CHAPTER 24
Credit Risk

Practice Questions

24.1
From equation (24.2), the average hazard rate over the three years is $0.0050/(1-0.3)=0.0071$ or 0.71% per year.

24.2
From equation (24.2), the average hazard rate over the five years is $0.0060/(1-0.3)=0.0086$ or 0.86% per year. Using the results in the previous question, the hazard rate is 0.71% per year for the first three years and

$$\frac{0.0086\times 5-0.0071\times 3}{2}=0.0107$$

or 1.07% per year in years 4 and 5.

24.3
Real-world probabilities of default should be used for calculating credit value at risk. Risk-neutral probabilities of default should be used for adjusting the price of a derivative for default.

24.4
The recovery rate for a bond is the value of the bond shortly after the issuer defaults as a percent of its face value.

24.5
The hazard rate, $h(t)$ at time t is defined so that $h(t)\Delta t$ is the probability of default between times t and $t+\Delta t$ conditional on no default prior to time t. The unconditional default probability density $q(t)$ is defined so that $q(t)\Delta t$ is the probability of default between times t and $t+\Delta t$ as seen at time zero.

24.6
The seven-year historical hazard rate for AAA-rated companies is

$$-[\ln(1-0.0051)]/7=0.00073$$

or 0.073% per year. Similarly, the historical hazard rates for AA, A, BBB, BB, B, and CCC/C companies are 0.072%, 0.113%, 0.337%, 1.349%, 3.252%, 9.413% , respectively. The spread to compensate for default are 60% of these. In basis points, the spreads for AAA, AA, A, BBB, BB, B, and CCC/C companies are 4.4, 4.3, 6.8, 20.2, 80.9, 195.1, and 564.8.

24.7
Suppose company A goes bankrupt when it has a number of outstanding contracts with company B. Netting means that the contracts with a positive value to A are netted against those with a negative value in order to determine how much, if anything, company A owes

company B. Company A is not allowed to "cherry pick" by keeping the positive-value contracts and defaulting on the negative-value contracts.
The new transaction will increase the bank's exposure to the counterparty if the contract tends to have a positive value whenever the existing contract has a positive value and a negative value whenever the existing contract has a negative value. However, if the new transaction tends to offset the existing transaction, it is likely to have the incremental effect of reducing credit risk.

24.8
When a bank is experiencing financial difficulties, its credit spread is likely to increase. This increases q_i^* and DVA increases. This is a benefit to the bank: the fact that it is more likely to default means that its derivatives are worth less.

24.9
(a) In the Gaussian copula model for time to default, a credit loss is recognized only when a default occurs. In CreditMetrics, it is recognized when there is a credit downgrade as well as when there is a default.
(b) In the Gaussian copula model of time to default, the default correlation arises because the value of the factor M . This defines the default environment or average default rate in the economy. In CreditMetrics, a copula model is applied to credit ratings migration and this determines the joint probability of particular changes in the credit ratings of two companies.

24.10
When the claim amount is the no-default value, the loss for a corporate bond arising from a default at time t is

$$v(t)(1-\hat{R})B^*$$

where $v(t)$ is the discount factor for time t and B^* is the no-default value of the bond at time t. Suppose that the zero-coupon bonds comprising the corporate bond have no-default values at time t of Z_1, Z_2, …, Z_n, respectively. The loss from the ith zero-coupon bond arising from a default at time t is

$$v(t)(1-\hat{R})Z_i$$

The total loss from all the zero-coupon bonds is

$$v(t)(1-\hat{R})\sum_i^n Z_i = v(t)(1-\hat{R})B^*$$

This shows that the loss arising from a default at time t is the same for the corporate bond as for the portfolio of its constituent zero-coupon bonds. It follows that the value of the corporate bond is the same as the value of its constituent zero-coupon bonds.
When the claim amount is the face value plus accrued interest, the loss for a corporate bond arising from a default at time t is

$$v(t)B^* - v(t)\hat{R}[L + a(t)]$$

where L is the face value and $a(t)$ is the accrued interest at time t. In general, this is not the same as the loss from the sum of the losses on the constituent zero-coupon bonds.

