Xi'an Jiaotong-Liverpool University
西交利物浦大学

CPT 302 Individual Project
Coursework/Assignment Submission Form

2021/22 Semester 2
Bachelor's degree – Year 4

Module Code	Module Leader	Module Title
CPT302	**Ka Lok Man**	**Multi-Agent Systems**

Section A: Your Details
To be completed by the student (in English using BLOCK CAPITALS)

Student's Name	Tianlei Shi
Student ID	**1824152**

Section B: Assignment Details
To be completed by the student (in English using BLOCK CAPITALS)

Coursework Assignment Number	**Assignment II**
Coursework (assignment) Title	**Assignment II**
Method of Working	**INDIVIDUAL**
Date and time of submission	**23 May 2022, 17:00**

Assignment details can be found in the assignment description.

Section C: Statement of Academic Honesty
To be completed by the student

By submitting this coursework for assessment, you are confirming that you have read and understood the University's policy on plagiarism and collusion and that the submitted work is your own.

(i) I confirm that I have read a copy of the current University's definitions of collusion and plagiarism on coursework and academic honesty, and that I fully understand the meaning of these terms.

(ii) I confirm that the submitted coursework has been created solely by me and that I have not been assisted, nor have copied part or all of somebody else's work, either with their explicit approval or without their knowledge or consent.

(iii) I confirm that this is my own work and that use of material from other sources, including the Internet, has been properly and fully acknowledged and referenced.

(iv) I confirm that the information I have given is correct to the best of my knowledge.

Student's signature	Tianlei Shi	Date	May. 23, 2022

If this form is submitted electronically, please type your name in English (BLOCK CAPTIALS)

Q1,

Sol:

a) 1, the payoff matrix of the above Prisoner's Dilemma problem:

	Tom confesses	Tom does not confess
Peter confesses	(X, X)	(0, Y)
Peter does not confess	(Y, 0)	(Z, Z)

2, Peter now has two choices: confess or does not confess. If Peter confesses, he will be jailed for 0-X years; but if Peter does not confess, he will be jailed for Z-Y years. Since Y > X > Z, so "confess" is the best choice (or dominant strategy) for Peter. Because if Peter expects Tom to confess, the best response is "confess" too, because he will be jailed for X years instead of Y years; and if Peter expects Tom does not to confess, the best response is "confess", because he will be jailed for 0 year instead of Z years. Therefore, Peter must confess.

3, the payoff matrix should be formed like below:

	Tom confesses	Tom does not confess
Peter confesses	(Z, Z)	(Y, Y)
Peter does not confess	(Y, Y)	(Z, Z)

For the above payoff matrix, there is no player has a dominant strategy. Because for Peter and Tom, they are both in the same situation. Take Tom as an example, whatever the choice of Peter is, Tom always will be jailed for Z-Y years, so does Peter. Therefore, there is no dominant strategy for both Peter and Tom.

b) To more convenient observation and discussion, we combine two separate matrices into one payoff matrix, shown in below:

	Agent i : D	Agent i : C
Agent j : D	(a, a)	(c, b)
Agent j : C	(b, c)	(d, d)

Only pair of (a, a) in pure strategy Nash equilibrium. Because if agent i choice D, now agent j has two choices: D (get a) and C (get b), and a > b, so agent j has no incentive to choose C; same as agent i, if agent j choice D, now agent i has two choices: D (get a) and C (get b), and a > b, so agent i also has no incentive to choose C. Moreover, other pairs all does not satisfy the definition of pure strategy Nash equilibrium. Therefore, only (a, a) in pure strategy Nash equilibrium.

c) According to the payoff matrix in question (b), there are pairs of (d, d), (b, c), and (c, b) are Pareto optimal. An outcome is said to be Pareto optimal if there is no other outcome that makes one agent better off without making another agent worse off. Thus, for pair (a, a), there exists a pair of (d, d) can make both agents better off, so it is not Pareto optimal. And for (d, d), (b, c), and (c, b), they all satisfy the definition of Pareto optimal. Therefore, (d, d), (b, c), and (c, b) are Pareto optimal.

d) According to the payoff matrix in question (b), we can calculate the social welfare for each pair. The formula of social welfare is $\sum_{i \in Ag} u_i(\omega)$. Thus:
For (a, a), social welfare is a + a = 2a.
For (c, b), social welfare is c + b.
For (b, c), social welfare is b + c.
For (d, d), social welfare is d + d = 2d.
Since we do not know the value of a, b, c, and d, so we can only know that the social welfare of (d, d) is larger than the social welfare of (a, a) because of 2d > 2a, and the social welfares of (c, b) and (b, c) are the same. However, we cannot compare b + c and 2d. Therefore, we cannot find the pair with maximum social welfare.

