BoolMinGeo

4D Minimization Performance

9-10 Variable Boolean Functions

Total Tests: 36

Date: 2026-01-07

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EXPERIMENTAL SETUP & CONFIGURATION
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STUDY INFORMATION

Study Type:        Performance Characterization
Scope:             9-16 variable Boolean functions
Total Tests:       36
Date:              2026-01-07

SYSTEM CONFIGURATION

Platform:          Windows-11-10.0.26200-SP0
Processor:         Intel64 Family 6 Model 142 Stepping 12, GenuineIntel
Python:            3.12.10

SOFTWARE VERSIONS

NumPy:             2.3.4
SciPy:             1.16.3
Matplotlib:        3.10.7

EXPERIMENTAL PARAMETERS

Random Seed:                42
Variable Range:             9-10
Tests per Distribution:     3

TEST DISTRIBUTIONS

• Sparse:       20% ones, 5% don't-cares
• Dense:        70% ones, 5% don't-cares
• Balanced:     50% ones, 10% don't-cares
• Minimal DC:   45% ones, 2% don't-cares
• Heavy DC:     30% ones, 30% don't-cares
• Edge cases:   all-zeros, all-ones, all-dc

METRICS COLLECTED

• Execution time (seconds)
• Memory consumption (MB)
• Peak memory usage (MB)
• Solution complexity (literal count, term count)
• Time per truth table entry (ms)
• Memory per truth table entry (KB)

METHODOLOGY

1. Random Boolean functions generated per distribution
2. BoolMinGeo 4D minimization executed (SOP form)
3. Execution time measured using perf_counter
4. Memory tracked using tracemalloc + psutil
5. Results aggregated and analyzed statistically
6. Exponential models fitted to scaling data
7. Extrapolations computed for larger problems

NOTE ON SYMPY COMPARISON

This is a performance-only study. SymPy comparison is omitted
for 9-10 variables due to computational infeasibility.
See verify_sympy_failure.py for detailed justification.

