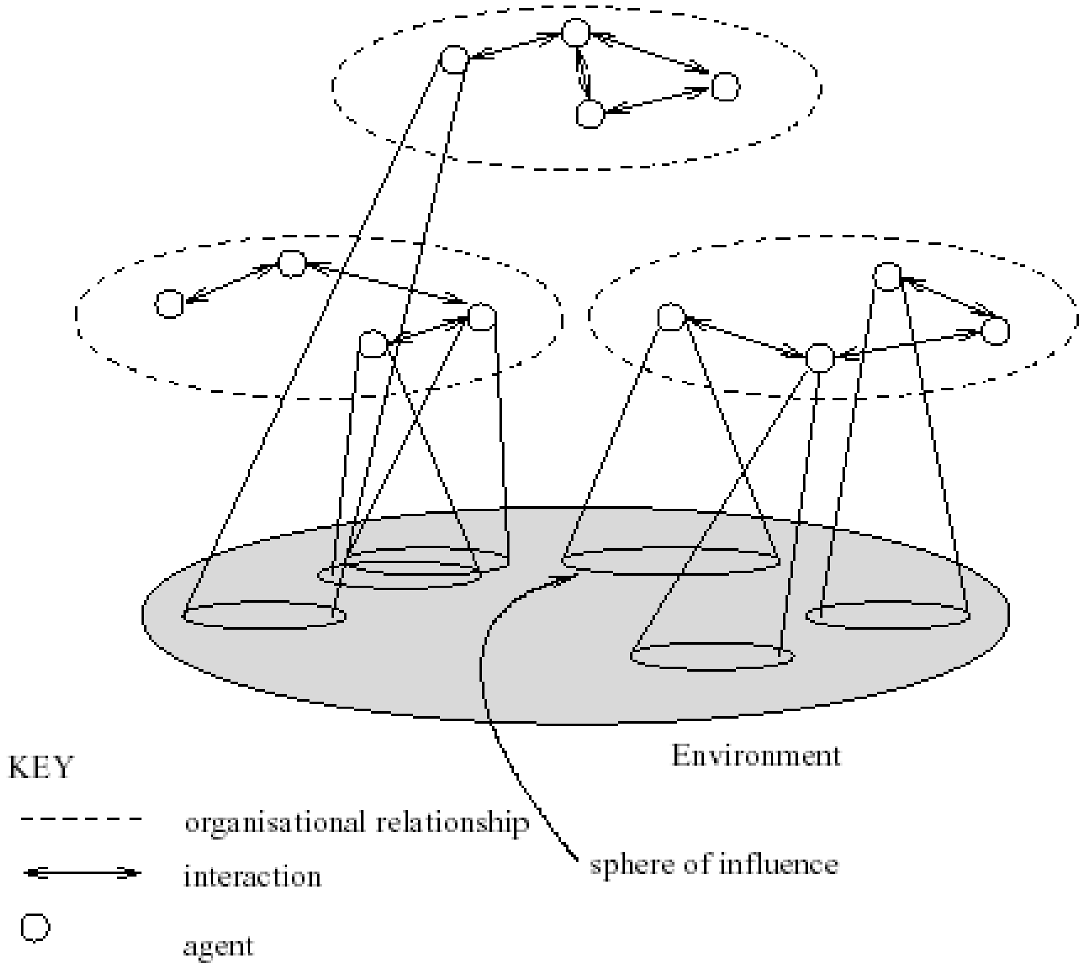
Multiagent Interactions

Outline

- Utilities and preferences
- Solution concepts
- The Prisoner’s Dilemma

The spheres of influence



MultiAgent Systems

Thus a multiagent system contains a number of agents…

- …which interact through communication…
- …are able to act in an environment…
- …have different “spheres of influence” (which may coincide)…
- …will be linked by other (organizational) relationships

Utilities and Preferences

- *Self-interested* agents:
  Each agent has its own *preferences* and *desires* over the states of the world *(non-cooperative game theory)*

