Social Choice

Last time . . .

- Discussed simple, abstract models of multiagent encounters
- Utilities, preferences and outcomes
- Game-theoretic models and solution concepts
- Examples: Prisoner's Dilemma, Coordination Game

Today . . .

- **Social Choice**

Making Group Decisions

- Previously we looked at agents acting strategically
- Outcome in normal-form games follows immediately from agents' choices
- Often a mechanism for deriving group decision is present
- What rules are appropriate to determine the joint decision given individual choices?
- **Social Choice Theory** is concerned with group decision making (basically analysis of mechanisms for voting)
- Basic setting:
  - Agents have preferences over outcomes
  - Agents vote to bring about their most preferred outcome

Preference Aggregation

- Setting:
  - $Ag = \{1, \ldots, n\}$ **voters** (finite, odd number)
  - $\Omega = \{\omega_1, \omega_2, \ldots\}$ possible **outcomes** or **candidates**
  - $\varpi_i \in \Pi(\Omega)$, preference ordering for agent i (e.g. $\omega \succ_i \omega'$)
- Preference Aggregation:

  How do we combine a collection of potentially different preference orders in order to derive a group decision?

- Voting Procedures:
  - **Social Welfare Function**: $f : \Pi(\Omega) \times \ldots \times \Pi(\Omega) \to \Pi(\Omega)$
  - **Social Choice Function**: $f : \Pi(\Omega) \times \ldots \times \Pi(\Omega) \to \Omega$
- Task is either to derive a globally acceptable preference ordering, or determine a winner

Plurality- Single-winner voting system

- Each voter votes for one alternative; the one with the most votes wins
- Example (9 voters):

3 voters	2 voters	4 voters
a	b	c
b	a	b
c	c	a

- Plurality voting has c winning, even though 5-4 majority rate c last
- Advantages: simple to implement and easy to understand
- Problems: Tactical voting(*voters are pressured to vote*)and tie vote

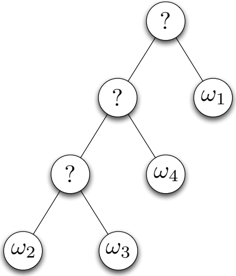
Condorcet's Paradox

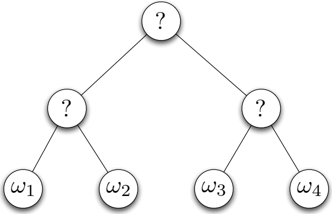
- Outcomes: $\Omega = \{\omega_1, \omega_2, \omega_3\}$
- Voters: $Ag = 1, 2, 3$ with preference orders
  - $\omega_1 \succ_1 \omega_2 \succ_1 \omega_3$
  - $\omega_3 \succ_2 \omega_1 \succ_2 \omega_2$
  - $\omega_2 \succ_3 \omega_3 \succ_3 \omega_1$
- With plurality voting, we obtain a tie
- For every candidate, $\frac{2}{3}$ of the voters prefers another outcome
- **Condorcet's Paradox**:

  There are scenarios in which no matter which outcome we choose the majority of voters will be unhappy with the outcome chosen

Sequential Majority Elections

- Instead of one-step protocol, voting can be done in several steps
- Candidates face each other in **pairwise elections**, the winner progresses to the next election
- **Election agenda** is the ordering of these elections (e.g. $\omega_2, \omega_3, \omega_4, \omega_1$)
- Can be organised as a **binary voting tree**

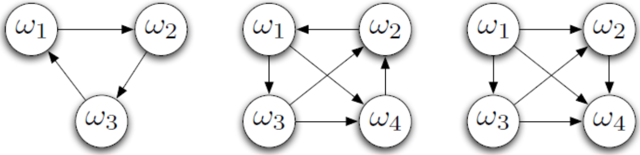




- Key Problem: The final outcome depends on the election agenda

Majority Graphs

- `Pairwise elections are easy to illustrate by using a majority graph`
- A **majority graph** is a succinct representation of voter preferences
- Nodes correspond to outcomes, e.g. $\omega_1, \omega_2, \ldots$
- There is an edge from ω to ω' if a majority of voters rank ω above ω'

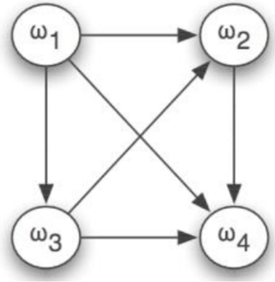


a b c

Condorcet Winners

- A Condorcet winner is a candidate that would beat every other candidate in a pairwise election

- Here, ω_1 is a Condorcet winner



The Borda Count

- Each voter submits preferences over the m alternatives

- Each alternative receives:

  - no points for being ranked last
  - 1 point for being ranked second-to-last

    ....

    ....

  - up to m-1 points for being ranked first

- Points for each alternative are summed across all voters, and the alternative with the highest total is the winner

Borda Count Example

1 voter	1 voter	1 voter	
a	c	b	3 points
b	a	d	2 points
d	b	c	1 point
c	d	a	0 points

- With Borda count, *a* gets 3 points from first voter, 2 points from the second, and 0 from the third
- Final Borda count totals: *a*:5, *b*:6, *c*:4, *d*:3
- *b* is the Borda winner

Desirable Properties (I)

- **Pareto Condition**
  - If every voter ranks ω_i above ω_j then $\omega_i \succ^* \omega_j$
  - Satisfied by plurality and Borda, but not by sequential majority
- **Condorcet winner condition**
  - The outcome would beat every other outcome in a pairwise election
  - Satisfied only by sequential majority elections

Desirable Properties (II)

- **Independence of irrelevant alternatives (IIA)**
  - The social ranking of two outcomes ω_i and ω_j should exclusively depend on their relevant ordering in the preference orders
  - Plurality, Borda and sequential majority elections do not satisfy IIA

- **Non-Dictatorship**
  - None of the voters can systematically impose his preferences on the other ones

Arrow's Theorem

- Overall vision in social choice theory: identify "good" social choice procedures
- Unfortunately, a fundamental theoretical result gets in the way
- **Arrow's Theorem:**

  For elections with more than two outcomes, the only voting procedures that satisfy the Pareto condition and IIA are dictatorships

- Disappointing, basically means we can never achieve combination of good properties without dictatorship

Summary

- Discussed procedures for making group decisions
- Plurality, Sequential Majority Elections, Borda Count
- Desirable properties
- Dictatorships