CHAPTER 21
Basic Numerical Procedures

Practice Questions

21.1
Delta, gamma, and theta can be determined from a single binomial tree. Vega is determined by making a small change to the volatility and recomputing the option price using a new tree. Rho is calculated by making a small change to the interest rate and recomputing the option price using a new tree.

21.2
In this case, $S_0 = 60$, $K = 60$, $r = 0.1$, $\sigma = 0.45$, $T = 0.25$, and $\Delta t = 0.0833$. Also

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.45\sqrt{0.0833}} = 1.1387$$

$$d = \frac{1}{u} = 0.8782$$

$$a = e^{r\Delta t} = e^{0.1\times 0.0833} = 1.0084$$

$$p = \frac{a-d}{u-d} = 0.4998$$

$$1-p = 0.5002$$

The output from DerivaGem for this example is shown in Figure S21.1. The calculated price of the option is \$5.16.

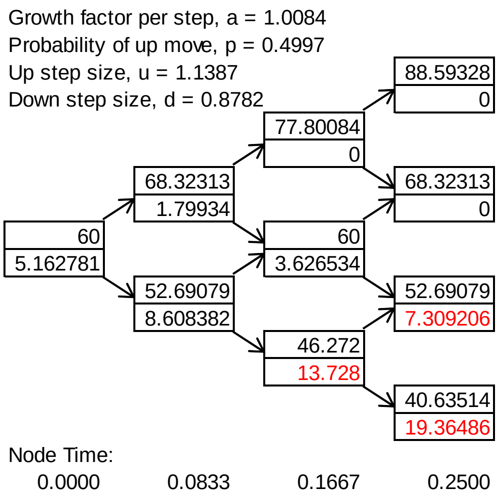


Figure S21.1: *Tree for Problem 21.2*

21.3
The control variate technique is implemented by:

1. Valuing an American option using a binomial tree in the usual way $(= f_A)$.
2. Valuing the European option with the same parameters as the American option using the same tree $(= f_E)$.
3. Valuing the European option using Black–Scholes–Merton $(= f_{BSM})$. The price of the American option is estimated as $f_A + f_{BSM} - f_E$.

21.4

In this case, $F_0 = 198$, $K = 200$, $r = 0.08$, $\sigma = 0.3$, $T = 0.75$, and $\Delta t = 0.25$. Also,

$$u = e^{0.3\sqrt{0.25}} = 1.1618$$

$$d = \frac{1}{u} = 0.8607$$

$$a = 1$$

$$p = \frac{a-d}{u-d} = 0.4626$$

$$1 - p = 0.5373$$

The output from DerivaGem for this example is shown in Figure S21.2. The calculated price of the option is 20.34 cents.

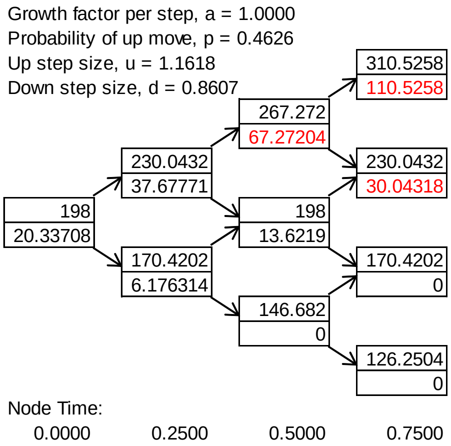


Figure S21.2: *Tree for Problem 21.4*

21.5

A binomial tree cannot be used in the way described in this chapter. This is an example of what is known as a history-dependent option. The payoff depends on the path followed by the stock price as well as its final value. The option cannot be valued by starting at the end of the tree and working backward since the payoff at the final branches is not known unambiguously. Chapter 27 describes an extension of the binomial tree approach that can be used to handle options where the payoff depends on the average value of the stock price.

21.6

Suppose a dividend equal to D is paid during a certain time interval. If S is the stock price at the beginning of the time interval, it will be either $Su - D$ or $Sd - D$ at the end of the time interval. At the end of the next time interval, it will be one of $(Su - D)u$, $(Su - D)d$, $(Sd - D)u$ and $(Sd - D)d$. Since $(Su - D)d$ does not equal $(Sd-D)u$, the tree does not recombine. If S is equal to the stock price less the present value of future dividends, this problem is avoided.

21.7

With the usual notation,

$$p = \frac{a-d}{u-d}$$

$$1-p = \frac{u-a}{u-d}$$

If $a < d$ or $a > u$, one of the two probabilities is negative. This happens when

$$e^{(r-q)\Delta t} < e^{-\sigma\sqrt{\Delta t}}$$

or

$$e^{(r-q)\Delta t} > e^{\sigma\sqrt{\Delta t}}$$

This in turn happens when $(q-r)\sqrt{\Delta t} > \sigma$ or $(r-q)\sqrt{\Delta t} > \sigma$. Hence, negative probabilities occur when

$$\sigma < |(r-q)\sqrt{\Delta t}\,|$$

This is the condition in footnote 8.

21.8

In Table 21.1, cells A1, A2, A3,..., A100 are random numbers between 0 and 1 defining how far to the right in the square the dart lands. Cells B1, B2, B3,...,B100 are random numbers between 0 and 1 defining how high up in the square the dart lands. For stratified sampling, we could choose equally spaced values for the A's and the B's and consider every possible combination. To generate 100 samples, we need ten equally spaced values for the A's and the B's so that there are $10 \times 10 = 100$ combinations. The equally spaced values should be 0.05, 0.15, 0.25,..., 0.95. We could therefore set the A's and B's as follows:

$$\text{A1} = \text{A2} = \text{A3} = \ldots = \text{A10} = 0.05$$

$$\text{A11} = \text{A12} = \text{A13} = \ldots = \text{A20} = 0.15$$

$$\ldots$$

$$\ldots$$

$$\text{A91} = \text{A92} = \text{A93} = \ldots = \text{A100} = 0.95$$

and

$$\text{B1} = \text{B11} = \text{B21} = \ldots. = \text{B91} = 0.05$$

$$\text{B2} = \text{B12} = \text{B22} = \ldots = \text{B92} = 0.15$$

$$\ldots$$

$$\ldots$$

$$\text{B10} = \text{B20} = \text{B30} = \ldots = \text{B100} = 0.95$$

We get a value for π equal to 3.2, which is closer to the true value than the value of 3.04 obtained with random sampling in Table 21.1. Because samples are not random, we cannot easily calculate a standard error of the estimate.