- Modelling preferences:
  - Outcomes (states of the world):

$$\Omega = \{w_1, w_2, \ldots\}$$

  - Utility function:

$$u_i\colon \Omega \rightarrow R \text{ (real numbers)}$$

Preference Ordering

Utility functions lead to preference orderings over outcomes

- Preference over *w* $u_i(w) \geq u_i(w') \Leftrightarrow w \succeq w'$
- Strict preference over *w* $u_i(w) > u_i(w') \Leftrightarrow w \succ w'$

Multiagent Encounter

- **Interaction as a game:** the *states* of the world can be seen as the *outcomes* of a game
  - Assume we have just two agents (players) Ag={i,j}
  - The final outcome in Ω depends on the *combination* of actions selected by each agent
- State transformer function:

$$\tau : \underbrace{Ac}_{\text{agent } i\text{'s action}} \times \underbrace{Ac}_{\text{agent } j\text{'s action}} \rightarrow \Omega$$

Multiagent Encounter

- **Normal-form game (or strategic-form game):**
  A strategic interaction is familiarly and generally represented in as a tuple *(N, A, u)*, where:
  - *N* is (finite) the set of player
  - $A = A_1 \times A_2 \times \ldots \times A_n$ where A_i is the set of actions available to player *i*
  - $U = (u_1, u_2, \ldots, u_n)$ utility functions of each player
- **Payoff matrix:**
  n-dimentional matrix with cells enclosing the values of the utility functions for each player

	i: C (strategy)	*i*: D (strategy)
j: C (strategy)	1, 1	1, 4
J: D (strategy)	4,1	4, 4

Multiagent Encounter

- Rational agent’s strategy:

  Given a game scenario, how a rational agent will act?

- Apparently: “just” maximize the expected payoff (single-agent point of view)
  - in most cases unfeasible because the individual best strategy depends on the choices of others (multi-agent point of view)
- In practice: answer in *solution concepts*:
  - *Dominant strategies*
  - *Pareto optimality*
  - *Nash equilibrium*

Solution Concepts

- **Best response:** given the player *j*'s strategy s_j, the player *i*'s best response to s_j is the strategy s_i that gives the highest payoff for player *i*.

	i: C	*i*: D
j: C	1, 1	1, 4
j: D	4,4	4, 1

Example:

j plays C -----> i's best response = D

j plays D -----> i's best response = C

Dominant Strategy

- **Dominant strategy:** A strategy s_i* is *dominant* for player *i* if no matter what strategy s_j agent *j* chooses, *i* will do at least as well playing s_i* as it would doing anything else.
  (s_i* is dominant if it is the *best response* to *all* of agent *j*'s strategies.)

	i: C	*i*: D
j: C	1, 4	1, 1
j: D	4,1	4, 4

Example:

- D is the dominant strategy for player j

-There are not dominant strategies for player i

Dominated strategy can be removed from the table

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	i: C	*i*: D
~~*j*: C~~	~~1, 4~~	~~1, 1~~
j: D	4,1	4, 4

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	i: C	*i*: D
j: D	4,1	4, 4

Example:

- D is the dominant strategy for player j

-There are not dominant strategies for player i

Dominated strategy can be removed from the table

Pareto Optimality

- **Pareto optimality (or Pareto efficiency):** An outcome for the game is said to be *Pareto optimal* (or *Pareto efficient*) if there is no other outcome that makes one agent better off *without* making another agent worse off.

If an outcome *w* is *not Pareto optimal*, then there is another outcome *w'* that makes *everyone as happy, if* not happier, than *w*.
"Reasonable" agents would agree to move to w' in this case. (Even if I don't directly benefit from w', you can benefit without me suffering.)

Example: Pareto Optimality

- **Pareto optimal:** if there is no way to make any agent better off without making some other agent worse off.

Agent 2 / Agent 1	C	D
C	3, 3	0, 5
D	5, 0	1, 1

Example: Pareto Optimality

- **Pareto optimal:** if there is no way to make any agent better off without making some other agent worse off.

