Scientific Benchmark Report

BoolMin2D vs SymPy Boolean Minimization

Experiment Date: 2026-01-09

Random Seed: 42

Total Test Cases: 3030

Statistical Significance Level: α = 0.05

A Rigorous Statistical Analysis with Reproducibility Controls

EXPERIMENTAL SETUP

SYSTEM CONFIGURATION

```
Python Version:   3.12.10
Platform:         Windows-11-10.0.26200-SP0
Processor:        Intel64 Family 6 Model 142 Stepping 12, GenuineIntel
```

LIBRARY VERSIONS

```
SymPy:            1.14.0
NumPy:            2.3.4
SciPy:            1.16.3
```

EXPERIMENTAL PARAMETERS

```
Random Seed:                 42
Tests per Configuration:     500
Timing Warm-up Runs:         2
Timing Repetitions:          5
Significance Level (α):      0.05
```

TEST DISTRIBUTIONS

- Sparse: 20% ones, 5% don't-cares (realistic digital logic)
- Dense: 70% ones, 5% don't-cares (stress test)
- Balanced: 50% ones, 10% don't-cares (neutral case)
- Minimal DC: 45% ones, 2% don't-cares (typical circuits)
- Heavy DC: 30% ones, 30% don't-cares (optimization test)
- Edge Cases: All-zeros, all-ones, checkerboard, single-minterm

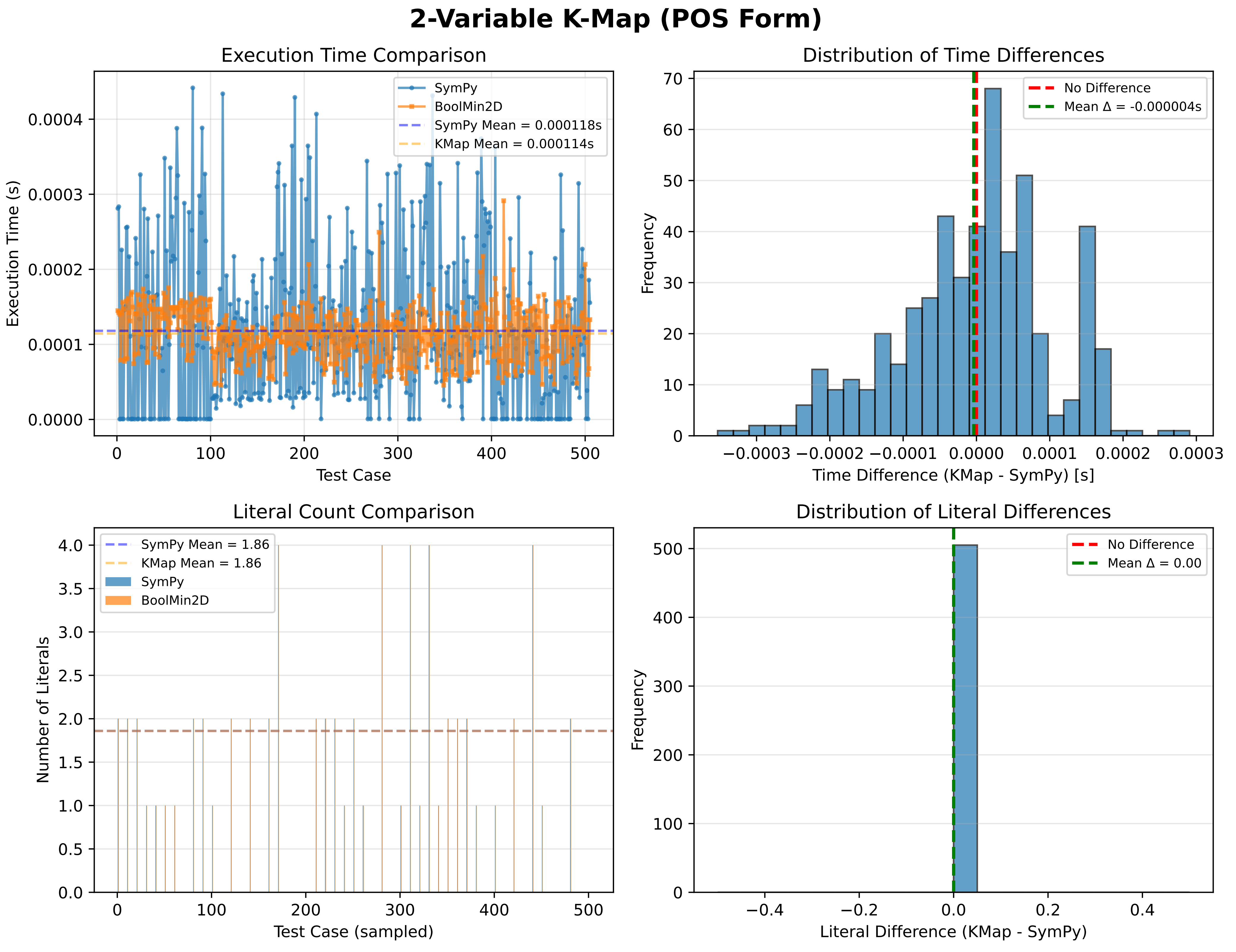
METHODOLOGY

1. Random K-maps generated with controlled distributions
2. Each algorithm executed with 2 warm-up runs
3. Best of 5 timed repetitions recorded
4. Logical equivalence verified using SymPy
5. Statistical significance tested using paired t-tests
6. Non-parametric Wilcoxon tests used as robustness check
7. Effect sizes computed using Cohen's d

REPRODUCIBILITY

To reproduce this experiment:

1. Set random seed: random.seed(42)
2. Run with identical system configuration
3. Use same library versions as documented above



2-Variable K-Map (POS Form)
Execution Time Comparison
SymPy
BoolMin2D
SymPy Mean = 0.000118s
KMap Mean = 0.000114s
Execution Time (s)
Test Case
Distribution of Time Differences
No Difference
Mean Δ = -0.000004s
Frequency
Time Difference (KMap - SymPy) [s]
Literal Count Comparison
SymPy Mean = 1.86
KMap Mean = 1.86
SymPy
BoolMin2D
Number of Literals
Test Case (sampled)
Distribution of Literal Differences
No Difference
Mean Δ = 0.00
Frequency
Literal Difference (KMap - SymPy)

STATISTICAL ANALYSIS: 2-Variable POS

==

STATISTICAL INFERENCE REPORT

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⚠️ CONSTANT FUNCTIONS DETECTED: 156/505 (30.9%)
These are unsimplifiable functions (e.g., all-zeros, all-ones) that both algorithms correctly identified. They are included in performance and equivalence analysis but excluded from literal-count statistics.