Q2,

Sol:

a) For Plurality vote, each voter votes for one alternative, and the one with the most votes wins. If we want to both Oliver and Micheal win, they must have same votes, and their votes larger than Nigel's. According to the preference table, the votes of Oliver are $A + E$, and the votes of Micheal are $B + D + F$, and the votes of Nigel are C. Therefore, we can design a Plurality vote protocol, and each voter votes for one alternative, and only the candidate in the first choice of alternative can get one vote. In this election protocol, to make sure both Oliver and Micheal be the winners, we only make $A + E = B + D + F > C$.

   For example, we can let $A = 1$, $B = 2$, $C = 5$, $D = 3$, $E = 8$, $F = 4$. So, the votes of Oliver are $A + E = 1 + 8 = 9$, and the votes of Micheal are $B + D + F = 2 + 3 + 4 = 9$, and the votes of Nigel are $C = 5$. Therefore, both Oliver and Micheal are the winners.

b) For Borda count, each voter submits preferences over the m alternatives, and for each alternative, the candidate ranked last get 1 point, … and the candidate ranked first get m points. Thus, we should first calculate the votes for each candidate.

   For Micheal, $A(1) + B(3) + C(1) + D(3) + E(1) + F(3) = A + 3B + C + 3D + E + 3F$.

   For Nigel, $A(2) + B(1) + C(3) + D(1) + E(2) + F(1) = 2A + B + 3C + D + 2E + F$.

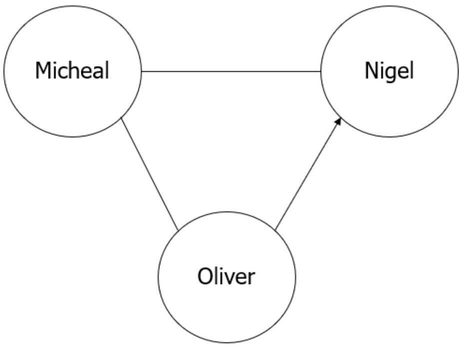
   For Oliver, $A(3) + B(2) + C(2) + D(2) + E(3) + F(2) = 3A + 2B + 2C + 2D + 3E + 2F$.

   Therefore, we can design a Borda count protocol, and to make both Oliver and Micheal are NOT the winner, we just make Nigel be the winner, i.e., make $2A + B + 3C + D + 2E + F > A + 3B + C + 3D + E + 3F$, and $2A + B + 3C + D + 2E + F > 3A + 2B + 2C + 2D + 3E + 2F$.

   For example, we can let $A = 2$, $B = 3$, $C = 20$, $D = 4$, $E = 1$, $F = 5$. So, the votes of Micheal are $A + 3B + C + 3D + E + 3F = 2 + 3\times3 + 20 + 3\times4 + 1 + 3\times5 = 59$, the votes of Nigel are $2A + B + 3C + D + 2E + F = 2\times2 + 3 + 3\times20 + 4 + 2\times1 + 5 = 78$, and the votes of Oliver are $3A + 2B + 2C + 2D + 3E + 2F = 3\times2 + 2\times3 + 2\times20 + 2\times4 + 3\times1 + 2\times5 = 73$. Therefore, both Oliver and Micheal are NOT the winner.

c) For Method of Pairwise Comparisons, candidates face each other in pairwise elections, and the winner progresses to the next election. Thus, we firstly compare Nigel and Oliver. The number of times Nigel beats Oliver is C, and the number of times Oliver beats Nigel is A + B + D + E + F. Because A > C > F, A=D, B=C and E=F, we can know that A + B + D + E + F > C, so Oliver wins. Next, we compare Oliver and Micheal. The number of times Oliver beats Micheal is A + C + E, and the number of times Micheal beats Oliver is B + D + F. Because A > C > F, A=D, B=C and E=F, we can know that A + C + E = B + D + F, so no winner. Finally, we compare Nigel and Micheal. The number of times Nigel beats Micheal is A + C + E, and the number of times Micheal beats Nigel is B + D + F. Because A > C > F, A=D, B=C and E=F, we can know that A + C + E = B + D + F, so no winner.
Therefore, Oliver is the winner, because he is the only one with a winning record. Assume that the winner gets 1 point, the tie gets 0.5 points, and the loser gets 0 points in Pairwise Comparisons. So, Oliver gets 1 + 0.5 = 1.5 points, Micheal gets 0.5 + 0.5 = 1 point, and Nigel gets 0 + 0.5 = 0.5 point. Therefore, Oliver is the winner.

d) According to the comparison in question (c), we can draw a majority graph shown in below (we use straight lines to connect the two sides of the tie):



The possible winners are Micheal and Oliver, because no other candidate beat them.

e) A Condorcet winner is a candidate that would beat every other candidate in a pairwise election. According to the comparison above, Oliver has beaten Nigel, and Oliver just needs to beat Micheal, then he can become a Condorcet winner. Since Oliver and Micheal both beat each other with the same number of times, so we only need to exchange the number votes so that make Oliver become a Condorcet winner.

