CHAPTER 36
Real Options

Practice Questions

36.1
In the net present value approach, cash flows are estimated in the real world and discounted at a risk-adjusted discount rate. In the risk-neutral valuation approach, cash flows are estimated in the risk-neutral world and discounted at the risk-free interest rate. The risk-neutral valuation approach is arguably more appropriate for valuing real options because it is very difficult to determine the appropriate risk-adjusted discount rate when options are valued.

36.2
In a risk-neutral world, the expected price of copper in six months is 75 cents. This corresponds to an expected growth rate of $2\ln(75/80) = -12.9\%$ per annum. The decrease in the growth rate when we move from the real world to the risk-neutral world is the volatility of copper times its market price of risk. This is $0.2 \times 0.5 = 0.1$ or 10% per annum. It follows that the expected growth rate of the price of copper in the real world is -2.9%.

36.3
We explained the concept of a convenience yield for a commodity in Chapter 5. It is a measure of the benefits realized from ownership of the physical commodity that are not realized by the holders of a futures contract. If y is the convenience yield and u is the storage cost, equation (5.17) shows that the commodity behaves like an investment asset that provides a return equal to $y - u$. In a risk-neutral world its growth is, therefore,

$$r - (y - u) = r - y + u$$

The convenience yield of a commodity can be related to its market price of risk. From Section 36.2, the expected growth of the commodity price in a risk-neutral world is $m - \lambda s$, where m is its expected growth in the real world, s its volatility, and λ is its market price of risk. It follows that

$$m - \lambda s = r - y + u$$

36.4
In equation (36.2), $\rho = 0.2$, $\mu_m - r = 0.06$, and $\sigma_m = 0.18$. It follows that the market price of risk lambda is

$$\frac{0.2 \times 0.06}{0.18} = 0.067$$

36.5
The option can be valued using Black's model. In this case, $F_0 = 24$, $K = 25$, $r = 0.05$, $\sigma = 0.2$, and $T = 3$. The value of an option to purchase one unit at \$25 is

$$e^{-rT}[F_0 N(d_1) - KN(d_2)]$$

where

$$d_1 = \frac{\ln(F_0 / K) + \sigma^2 T / 2}{\sigma\sqrt{T}}$$
$$d_2 = \frac{\ln(F_0 / K) - \sigma^2 T / 2}{\sigma\sqrt{T}}$$
This is 2.489. The value of the option to purchase one million units is therefore $2,489,000.

36.6

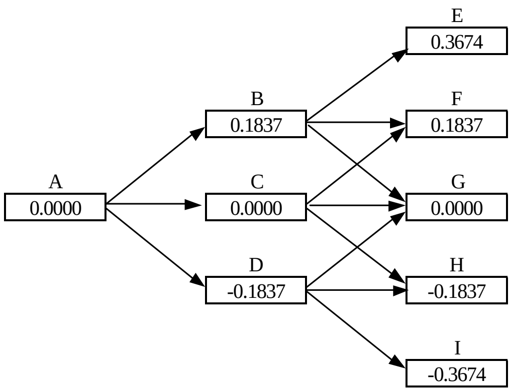
The expected growth rate of the car price in a risk-neutral world is $-0.25 - (-0.1 \times 0.15) = -0.235$ The expected value of the car in a risk-neutral world in four years, $\hat{E}(S_T)$, is therefore $30{,}000e^{-0.235\times 4} = \$11{,}719$. Using the result in the appendix to Chapter 15, the value of the option is
$$e^{-rT}[\hat{E}(S_T)N(d_1) - KN(d_2)]$$
where
$$d_1 = \frac{\ln(\hat{E}(S_T) / K) + \sigma^2 T / 2}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(\hat{E}(S_T) / K) - \sigma^2 T / 2}{\sigma\sqrt{T}}$$
$r = 0.06$, $\sigma = 0.15$, $T = 4$, and $K = 10{,}000$. It is $1,832.