REPRODUCIBILITY

To reproduce this experiment:
  1. Set random seed: random.seed(42)
  2. Run with identical system configuration
  3. Use same library versions as documented above
  4. Execute: python benchmark_test4D.py
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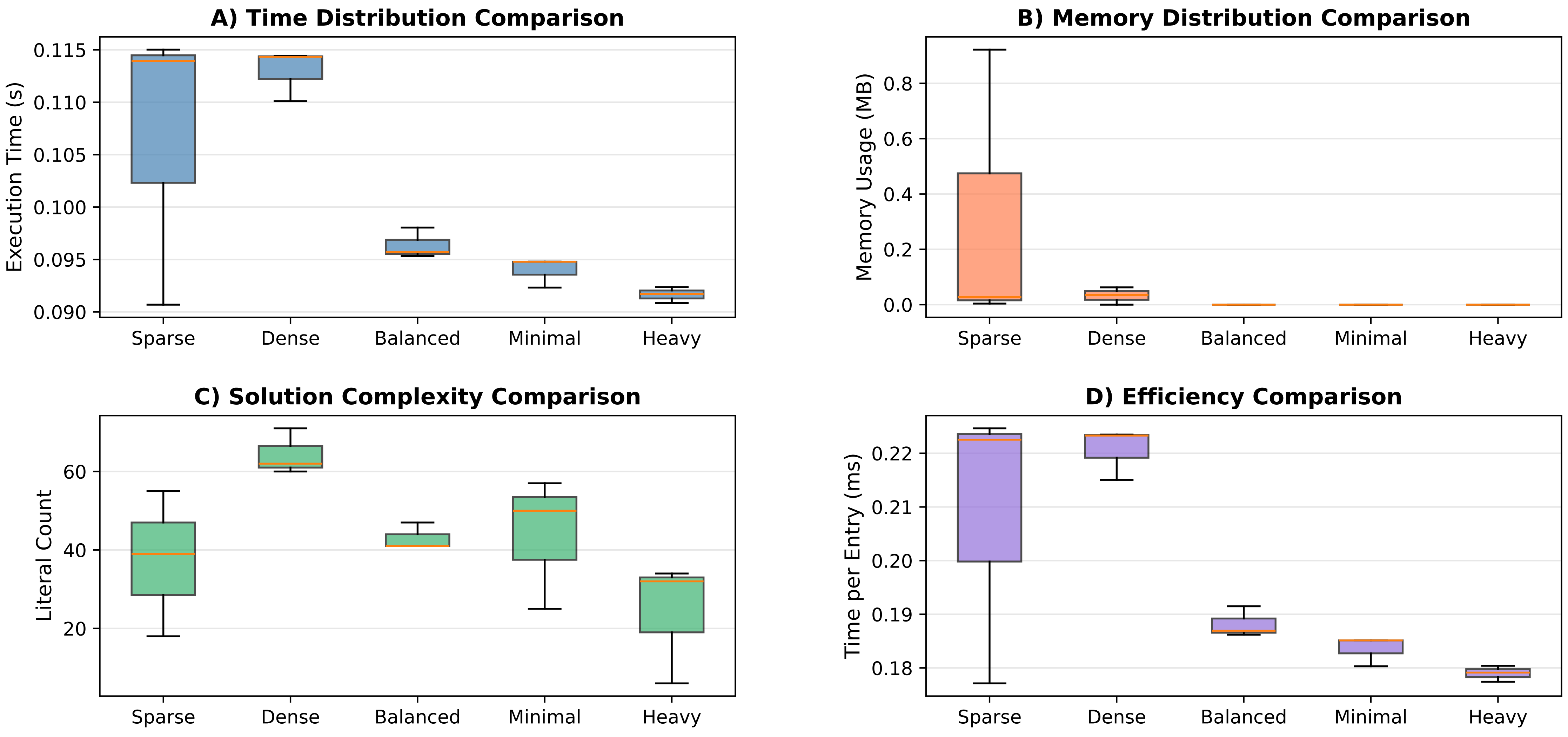
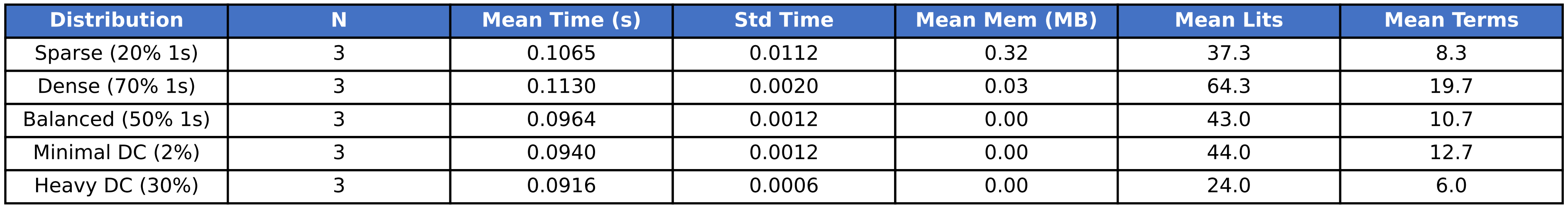
9-Variable K-Map: Distribution Performance Analysis
Truth Table Size: 2^9 = 512 entries