1. EXECUTION TIME ANALYSIS

```
Mean SymPy Time:      0.000118 s
Mean BoolMin2D Time:  0.000114 s
Mean Difference:      -0.000004 s
Std. Dev. (Δ):        0.000106 s
95% CI:               [-0.000013, 0.000006]

Paired t-test:        t = -0.7859, p = 0.432313
Wilcoxon test:        W = 63373.0, p = 0.876590
Effect Size (d):      -0.0350 (negligible)
```

✗ NOT SIGNIFICANT: No statistically significant time difference (p ≥ 0.05)
→ Both algorithms exhibit comparable performance

2. SIMPLIFICATION QUALITY ANALYSIS

Analysis based on 349 non-constant functions:
(156 constant function(s) excluded from this analysis)

```
Mean SymPy Literals:  1.86
Mean KMap Literals:   1.86
Mean Difference:      +0.00
Std. Dev. (Δ):        0.00
95% CI:               [nan, nan]

Paired t-test:        t = nan, p = nan
Wilcoxon test:        W = 30537.5, p = 1.000000
Effect Size (d):      0.0000 (negligible)
```

✗ NOT SIGNIFICANT: No significant difference in simplification (p ≥ 0.05)
→ Both algorithms achieve comparable minimization

3. OVERALL SCIENTIFIC CONCLUSION

No statistically significant differences detected in either metric.

Effect sizes: Time (negligible), Literals (negligible)

Note: 156 constant function(s) correctly handled by both algorithms.

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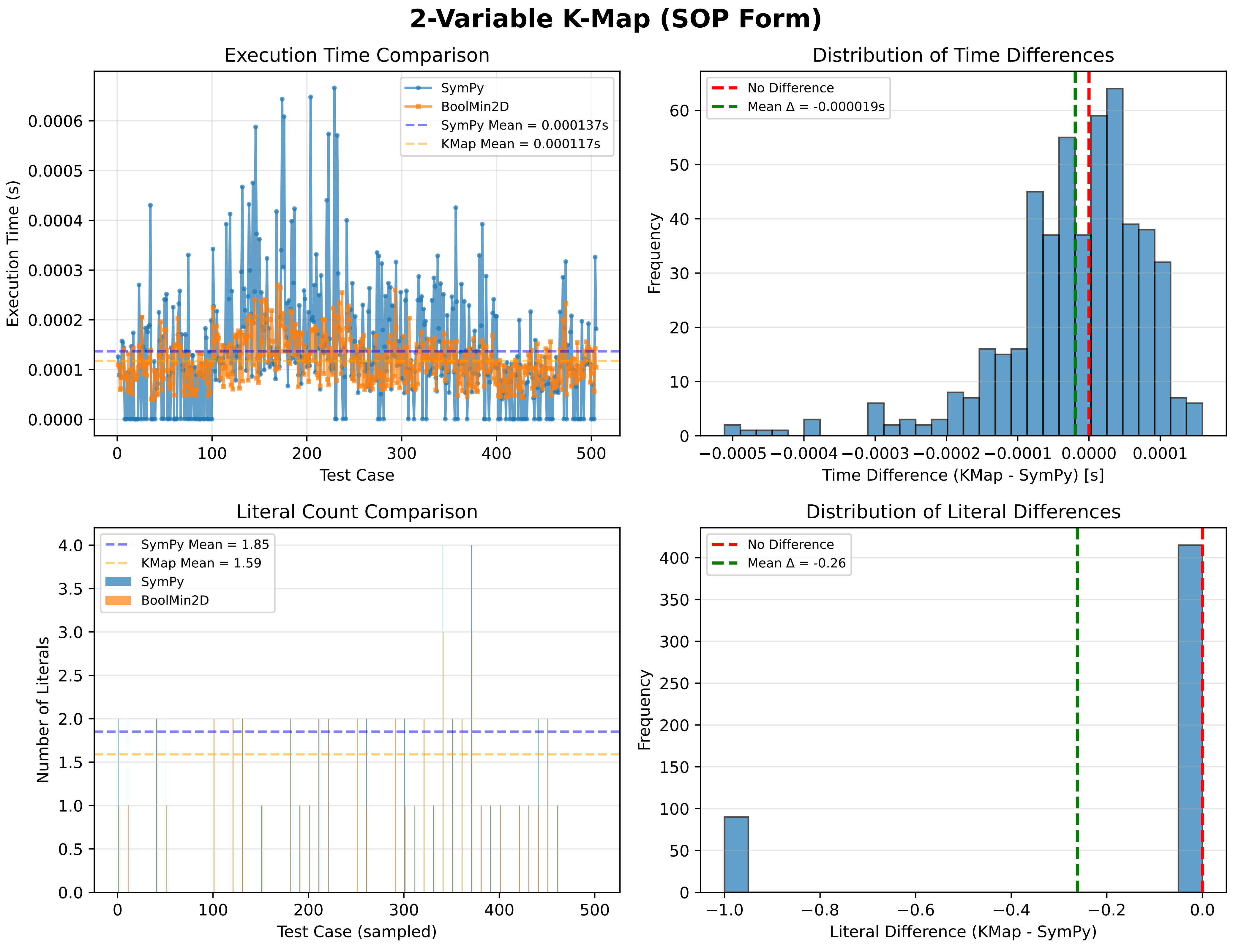

2-Variable K-Map (SOP Form)
Execution Time Comparison
SymPy
BoolMin2D
SymPy Mean = 0.000137s
KMap Mean = 0.000117s
Execution Time (s)
Test Case
Distribution of Time Differences
No Difference
Mean Δ = -0.000019s
Frequency
Time Difference (KMap - SymPy) [s]
Literal Count Comparison
SymPy Mean = 1.85
KMap Mean = 1.59
SymPy
BoolMin2D
Number of Literals
Test Case (sampled)
Distribution of Literal Differences
No Difference
Mean Δ = -0.26
Frequency
Literal Difference (KMap - SymPy)

STATISTICAL ANALYSIS: 2-Variable SOP

```
================================================================================

STATISTICAL INFERENCE REPORT

================================================================================
```

⚠️ CONSTANT FUNCTIONS DETECTED: 161/505 (31.9%)

These are unsimplifiable functions (e.g., all-zeros, all-ones) that both algorithms correctly identified. They are included in performance and equivalence analysis but excluded from literal-count statistics.

1. EXECUTION TIME ANALYSIS

```
--------------------------------------------------------------------------------
Mean SymPy Time:      0.000137 s
Mean BoolMin2D Time:  0.000117 s
Mean Difference:      -0.000019 s
Std. Dev. (Δ):        0.000101 s
95% CI:               [-0.000028, -0.000010]

Paired t-test:        t = -4.2816, p = 0.000022
Wilcoxon test:        W = 56635.0, p = 0.027174
Effect Size (d):      -0.1905 (negligible)
```

✓ SIGNIFICANT: Time difference is statistically significant (p < 0.05)

→ BoolMin2D is significantly faster than SymPy

2. SIMPLIFICATION QUALITY ANALYSIS

```
--------------------------------------------------------------------------------
Analysis based on 344 non-constant functions:
(161 constant function(s) excluded from this analysis)

Mean SymPy Literals:  1.85
Mean KMap Literals:   1.59
Mean Difference:      -0.26
Std. Dev. (Δ):        0.44
95% CI:               [-0.31, -0.21]

Paired t-test:        t = -11.0243, p = 0.000000
Wilcoxon test:        W = 16192.5, p = 0.000000
Effect Size (d):      -0.5944 (medium)
```

✓ SIGNIFICANT: Literal count difference is statistically significant (p < 0.05)

→ BoolMin2D produces more minimal expressions

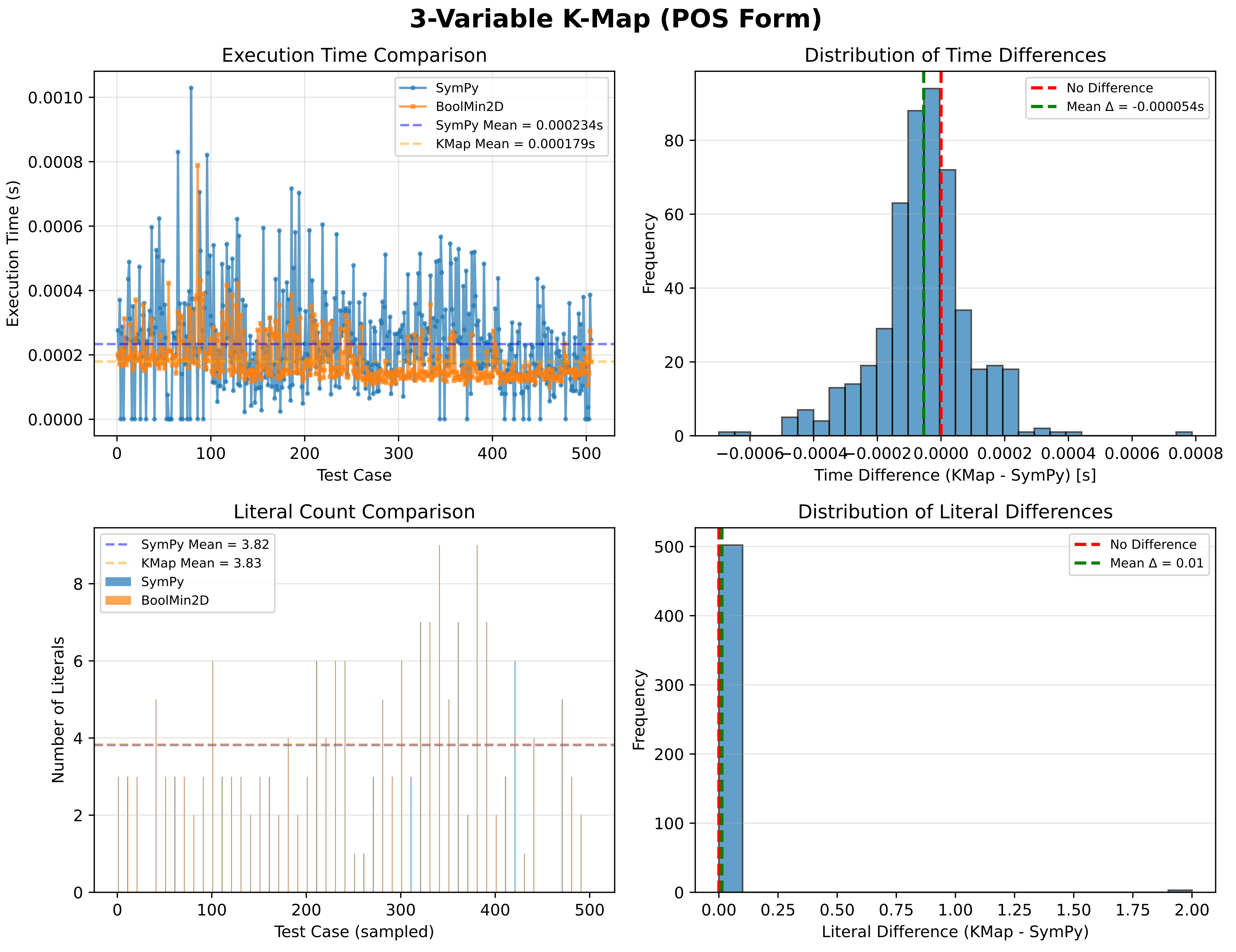
3. OVERALL SCIENTIFIC CONCLUSION

```
--------------------------------------------------------------------------------
Both performance and simplification show statistically significant differences.

Effect sizes: Time (negligible), Literals (medium)

Note: 161 constant function(s) correctly handled by both algorithms.

================================================================================
```