21.9

In Monte Carlo simulation, sample values for the derivative security in a risk-neutral world are obtained by simulating paths for the underlying variables. On each simulation run, values for the underlying variables are first determined at time Δt, then at time $2\Delta t$, then at time $3\Delta t$, etc. At time $i\Delta t (i = 0,1,2\ldots)$, it is not possible to determine whether early exercise is optimal since the range of paths which might occur after time $i\Delta t$ have not been investigated. In short, Monte Carlo simulation works by moving forward from time t to time T. Other numerical procedures which accommodate early exercise work by moving backwards from time T to time t.

21.10

In this case, $S_0 = 50$, $K = 49$, $r = 0.05$, $\sigma = 0.30$, $T = 0.75$, and $\Delta t = 0.25$. Also,

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.30\sqrt{0.25}} = 1.1618$$

$$d = \frac{1}{u} = 0.8607$$

$$a = e^{r\Delta t} = e^{0.05\times 0.25} = 1.0126$$

$$p = \frac{a-d}{u-d} = 0.5043$$

$$1 - p = 0.4957$$

The output from DerivaGem for this example is shown in Figure S21.3. The calculated price of the option is \$4.29. Using 100 steps, the price obtained is \$3.91.

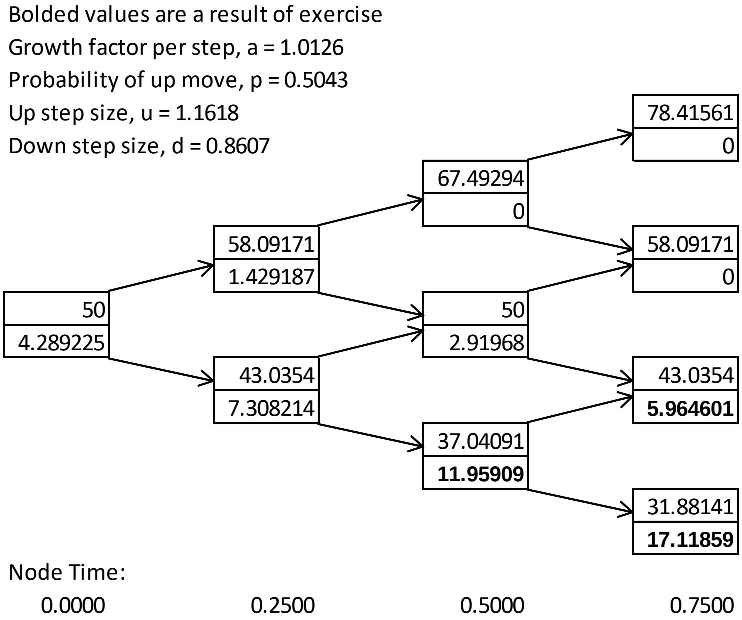


Figure S21.3: *Tree for Problem 21.10*

21.11

In this case, $F_0 = 400$, $K = 420$, $r = 0.06$, $\sigma = 0.35$, $T = 0.75$, and $\Delta t = 0.25$. Also,

$$u = e^{0.35\sqrt{0.25}} = 1.1912$$

$$d = \frac{1}{u} = 0.8395$$

$$a = 1$$

$$p = \frac{a-d}{u-d} = 0.4564$$

$$1 - p = 0.5436$$

The output from DerivaGem for this example is shown in Figure S21.4. The calculated price of the option is 42.07 cents. Using 100 time steps, the price obtained is 38.64. The option's delta is calculated from the tree is

$$(79.971 - 11.419)/(476.498 - 335.783) = 0.487$$

When 100 steps are used the estimate of the option's delta is 0.483.

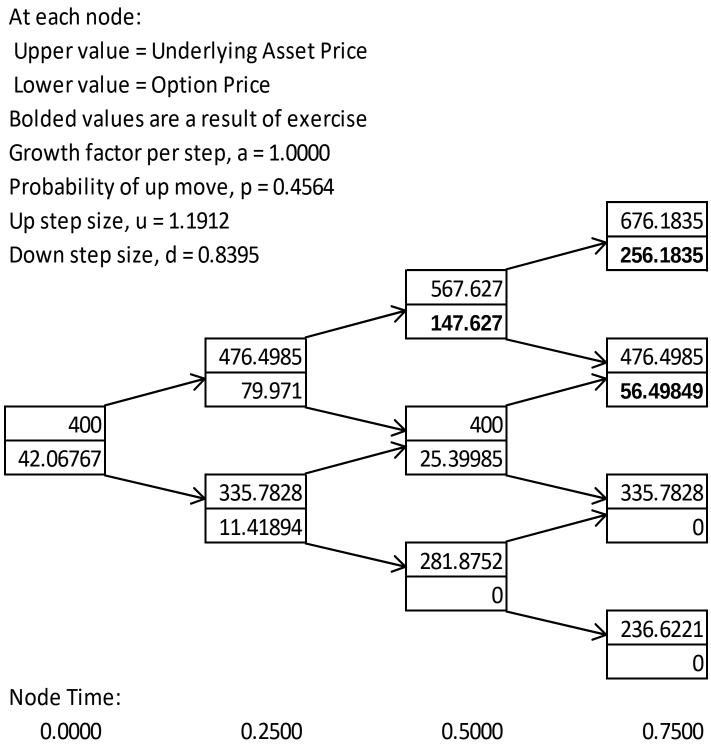


Figure S21.4: *Tree for Problem 21.11*

21.12

In this case, the present value of the dividend is $2e^{-0.03\times0.125} = 1.9925$. We first build a tree for $S_0 = 20 - 1.9925 = 18.0075$, $K = 20$, $r = 0.03$, $\sigma = 0.25$, and $T = 0.25$ with $\Delta t = 0.08333$. This gives Figure S21.5. For nodes between times 0 and 1.5 months, we then add the present value of the dividend to the stock price. The result is the tree in Figure S21.6. The price of the option calculated from the tree is 0.674. When 100 steps are used, the price obtained is 0.690.