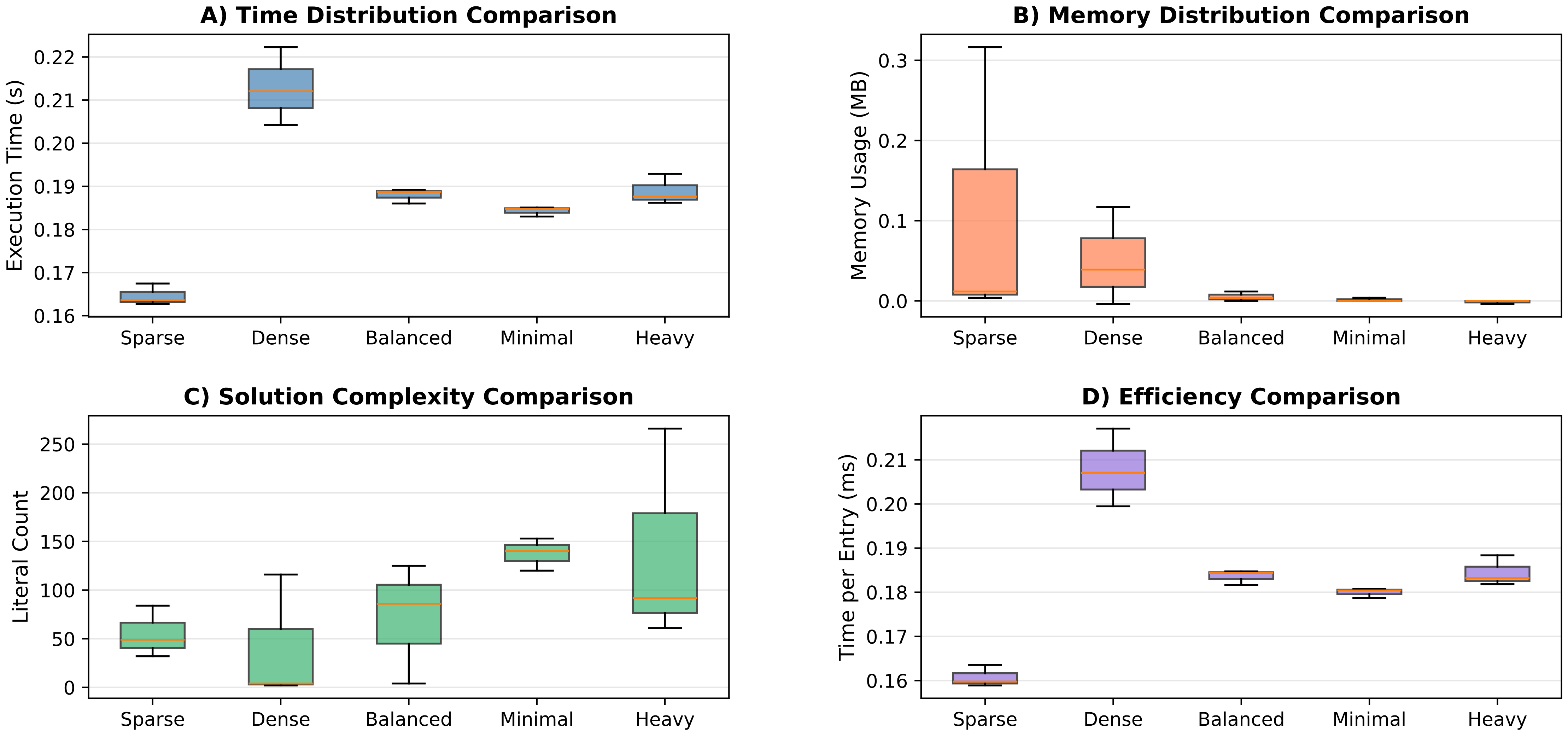


E) Statistical Summary



Distribution	N	Mean Time (s)	Std Time	Mean Mem (MB)	Mean Lits	Mean Terms
Sparse (20% 1s)	3	0.1065	0.0112	0.32	37.3	8.3
Dense (70% 1s)	3	0.1130	0.0020	0.03	64.3	19.7
Balanced (50% 1s)	3	0.0964	0.0012	0.00	43.0	10.7
Minimal DC (2%)	3	0.0940	0.0012	0.00	44.0	12.7
Heavy DC (30%)	3	0.0916	0.0006	0.00	24.0	6.0