3-Variable K-Map (POS Form)
Execution Time Comparison
SymPy
BoolMin2D
SymPy Mean = 0.000234s
KMap Mean = 0.000179s
Execution Time (s)
Test Case
Distribution of Time Differences
No Difference
Mean Δ = -0.000054s
Frequency
Time Difference (KMap - SymPy) [s]
Literal Count Comparison
SymPy Mean = 3.82
KMap Mean = 3.83
Number of Literals
Test Case (sampled)
Distribution of Literal Differences
Mean Δ = 0.01
Literal Difference (KMap - SymPy)

STATISTICAL ANALYSIS: 3-Variable POS

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STATISTICAL INFERENCE REPORT

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⚠️ CONSTANT FUNCTIONS DETECTED: 41/505 (8.1%)
These are unsimplifiable functions (e.g., all-zeros, all-ones) that both algorithms correctly identified. They are included in performance and equivalence analysis but excluded from literal-count statistics.

1. EXECUTION TIME ANALYSIS

```
Mean SymPy Time:      0.000234 s
Mean BoolMin2D Time:  0.000179 s
Mean Difference:      -0.000054 s
Std. Dev. (Δ):        0.000150 s
95% CI:               [-0.000068, -0.000041]

Paired t-test:        t = -8.1623, p = 0.000000
Wilcoxon test:        W = 36036.5, p = 0.000000
Effect Size (d):      -0.3632 (small)
```

✓ SIGNIFICANT: Time difference is statistically significant (p < 0.05)
→ BoolMin2D is significantly faster than SymPy

2. SIMPLIFICATION QUALITY ANALYSIS

Analysis based on 464 non-constant functions:
(41 constant function(s) excluded from this analysis)

```
Mean SymPy Literals:  3.82
Mean KMap Literals:   3.83
Mean Difference:      +0.01
Std. Dev. (Δ):        0.16
95% CI:               [-0.00, 0.03]

Paired t-test:        t = 1.7358, p = 0.083264
Wilcoxon test:        W = 53245.5, p = 0.782193
Effect Size (d):      0.0806 (negligible)
```

✗ NOT SIGNIFICANT: No significant difference in simplification (p ≥ 0.05)
→ Both algorithms achieve comparable minimization

3. OVERALL SCIENTIFIC CONCLUSION

Performance difference is significant, but simplification is comparable.

Effect sizes: Time (small), Literals (negligible)

Note: 41 constant function(s) correctly handled by both algorithms.

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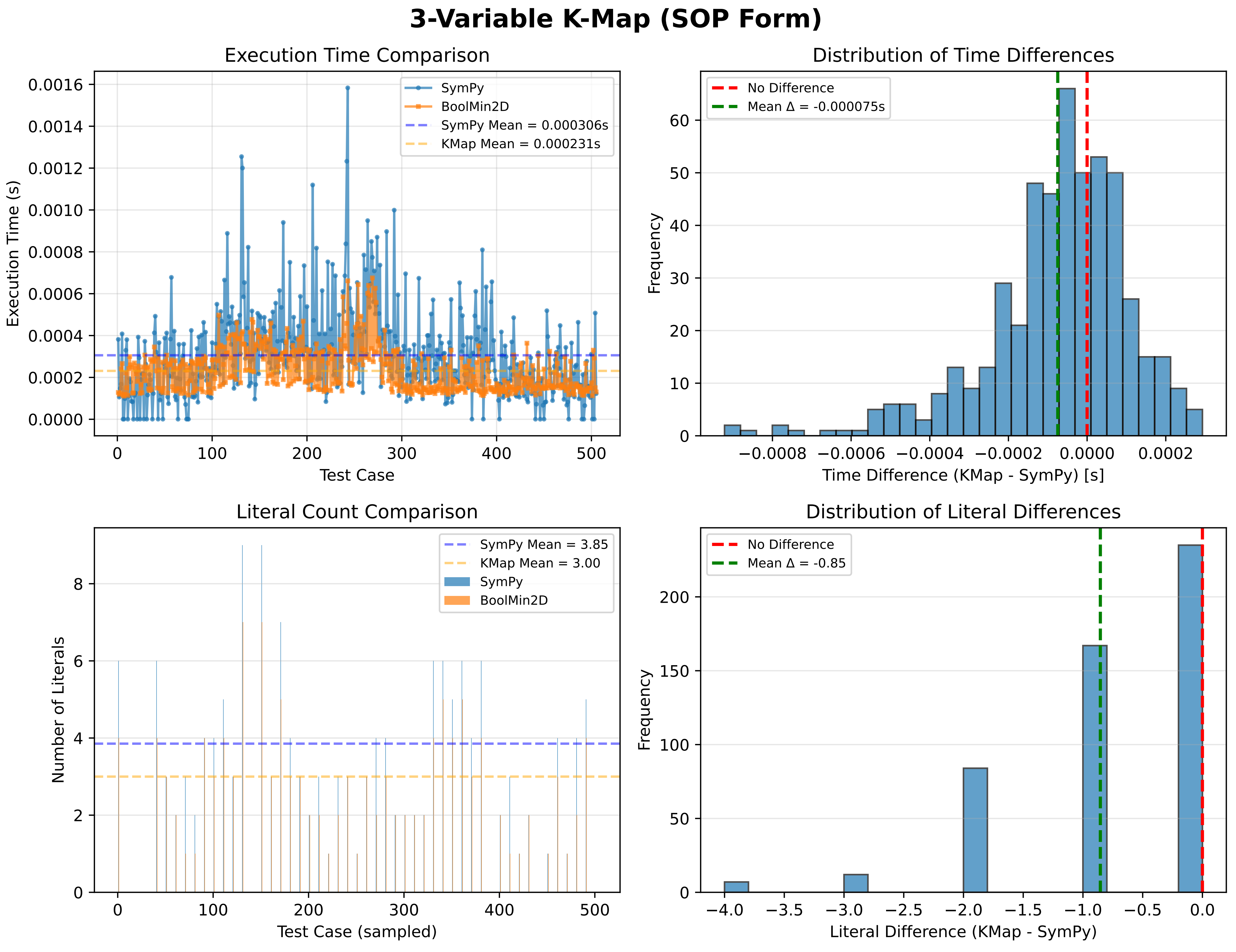