For example, we can exchange values for columns 5 () and 6 (). Now, the preference table is shown in below:

Number of votes	A	B	C	E	D	F
1st choice	Oliver	Micheal	Nigel	Micheal	Oliver	Micheal
2nd choice	Nigel	Oliver	Oliver	Oliver	Nigel	Oliver
3rd choice	Micheal	Nigel	Micheal	Nigel	Micheal	Nigel

Thus, we firstly compare Nigel and Oliver. The number of times Nigel beats Oliver is C, and the number of times Oliver beats Nigel is A + B + D + E + F. Because A > C > F, A=D, B=C and E=F, we can know that A + B + D + E + F > C, so Oliver wins.

Next, we compare Oliver and Micheal. The number of times Oliver beats Micheal is A + C + D, and the number of times Micheal beats Oliver is B + E + F. Because A > C > F, A=D, B=C and E=F, we can know that A + C + D > B + E + F, because A + C + D = 2A + B, and B + E + F = 2F + B. So, Oliver wins.

Therefore, change preferences of voters like this can make Oliver become a Condorcet winner.

Q3,

Sol:

In this question, there are three agents Ag = {w1, w2, w3}, each subset C ⊆ Ag is a possible coalition. And we are required to verify whether it is fair to distribute the estate according to the way described in question when the estate is less than the amount mentioned in the will.

Let d_1, d_2, d_3 represents the units allotted to the three wives in the will, and $d_1 < d_2 < d_3$, and E represents actual estate. Moreover, we view this question as a cumulative game, which means that we can calculate the value of the generalized characteristic function v for each subset of agents Ag by the following equation:

$$v(\{w_1, \dots, w_k\}) = min\{v(\{w_1\}) + \cdots + v(\{w_k\}), E\}$$

and for all $k = 2, \dots, n$. If the claim does not exceed the estate E, the claim of each coalition is the sum of the claims of its members. This is the implication of cumulative game: agents stick to their own claims and are not willing to compromise and share the claims.

However, we also need to consider whether the agent's claim is greater than the estate, because if claim larger than estate, agent must share the claim. Therefore, we can get the generalized characteristic function:

$$v(w) = min\{d_i, E\}, (w = 1, \dots, n)$$

$$v(w) = d_{m-1} + (E - d_{m-1})/(n - m + 1), (w = 1, \dots, n)$$

$$v(\{w_1, \dots, w_k\}) = min\{v(\{w_1\}) + \cdots + v(\{w_k\}), E\} = E, (k = m, \dots, n)$$

Next, we will analyze the three cases mentioned in the question.

For α = 100:

Since $E \leq d_1 < d_2 < d_3$, we have n = 3 and m = 1. Thus, the characteristic function is:

$$v(C) = \begin{cases} min\{100, 100\} = 100, & \text{if } C = \{w_1\} \\ min\{200, 100\} = 100, & \text{if } C = \{w_2\} \\ min\{300, 100\} = 100, & \text{if } C = \{w_3\} \\ min\{100 + 200, 100\} = 100, & \text{if } C = \{w_1, w_2\} \\ min\{100 + 300, 100\} = 100, & \text{if } C = \{w_1, w_3\} \\ min\{200 + 300, 100\} = 100, & \text{if } C = \{w_2, w_3\} \\ min\{100 + 200 + 300, 100\} = 100, & \text{if } C = \{w_1, w_2, w_3\} \end{cases}$$

Moreover, we can find the way to allocating fair payoffs to each wife by calculating the Shapley value.

Probability	Order	w_1	w_2	w_3
$\frac{1}{6}$	w_1 -> w_2 -> w_3	100	0	0
$\frac{1}{6}$	w_1 -> w_3 -> w_2	100	0	0
$\frac{1}{6}$	w_2 -> w_1 -> w_3	0	100	0
$\frac{1}{6}$	w_2 -> w_3 -> w_1	0	100	0
$\frac{1}{6}$	w_3 -> w_1 -> w_2	0	0	100
$\frac{1}{6}$	w_3 -> w_2 -> w_1	0	0	100

So, we can get the Shapley value for each agent:

$\mathrm{Sh}\ (S, w_1)/\varphi_i = \frac{1}{6}\ (100 + 100 + 0 + 0 + 0 + 0) = \frac{100}{3}$,

$\mathrm{Sh}\ (S, w_2)/\varphi_i = \frac{1}{6}\ (0 + 0 + 100 + 100 + 0 + 0) = \frac{100}{3}$,

$\mathrm{Sh}\ (S, w_3)/\varphi_i = \frac{1}{6}\ (0 + 0 + 0 + 0 + 100 + 100) = \frac{100}{3}$.