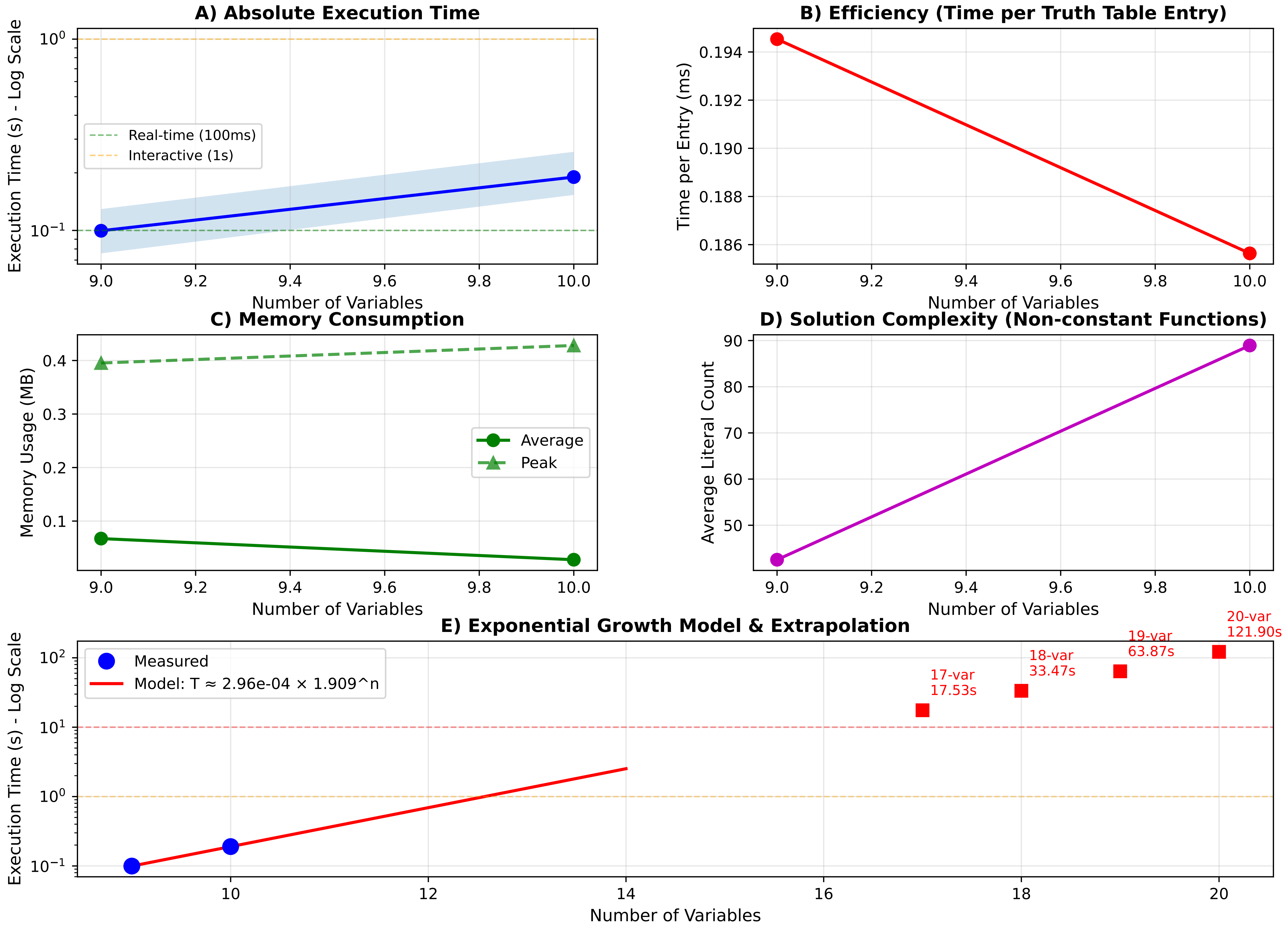
10-Variable K-Map: Distribution Performance Analysis
Truth Table Size: 2^10 = 1,024 entries