24.11

Define Q as the risk-free rate. The calculations are as follows:

Time (yrs)	*Def. Prob.*	*Recovery Amount ($)*	*Risk-free Value ($)*	*Loss Given Default ($)*	*Discount Factor*	*PV of Expected Loss ($)*
1.0	Q	30	104.78	74.78	0.9704	$72.57Q$
2.0	Q	30	103.88	73.88	0.9418	$69.58Q$
3.0	Q	30	102.96	72.96	0.9139	$66.68Q$
4.0	Q	30	102.00	72.00	0.8869	$63.86Q$
Total						$272.69Q$

The bond pays a coupon of 2 every six months and has a continuously compounded yield of 5% per year. Its market price is 96.19. The risk-free value of the bond is obtained by discounting the promised cash flows at 3%. It is 103.66. The total loss from defaults should therefore be equated to $103.66-96.19=7.46$. The value of Q implied by the bond price is therefore given by $272.69Q=7.46$. or $Q=0.0274$. The implied probability of default is 2.74% per year.

24.12

The table for the first bond is as follows:

Time (yrs)	*Def. Prob.*	*Recovery Amount ($)*	*Risk-free Value ($)*	*Loss Given Default ($)*	*Discount Factor*	*PV of Expected Loss ($)*
0.5	Q_1	40	103.01	63.01	0.9827	$61.92Q_1$
1.5	Q_1	40	102.61	62.61	0.9489	$59.41Q_1$
2.5	Q_1	40	102.20	62.20	0.9162	$56.98Q_1$
Total						$178.31Q_1$

The market price of the bond is 98.35 and the risk-free value is 101.23. It follows that Q_1 is given by

$$178.31Q_1=101.23-98.35$$

so that $Q_1=0.0161$.

The table for the second bond is as follows:

Time (yrs)	*Def. Prob.*	*Recovery Amount ($)*	*Risk-free Value ($)*	*Loss Given Default ($)*	*Discount Factor*	*PV of Expected Loss ($)*
0.5	Q_1	40	103.77	63.77	0.9827	$62.67Q_1$
1.5	Q_1	40	103.40	63.40	0.9489	$60.16Q_1$
2.5	Q_1	40	103.01	63.01	0.9162	$57.73Q_1$
3.5	Q_2	40	102.61	62.61	0.8847	$55.39Q_2$
4.5	Q_2	40	102.20	62.20	0.8543	$53.13Q_2$
Total						$180.56Q_1 + 108.53Q_2$

The market price of the bond is 96.24 and the risk-free value is 101.97. It follows that

$$180.56Q_1 + 108.53Q_2 = 101.97 - 96.24$$

From which we get $Q_2 = 0.0260$. The bond prices therefore imply a probability of default of 1.61% per year for the first three years and 2.60% for the next two years.

24.13
The statements in (a) and (b) are true. The statement in (c) is not. Suppose that v_X and v_Y are the exposures to X and Y. The expected value of $v_X + v_Y$ is the expected value of v_X plus the expected value of v_Y. The same is not true of 95% confidence limits.

24.14
Assume that defaults happen only at the end of the life of the forward contract. In a default-free world, the forward contract is the combination of a long European call and a short European put where the strike price of the options equals the delivery price and the maturity of the options equals the maturity of the forward contract. If the no-default value of the contract is positive at maturity, the call has a positive value and the put is worth zero. The impact of defaults on the forward contract is the same as that on the call. If the no-default value of the contract is negative at maturity, the call has a zero value and the put has a positive value. In this case, defaults have no effect. Again, the impact of defaults on the forward contract is the same as that on the call. It follows that the contract has a value equal to a long position in a call that is subject to default risk and short position in a default-free put.

24.15
Suppose that the forward contract provides a payoff at time T. With our usual notation, the

value of a long forward contract is $S_T - Ke^{-rT}$. The credit exposure on a long forward contract is therefore $\max(S_T - Ke^{-rT}, 0)$; that is, it is a call on the asset price with strike price Ke^{-rT}. Similarly, the credit exposure on a short forward contract is $\max(Ke^{-rT} - S_T, 0)$; that is, it is a put on the asset price with strike price Ke^{-rT}. The total credit exposure is, therefore, a straddle with strike price Ke^{-rT}.

24.16

The credit risk on a matched pair of interest rate swaps is $|B_{\text{fixed}} - B_{\text{floating}}|$. As maturity is approached, all bond prices tend to par and this tends to zero. The credit risk on a matched pair of currency swaps is $|SB_{\text{foreign}} - B_{\text{fixed}}|$ where S is the exchange rate. The expected value of this tends to increase as the swap maturity is approached because of the uncertainty in S.

