統計學習初論 **HW1 Q2(a)** **b03705002** 林軒逸

$$\sum_{ML}^{N} = \frac{1}{N}\sum_{i=1}^{N}(xi - \mu_{ML}^{N})(xi - \mu_{ML}^{N})^T$$

$$= \frac{1}{N}(xN - \mu_{ML}^{N})(xN - \mu_{ML}^{N})^T + \frac{1}{\mathrm{N}}\sum_{i=1}^{N-1}(xi - \mu_{ML}^{N})(xi - \mu_{ML}^{N})^T$$

$$= \frac{1}{N}(xN - \mu_{ML}^{N})(xN - \mu_{ML}^{N})^T + \frac{1}{N}\sum_{i=1}^{N-1}\left(xi - \mu_{ML}^{N-1} - \frac{1}{N}(xN - \mu_{ML}^{N-1})\right)\left(xi - \mu_{ML}^{N-1} - \frac{1}{N}(xN - \mu_{ML}^{N-1})\right)^T$$

$$= \frac{1}{N}(xN - \mu_{ML}^{N})(xN - \mu_{ML}^{N})^T + \frac{1}{\mathrm{N}}\sum_{i=1}^{N-1}(xi - \mu_{ML}^{N-1})(xi - \mu_{ML}^{N-1})^T$$

$$-\frac{1}{N^2}\sum_{i=1}^{N-1}(xi - \mu_{ML}^{N-1})(xN - \mu_{ML}^{N-1})^T - \frac{1}{N^2}\sum_{i=1}^{N-1}(xN - \mu_{ML}^{N-1})(xi - \mu_{ML}^{N-1})^T + \frac{1}{N^3}\sum_{i=1}^{N-1}(xN - \mu_{ML}^{N-1})(xN - \mu_{ML}^{N-1})^T$$

$$= \frac{1}{N}(xN - \mu_{ML}^{N})(xN - \mu_{ML}^{N})^T + \frac{N-1}{N}\sum_{ML}^{N-1} - \frac{1}{N^2}\sum_{i=1}^{N-1}(xi - \mu_{ML}^{N-1})(xN - \mu_{ML}^{N-1})^T$$

$$-\frac{1}{N^2}\sum_{i=1}^{N-1}(xN - \mu_{ML}^{N-1})(xi - \mu_{ML}^{N-1})^T + \frac{1}{N^3}\sum_{i=1}^{N-1}(xN - \mu_{ML}^{N-1})(xN - \mu_{ML}^{N-1})^T$$

$$= \frac{1}{N}(xN - \mu_{ML}^{N})(xN - \mu_{ML}^{N})^T + \frac{N-1}{N}\sum_{ML}^{N-1} + \frac{1}{N^3}\sum_{i=1}^{N-1}(xN - \mu_{ML}^{N-1})(xN - \mu_{ML}^{N-1})^T$$

$$-\frac{1}{N^2}\sum_{i=1}^{N-1}xi(xN - \mu_{ML}^{N-1})^T - \mu_{ML}^{N-1}(xN - \mu_{ML}^{N-1})^T - \frac{1}{N^2}\sum_{i=1}^{N-1}xi^T(xN - \mu_{ML}^{N-1}) - \mu_{ML}^{N-1^T}(xN - \mu_{ML}^{N-1})$$

$$= \frac{1}{N}(xN - \mu_{ML}^{N})(xN - \mu_{ML}^{N})^T + \frac{N-1}{N}\sum_{ML}^{N-1} + \frac{N-1}{N^3}(xN - \mu_{ML}^{N-1})(xN - \mu_{ML}^{N-1})^T$$

$$-\frac{1}{\mathrm{N}^2}(\sum_{i=1}^{N-1}xi)(xN - \mu_{ML}^{N-1})^T - \frac{1}{N^2}\sum_{i=1}^{N-1}\mu_{ML}^{N-1}(xN - \mu_{ML}^{N-1})^T$$

$$(\mu_{ML}^{N-1^T} = \mu_{ML}^{N-1} \quad)$$

$$-\frac{1}{N^2}\sum_{i=1}^{N-1}(xN - \mu_{ML}^{N-1}) - \mu_{ML}^{N-1^T}(xN - \mu_{ML}^{N-1})$$

$$= \frac{1}{N}(xN - \mu_{ML}^{N}\ )(xN - \mu_{ML}^{N}\ )^T + \frac{N-1}{N}\sum_{ML}^{N-1} + \frac{N-1}{N^3}(xN - \mu_{ML}^{N-1})(xN - \mu_{ML}^{N-1})^T$$

$$-\left(\ \frac{\mathrm{N}-1}{\mathrm{N}^2}\mu_{ML}^{N-1}(xN - \mu_{ML}^{N}\ )^T - \frac{\mathrm{N}-1}{\mathrm{N}^2}\mu_{ML}^{N-1}(xN - \mu_{ML}^{N}\ )^T\right)$$

$$-\left(\ \frac{\mathrm{N}-1}{\mathrm{N}^2}\mu_{ML}^{N-1}(xN - \mu_{ML}^{N}\ ) \quad - \frac{\mathrm{N}-1}{\mathrm{N}^2}\mu_{ML}^{N-1}(xN - \mu_{ML}^{N}\ )\ \right)$$

$$= \frac{1}{N}(xN - \mu_{ML}^{N}\ )(xN - \mu_{ML}^{N}\ )^T + \frac{N-1}{N}\sum_{ML}^{N-1} + \frac{N-1}{N^3}(xN - \mu_{ML}^{N-1})(xN - \mu_{ML}^{N-1})^T - 0 - 0$$

$$= \frac{1}{N}(xN - \mu_{ML}^{N}\ )(xN - \mu_{ML}^{N}\ )^T + \frac{N-1}{N}\sum_{ML}^{N-1} + \frac{N-1}{N^3}(xN - \mu_{ML}^{N-1})(xN - \mu_{ML}^{N-1})^T \quad \#$$