Therefore, each wife gets $\frac{100}{3}$.

For α = 200:

Since $d_1 < E \leq d_2 < d_3$, we have n = 3 and m = 2. Thus, the characteristic function is:

$$v(C) = \begin{cases} min\{100, 200\} = 100, & \text{if } C = \{w_1\} \\ 100 + (200 - 100)/2 = 150, & \text{if } C = \{w_2\} \\ 100 + (200 - 100)/2 = 150, & \text{if } C = \{w_3\} \\ min\{100 + 150, 200\} = 200, & \text{if } C = \{w_1, w_2\} \\ min\{100 + 150, 200\} = 200, & \text{if } C = \{w_1, w_3\} \\ min\{150 + 150, 200\} = 200, & \text{if } C = \{w_2, w_3\} \\ min\{100 + 150 + 150, 200\} = 200, & \text{if } C = \{w_1, w_2, w_3\} \end{cases}$$

Moreover, we can find the way to allocating fair payoffs to each wife by calculating the Shapley value.

Probability	Order	w_1	w_2	w_3
$\frac{1}{6}$	w_1 -> w_2 -> w_3	100	100	0
$\frac{1}{6}$	w_1 -> w_3 -> w_2	100	0	100
$\frac{1}{6}$	w_2 -> w_1 -> w_3	50	150	0
$\frac{1}{6}$	w_2 -> w_3 -> w_1	0	150	50
$\frac{1}{6}$	w_3 -> w_1 -> w_2	50	0	150
$\frac{1}{6}$	w_3 -> w_2 -> w_1	0	50	150

So, we can get the Shapley value for each agent:

$\mathrm{Sh}\ (\mathrm{S}, w_1)/\varphi_i = \frac{1}{6}\,(100 + 100 + 50 + 0 + 50 + 0) \ = \ 50$,

$\mathrm{Sh}\ (\mathrm{S}, w_2)/\varphi_i = \frac{1}{6}\,(100 + 0 + 150 + 150 + 0 + 50) \ = \ 75$,

$\mathrm{Sh}\ (\mathrm{S}, w_3)/\varphi_i = \frac{1}{6}\,(0 + 100 + 0 + 50 + 150 + 150) \ = \ 75$.

Therefore, wife 1 gets 50, and the other two get 75 each.

For α = 300:

Since $d_1 < d_2 < E \leq d_3$, we have n = 3 and m = 3. Thus, the characteristic function is:

$$v(C) = \begin{cases} min\{100, 300\} = 100, & \text{if } C = \{w_1\} \\ min\{200, 300\} = 200, & \text{if } C = \{w_2\} \\ 200 + (300 - 200)/1 = 300, & \text{if } C = \{w_3\} \\ min\{100 + 200, 300\} = 300, & \text{if } C = \{w_1, w_2\} \\ min\{100 + 300, 300\} = 300, & \text{if } C = \{w_1, w_3\} \\ min\{200 + 300, 300\} = 300, & \text{if } C = \{w_2, w_3\} \\ min\{100 + 200 + 300, 300\} = 300, & \text{if } C = \{w_1, w_2, w_3\} \end{cases}$$

Moreover, we can find the way to allocating fair payoffs to each wife by calculating the Shapley value.

Probability	Order	w_1	w_2	w_3
$\frac{1}{6}$	w_1 -> w_2 -> w_3	100	200	0
$\frac{1}{6}$	w_1 -> w_3 -> w_2	100	0	200
$\frac{1}{6}$	w_2 -> w_1 -> w_3	100	200	0
$\frac{1}{6}$	w_2 -> w_3 -> w_1	0	200	100
$\frac{1}{6}$	w_3 -> w_1 -> w_2	0	0	300
$\frac{1}{6}$	w_3 -> w_2 -> w_1	0	0	300

So, we can get the Shapley value for each agent:

$\mathrm{Sh}\ (\mathrm{S}, w_1)/\varphi_i = \frac{1}{6}\,(100 + 100 + 100 + 0 + 0 + 0)\ =\ 50$,

$\mathrm{Sh}\ (\mathrm{S}, w_2)/\varphi_i = \frac{1}{6}\,(200 + 0 + 200 + 200 + 0 + 0)\ =\ 100$,

$\mathrm{Sh}\ (\mathrm{S}, w_3)/\varphi_i = \frac{1}{6}\,(0 + 200 + 0 + 100 + 300 + 300)\ =\ 150$.