24.17

As time passes, there is a tendency for the currency which has the lower interest rate to strengthen. This means that a swap where we are receiving this currency will tend to move in the money (i.e., have a positive value). Similarly, a swap where we are paying the currency will tend to move out of the money (i.e., have a negative value). From this, it follows that our expected exposure on the swap where we are receiving the low-interest currency is much greater than our expected exposure on the swap where we are receiving the high-interest currency. We should therefore look for counterparties with a low credit risk on the side of the swap where we are receiving the low-interest currency. On the other side of the swap, we are far less concerned about the creditworthiness of the counterparty.

24.18

No, put–call parity does not hold when there is default risk. Suppose c^* and p^* are the no-default prices of a European call and put with strike price K and maturity T on a non-dividend-paying stock whose price is S, and that c and p are the corresponding values when there is default risk. The text shows that when we make the independence assumption (that is, we assume that the variables determining the no-default value of the option are independent of the variables determining default probabilities and recovery rates), $c = c^* e^{-[y(T)-y^*(T)]T}$ and $p = p^* e^{-[y(T)-y^*(T)]T}$. The relationship

$$c^* + Ke^{-y^*(T)T} = p^* + S$$

which holds in a no-default world therefore becomes

$$c + Ke^{-y(T)T} = p + Se^{-[y(T)-y^*(T)]T}$$

when there is default risk. This is not the same as regular put–call parity. What is more, the relationship depends on the independence assumption and cannot be deduced from the same sort of simple no-arbitrage arguments that we used in Chapter 11 for the put–call parity relationship in a no-default world.

24.19

We can assume that the principal is paid and received at the end of the life of the swap without changing the swap's value. Because the coupons on the bond are paid regardless of whether the bond defaults, the value of what one side pays is the default-free value of the bond. This is the current market value of the bond plus the present value of defaults. The value of what the other side pays is the value of a floating rate bond plus the present value of the spreads. Hence, the current market value of the bond plus present value of cost of defaults equals value of floating rate bond plus present value of the spreads. We are told that the bond is worth par. The floating rate bond is also worth par. It follows that the present value of the cost of defaults equals the present value of the spread. (Note that if the bond is not worth par, the asset swap is structured so that one side initially pays the difference between its value and par to the other side. This preserves our result that the present value of the spreads equals the present value of the cost of defaults.)

24.20

The value of the debt in Merton's model is $V_0 - E_0$ or

$$De^{-rT}N(d_2) - V_0N(d_1) + V_0 = De^{-rT}N(d_2) + V_0N(-d_1)$$

If the credit spread is *s*, this should equal $De^{-(r+s)T}$ so that

$$De^{-(r+s)T} = De^{-rT}N(d_2) + V_0N(-d_1)$$

Substituting $De^{-rT} = LV_0$

$$LV_0e^{-sT} = LV_0N(d_2) + V_0N(-d_1)$$

or

$$Le^{-sT} = LN(d_2) + N(-d_1)$$

so that

$$s = -\ln[N(d_2) + N(-d_1)/L]/T$$

24.21

When the default risk of the seller of the option is taken into account, the option value is the Black–Scholes price multiplied by $e^{-0.01\times3} = 0.9704$. Black–Scholes overprices the option by about 3%.

24.22

Right way risk describes the situation when a default by the counterparty is most likely to occur when the contract has a positive value to the counterparty. An example of right way risk would be when a counterparty's future depends on the price of a commodity and it enters into a contract to partially hedging that exposure.

Wrong way risk describes the situation when a default by the counterparty is most likely to occur when the contract has a negative value to the counterparty. An example of right way risk would be when a counterparty is a speculator and the contract has the same exposure as the rest of the counterparty's portfolio.