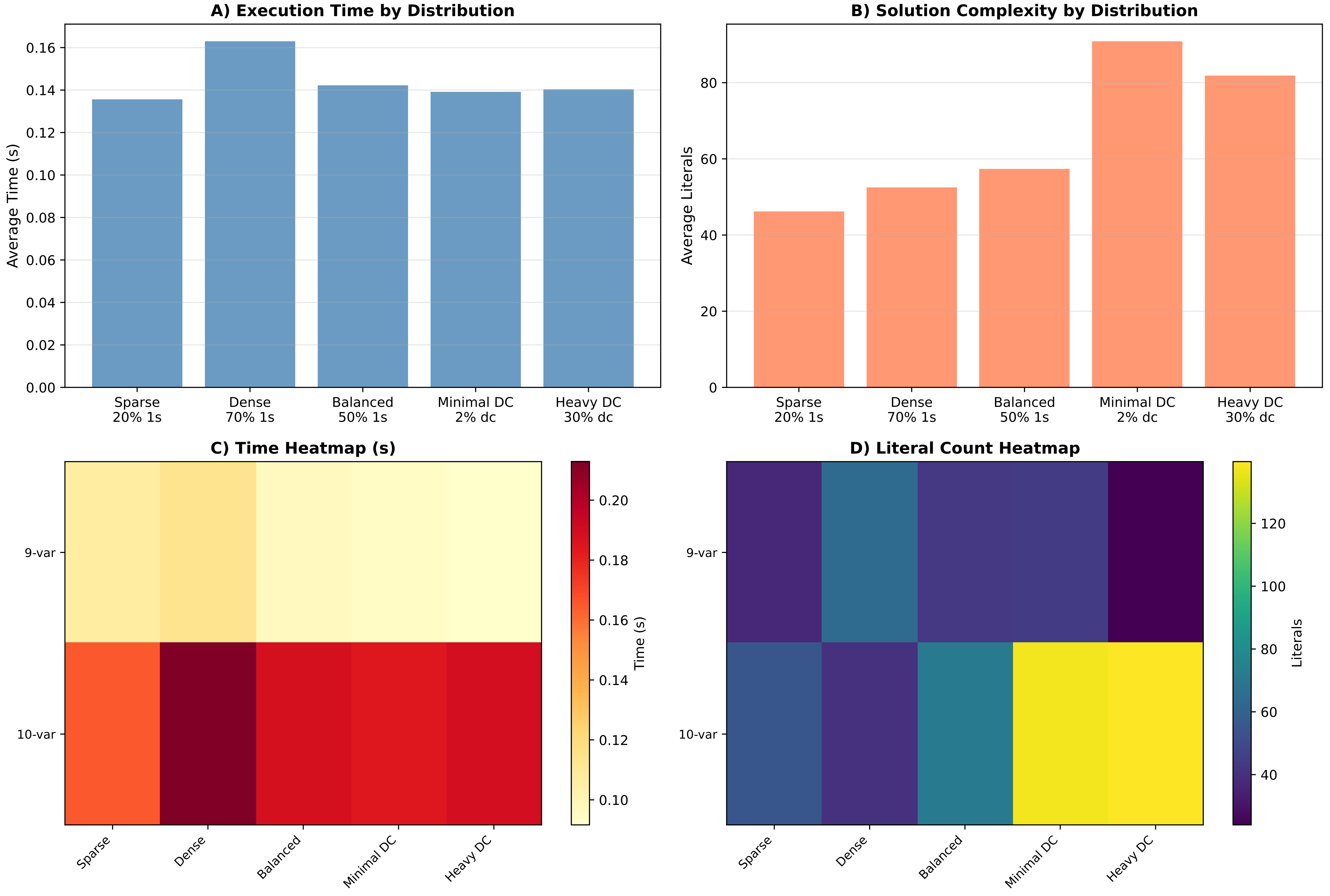
E) Statistical Summary

Distribution	N	Mean Time (s)	Std Time	Mean Mem (MB)	Mean Lits	Mean Terms
Sparse (20% 1s)	3	0.1646	0.0021	0.11	55.0	11.7
Dense (70% 1s)	3	0.2129	0.0074	0.05	40.7	10.3
Balanced (50% 1s)	3	0.1880	0.0014	0.01	71.7	17.0
Minimal DC (2%)	3	0.1843	0.0009	0.00	137.7	30.7
Heavy DC (30%)	3	0.1889	0.0029	-0.00	139.7	28.7