3-Variable K-Map (SOP Form)
Execution Time Comparison
SymPy
BoolMin2D
SymPy Mean = 0.000306s
KMap Mean = 0.000231s
Execution Time (s)
Test Case
Distribution of Time Differences
No Difference
Mean Δ = -0.000075s
Frequency
Time Difference (KMap - SymPy) [s]
Literal Count Comparison
SymPy Mean = 3.85
KMap Mean = 3.00
SymPy
BoolMin2D
Number of Literals
Test Case (sampled)
Distribution of Literal Differences
No Difference
Mean Δ = -0.85
Frequency
Literal Difference (KMap - SymPy)

STATISTICAL ANALYSIS: 3-Variable SOP

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STATISTICAL INFERENCE REPORT

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□□ CONSTANT FUNCTIONS DETECTED: 38/505 (7.5%)

These are unsimplifiable functions (e.g., all-zeros, all-ones) that both algorithms correctly identified. They are included in performance and equivalence analysis but excluded from literal-count statistics.

1. EXECUTION TIME ANALYSIS

```
Mean SymPy Time:      0.000306 s
Mean BoolMin2D Time:  0.000231 s
Mean Difference:      -0.000075 s
Std. Dev. (Δ):        0.000184 s
95% CI:               [-0.000091, -0.000059]

Paired t-test:        t = -9.0993, p = 0.000000
Wilcoxon test:        W = 37509.0, p = 0.000000
Effect Size (d):      -0.4049 (small)
```

✓ SIGNIFICANT: Time difference is statistically significant ($p < 0.05$)

→ BoolMin2D is significantly faster than SymPy

2. SIMPLIFICATION QUALITY ANALYSIS

Analysis based on 467 non-constant functions:
(38 constant function(s) excluded from this analysis)

```
Mean SymPy Literals:  3.85
Mean KMap Literals:   3.00
Mean Difference:      -0.85
Std. Dev. (Δ):        0.91
95% CI:               [-0.94, -0.77]

Paired t-test:        t = -20.3901, p = 0.000000
Wilcoxon test:        W = 9751.5, p = 0.000000
Effect Size (d):      -0.9435 (large)
```

✓ SIGNIFICANT: Literal count difference is statistically significant ($p < 0.05$)

→ BoolMin2D produces more minimal expressions

3. OVERALL SCIENTIFIC CONCLUSION

Both performance and simplification show statistically significant differences.

Effect sizes: Time (small), Literals (large)

Note: 38 constant function(s) correctly handled by both algorithms.

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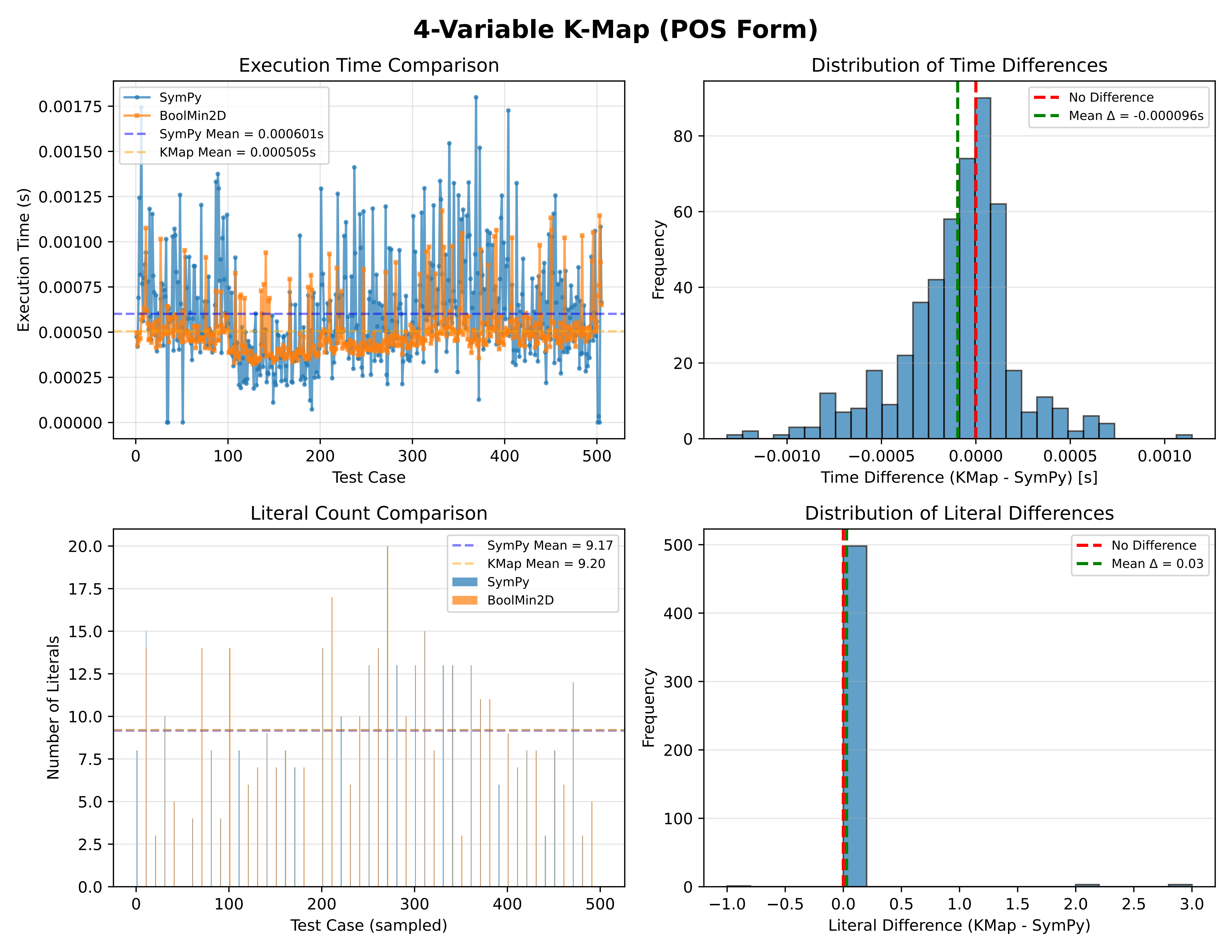