24.23

Year	*Cumulative average hazard rate (%)*	*Average hazard rate during year (%)*
1	0.77	0.77
2	0.92	1.08
3	1.08	1.38
4	1.23	1.69
5	1.33	1.77

24.24 (Excel file)

The market price of the bond is 105.51. The risk-free price is 108.40. The expected cost of defaults is therefore 2.89. We need to find the hazard rate λ that leads to the expected cost of defaults being 2.89. We need to make an assumption about how the probability of default at a time is calculated from the hazard rate. The following table shows the calculations:

Time (yrs)	*Def. Prob.*	*Recovery Amount ($)*	*Risk-free Value ($)*	*Loss Given Default ($)*	*Discount Factor*	*PV of Loss Given Default ($)*
0.5	$1-e^{-0.5\lambda}$	45	110.57	65.57	0.9804	64.28
1.0	$e^{-0.5\lambda}-e^{-1.0\lambda}$	45	109.21	64.21	0.9612	61.73
1.5	$e^{-1.0\lambda}-e^{-1.5\lambda}$	45	107.83	62.83	0.9423	59.20
2.0	$e^{-1.5\lambda}-e^{-2.0\lambda}$	45	106.41	61.41	0.9238	56.74
2.5	$e^{-2.0\lambda}-e^{-2.5\lambda}$	45	104.97	59.97	0.9057	54.32
3.0	$e^{-2.5\lambda}-e^{-3.0\lambda}$	45	103.50	58.50	0.8880	51.95

Solver can be used to determine the value of λ such that
$(1-e^{-0.5\lambda})\times 64.28 + (e^{-1.0\lambda}-e^{-0.5\lambda})\times 61.73 + (e^{-1.5\lambda}-e^{-1.0\lambda})\times 59.20 + (e^{-2.0\lambda}-e^{-1.5\lambda})\times 56.74$
$+ (e^{-2.5\lambda}-e^{-2.0\lambda})\times 54.32 + (e^{-3.0\lambda}-e^{-2.5\lambda})\times 51.95 = 2.89$
It is 1.70%.

24.25

Real world default probabilities are the true probabilities of defaults. They can be estimated from historical data. Risk-neutral default probabilities are the probabilities of defaults in a world where all market participants are risk neutral. They can be estimated from bond prices. Risk-neutral default probabilities are higher. This means that returns in the risk-neutral world are lower. From Table 24.4, the probability of a company moving from A to BBB or lower in one year is 5.74%. An estimate of the value of the derivative is therefore $0.0574\times\$100\times e^{-0.05\times 1} = \$5.46$. The approximation in this is that we are using the real-world probability of a downgrade. To value the derivative correctly, we should use the risk-neutral probability of a downgrade. Since the risk-neutral probability of a default is higher than the real-world probability, it seems likely that the same is true of a downgrade. This means that $5.46 is likely to be too low as an estimate of the value of the derivative.

24.26

In this case, $E_0 = 4$, $\sigma_E = 0.60$, $D = 15$, $r = 0.06$. Setting up the data in Excel, we can solve equations (24.3) and (24.4) by using the approach in footnote 10. The solution to the equations proves to be $V_0 = 17.084$ and $\sigma_V = 0.1576$. The probability of default is $N(-d_2)$ or 15.61%. The market value of the debt is $17.084 - 4 = 13.084$. The present value of the promised payment on the debt is $15e^{-0.06\times 2} = 13.304$. The expected loss on the debt is, therefore, $(13.304 - 13.084)/13.304$ or 1.65% of its no-default value. The expected recovery rate in the event of default is therefore $(15.61 - 1.65)/15.61$ or about 89%. The reason the recovery rate is so high is as follows. There is a default if the value of the assets moves from 17.08 to below 15. A value for the assets significantly below 15 is unlikely. Conditional on a default, the expected value of the assets is, therefore, not a huge amount below 15. In practice, it is likely that companies manage to delay defaults until asset values are well below the face value of the debt.

24.27
From equation (24.10), the 99.5% worst case probability of default is

$$N\left(\frac{N^{-1}(0.01) + \sqrt{0.2}N^{-1}(0.995)}{\sqrt{0.8}}\right) = 0.0946$$

This gives the 99.5% credit VaR as $10 \times (1 - 0.4) \times 0.0946 = 0.568$ millions of dollars or \$568,000.

24.28 (Excel file)
The spreadsheet shows the answer is 5.73.

24.29 (Excel file)
In this case, we look at the exposure from the point of view of the counterparty. The exposure at time *t* is

$$e^{-r(T-t)}\max[K - F_t,\ 0]$$

The expected exposure at time *t* is therefore

$$e^{-r(T-t)}\left[KN(-d_2(t)) - F_0 N(-d_1(t))\right]$$

The spreadsheet shows that the DVA is 1.134.