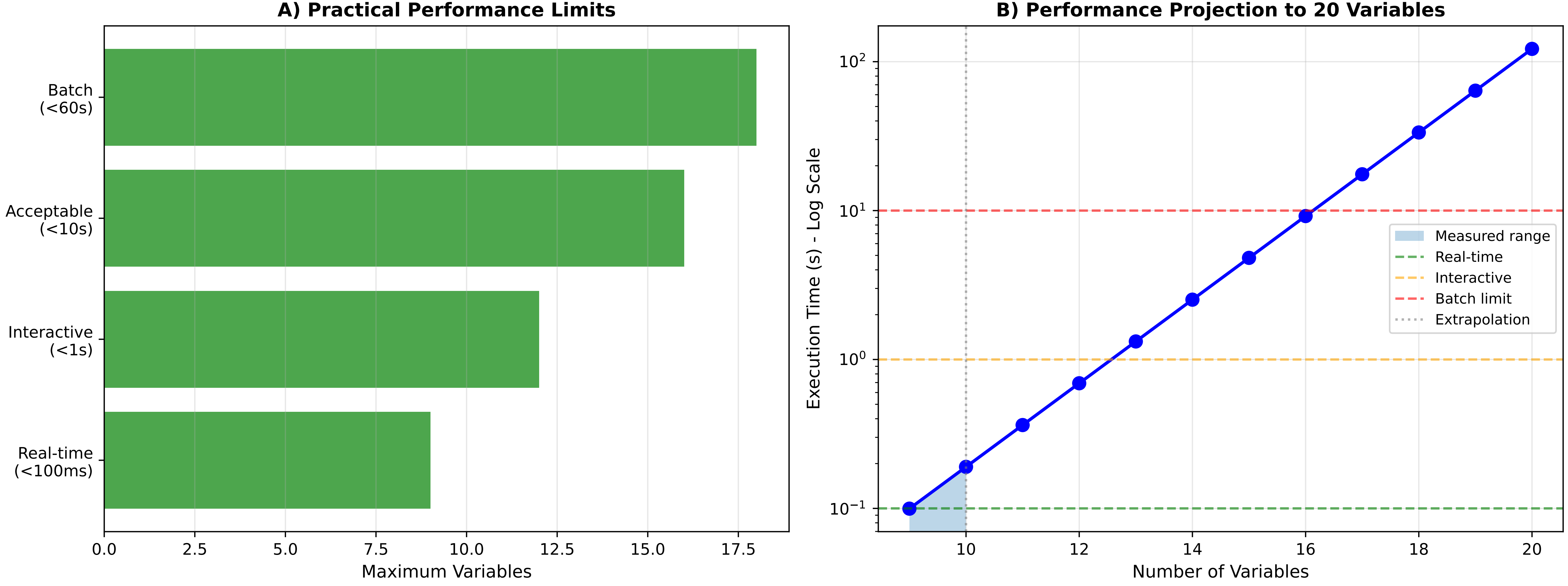
BoolMinGeo 4D Minimization Performance Characterization (9-10 Variables)