Agent 2 / Agent 1	C	D
C	3, 3	0, 5
D	5, 0	1, 1

- (*C*,*C*), (*D*,*C*) and (*C*,*D*) are Pareto optimal : because there is no other spot to choose that does not make at least one player worse off.
- (*D*,*D*) is NOT Pareto optimal (why?)

Nash Equilibrium

Nash equilibrium for pure strategies:
Two strategies *s1* and *s2* are in Nash equilibrium if:

1. under the assumption that agent *i* plays *s1*, agent *j* can do no better than play *s2;*

   AND

2. under the assumption that agent *j plays s2*, agent *i* can do no better than play *s1*.

- Neither agent has any incentive to deviate from a Nash equilibrium.
- Nash equilibrium represents the "Rational" outcome of a game played by self-interested agents.

Unfortunately:

- Not every interaction scenario has a Nash equilibrium.
- Some interaction scenarios have more than one Nash equilibrium.

John Forbes Nash, Jr.
Bluefield. 1928
Nobel Prize for
Economics, 1994
"A beautiful mind"

Finding Nash Equilibria

- **Battle of the sexes game:** husband and wife wish to go to the movies and they are undecided between "FilmA" and "FilmB". They much prefer to go together rather than separate to the movie, althought the wife prefers FilmA and the husband prefers FilmB.

	husband: FilmA	*husband*: FilmB
wife: FilmA	2, 1	0, 0
wife: FilmB	0, 0	1, 2

Finding Nash Equilibria

- **Easy way to find *pure-strategy* Nash equilibria in a payoff matrix:** Take the cell where the first number is the maximum of the column and check if the second number is the maximum of that row.

	husband: FilmA	*husband*: FilmB
wife: FilmA	$\boxed{2, 1}$	0, 0
wife: FilmB	0, 0	$\boxed{1, 2}$

Social Welfare

The social welfare of an outcome ω is the sum of the utilities that each agent gets from ω:

$$\sum_{i \in Ag} u_i(\omega)$$

Think of it as the "total amount of money in the system"

As a solution concept, may be appropriate when the whole system (all agents) has a single owner (then overall benefit of the system is important, not individuals).

Classical Example: The Prisoner's Dilemma

Two men are collectively charged with a crime and held in separate cells, with no way of meeting or communicating. They are told that:

- if one confesses and the other does not, the confessor will be freed, and the other will be jailed for ten years;
- if both confess, then each will be jailed for five years;
- both prisoners know that if neither confesses, then they will each be jailed for one year.

Payoff matrix for prisoner's dilemma:

	Player 2 confesses	*Player 2 does not confess*
Player 1 confesses	5, 5	0, 10
Player 1 does not confess	10, 0	1, 1

Classical Example: The Prisoner's Dilemma

	Player 2 confesses	Player 2 does not confess
Player 1 confesses	(5,5)	(0,10)
Player 1 does not confess	(10,0)	(1,1)

- the best answer is for neither Player 1 nor Player 2 to confess
  - both would get only 1 year in jail
  - but this would require cooperation
- suppose Player 1 decides not to confess
  - if Player 2 confesses, Player 1 is in trouble
- on the other hand, suppose Player 1 confesses, reasoning that Player 2 will confess, too
  - then if Player 2 does not confess, Player 1's outcome is better than he/she was expecting

Classical Example: The Prisoner’s Dilemma (key ideas behind)

Nash Equilibrium

- the “both confess” strategy pair (or solution concept) has the property that neither Play1 nor Play2 can do better by changing his strategy
  - that is, neither can do better without coordinating a change in strategy with the other player

Pareto Optimality

- Pareto optimal solution concept: any change in strategy that causes improvement for one player must make it worse for the other player

Classical Example: The Prisoner’s Dilemma

	Player 2 confesses	Player 2 does not confess
Player 1 confesses	(5,5) ***<u>NASH</u>***	(0,10) ***<u>PARETO</u>***
Player 1 does not confess	(10,0) ***<u>PARETO</u>***	(1,1) ***<u>PARETO</u>***

Summary

- Payoff matrix and solution concepts
- Dominant strategies
- Pareto optimality
- Nash equilibria
- Social welfare
- Classical example: The Prisoner’s dilemma