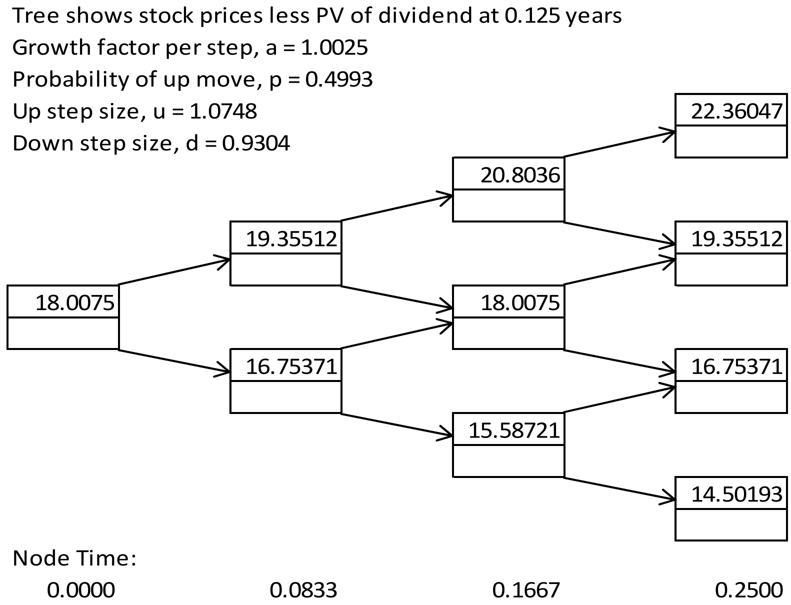


Figure S21.5: *First Tree for Problem 21.12*

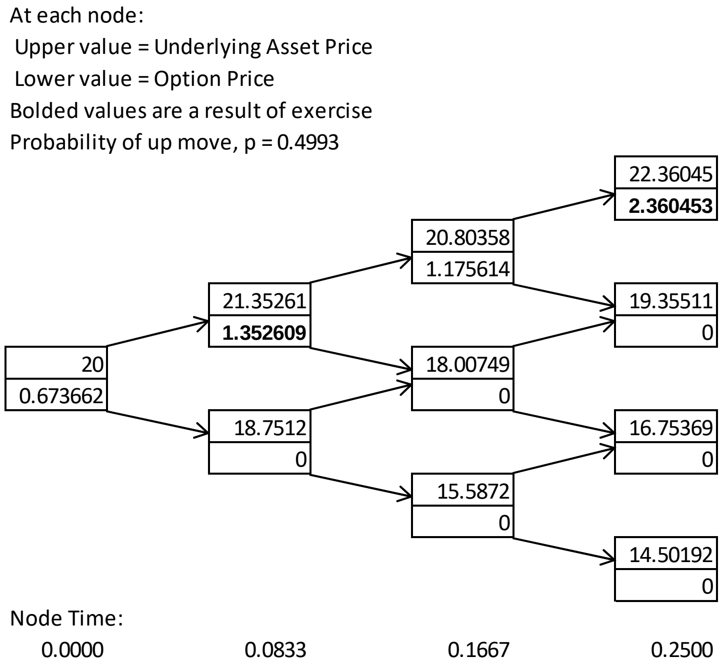


Figure S21.6: *Final Tree for Problem 21.12*

21.13

In this case, $S_0 = 20$, $K = 18$, $r = 0.15$, $\sigma = 0.40$, $T = 1$, and $\Delta t = 0.25$. The parameters for the tree are:

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.4\sqrt{0.25}} = 1.2214$$
$$d = 1/u = 0.8187$$
$$a = e^{r\Delta t} = 1.0382$$
$$p = \frac{a-d}{u-d} = \frac{1.0382-0.8187}{1.2214-0.8187} = 0.545$$

The tree produced by DerivaGem for the American option is shown in Figure S21.7. The estimated value of the American option is \$1.29.