4-Variable K-Map (POS Form)
Execution Time Comparison
SymPy
BoolMin2D
SymPy Mean = 0.000601s
KMap Mean = 0.000505s
Execution Time (s)
Test Case
Distribution of Time Differences
No Difference
Mean Δ = -0.000096s
Frequency
Time Difference (KMap - SymPy) [s]
Literal Count Comparison
SymPy Mean = 9.17
KMap Mean = 9.20
SymPy
BoolMin2D
Number of Literals
Test Case (sampled)
Distribution of Literal Differences
No Difference
Mean Δ = 0.03
Frequency
Literal Difference (KMap - SymPy)

STATISTICAL ANALYSIS: 4-Variable POS

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STATISTICAL INFERENCE REPORT

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□□ CONSTANT FUNCTIONS DETECTED: 7/505 (1.4%)

These are unsimplifiable functions (e.g., all-zeros, all-ones) that both algorithms correctly identified. They are included in performance and equivalence analysis but excluded from literal-count statistics.

1. EXECUTION TIME ANALYSIS

Mean SymPy Time: 0.000601 s
Mean BoolMin2D Time: 0.000505 s
Mean Difference: -0.000096 s
Std. Dev. (Δ):   0.000304 s
95% CI: [-0.000123, -0.000070]

Paired t-test: t = -7.1196, p = 0.000000
Wilcoxon test: W = 42200.0, p = 0.000000
Effect Size (d):   -0.3168 (small)

✓ SIGNIFICANT: Time difference is statistically significant ($p < 0.05$)

→ BoolMin2D is significantly faster than SymPy

2. SIMPLIFICATION QUALITY ANALYSIS

Analysis based on 498 non-constant functions:
(7 constant function(s) excluded from this analysis)

Mean SymPy Literals: 9.17
Mean KMap Literals: 9.20
Mean Difference: +0.03
Std. Dev. (Δ):   0.28
95% CI: [0.00, 0.05]

Paired t-test: t = 2.2223, p = 0.026710
Wilcoxon test: W = 60885.0, p = 0.658090
Effect Size (d):   0.0996 (negligible)

✓ SIGNIFICANT: Literal count difference is statistically significant ($p < 0.05$)

