Shapley Example

- Suppose we have $Ag = \{1, 2\}$, with the following characteristic function

$$\nu(\{1\}) = 5$$
$$\nu(\{2\}) = 10$$
$$\nu(\{1, 2\}) = 20$$

- We can now calculate the marginal contribution $\delta_i(C)$ of each agent $i \in C$, for each coalition $C \subseteq Ag$

$$\begin{array}{llll} \delta_1(\varnothing) & = \nu(\varnothing \cup \{1\}) - \nu(\varnothing) & = (5 - 0) & = 5 \\ \delta_1(\{2\}) & = \nu(\{2\} \cup \{1\}) - \nu(\{2\}) & = (20 - 10) & = 10 \\ \delta_2(\varnothing) & = \nu(\varnothing \cup \{2\}) - \nu(\varnothing) & = (10 - 0) & = 10 \\ \delta_2(\{1\}) & = \nu(\{1\} \cup \{2\}) - \nu(\{1\}) & = (20 - 5) & = 15 \end{array}$$

- Finally, we can calculate the individual Shapley values for each i:

$$\varphi_1 = \frac{\delta_1(\varnothing) + \delta_1(\{2\})}{|Ag|!} = \frac{5 + 10}{2} = 7.5$$

$$\varphi_2 = \frac{\delta_2(\varnothing) + \delta_2(\{1\})}{|Ag|!} = \frac{10 + 15}{2} = 12.5$$

Shapley Value (reminder)

Marginal Contribution:

$$\delta_i(C) = \boldsymbol{\nu}(C \cup \{i\}) - \boldsymbol{\nu}(C)$$

Shapley value:

$$\varphi_i = \frac{\sum_{o \in \Pi(Ag)} \delta_i(C_i(o))}{|Ag|!}$$

Representation 3: Marginal Contribution Nets

- Characteristic function represented as rules:

$$\text{patern} \longrightarrow \text{value}$$

- Pattern is conjunction of agents, a rule applies to a group of agents C if C is a superset of the agents in the pattern.
  - Value of a coalition is then sum over the values of all the rules that apply to the coalition.
  - Example (rule set 1):

$$a \wedge b \rightarrow 5$$
$$b \rightarrow 2$$

  - We have: $v_{rs1}(\{a\}) = 0$, $v_{rs1}(\{b\}) = 2$, and $v_{rs1}(\{a, b\}) = 5+2 = 7$.

- We can also allow negations in rules (i.e. for when an agent is not present).

rule set 2:

$$a \wedge b \rightarrow 5$$
$$b \rightarrow 2$$
$$c \rightarrow 4$$
$$b \wedge \neg c \rightarrow -2$$

$v_{rs2}(\{a\}) = 0$	no rules apply
$v_{rs2}(\{b\}) = 2+ -2 = 0$	2nd and 4th rules
$v_{rs2}(\{c\}) = 4$	3rd rule
$v_{rs2}(\{a, b\}) = 5+2+-2 = 5$	1st, 2nd and 4th rules
$v_{rs2}(\{a, c\}) = 4$	3rd rule
$v_{rs2}(\{b, c\}) = 2+4 = 6$	2nd and 3rd rules
$v_{rs2}(\{a, b, c\}) = 5+2+4 = 11$	1st, 2nd and 3rd rules