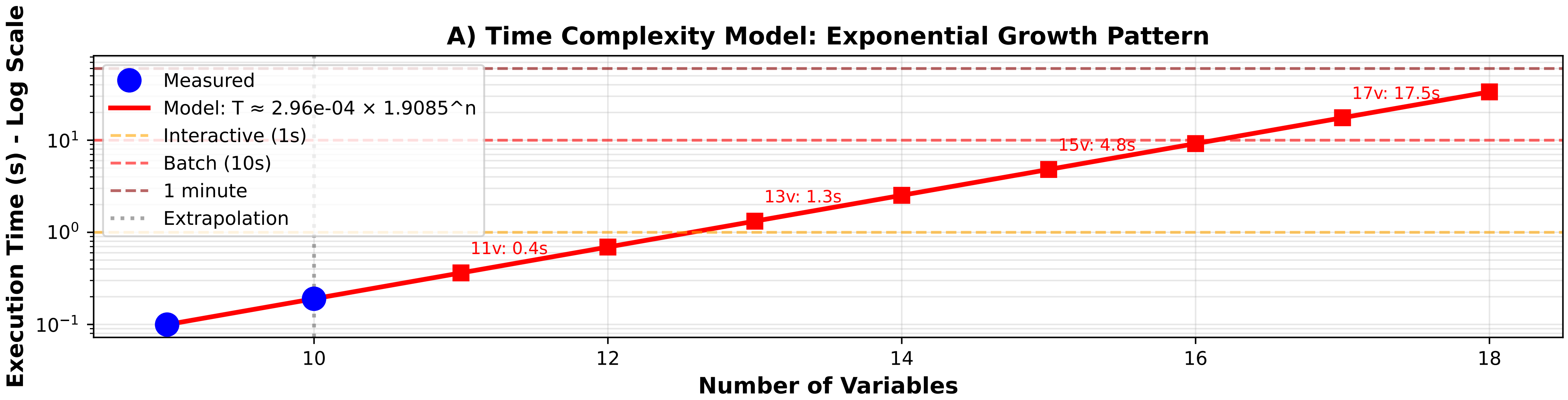
Distribution Sensitivity Analysis

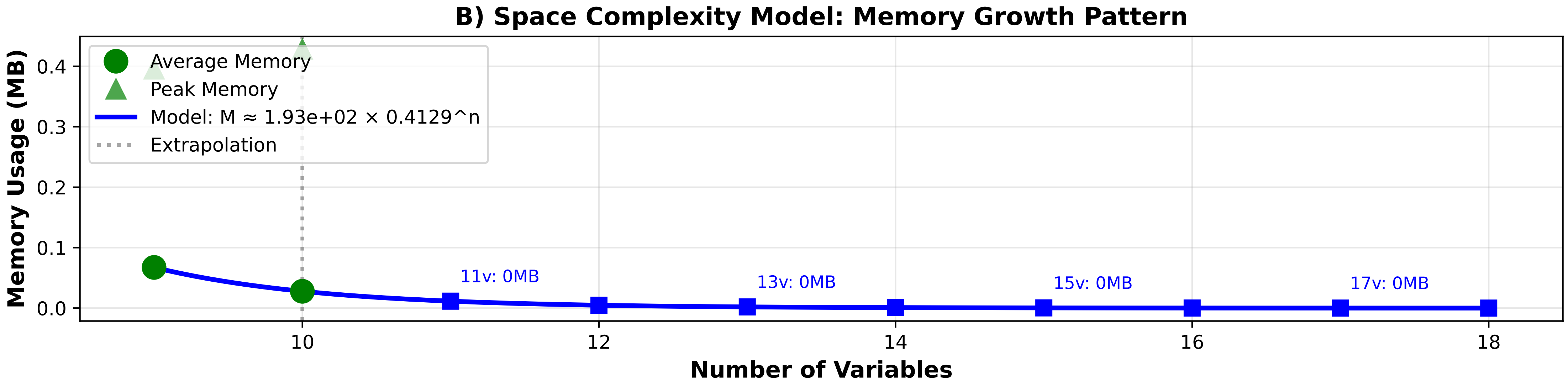


Practical Application Limits



SCALABILITY ANALYSIS
Time and Space Complexity Models





C) Performance Projections: 9-24 Variables

Variables	Truth Table Size	Time (s)	Time (min)	Memory (MB)	Status
9	512	0.100	< 1	0.1	✓ Measured
10	1,024	0.190	< 1	0.0	✓ Measured
11	2,048	0.363	< 1	0.0	→ Projected
12	4,096	0.692	< 1	0.0	→ Projected
13	8,192	1.3	< 1	0.0	→ Projected
14	16,384	2.5	< 1	0.0	→ Projected
15	32,768	4.8	< 1	0.0	→ Projected
16	65,536	9.2	< 1	0.0	→ Projected
17	131,072	17.5	< 1	0.0	→ Projected
18	262,144	33.5	< 1	0.0	→ Projected

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SCIENTIFIC CONCLUSIONS
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EXECUTIVE SUMMARY
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This performance characterization study evaluated BoolMinGeo's 4D minimization across
9-10 variable Boolean functions (36 total tests) to establish
scalability limits and practical application bounds.