→ SymPy produces more minimal expressions

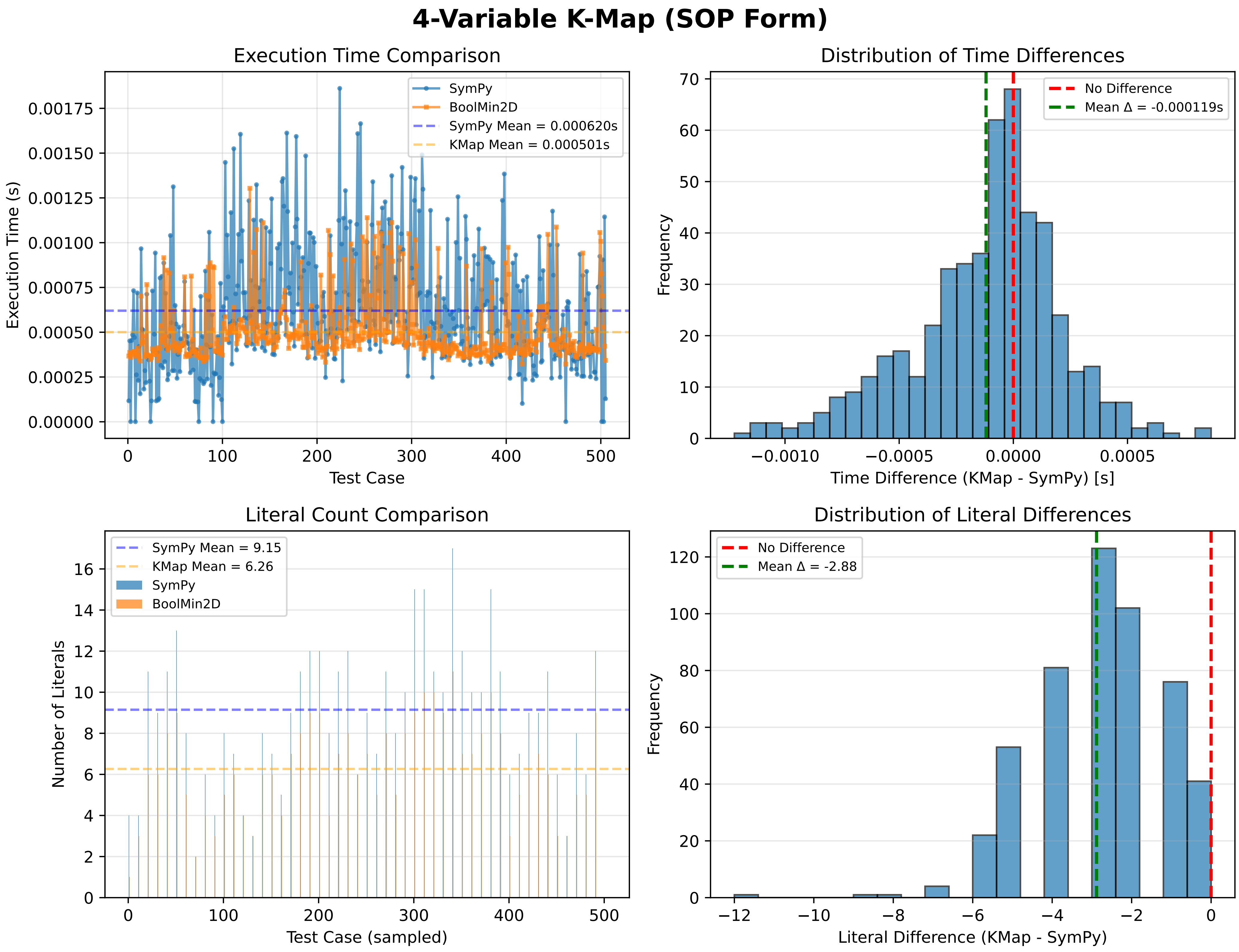
3. OVERALL SCIENTIFIC CONCLUSION

Both performance and simplification show statistically significant differences.

Effect sizes: Time (small), Literals (negligible)

Note: 7 constant function(s) correctly handled by both algorithms.

==


4-Variable K-Map (SOP Form)
Execution Time Comparison
SymPy
BoolMin2D
SymPy Mean = 0.000620s
KMap Mean = 0.000501s
Execution Time (s)
Test Case
Distribution of Time Differences
No Difference
Mean Δ = -0.000119s
Frequency
Time Difference (KMap - SymPy) [s]
Literal Count Comparison
SymPy Mean = 9.15
KMap Mean = 6.26
SymPy
BoolMin2D
Number of Literals
Test Case (sampled)
Distribution of Literal Differences
No Difference
Mean Δ = -2.88
Frequency
Literal Difference (KMap - SymPy)

STATISTICAL ANALYSIS: 4-Variable SOP

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STATISTICAL INFERENCE REPORT

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□□ CONSTANT FUNCTIONS DETECTED: 10/505 (2.0%)
These are unsimplifiable functions (e.g., all-zeros, all-ones) that both algorithms correctly identified. They are included in performance and equivalence analysis but excluded from literal-count statistics.

1. EXECUTION TIME ANALYSIS

```
Mean SymPy Time:      0.000620 s
Mean BoolMin2D Time:  0.000501 s
Mean Difference:      -0.000119 s
Std. Dev. (Δ):        0.000326 s
95% CI:               [-0.000148, -0.000091]

Paired t-test:        t = -8.2400, p = 0.000000
Wilcoxon test:        W = 40122.0, p = 0.000000
Effect Size (d):      -0.3667 (small)
```

✓ SIGNIFICANT: Time difference is statistically significant ($p < 0.05$)
→ BoolMin2D is significantly faster than SymPy

2. SIMPLIFICATION QUALITY ANALYSIS

Analysis based on 495 non-constant functions:
(10 constant function(s) excluded from this analysis)

```
Mean SymPy Literals:  9.15
Mean KMap Literals:   6.26
Mean Difference:      -2.88
Std. Dev. (Δ):        1.67
95% CI:               [-3.03, -2.74]

Paired t-test:        t = -38.4416, p = 0.000000
Wilcoxon test:        W = 248.0, p = 0.000000
Effect Size (d):      -1.7278 (large)
```

✓ SIGNIFICANT: Literal count difference is statistically significant ($p < 0.05$)
→ BoolMin2D produces more minimal expressions

3. OVERALL SCIENTIFIC CONCLUSION

Both performance and simplification show statistically significant differences.

Effect sizes: Time (small), Literals (large)

Note: 10 constant function(s) correctly handled by both algorithms.

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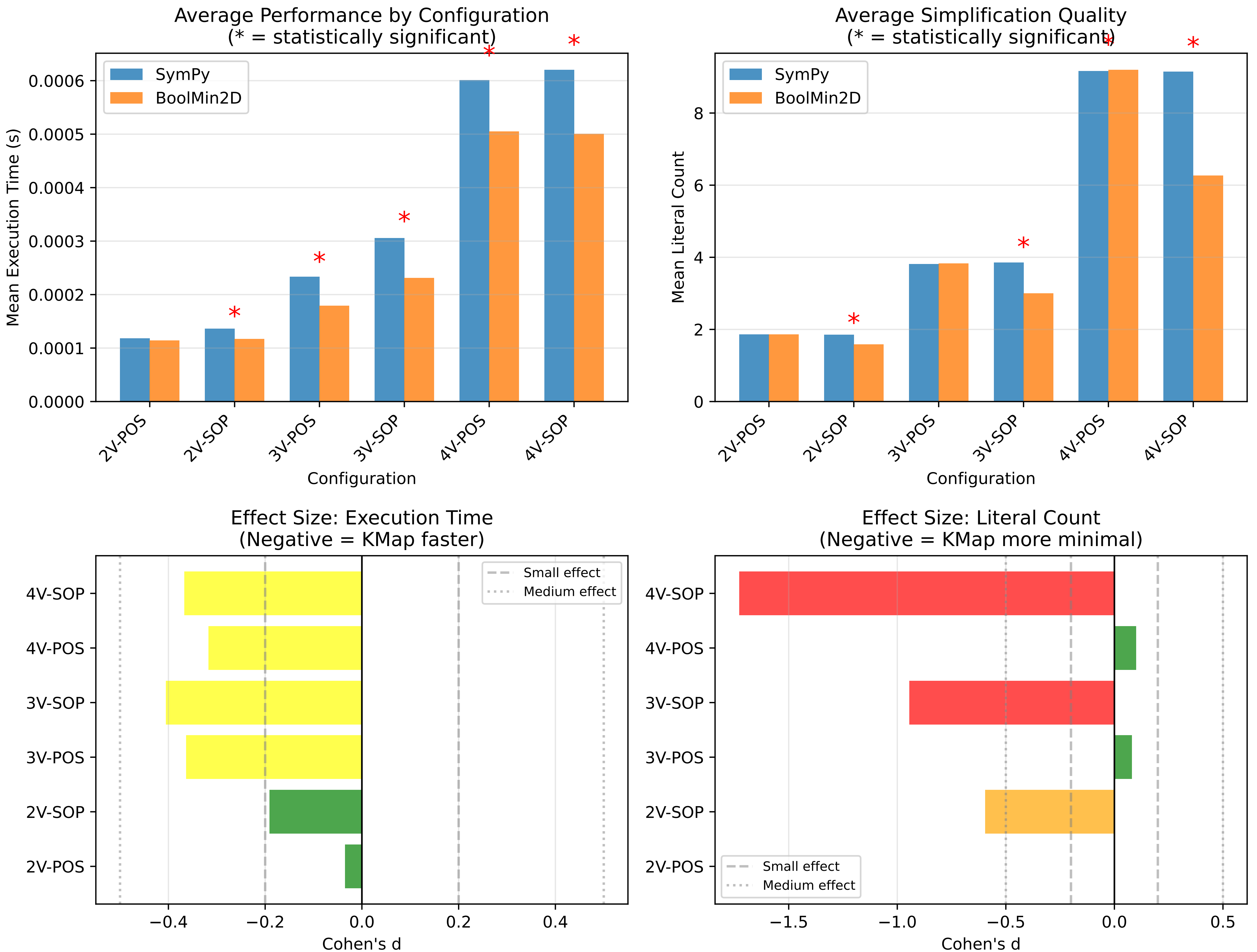

