} W_i \, \Delta y \tag{10.35}$$

and

$$M = \sum_{i}^{N} V_i \, \Delta y \quad , \tag{10.36}$$

where N is the number of elements over which the wing span is divided. Of course, the sums approximate the integrals better as the number of elements becomes large; however, a reasonably good estimate for the conceptual design can be obtained with approximately 20 elements over the half-span of the wing.

In order to make these definite integrals, the integration (summation) needs to be started where the shear and moment are known. With the wing, this location is at the wing tip (y = b/2), where V(b/2) = M(b/2) = 0.

Note that in this case, the resultant load on an element is $W_i = W(y)\,\Delta y$ which is the quantity inside the sum in Eq. [10.35]. If the index, i, in Eq. [10.35] indicates the elements along the wing span, with i = 1 signifying the one at the wing tip, then

$$V_1 = 0;$$

$$V_2 = W_1 + W_2;$$

$$V_3 = W_1 + W_2 + W_3 = V_2 + W_3;$$

$$V_4 = V_3 + W_4;$$

$$.$$

$$.$$

$$.$$

$$V_N = V_{N-1} + W_N. \tag{10.37}$$

Note that the shear on element N must equal the sum of the resultant loads on the wing. In reality, there might be a small discrepancy due to the finite number of elements in which the wing span is subdivided. However, with a large enough number of elements (for example 20), the difference should be small.

The bending moment on the wing is given by Eq. [10.36]. For the moments along the wing span, one should also start at the wing tip where the moment on that element is zero. Then following the format in Eq. [10.37],

$$M_1 = 0;$$

$$M_2 = V_1 + \Delta y\, V_2;$$

$$M_3 = V_1 + \Delta y\, V_2 + \Delta y\, V_3 = M_2 + \Delta y\, V_3;$$

$$M_4 = M_3 + \Delta y\, V_4;$$

$$.$$

$$.$$

$$.$$

$$M_N = M_{N-1} + \Delta y\, V_N. \tag{10.38}$$

These formulas provide a good approximation of the distribution of the shear and moment along the span of the wing. An example of their use is given in the spreadsheet that accompanies this chapter.

An example of the application of these equations is shown in Figure 10.9. The loads correspond to those listed in Table 10.5 and illustrated in Figure 10. 7.

The top plot in Figure 10.9 illustrates Schrenk's approximation of the spanwise lift distribution for the finite span wing. The solid curve corresponds to the lift distribution for the trapezoidal wing. This is constant along the span because the taper ratio (λ) in this example is 1. (See Eq. [10.28].) The long-dashed curve corresponds to the spanwise lift distribution for an equivalent elliptic planform airfoil given by Eq. [10.22]. The short-dashed curve is the average of the trapezoidal and elliptic distributions given by Eq. [10.29]. This is the lift distribution that is used in evaluating the shear and moment distribution for the wing.

The total spanwise load distribution, W, shown in Figure 10.9 for a generic wing includes all of the weight and lift components. For this, the wing was divided into 20 spanwise elements. The sharp negative spike in the load distribution marks the location of the engine. The more gradual dip in the loads near y/(b/2) = 0.4 corresponds to the outboard edge of the flaps.

The span wise distribution of the shear load, V, comes from Eq. [10.37]. This shows that the largest shear is at the wing root, with the second largest shear being at the location of the engine. The moment distribution, M, in Figure 10.9 is based on Eq. [10.38]. It reflects the wing cantilever structure, whereby the largest moment is at the wing root The small peak in the moment distribution near y / (b /2) is due to the engine.

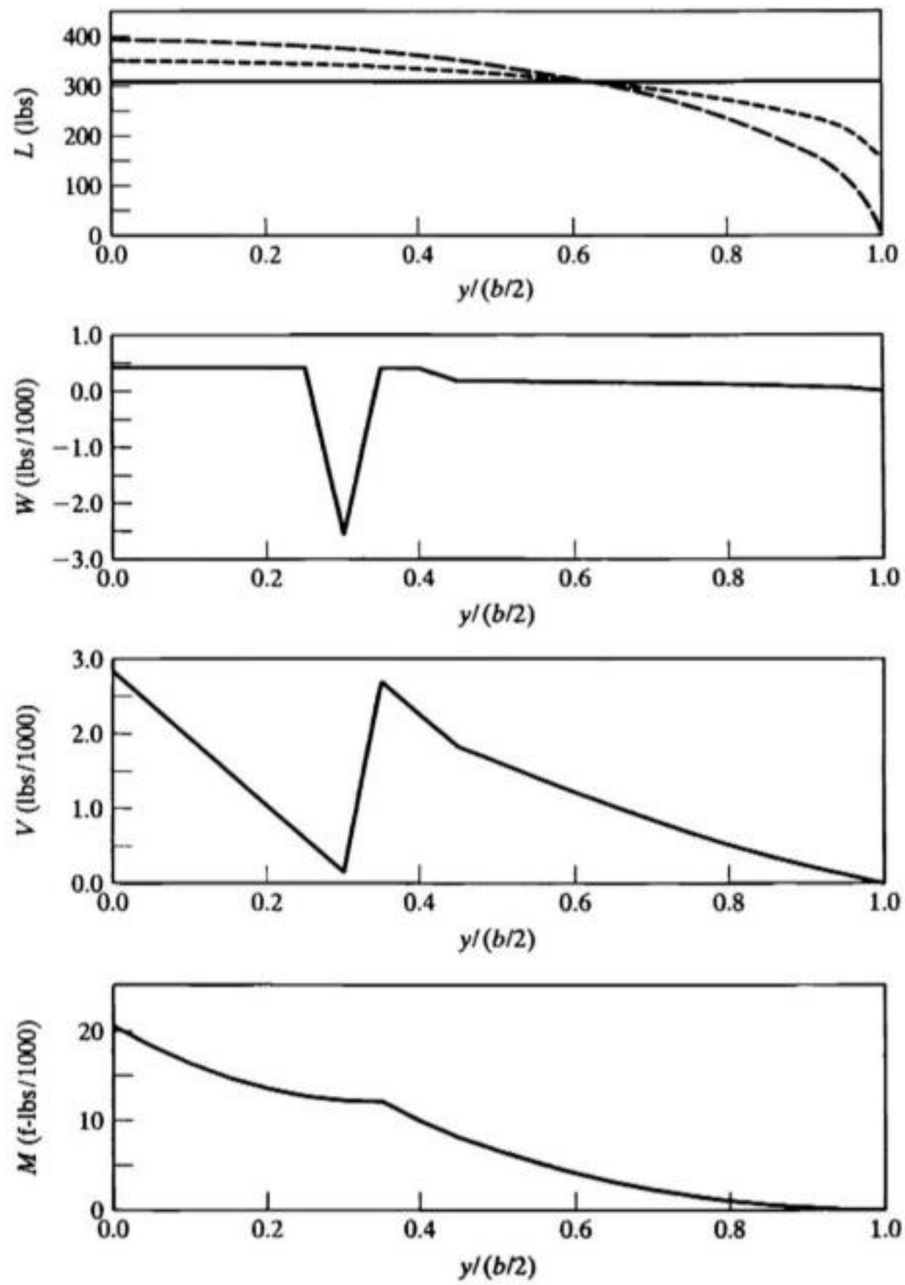

Figure 10.9: Spanwise distributions of lift force, L; weight, W; shear loads, V; and bending moment, M, for the load distribution listed in Table 10.5 and lustrated in Figure 10.7.