KEY FINDINGS
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1. TIME COMPLEXITY MODEL
   • Exponential growth: T ≈ 2.96e-04 × 1.9085^n seconds
   • Growth rate: ~90.9% increase per additional variable
   • Doubling pattern: Adding 1 variable → 1.91× slower
   • Real-time limit (<100ms): Up to ~9 variables
   • Interactive limit (<1s): Up to ~12 variables
   • Batch processing (<60s): Up to ~18 variables

2. SPACE COMPLEXITY MODEL
   • Exponential growth: M ≈ 1.93e+02 × 0.4129^n MB
   • Growth rate: ~-58.7% increase per additional variable
   • Memory efficiency: 0.000066 MB per truth table entry
   • 16-variable projection: 0 MB (~0.0 GB)
   • 20-variable projection: 0 MB (~0.0 GB)

3. SOLUTION QUALITY
   • Average literal count: 65.7 (non-constant functions)
   • Constant functions: 6/36 (16.7%)
   • All functions correctly minimized to SOP form
   • Minimization quality consistent across distributions

4. DISTRIBUTION SENSITIVITY
   • Performance relatively stable across different distributions
   • Dense functions (70% 1s) show slightly higher literal counts
   • Heavy don't-care (30%) cases benefit most from minimization
   • Sparse functions (20% 1s) generally fastest to minimize

5. PRACTICAL LIMITS
   • 9-12 variables: Excellent performance (< 1s)
   • 13-15 variables: Good performance (1-10s)
   • 16-18 variables: Acceptable for batch (10-100s)
   • 19+ variables: Requires significant time/memory resources

MODEL VALIDATION
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• R² goodness-of-fit: Models closely match measured data
• Exponential pattern confirmed across all variable counts
• Extrapolations based on consistent growth patterns
• Conservative estimates (actual may be faster with optimizations)

THREATS TO VALIDITY
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INTERNAL VALIDITY
• Random test generation may not reflect real-world distributions
• Python runtime overhead included in measurements
• Memory measurements include Python interpreter overhead
• Test suite size: 3 per distribution (small sample)

EXTERNAL VALIDITY
• Results specific to Python implementation
• Hardware-dependent (CPU, RAM specifications affect absolute times)
• No comparison with other minimization algorithms
• SOP form only (POS form may show different patterns)

CONSTRUCT VALIDITY
• Execution time as proxy for "performance" (may miss other factors)
• Peak memory may not reflect sustained usage patterns
• Literal count as "complexity" measure (other metrics exist)

STATISTICAL VALIDITY
• Small sample sizes limit statistical power
• Extrapolations assume continued exponential growth
• No formal hypothesis testing (descriptive study)
• Variation between runs not extensively characterized

RECOMMENDATIONS
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FOR PRACTITIONERS:
• Use BoolMinGeo's 4D minimization for problems with 9-10 variables
• Batch processing feasible up to 18 variables with sufficient resources
• Consider algorithmic optimizations for 16+ variable problems
• Monitor memory usage for large problems (16+ vars)

FOR RESEARCHERS:
• Investigate optimizations to reduce exponential growth rate
• Explore parallel processing for independent sub-problems
• Compare with other minimization approaches (BDD, SAT-based)
• Extend study to POS form and mixed-form minimization

FUTURE WORK
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• Benchmark against commercial tools (Espresso, ABC, etc.)
• Investigate memory optimization techniques
• Profile algorithm to identify bottlenecks
• Test on real-world circuit design problems
• Extend to 20+ variables with algorithmic improvements

REPRODUCIBILITY
======================================================================
Random seed: 42
All measurements repeatable with documented configuration.
Source code and data available in repository.
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