36.7

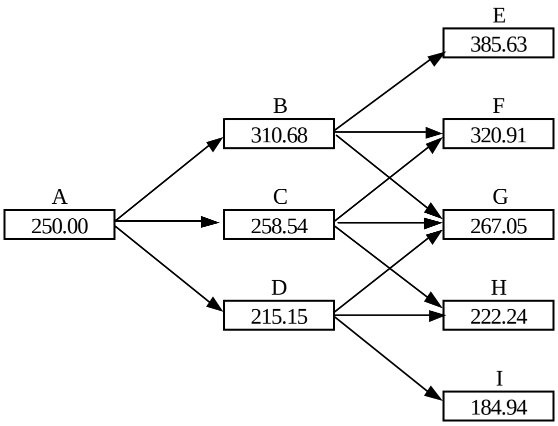
In this case, $a = 0.05$ and $\sigma = 0.15$. We first define a variable X that follows the process
$$dX = -a\,dt + \sigma\,dz$$
A tree for X constructed in the way described in Chapters 32 and 35 is shown in Figure S36.1. We now displace nodes so that the tree models $\ln S$ in a risk-neutral world where S is the price of wheat. The displacements are chosen so that the initial price of wheat is 250 cents and the expected prices at the ends of the first and second time steps are 260 and 270 cents, respectively. Suppose that the displacement to give $\ln S$ at the end of the first time step is α_1. Then
$$0.1667e^{\alpha_1+0.1837} + 0.6666e^{\alpha_1} + 0.1667e^{\alpha_1-0.1837} = 260$$
so that $\alpha_1 = 5.5551$ The probabilities of nodes E, F, G, H, and I being reached are 0.0257, 0.2221, 0.5043, 0.2221, and 0.0257, respectively. Suppose that the displacement to give $\ln S$ at the end of the second step is α_2. Then
$$0.0257e^{\alpha_2+0.3674} + 0.2221e^{\alpha_2+0.1837} + 0.5043e^{\alpha_2} + 0.2221e^{\alpha_2-0.1837}$$

$$+0.0257e^{\alpha_2-0.3674} = 270$$
so that $\alpha_2 = 5.5874$. This leads to the tree for the price of wheat shown in Figure S36.2. Using risk-neutral valuation, the value of the project (in thousands of dollars) is
$$-10 - 90e^{-0.05\times 0.5} + 2.70 \times 40e^{-0.05\times 1} = 4.94$$
This shows that the project is worth undertaking. Figure S36.3 shows the value of the project on a tree. The project should be abandoned at node D for a saving of 2.41. Figure S36.4 shows that the abandonment option is worth 0.39.



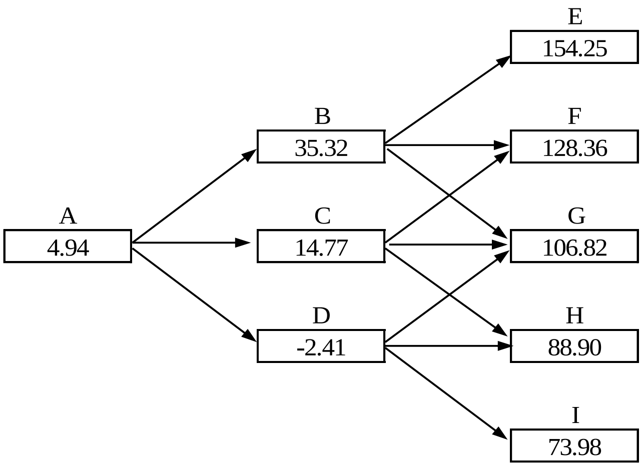
Node	A	B	C	D
p_u	0.1667	0.1545	0.1667	0.1795
p_m	0.6666	0.6660	0.6666	0.6660
p_d	0.1667	0.1795	0.1667	0.1545

Figure S36.1: *Tree for X in Problem 36.7*



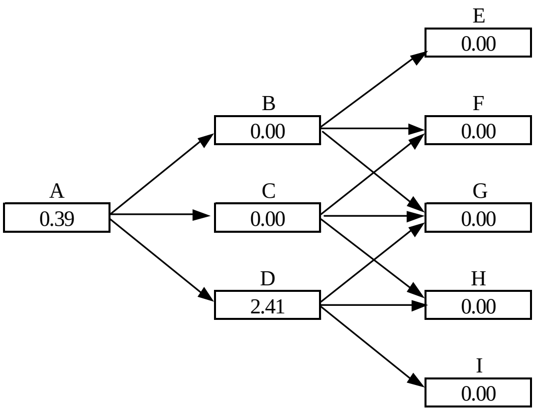
Node	A	B	C	D
p_u	0.1667	0.1545	0.1667	0.1795
p_m	0.6666	0.6660	0.6666	0.6660
p_d	0.1667	0.1795	0.1667	0.1545

Figure S36.2: *Tree for price of wheat in Problem 36.7*



Node	A	B	C	D
p_u	0.1667	0.1545	0.1667	0.1795
p_m	0.6666	0.6660	0.6666	0.6660
p_d	0.1667	0.1795	0.1667	0.1545

Figure S36.3: *Tree for value of project in Problem 36.7*



Node	A	B	C	D
p_u	0.1667	0.1545	0.1667	0.1795
p_m	0.6666	0.6660	0.6666	0.6660
p_d	0.1667	0.1795	0.1667	0.1545

Figure S36.4: *Tree for abandonment option in Problem 36.7*