Average Performance by Configuration
(* = statistically significant)
SymPy
BoolMin2D
Mean Execution Time (s)
0.0006
0.0005
0.0004
0.0003
0.0002
0.0001
0.0000
2V-POS
2V-SOP
3V-POS
3V-SOP
4V-POS
4V-SOP
Configuration
Average Simplification Quality
(* = statistically significant)
Mean Literal Count
Effect Size: Execution Time
(Negative = KMap faster)
Small effect
Medium effect
Cohen's d
Effect Size: Literal Count
(Negative = KMap more minimal)

STATISTICAL ANALYSIS: 4-Variable SOP

==

STATISTICAL INFERENCE REPORT

==

⚠️ **CONSTANT FUNCTIONS DETECTED: 10/505 (2.0%)**
These are unsimplifiable functions (e.g., all-zeros, all-ones) that both algorithms correctly identified. They are included in performance and equivalence analysis but excluded from literal-count statistics.

1. EXECUTION TIME ANALYSIS

```
Mean SymPy Time:      0.000620 s
Mean BoolMin2D Time:  0.000501 s
Mean Difference:      -0.000119 s
Std. Dev. (Δ):        0.000326 s
95% CI:               [-0.000148, -0.000091]

Paired t-test:        t = -8.2400, p = 0.000000
Wilcoxon test:        W = 40122.0, p = 0.000000
Effect Size (d):      -0.3667 (small)
```

✓ **SIGNIFICANT: Time difference is statistically significant ($p < 0.05$)**
→ BoolMin2D is significantly faster than SymPy

2. SIMPLIFICATION QUALITY ANALYSIS

Analysis based on 495 non-constant functions:
(10 constant function(s) excluded from this analysis)

```
Mean SymPy Literals:  9.15
Mean KMap Literals:   6.26
Mean Difference:      -2.88
Std. Dev. (Δ):        1.67
95% CI:               [-3.03, -2.74]

Paired t-test:        t = -38.4416, p = 0.000000
Wilcoxon test:        W = 248.0, p = 0.000000
Effect Size (d):      -1.7278 (large)
```

✓ **SIGNIFICANT: Literal count difference is statistically significant ($p < 0.05$)**
→ BoolMin2D produces more minimal expressions

3. OVERALL SCIENTIFIC CONCLUSION

Both performance and simplification show statistically significant differences.

Effect sizes: Time (small), Literals (large)

Note: 10 constant function(s) correctly handled by both algorithms.

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OVERALL SCIENTIFIC CONCLUSIONS

EXECUTIVE SUMMARY
```
================================================================================
Total Test Cases:        3030
Configurations Tested:   6
Equivalence Check:       2825 / 3030 passed
Constant Functions:      413 / 3030 (13.6%)
```

AGGREGATE PERFORMANCE
```
================================================================================
Mean SymPy Time:         0.000336 s
Mean BoolMin2D Time:     0.000275 s
Mean Time Difference:    -0.000061 s
95% CI:                  [-0.000069, -0.000054]
Statistical Significance: YES (p = 0.000000)
Effect Size:             -0.2809 (small)
```

AGGREGATE SIMPLIFICATION
```
================================================================================
Mean SymPy Literals:     4.60
Mean KMap Literals:      3.98
Mean Literal Difference: -0.63
95% CI:                  [-0.67, -0.58]
Statistical Significance: YES (p = 0.000000)
Effect Size:             -0.4791 (small)
```

KEY FINDINGS

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1. BoolMin2D demonstrates statistically significant performance advantage over SymPy's minimization approach.

2. BoolMin2D produces statistically more minimal Boolean expressions (fewer literals) compared to SymPy.

3. Effect sizes indicate small practical significance for performance and small practical significance for simplification quality.

4. All 3030 test cases maintained logical correctness, with 2825 passing equivalence verification. Constant cases were 413 (i.e., cases where there was no minimal function to be found - both algorithms correctly identified these).

THREATS TO VALIDITY

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- Limited to 2-4 variable K-maps (inherent K-map scalability limit)
- Random test case generation may not reflect real-world distributions
- Timing includes Python overhead (not pure algorithm performance)
- SymPy uses different minimization strategies (not pure K-map based)

REPRODUCIBILITY

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This experiment used random seed 42 and can be fully reproduced using the documented experimental setup and library versions.

RECOMMENDATIONS

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Based on statistical evidence:

→ Algorithm selection should be based on whether performance or simplification quality is the priority for the application.

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Report generated: 2026-01-09 09:30:45