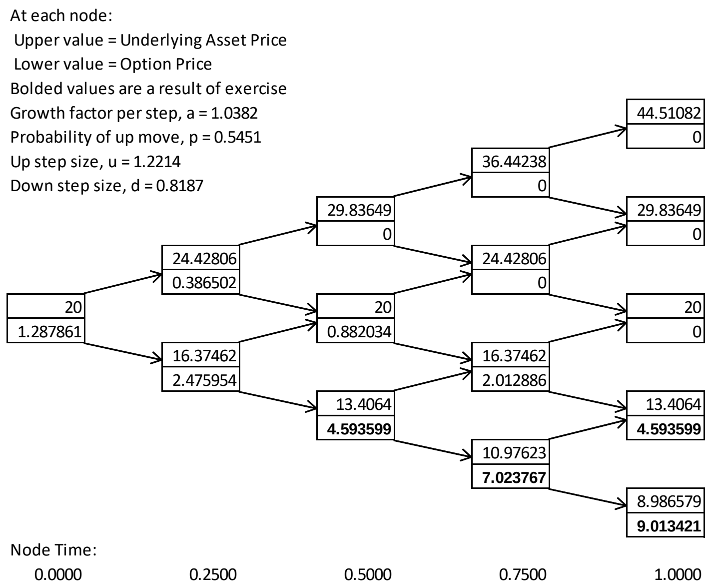


Figure S21.7: *Tree to evaluate American option for Problem 21.13*

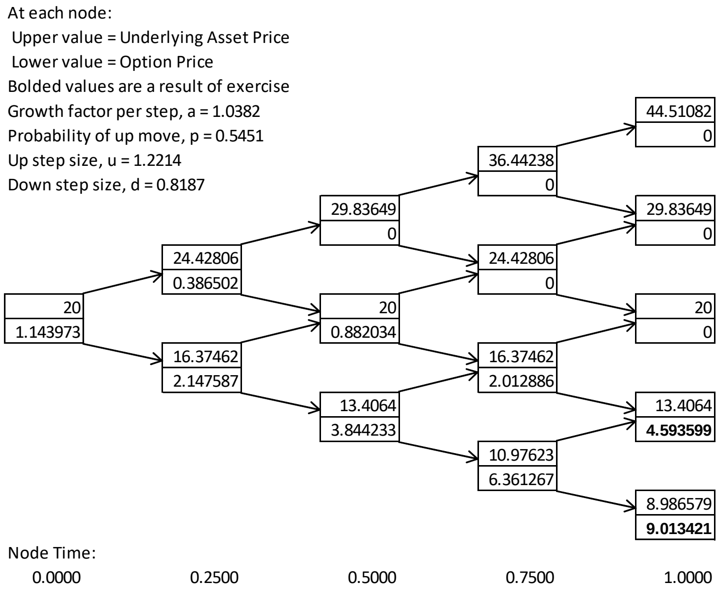


Figure S21.8: *Tree to evaluate European option in Problem 21.13*

As shown in Figure S21.8, the same tree can be used to value a European put option with the same parameters. The estimated value of the European option is \$1.14. The option parameters are $S_0 = 20$, $K = 18$, $r = 0.15$, $\sigma = 0.40$ and $T = 1$

$$d_1 = \frac{\ln(20/18) + 0.15 + 0.40^2/2}{0.40} = 0.8384$$
$$d_2 = d_1 - 0.40 = 0.4384$$

$$N(-d_1) = 0.2009; \quad N(-d_2) = 0.3306$$

The true European put price is therefore,

$$18e^{-0.15} \times 0.3306 - 20 \times 0.2009 = 1.10$$

This can also be obtained from DerivaGem. The control variate estimate of the American put price is therefore 1.29 + 1.10 − 1.14 = \$1.25.

21.14

In this case, $S_0 = 484$, $K = 480$, $r = 0.10$, $\sigma = 0.25$ $q = 0.03$, $T = 0.1667$, and $\Delta t = 0.04167$

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.25\sqrt{0.04167}} = 1.0524$$
$$d = \frac{1}{u} = 0.9502$$
$$a = e^{(r-q)\Delta t} = 1.00292$$
$$p = \frac{a-d}{u-d} = \frac{1.0029 - 0.9502}{1.0524 - 0.9502} = 0.516$$

The tree produced by DerivaGem is shown in the Figure S21.9. The estimated price of the option is \$14.93.

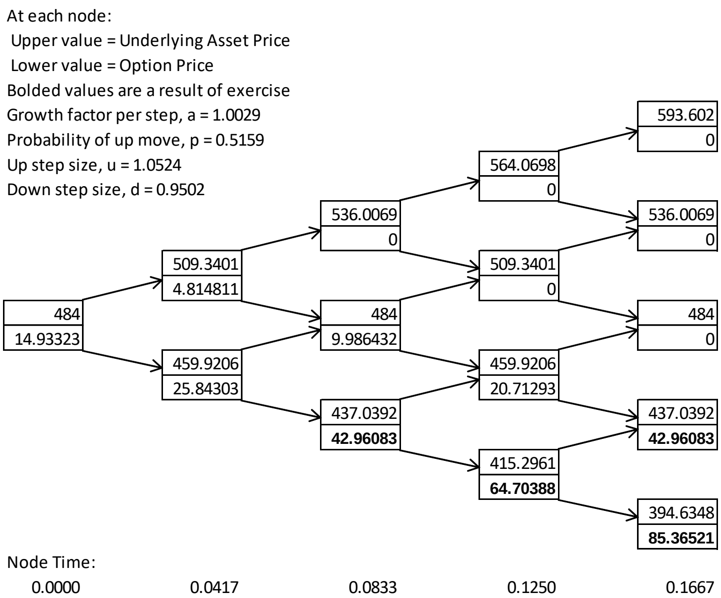


Figure S21.9: *Tree to evaluate option in Problem 21.14*

21.15
First the delta of the American option is estimated in the usual way from the tree. Denote this by Δ_A^*. Then the delta of a European option which has the same parameters as the American option is calculated in the same way using the same tree. Denote this by Δ_B^*. Finally the true European delta, Δ_B, is calculated using the formulas in Chapter 19. The control variate estimate of delta is then:

$$\Delta_A^* - \Delta_B^* + \Delta_B$$

21.16.
In this case, a simulation requires two sets of samples from standardized normal distributions. The first is to generate the volatility movements. The second is to generate the stock price movements once the volatility movements are known. The control variate technique involves carrying out a second simulation on the assumption that the volatility is constant. The same random number stream is used to generate stock price movements as in the first simulation. An improved estimate of the option price is

$$f_A^* - f_B^* + f_B$$

where f_A^* is the option value from the first simulation (when the volatility is stochastic), f_B^* is the option value from the second simulation (when the volatility is constant) and f_B is the true Black–Scholes–Merton value when the volatility is constant.
To use the antithetic variable technique, two sets of samples from standardized normal distributions must be used for each of volatility and stock price. Denote the volatility samples by $\{V_1\}$ and $\{V_2\}$ and the stock price samples by $\{S_1\}$ and $\{S_2\}$. $\{V_1\}$ is antithetic to $\{V_2\}$ and $\{S_1\}$ is antithetic to $\{S_2\}$. Thus, if

$$\{V_1\} = +0.83, +0.41, -0.21\ldots$$

then

$$\{V_2\} = -0.83, -0.41, +0.21\ldots$$

Similarly for $\{S_1\}$ and $\{S_2\}$.
An efficient way of proceeding is to carry out six simulations in parallel:

Simulation 1: Use $\{S_1\}$ with volatility constant
Simulation 2: Use $\{S_2\}$ with volatility constant
Simulation 3: Use $\{S_1\}$ and $\{V_1\}$
Simulation 4: Use $\{S_1\}$ and $\{V_2\}$
Simulation 5: Use $\{S_2\}$ and $\{V_1\}$
Simulation 6: Use $\{S_2\}$ and $\{V_2\}$

If f_i is the option price from simulation i, simulations 3 and 4 provide an estimate $0.5(f_3 + f_4)$ for the option price. When the control variate technique is used, we combine this estimate with the result of simulation 1 to obtain $0.5(f_3 + f_4) - f_1 + f_B$ as an estimate of the price where f_B is, as above, the Black–Scholes–Merton option price. Similarly, simulations 2, 5 and 6 provide an estimate $0.5(f_5 + f_6) - f_2 + f_B$. Overall the best estimate is:

$$0.5[0.5(f_3 + f_4) - f_1 + f_B + 0.5(f_5 + f_6) - f_2 + f_B]$$

21.17
For an American call option on a currency,

$$\frac{\partial f}{\partial t}+(r-r_f)S\frac{\partial f}{\partial S}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 f}{\partial S^2}=rf$$

With the notation in the text, this becomes

$$\frac{f_{i+1,j}-f_{ij}}{\Delta t}+(r-r_f)j\Delta S\frac{f_{i,j+1}-f_{i,j-1}}{2\Delta S}+\frac{1}{2}\sigma^2 j^2\Delta S^2\frac{f_{i,j+1}-2f_{i,j}+f_{i,j-1}}{\Delta S^2}=rf_{ij}$$

for $j=1,2\ldots\ M-1$ and $i=0,1\ldots\ N-1$. Rearranging terms, we obtain

$$a_j f_{i,j-1}+b_j f_{ij}+c_j f_{i,j+1}=f_{i+1,j}$$

where

$$a_j=\frac{1}{2}(r-r_f)j\Delta t-\frac{1}{2}\sigma^2 j^2\Delta t$$

$$b_j=1+\sigma^2 j^2\Delta t+r\Delta t$$

$$c_j=-\frac{1}{2}(r-r_f)j\Delta t-\frac{1}{2}\sigma^2 j^2\Delta t$$

Equations (21.28), (21.29) and (21.30) become

$$f_{Nj}=\max[j\Delta S-K,0]\quad j=0,1\ldots M$$

$$f_{i0}=0\qquad i=0,1\ldots N$$

$$f_{iM}=M\Delta S-K\quad i=0,1\ldots N$$

21.18

We consider stock prices of \$0, \$4, \$8, \$12, \$16, \$20, \$24, \$28, \$32, \$36 and \$40. Using equation (21.34) with $r=0.10$, $\Delta t=0.0833$, $\Delta S=4$, $\sigma=0.30$, $K=21$, $T=0.3333$ we obtain the grid shown below. The option price is \$1.56.

Stock Price	*Time to Maturity (Months)*				
(\$)	4	3	2	1	0
40	0.00	0.00	0.00	0.00	0.00
36	0.00	0.00	0.00	0.00	0.00
32	0.01	0.00	0.00	0.00	0.00
28	0.07	0.04	0.02	0.00	0.00
24	0.38	0.30	0.21	0.11	0.00
20	1.56	1.44	1.31	1.17	1.00
16	5.00	5.00	5.00	5.00	5.00
12	9.00	9.00	9.00	9.00	9.00
8	13.00	13.00	13.00	13.00	13.00
4	17.00	17.00	17.00	17.00	17.00
0	21.00	21.00	21.00	21.00	21.00

21.19

In this case, $\Delta t=0.25$ and $\sigma=0.4$ so that

$$u=e^{0.4\sqrt{0.25}}=1.2214$$

$$d=\frac{1}{u}=0.8187$$

The futures prices provide estimates of the growth rate in copper in a risk-neutral world. During the first three months, this growth rate (with continuous compounding) is

$$4\ln\frac{0.59}{0.60} = -6.72\% \text{ per annum}$$

The parameter p for the first three months is therefore,

$$\frac{e^{-0.0672\times 0.25} - 0.8187}{1.2214 - 0.8187} = 0.4088$$

The growth rate in copper is equal to -13.79%, -21.63% and -30.78% in the following three quarters. Therefore, the parameter p for the second three months is

$$\frac{e^{-0.1379\times 0.25} - 0.8187}{1.2214 - 0.8187} = 0.3660$$

For the third quarter, it is

$$\frac{e^{-0.2163\times 0.25} - 0.8187}{1.2214 - 0.8187} = 0.3195$$

For the final quarter, it is

$$\frac{e^{-0.3078\times 0.25} - 0.8187}{1.2214 - 0.8187} = 0.2663$$

The tree for the movements in copper prices in a risk-neutral world is shown in Figure S21.10. The value of the option is $0.062.

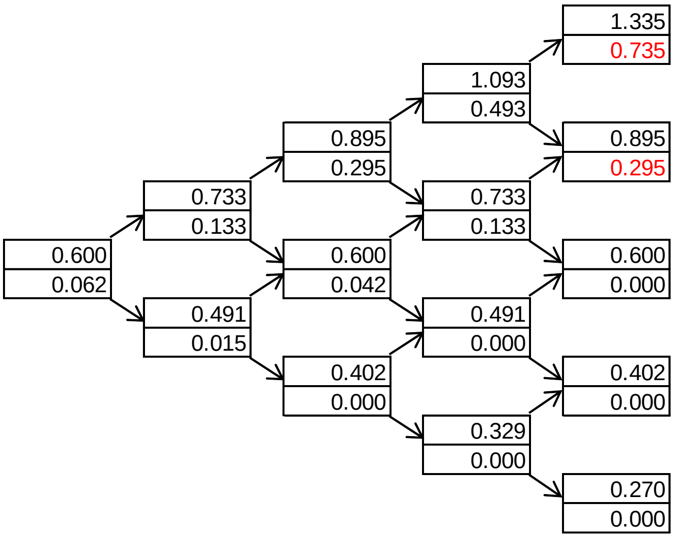


Figure S21.10: *Tree to value option in Problem 21.19: At each node, upper number is price of copper and lower number is option price.*

21.20

In this problem, we use exactly the same tree for copper prices as in Problem 21.19. However, the values of the derivative are different. On the final nodes, the values of the derivative equal the square of the price of copper. On other nodes, they are calculated in the usual way. The current value of the security is $0.275 (see Figure S21.11).

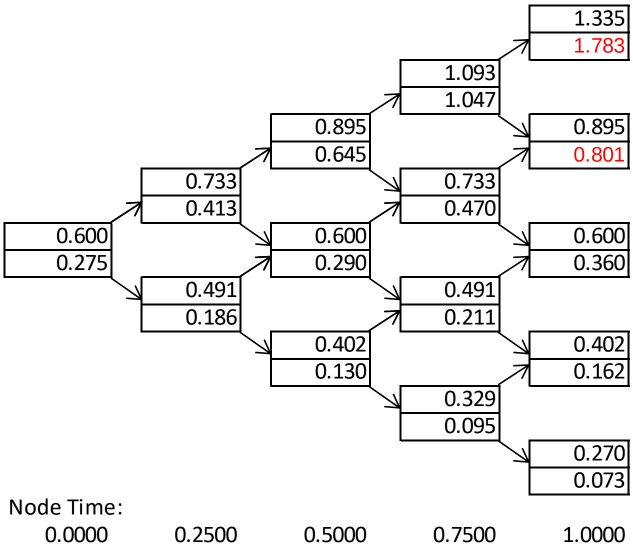


Figure S21.11: *Tree to value derivative in Problem 21.20. At each node, upper number is price of copper and lower number is derivative's price.*

21.21

Define S_t as the current asset price, $S_{\max}$ as the highest asset price considered and $S_{\min}$ as the lowest asset price considered. (In the example in the text $S_{\min} = 0$). Let

$$Q_1 = \frac{S_{\max} - S_t}{\Delta S} \quad \text{and} \quad Q_2 = \frac{S_t - S_{\min}}{\Delta S}$$

and let N be the number of time intervals considered. From the triangular structure of the calculations in the explicit version of the finite difference method, we can see that the values assumed for the derivative security at $S = S_{\min}$ and $S = S_{\max}$ affect the derivative's value if

$$N \geq \max(Q_1, Q_2)$$

21.22

The following changes could be made. Set LI as

= NORMSINV(RAND())

A1 as

=C*EXP(($E$2-$F$2*$F$2/2)*$G$2+$F$2*L2*SQRT($G$2))

H1 as

=C*EXP(($E$2-$F$2*$F$2/2)*$G$2-$F$2*L2*SQRT($G$2))

I1 as

= EXP(-$E$2*$G$2)*MAX(H1-$D$2,0)

and J1 as

=0.5*(B1+J1)

Other entries in columns L, A, H, and I are defined similarly. The estimate of the value of the option is the average of the values in the J column.

21.23

The basic approach is similar to that described in Section 21.8. The only difference is the boundary conditions. For a sufficiently small value of the stock price, $S_{\min}$, it can be assumed that conversion will never take place and the convertible can be valued as a straight bond. The highest stock price which needs to be considered, $S_{\max}$, is $18. When this is reached, the value of the convertible bond is \$36. At maturity, the convertible is worth the greater of $2S_T$ and \$25 where S_T is the stock price.

The convertible can be valued by working backwards through the grid using either the explicit or the implicit finite difference method in conjunction with the boundary conditions. In formulas (21.25) and (21.32), the present value of the income on the convertible between time $t+i\,\Delta t$ and $t+(i+1)\,\Delta t$ discounted to time $t+i\,\Delta t$ must be added to the right-hand side. Chapter 27 considers the pricing of convertibles in more detail.

21.24

Suppose x_1, x_2, and x_3 are random samples from three independent normal distributions. Random samples with the required correlation structure are ε_1, ε_2, ε_3 where

$$\varepsilon_1 = x_1$$

$$\varepsilon_2 = \rho_{12}x_1 + x_2\sqrt{1-\rho_{12}^2}$$

and

$$\varepsilon_3 = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3$$

where

$$\alpha_1 = \rho_{13}$$

$$\alpha_1\rho_{12} + \alpha_2\sqrt{1-\rho_{12}^2} = \rho_{23}$$

and

$$\alpha_1^2 + \alpha_2^2 + \alpha_3^2 = 1$$

This means that

$$\alpha_1 = \rho_{13}$$

$$\alpha_2 = \frac{\rho_{23} - \rho_{13}\rho_{12}}{\sqrt{1-\rho_{12}^2}}$$

$$\alpha_3 = \sqrt{1-\alpha_1^2-\alpha_2^2}$$

21.25

The tree is shown in Figure S21.12. The value of the option is estimated as 0.0207 and its delta is estimated as

$$\frac{0.006221-0.041153}{0.858142-0.764559} = -0.373$$

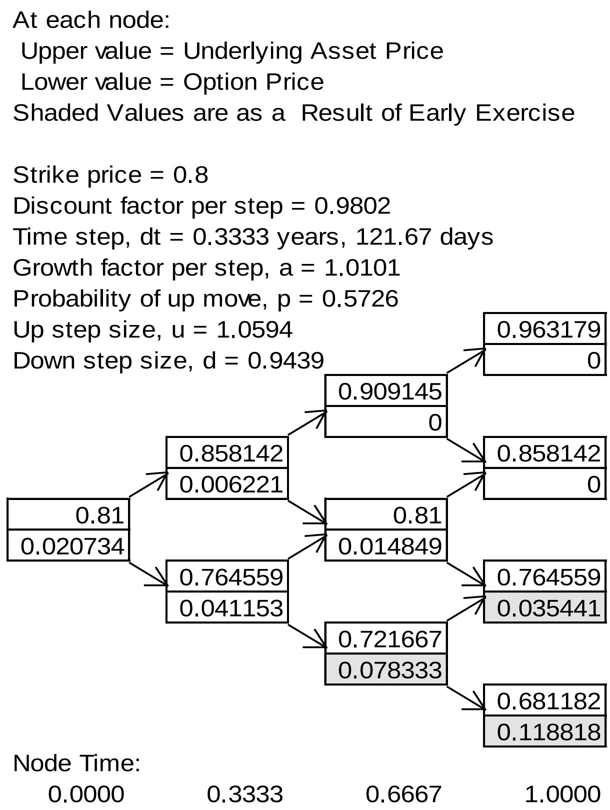


Figure S21.12: *Tree for Problem 21.25*

21.26

In this case, $F_0 = 8.5$, $K = 9$, $r = 0.12$, $T = 1$, $\sigma = 0.25$, and $\Delta t = 0.25$. The parameters for the tree are

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.25\sqrt{0.25}} = 1.1331$$

$$d = \frac{1}{u} = 0.8825$$

$$a = 1$$

$$p = \frac{a-d}{u-d} = \frac{1-0.8825}{1.1331-0.8825} = 0.469$$

The tree output by DerivaGem for the American option is shown in Figure S21.13. The estimated value of the option is \$0.596. The tree produced by DerivaGem for the European version of the option is shown in Figure S21.14. The estimated value of the option is \$0.586. The Black–Scholes–Merton price of the option is \$0.570. The control variate estimate of the price of the option is therefore,

$$0.596 + 0.570 - 0.586 = 0.580$$

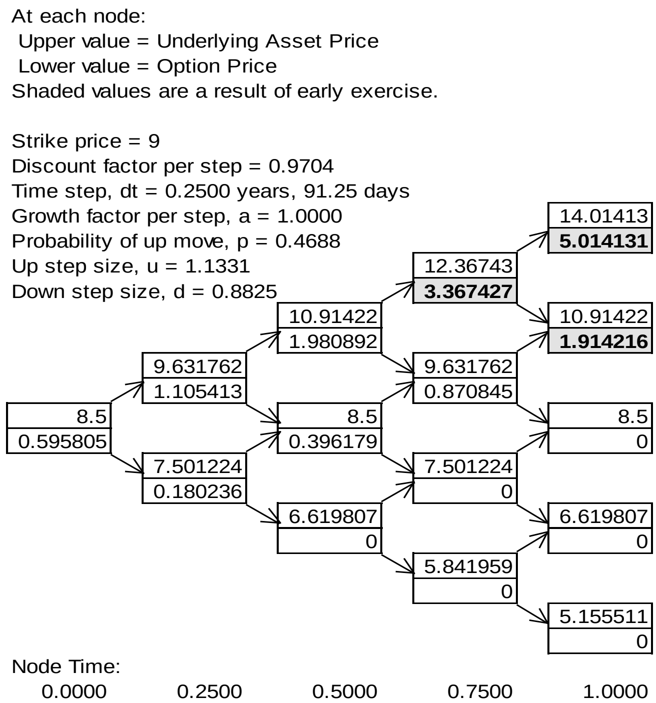


Figure S21.13: *Tree for American option in Problem 21.26*

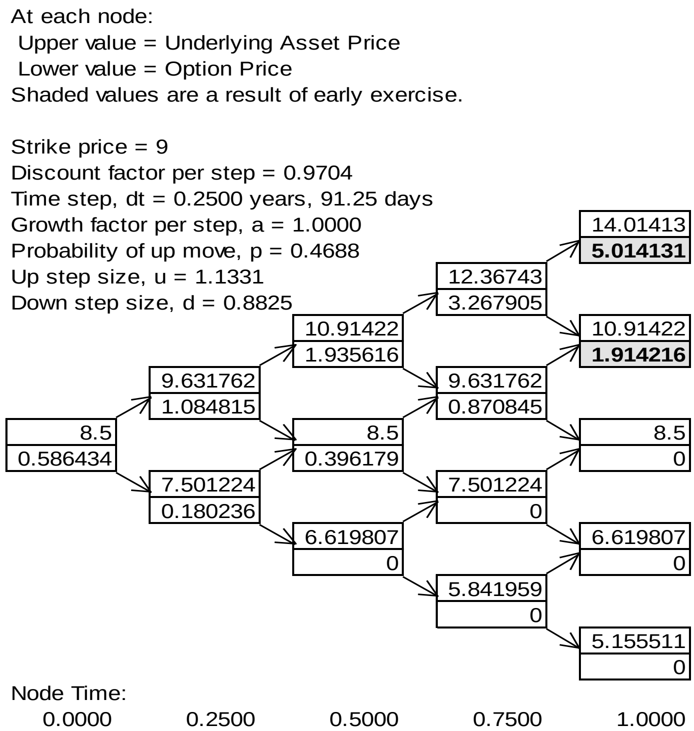


Figure S21.14: *Tree for European option in Problem 21.26*

21.27

(a) For the binomial model in Section 21.4, there are two equally likely changes in the logarithm of the stock price in a time step of length Δt. These are $(r-\sigma^2/2)\Delta t+\sigma\sqrt{\Delta t}$ and $(r-\sigma^2/2)\Delta t-\sigma\sqrt{\Delta t}$. The expected change in the logarithm of the stock price is

$0.5[(r-\sigma^2/2)\Delta t+\sigma\sqrt{\Delta t}]+0.5[(r-\sigma^2/2)\Delta t-\sigma\sqrt{\Delta t}]=(r-\sigma^2/2)\Delta t$

This is correct. The variance of the change in the logarithm of the stock price is

$0.5\sigma^2\Delta t+0.5\sigma^2\Delta t=\sigma^2\Delta t$

This is correct.

(b) For the trinomial tree model in Section 21.4, the change in the logarithm of the stock price in a time step of length Δt is $+\sigma\sqrt{3\Delta t}$, 0, and $-\sigma\sqrt{3\Delta t}$ with probabilities

$$\sqrt{\frac{\Delta t}{12\sigma^2}}\left(r-\frac{\sigma^2}{2}\right)+\frac{1}{6},\qquad \frac{2}{3},\qquad -\sqrt{\frac{\Delta t}{12\sigma^2}}\left(r-\frac{\sigma^2}{2}\right)+\frac{1}{6}$$

The expected change is

$$\left(r-\frac{\sigma^2}{2}\right)\Delta t$$

It variance is $\sigma^2\Delta t$ plus a term of order $(\Delta t)^2$. These are correct.

(c) To get the expected change in the logarithm of the stock price in time Δt correct, we require

$$\frac{1}{6}(\ln u)+\frac{2}{3}(\ln m)+\frac{1}{6}(\ln d)=\left(r-\frac{\sigma^2}{2}\right)\Delta t$$

