4.25

$$\frac{d\sigma}{da} = \sigma(a)(1-\sigma(a)) \rightarrow \text{by } 4.88$$

$$\xrightarrow{a=0} \sigma(0)\cdot(1-\sigma(0)) = \frac{1}{2}\left(1-\frac{1}{2}\right) = \frac{1}{4} \#$$

$$\frac{d\phi\lambda a}{da} = \frac{\int_{-\infty}^{\lambda a} N(\theta|0,1)\,d\theta}{da} = \lambda N(a|0,1)$$

$$\xrightarrow{a=0} \lambda N(0|0,1) = \lambda\frac{1}{(2\pi 1^2)^{1/2}} \exp\left\{-\frac{1}{2\cdot 1^2}(0-0)^2\right\}$$

$$= \lambda\cdot\frac{1}{\sqrt{2\pi}} = \frac{\lambda}{\sqrt{2\pi}}$$

$$\rightarrow \frac{\lambda}{\sqrt{2\pi}} = \frac{1}{4} \rightarrow \lambda = \frac{\sqrt{2\pi}}{4} \rightarrow \lambda^2 = \frac{2\pi}{16} = \frac{\pi}{8} \#$$

4.26

$$4.152: \int \Phi(\lambda a) N(a|\mu,\sigma^2)\,da = \Phi\left(\frac{\mu}{(\lambda^{-2}+\sigma^2)^{1/2}}\right)$$

right hand size:

$$\Phi\left(\frac{\mu}{(\lambda^{-2}+\sigma^2)^{\frac{1}{2}}}\right) = \int_{-\infty}^{\frac{\mu}{(\lambda^{-2}+\sigma^2)^{\frac{1}{2}}}} N(\theta|0,1)\,d\theta \xrightarrow[\text{derivative}]{\text{Gaussian}} \left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \exp\left\{-\frac{\mu^2}{2(\lambda^{-2}+\sigma^2)}\right\}\frac{1}{(\lambda^{-2}+\sigma^2)^{\frac{1}{2}}}$$

left hand size, substicute $a \rightarrow \mu+\sigma(z)$

$$\int_{-\infty}^{\infty} \Phi(\lambda\mu+\lambda\sigma z)\frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left\{-\frac{1}{2}\frac{(\mu+\sigma z-\mu)^2}{\sigma^2}\right\}\sigma\,dz$$

$$= \int_{-\infty}^{\infty} \Phi(\lambda\mu+\lambda\sigma z)\frac{1}{(2\pi\sigma)^{\frac{1}{2}}} \exp\left\{-\frac{z^2}{2}\right\}\sigma\,dz$$

$\downarrow \frac{d}{d\mu}$ $\quad\Phi(\lambda\mu+\lambda\sigma z)$ derivative to μ

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp\left\{-\frac{z^2}{2}-\frac{\lambda^2}{2}(\mu+\sigma z)^2\right\}\sigma\,dz$$

Expand the part in exponential :

$$-\frac{1}{2}z^2 - \frac{\lambda^2}{2}(\mu+\sigma z)^2$$

$$= -\frac{1}{2}z^2 - \frac{\lambda^2}{2}(\mu^2 + 2\mu\sigma z + \sigma^2 z^2)$$

$$= -\frac{1}{2}z^2(1+\lambda^2\sigma^2) - z\lambda^2\mu\sigma - \frac{1}{2}\lambda^2\mu^2$$

$$= -\frac{1}{2}\left[z + \lambda^2\mu\sigma(1+\lambda^2\sigma^2)^{-1}\right]^2(1+\lambda^2\sigma^2) + \frac{1}{2}\frac{\lambda^4\mu^2\sigma^2}{(1+\lambda^2\sigma^2)} - \frac{1}{2}\lambda^2\mu^2$$

$\downarrow$ Integrate over z

$$\frac{1}{(2\pi)^{\frac{1}{2}}}\cdot\frac{1}{(1+\lambda^2\sigma^2)^{\frac{1}{2}}}\cdot\exp\left\{-\frac{1}{2}\lambda^2\mu^2+\frac{1}{2}\frac{\lambda^4\mu^2\sigma^2}{(1+\lambda^2\sigma^2)}\right\}$$

$$= \frac{1}{(2\pi)^{\frac{1}{2}}}\frac{1}{(1+\lambda^2\sigma^2)^{\frac{1}{2}}}\exp\left\{-\frac{1}{2}\frac{\lambda^2\mu^2+\cancel{\lambda^4\mu^2\sigma^2}-\cancel{\lambda^4\mu^2\sigma^2}}{(1+\lambda^2\sigma^2)^2}\right\}$$

$$= \frac{1}{\sqrt{2\pi}}\cdot\frac{1}{\sqrt{1+\lambda^2\sigma^2}}\exp\left\{-\frac{1}{2}\frac{\lambda^2\mu^2}{(1+\lambda^2\sigma^2)^2}\right\}$$

Thus, left hand = right hand " in respect to μ ".

when $\mu \to -\infty$

left hand size $\to 0$ $\left(\frac{1}{\sqrt{2\pi}}\cdot\frac{1}{\sqrt{1+\lambda^2\sigma^2}}\cdot\exp(-\infty)\right)$

right hand size $\to 0$ $\left(\left(\frac{1}{2\pi}\right)^{\frac{1}{2}}\exp(-\infty)\cdot\frac{1}{(\lambda^{-2}+\sigma^2)^{\frac{1}{2}}}\right)$

$\to$ constant integration $\to 0$