CPT302 Week 6 In-Class Exercises - bis

Name and Surname:___________________________

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Sample of Prisoner's Dilemma

Figure 1: Contribution towards a Bridge

Player 1 \ Player 2	Contribute	Do Not Contribute
Contribute	32, 32 (C,C)	28, 35 (C,NC)
Do Not Contribute	35, 28 (NC,C)	30, 30 (NC,NC)

Figure 1 shows another example of a Prisoner's Dilemma type game. The situation involves construction of a bridge by two different individuals. If they both contribute to the building of this bridge, then they both receive a utility of 32. However, if they both fail to contribute, they are left with a utility of just 30. If one player contributes and the other one does not, then the player who does not contribute is a 'free rider' and will receive a utility of 35. The contributing player is left with 28.

Pareto Efficiency and Nash Equilibrium

Pareto efficiency is a term that can be used when analyzing prisoner dilemma games. An outcome (of the game) is said to be Pareto efficient if there is no other outcome in which some other individual is better off and no individual is worst off. In Figure 1, there are three Pareto efficient outcomes: (contribute, contribute), (do not contribute, contribute), and (contribute, do not contribute). At these three payoffs, there is no other spot to choose that doesn't make at least one player worst off. At (35, 28), if you move to any other space, one player will be worst off. However, if you are at (do not con- tribute, do not contribute), each player will get a payoff of 30. This outcome is said to be Pareto dominated by (contribute, contribute) because you can move to that outcome and make both players better off. Also, do not contribute (NC) is the dominant strategy for both players.

A Nash equilibrium, named after John Nash, is a set of strategies, one for each player, such that no player has incentive to unilaterally change her action. Players are in equilibrium if a change in strategies by any one of them would lead that player to earn less than if she/he remained with her/his current strategy. So (NC,NC) is the only Nash equilibrium. This can also be verified using the method: Finding Nash Equilibria presented in the Lecture.

Price Setting Oligopoly Game

Assume that Ford and GM build cars that are almost identical so that price is the variable that consumers look at when deciding which type of car to buy. If both Ford and GM are colluding and setting a high price, each reaps a good profit of $500 million. If one firm sets a high price, then the other firm can achieve an advantage by cheating and setting a low price. The firm with the low price will steal virtually the entire market and earn a profit of $700 million for itself, while leaving its competitor with a profit of only $100 million. If both firms set a low price, then they share equally in an expanded market, but because of the low price, each earns a profit of only $300 million. The payoff matrix of the game is as follows:

		GM	
		High Price	Low Price
Ford	High Price	500, 500	100, 700
	Low Price	700, 100	300, 300

Is there a dominant strategy for each player? Justify your answer.

Ans: If Ford expects GM to set a high price, then its best response is to cheat and set a low price because it will then earn $700 million instead of $500 million. On the other hand, if Ford expects GM to set a low price, its best response is again to set a low price but for a different reason – to avoid setting a high price and losing a large portion of the market. In this case, Ford's payoff will be $300 million instead of $100 million. Clearly, no matter what Ford expects GM to do, it is better off setting a low price. When one strategy is best for a player no matter what strategy the other player uses, that strategy is said to dominate all other strategies and is called a dominant strategy. In the game, both firms have a dominant strategy, which is to set a low price.

The Game of Performance-enhancing Drugs in Professional Sports

Here, the athletes are the players, and the two possible strategies are to use performance-enhancing drugs or not. If you uses drugs while your opponent does not, you will get an advantage in the competition. We consider a sport where it is difficult to detect the use of such drugs, and we capture the situation with numerical payoffs that might look as follows (the numbers are arbitrary here; we are only interested in their relative sizes):

		Athlete 2	
		Don't Use Drugs	*Use Drugs*
Athlete 1	*Don't Use Drugs*	3, 3	1, 4
	Use Drugs	4, 1	2, 2

Is there a dominant strategy for each player? Justify your answer.

Ans: Here, the best outcome (with a payoff of 4) is to use drugs when your opponent does not, since then you maximize your chances of winning. However, the payoff to both using drugs (2) is worse than the payoff to both not using drugs (3). Thus, we can now see that using drugs is a strictly dominant strategy for both of them.

Q1. Define the following notions/concepts:

- Dominant Strategy
- Pure Nash Equilibrium
- Pareto Optimality

Ans:

Q2. The payoff matrices for agent i and agent j are shown in separate matrices below:

Agent i	$i : D$	$i : C$
$j : D$	4	3
$j : C$	6	5

Agent j	$i : D$	$i : C$
$j : D$	4	6
$j : C$	3	5

With reference to the matrices above answer the following questions:

- Identify with justification, if any, the pairs that are in Nash equilibrium.
- State whether any outcomes are Pareto optimal. Justify your answer.
- What is the social welfare of an outcome? Identify with justification, if any, the pairs that maximize the social welfare.

Ans: