CHAPTER 35
Energy and Commodity Derivatives

Practice Questions

35.1
A day's HDD is $\max(0, 65 - A)$ and a day's CDD is $\max(0, A - 65)$ where A is the average of the highest and lowest temperature during the day at a specified weather station, measured in degrees Fahrenheit.

35.2
It is an agreement by one side to deliver a specified amount of gas at a roughly uniform rate during a month to a particular hub for a specified price.

35.3
The historical data approach to valuing an option involves calculating the expected payoff using historical data and discounting the payoff at the risk-free rate. The risk-neutral approach involves calculating the expected payoff in a risk-neutral world and discounting at the risk-free rate. The two approaches give the same answer when percentage changes in the underlying market variables have zero correlation with stock market returns. (In these circumstances, all risks can be diversified away.)

35.4
The average temperature each day is 75°. The CDD each day is therefore 10 and the cumulative CDD for the month is $10 \times 31 = 310$. The payoff from the call option is therefore $(310 - 250) \times 5{,}000 = \$300{,}000$.

35.5
Unlike most commodities, electricity cannot be stored easily. If the demand for electricity exceeds the supply, as it sometimes does during the air conditioning season, the price of electricity in a deregulated environment will skyrocket. When supply and demand become matched again, the price will return to former levels.

35.6
There is no systematic risk (i.e., risk that is priced by the market) in weather derivatives and CAT bonds.

35.7
HDD is $\max(65 - A, 0)$ where A is the average of the maximum and minimum temperature during the day. This is the payoff from a put option on A with a strike price of 65. CDD is $\max(A - 65, 0)$. This is the payoff from call option on A with a strike price of 65.

35.8
It would be useful to calculate the cumulative CDD for July of each year of the last 50 years. A linear regression relationship

$$\text{CDD} = a + bt + e$$

could then be estimated where a and b are constants, t is the time in years measured from the start of the 50 years, and e is the error. This relationship allows for linear trends in temperature through time. The expected CDD for next year (year 51) is then $a+51b$. This could be used as an estimate of the forward CDD.

35.9
The volatility of the one-year forward price will be less than the volatility of the spot price. This is because, when the spot price changes by a certain amount, mean reversion will cause the forward price will change by a lesser amount.

35.10
The price of the energy source will show big changes, but will be pulled back to its long-run average level fast. Electricity is an example of an energy source with these characteristics.

35.11
The energy producer faces quantity risks and price risks. It can use weather derivatives to hedge the quantity risks and energy derivatives to hedge against the price risks.

35.12
A 5×8 contract is a contract to provide electricity for five days per week during the off-peak period (11PM to 7AM). When daily exercise is specified, the holder of the option is able to choose each weekday whether to buy electricity at the strike price at the agreed rate. When there is monthly exercise, there is one choice at the beginning of the month concerning whether electricity is to be bought at the strike price at the agreed rate for the whole month. The option with daily exercise is worth more.

35.13
CAT bonds (catastrophe bonds) are an alternative to reinsurance for an insurance company that has taken on a certain catastrophic risk (e.g., the risk of a hurricane or an earthquake) and wants to get rid of it. CAT bonds are issued by the insurance company. They provide a higher rate of interest than government bonds. However, the bondholders agree to forego interest, and possibly principal, to meet any claims against the insurance company that are within a prespecified range.

35.14
The CAT bond has very little systematic risk. Whether a particular type of catastrophe occurs is independent of the return on the market. The risks in the CAT bond are likely to be largely “diversified away” by the other investments in the portfolio. A B-rated bond does have systematic risk that cannot be diversified away. It is likely therefore that the CAT bond is a better addition to the portfolio.

35.15
In this case,

$$\frac{dS}{S} = \mu(t)\,dt + \sigma\,dz$$

or

$$d\ln S = [\mu(t) - \sigma^2/2]\,dt + \sigma\,dz$$

so that $\ln S_T$ is normal with mean

$$\ln S_0 + \int_{t=0}^{T} \mu(t)dt - \sigma^2 T / 2$$

and standard deviation $\sigma\sqrt{T}$. Section 35.4 shows that

$$\mu(t) = \frac{\partial}{\partial t}[\ln F(t)]$$

so that

$$\int_{t=0}^{T} \mu(t)dt = \ln F(T) - \ln F(0)$$

Since $F(0) = S_0$ the result follows.

35.16

To find the nodes at the end of one year, we must solve

$0.1667e^{0.3464+\alpha_1} + 0.6666e^{\alpha_1} + 0.1667e^{-0.3464+\alpha_1} = 21$

The solution is α_1 =3.025. To find the nodes at the end of two years, we must solve

$0.0203e^{0.6928+\alpha_2} + 0.2206e^{0.3464+\alpha_2} + 0.5183e^{\alpha_2} + 0.2206e^{-0.3464+\alpha_2} + 0.0203e^{-0.6928+\alpha_2} = 22$

The solution is α_2 = 3.055. This gives the tree in Figure S35.1. The probabilities on branches are unchanged. Rolling back through the tree, the value of a three-year put option with a strike price of 20 is shown in Figure S35.2 to be 1.68.

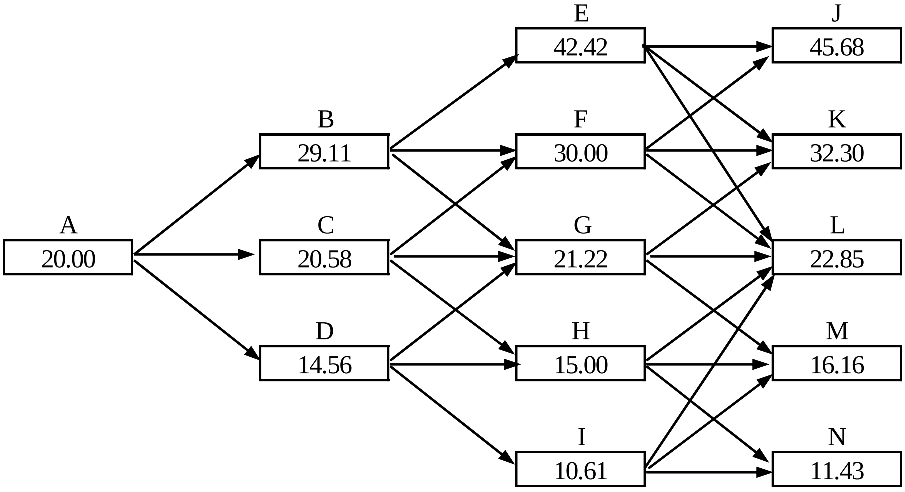

Figure S35.1: *Commodity Prices in Problem 35.16*

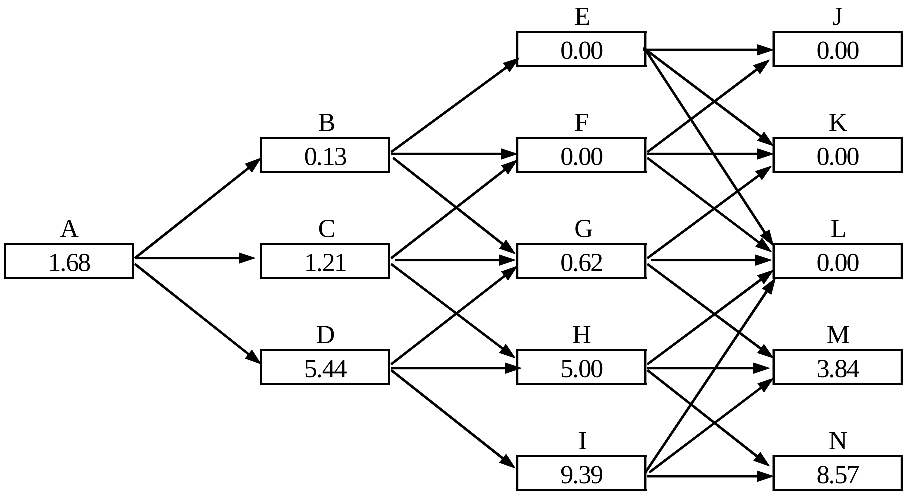

Figure S35.2: *American option prices in Problem 35.16*