Therefore, wife 1 gets 50, wife 2 gets 100 and wife 3 gets 150.

In conclusion, the proposed way of allocating payoffs to each wife in question is fair.

Q4,

Sol:

I think the above statement is true. Because compare to other types of auction mechanisms, it has the following advantages:

1. second-price sealed-bid (SPSB) auctions are more efficient and safer: auctions don't take as much time as English auction, and the price is known only to the auctioneer, which protects the bidder's privacy.
2. SPSB auctions induce bidders to bid higher prices: if the bidder wins, he/she only has to pay the second highest price, so he/she gives a truthful valuation. However, in other types of auctions, bidders usually bid below their valuation.
3. SPSB auctions has no incentive for counter-speculation: bidders have no reason to speculation, because if the competitor's valuation is higher than bidder, winning becomes unworthy; and if the competitor's valuation is lower than bidder, bidder don't have to speculate.

For example, there are three companies wishing to bid on a project without cheating and collusion. Take one of them as an example, like company C. C wants to get this project, it has a valuation, and this is the **highest price** it was willing to pay for winning the bid, and if the transaction price exceeds the valuation, it will give up because it may lose money. At the same, it also doesn't have to inquire about competitors' bids (**speculation**), because if bids higher than its valuation, it will give up; and if bids lower than its valuation, there is no need to waste time. Therefore, C only needs to quote its valuation, and no need to speculate, and if it wins the bid, the price it pays must be lower than its valuation, which makes it profitable. Moreover, this also **saves time** of C from wasting in bidding (unlike the English auction), and protects the bid of C from being known to competitors (**safer**).

Q5,

Sol:

Cooperative Distributed Problem Solving (CDPS) focus on how to solve a problem by modular problems and assign tasks, and control component agents to let them work together by efficient protocol, and finally construct a loose coupling network.

For instance, agents now face a problem, they are required to building a self-driving electric car. For the manager agent, it realized it cannot achieve the goal individually, because this project requires many specialized knowledge and involves many engineering. So, cooperation is necessary because no single agent has sufficient expertise, resources, and information to solve a problem, and different agents might have expertise for solving different parts of the problem. Therefore, the manager agent will decompose problem, and then broadcast announcements to invite bids, and announcements include description of task, constraints, and meta-task information. Next, agents that receive the announcement decide for themselves whether they wish to bid for the task, and if they choose to bid, then they submit a tender, and manager agent screens them to determine who to "award the contract" to. Finally, the successful contractor then expedites the task (become component agent), or conduct further sub-contracting (become manager agent). At the end, agents will form manager agents and contractor agents. Manager agents are responsible for management and control, and contractor agents are responsible for using their abilities and resources to solve problems. For example, the task of building cars may be divided into sub-tasks such as battery, automatic drive, and car structure, and the agent who is good at the sub-task will be responsible for the task and communicate the progress with its manager.

The key problems that need to be fixed in CDPS include the following:

1. How can a problem be divided into smaller tasks for distribution among agents?
2. How can a problem solution be effectively synthesized from subproblem results?
3. How can the overall problem-solving activities of the agents be optimized so as to produce a solution that maximizes the coherence metric?
4. What techniques can be used to coordinate the activity of the agents, thus avoiding destructive (and therefore unhelpful) interactions, and maximizing effectiveness (by exploiting any positive interactions)?

We can use the example mentioned before to solve those key problems.

For the problem 1, a whole task can be divided into smaller tasks by managers based on similarities or modules, and those smaller tasks can be divided further if contractor thinks that it is incapable of completing, until an agent can solve it individually.

For the problem 2, during the problem-solving process, the contractor communicates progress with the manager such as interim report and final report. The results of subproblem will be aggregated to manager agent, and if this manager agent thinks it cannot solve the problem, it continues to report the results. Eventually, a manager will generate the solution by synthesizing all related results of subproblem.

For the problem 3, the reason for not satisfying coherence metric may be tasks are not properly assigned, resulting in different manager agents having different criteria. Therefore, manager agents should achieve loose coupling when they decompose tasks, and should not have too many subcontractors.

For the problem 4, we can avoid destructive interactions and maximizing effectiveness by using techniques such as focused addressing and directed contracts, request-response mechanism, and node-available message.