CSE304 Week 8 In-Class Exercises

Name and Surname:_____

Student ID:_____

Q1. Consider the following characteristic function:

$v(\{1\}) = 100, \ v(\{2\}) = 125, \ v(\{3\}) = 50,$

$v(\{1,2\}) = 270, \ v(\{1,3\}) = 375, \ v(\{2,3\}) = 350$ and $v(\{1,2,3\}) = 500$

Compute the Shapley values for the agents 1, 2 and 3.
Ans:

Probability	Order of arrival	1's marginal contribution	2's marginal contribution	3's marginal contribution
$\frac{1}{6}$	first 1 then 2 then 3: 123	$v(\{1\}) = 100$	$v(\{1,2\}) - v(\{1\}) = 270 - 100$ $= 170$	$v(\{1,2,3\}) - v(\{1,2\}) =$ $500 - 270 = 230$
$\frac{1}{6}$	first 1 then 3 then 2: 132	$v(\{1\}) = 100$	$v(\{1,2,3\}) - v(\{1,3\}) =$ $500 - 375 = 125$	$v(\{1,3\}) - v(\{1\}) = 375 - 100$ $= 275$
$\frac{1}{6}$	first 2 then 1 then 3: 213	$v(\{1,2\}) - v(\{2\}) = 270$ $- 125 = 145$	$v(\{2\}) = 125$	$v(\{1,2,3\}) - v(\{1,2\}) =$ $500 - 270 = 230$
$\frac{1}{6}$	first 2 then 3 then 1: 231	$v(\{1,2,3\}) - v(\{2,3\}) =$ $500 - 350 = 150$	$v(\{2\}) = 125$	$v(\{2,3\}) - v(\{2\}) = 350 - 125$ $= 225$
$\frac{1}{6}$	first 3 then 1 then 2: 312	$v(\{1,3\}) - v(\{3\}) = 375$ $- 50 = 325$	$v(\{1,2,3\}) - v(\{1,3\}) =$ $500 - 375 = 125$	$v(\{3\}) = 50$
$\frac{1}{6}$	first 3 then 2 then 1: 321	$v(\{1,2,3\}) - v(\{2,3\}) =$ $500 - 350 = 150$	$v(\{2,3\}) - v(\{3\}) = 350 - 50 =$ 300	$v(\{3\}) = 50$

Thus 1's expected marginal contribution is: $\frac{1}{6}(100+100+145+150+325+150) = \frac{970}{6}$

2's expected marginal contribution is $\frac{1}{6}170 + \frac{1}{6}125 + \frac{1}{6}125 + \frac{1}{6}125 + \frac{1}{6}125 + \frac{1}{6}300 = \frac{970}{6}$

3's expected marginal contribution is $\frac{1}{6}230 + \frac{1}{6}275 + \frac{1}{6}230 + \frac{1}{6}225 + \frac{1}{6}50 + \frac{1}{6}50 = \frac{1060}{6}$

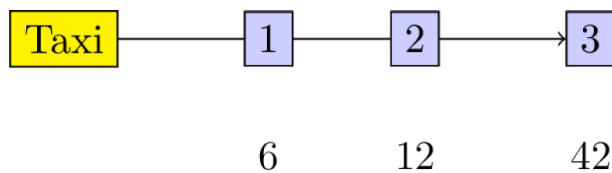The sum, of course, is $\frac{3000}{6} = 500 = v(\{1,2,3\})$

Q2. Consider the following game:

Three students share a taxi. Here are the costs for each individual journey:
- Player 1 - Zhao: 6 (rmb)
- Player 2 - Wang: 12 (rmb)
- Player 3 - Xu: 42 (rmb)

(a). Construct the characteristic function of the above game.
Ans:

$$v(C) = \begin{cases} 6, & \text{if } C = \{1\} \\ 12, & \text{if } C = \{2\} \\ 42, & \text{if } C = \{3\} \\ 12, & \text{if } C = \{1,2\} \\ 42, & \text{if } C = \{1,3\} \\ 42, & \text{if } C = \{2,3\} \\ 42, & \text{if } C = \{1,2,3\} \end{cases}$$

(b). Find a fair way of sharing taxi fare for Zhao, Wang and Xu.
Ans:

First, we make the Shapley value calculation for the above taxi sharing game. We obtain $\phi(C) = (2, 5, 35)$. Thus the fair way of sharing the taxi fare is for Zhao to pay 2, Wang to pay 5 and Xu to pay 35.