Most probably transition pathway of biology-modelling SDEs

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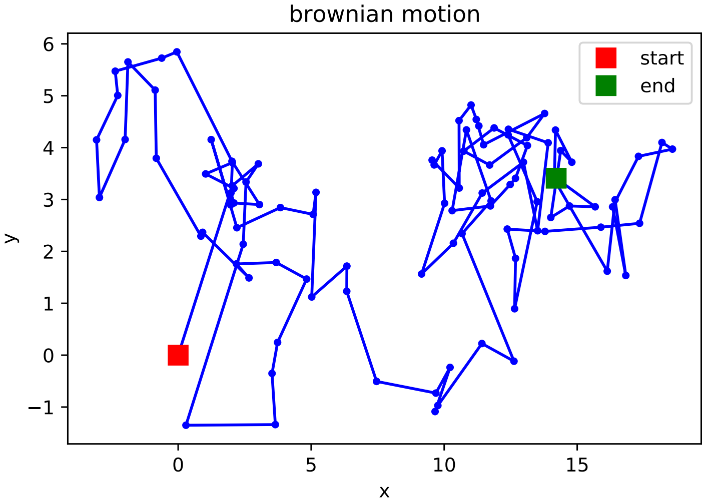
- All physical systems are "noisy"
- Noise: think of Brownian motion
- Accounts for sudden changes in system
- Many biological processes are stochastic: mutation, gene regulation, etc.

Brownian Motion



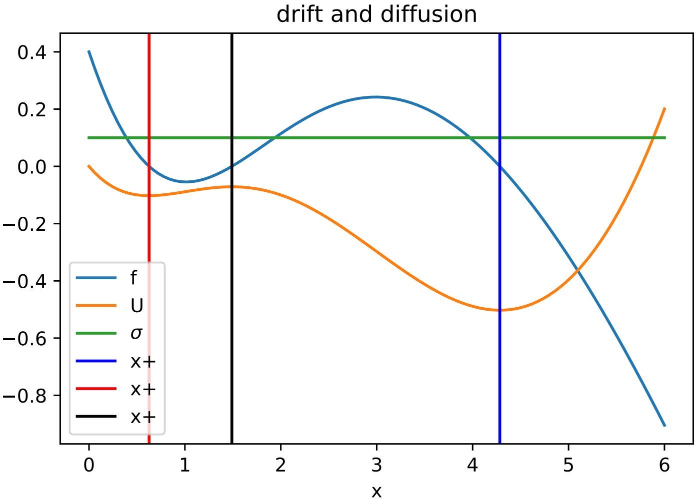
What's an SDE?

The definition of a *stochastic differential equation* given a stochastic process X_t, drift function $f(X_t, t)$, and diffusion function $\sigma(X_t, t)$:

$$dX_t = f(X_t, t)dt + \sigma(X_t, t)dB_t \tag{1}$$

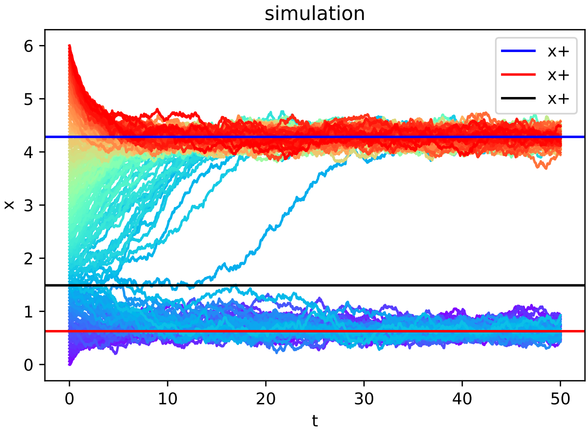
Think: f determines direction, σ denotes noise intensity

Our Model



Simulation

Given f and σ, we simulate the SDE with the Euler-Maruyama method.