The relationship $m^2=ud$ implies $\ln m=0.5(\ln u+\ln d)$ so that the requirement becomes

$$\ln m=\left(r-\frac{\sigma^2}{2}\right)\Delta t$$

or

$$m=e^{(r-\sigma^2)\Delta t}$$

The expected change in $\ln S$ is $\ln m$. To get the variance of the change in the logarithm of the stock price in time Δt correct, we require

$$\frac{1}{6}(\ln u-\ln m)^2+\frac{1}{6}(\ln d-\ln m)^2=\sigma^2\Delta t$$

Because $\ln u-\ln m=-(\ln d-\ln m)$ it follows that

$$\ln u=\ln m+\sigma\sqrt{3\Delta t}$$
$$\ln d=\ln m-\sigma\sqrt{3\Delta t}$$

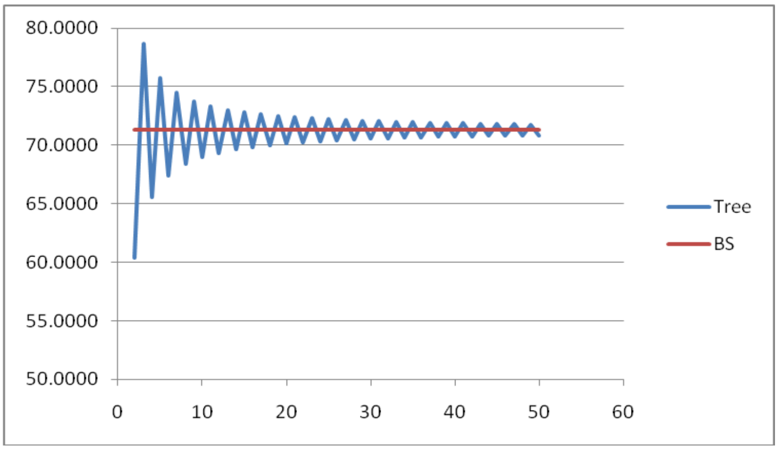
These results imply that

$$m=e^{(r-\sigma^2)\Delta t}$$
$$u=e^{(r-\sigma^2)\Delta t+\sigma\sqrt{3\Delta t}}$$
$$d=e^{(r-\sigma^2)\Delta t-\sigma\sqrt{3\Delta t}}$$

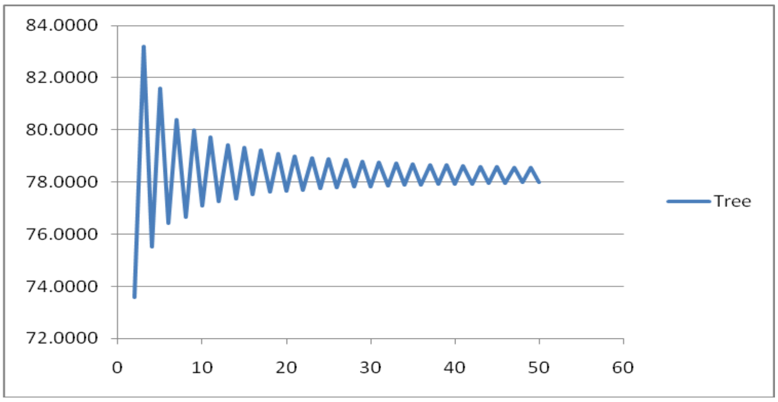
21.28 (Excel file)

The results, produced by making small modifications to Sample Application A, are shown in Figure S21.15.

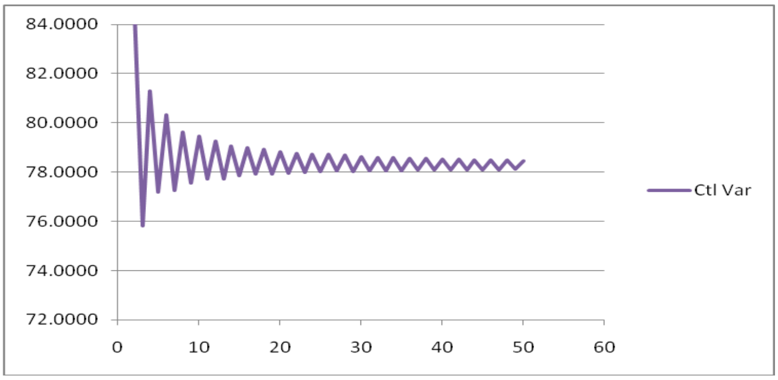
(a)


80.0000
75.0000
70.0000
65.0000
60.0000
55.0000
50.0000
0
10
20
30
40
50
60
Tree
BS

(b)


84.0000
82.0000
80.0000
78.0000
76.0000
74.0000
72.0000
0
10
20
30
40
50
60
Tree

(c)


84.0000
82.0000
80.0000
78.0000
76.0000
74.0000
72.0000
0
10
20
30
40
50
60
Ctl Var

(d)

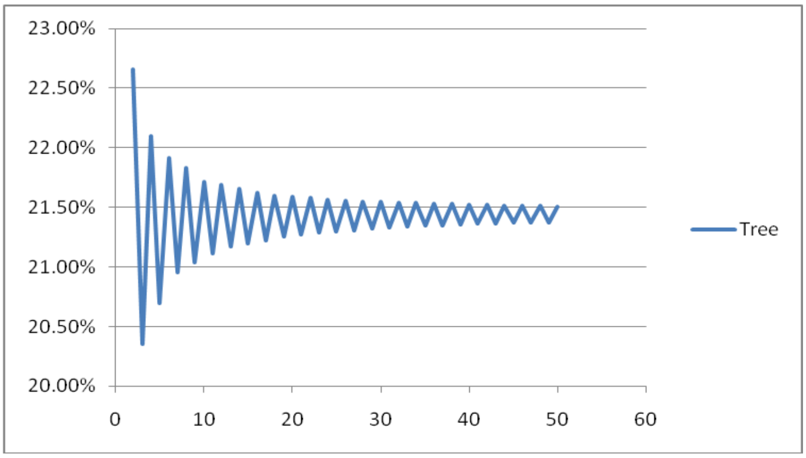


Figure S21.15: *Convergence Charts for Problem 21.28*

21.29

From Figure 21.5, delta is (33.64−6.13)/(327.14−275.11) = 0.5288. This is the rate of change of the option price with respect to the futures price. Gamma is

$$\frac{(56.73-12.90)/(356.73-300)-(12.90-0)/(300-252.29)}{0.5\times(356.73-252.29)}=0.009$$

This is the rate of change of delta with respect to the futures price. Theta is (12.9−19.16)/0.16667=−37.59 per year or −0.1029 per calendar day.

21.30

Without early exercise, the option is worth 0.2535 at the lowest node at the 9 month point. With early exercise, it is worth 0.2552. The gain from early exercise is therefore 0.0017.