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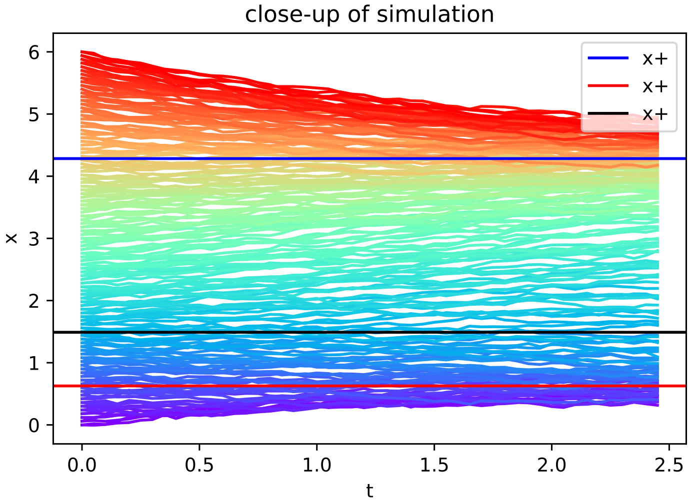
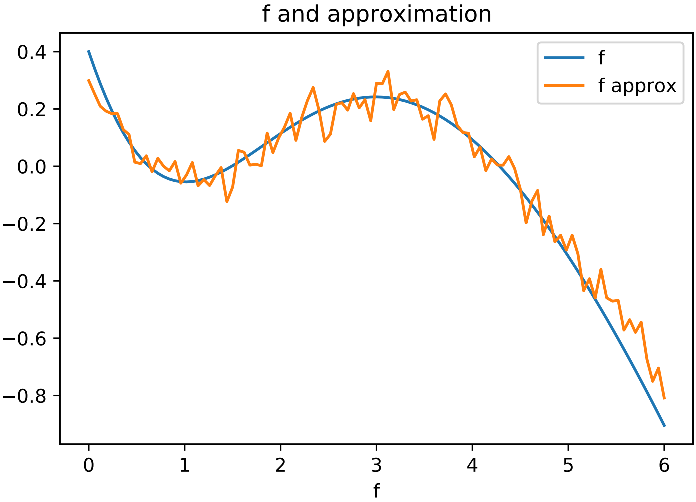
$$f(x) = \lim_{\Delta t \to 0} \mathbb{E}\left(\frac{X_{\Delta t} - X_0}{\Delta t} \middle| X_0 = x \right) \tag{2}$$

$$\sigma^2(x) = \lim_{\Delta t \to 0} \mathbb{E}\left(\frac{(X_{\Delta t} - X_0)^2}{\Delta t} \middle| X_0 = x \right) \tag{3}$$

Simulation Zoom-In



Learned f



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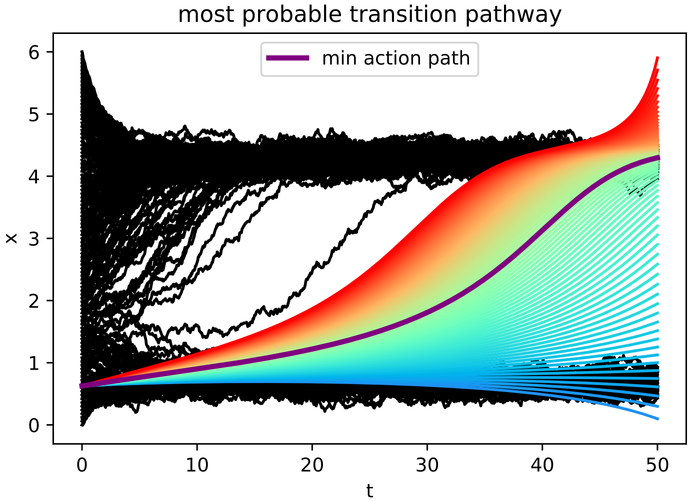
- Solving analytically is very difficult
- The shooting method lets us find the "velocity" that minimizes the "loss"

Learned Probable Pathway



Future Work

- Fine tune/optimize specific data extraction
- Applying machine learning techniques to pathway distribution