

## CHAPTER 34

### Swaps Revisited

#### Practice Questions

##### 34.1.

Results are as follows:

<i>Target payment date</i>	<i>Day of week</i>	<i>Actual payment date</i>	<i>Days from previous to current target pmt dates</i>	<i>Fixed Payment (\$)</i>
Jul 11, 2021	Sunday	Jul 12, 2021	181	991,781
Jan 11, 2022	Tuesday	Jan 11, 2022	184	1,008,219
Jul 11, 2022	Monday	Jul 11, 2022	181	991,971
Jan 11, 2023	Wednesday	Jan 11, 2023	184	1,008,219
Jul 11, 2023	Tuesday	Jul 11, 2023	181	991,781
Jan 11, 2024	Thursday	Jan 11, 2024	184	1,008,219
Jul 11, 2024	Thursday	Jul 11, 2024	182	997,260
Jan 11, 2025	Saturday	Jan 13, 2025	184	3,024,658
Jul 11, 2025	Friday	Jul 11, 2025	181	991,781
Jan 11, 2026	Sunday	Jan 12, 2026	184	1,008,219

The fixed rate day count convention is Actual/365. There are 181 days between January 11, 2021, and July 11, 2021. This means that the fixed payments on July 12, 2021, is

$$\frac{181}{365} \times 0.02 \times 100,000,000 = \$991,781$$

Other fixed payments are calculated similarly.

##### 34.2

Yes. The swap is the same as one on twice the principal where half the fixed rate is exchanged for floating.

##### 34.3

The final fixed payment is in millions of dollars:

$$[(1.5 \times 1.0165 + 1.5) \times 1.0165 + 1.5] \times 1.0165 + 1.5 = 6.1501$$

The final floating payment assuming forward rates are realized is

$$[(1.55 \times 1.016 + 1.55) \times 1.016 + 1.55] \times 1.016 + 1.55 = 6.3504$$

The value of the swap is therefore the present value of  $-0.2003$  or  $-0.2003/(1.015^4) = -0.1887$ . This makes the small approximation discussed in footnote 1 of Chapter 34.

##### 34.4

The value is zero. The receive side is the same as the pay side with the cash flows compounded forward at the risk-free rate. Compounding cash flows forward at the risk-free rate does not change their value.

### 34.5

Suppose that the fixed rate accrues only when the floating reference rate is below  $R_X$  and above  $R_Y$  where  $R_Y < R_X$ . In this case, the swap is a regular swap plus two series of binary options, one for each day of the life of the swap. Using the notation in the text, the risk-neutral probability that the floating reference rate will be above  $R_X$  on day  $i$  is  $N(d_2)$  where

$$d_2 = \frac{\ln(F_i / R_X) - \sigma_i^2 t_i / 2}{\sigma_i \sqrt{t_i}}$$

The probability that it will be below  $R_Y$  where  $R_Y < R_X$  is  $N(-d'_2)$  where

$$d'_2 = \frac{\ln(F_i / R_Y) - \sigma_i^2 t_i / 2}{\sigma_i \sqrt{t_i}}$$

From the viewpoint of the party paying fixed, the swap is a regular swap plus binary options. The binary options corresponding to day  $i$  have a total value of

$$\frac{QL}{n_2} P(0, s_i) [N(d_2) + N(-d'_2)]$$

(This ignores the small timing adjustment mentioned in Section 34.6.)

### 34.6

There are four payments of USD 0.4 million. The present value in millions of dollars is

$$0.4/1.02 + 0.4/1.02^2 + 0.4/1.02^3 + 0.4/1.02^4 = 1.5231$$

The forward Australian floating rate is 5% with annual compounding. The quanto adjustment to the floating payment at time  $t_i + 1$  is

$$0.3 \times 0.15 \times 0.25 t_i = 0.01125 t_i$$

The value of the floating payments received is therefore

$$0.5/1.02 + 0.5 \times 1.01125/1.02^2 + 0.5 \times 1.0225/1.02^3 + 0.5 \times 1.03375/1.02^4 = 1.9355$$

The value of the swap is  $1.9355 - 1.5231 = 0.4124$  million.

### 34.7

When the CP rate is 6.5% and Treasury rates are 6% with semiannual compounding, the CMT% is 6% and an Excel spreadsheet can be used to show that the price of a 30-year bond with a 6.25% coupon is about 103.46. The spread is zero and the rate paid by P&G is 5.75%. When the CP rate is 7.5% and Treasury rates are 7% with semiannual compounding, the CMT% is 7% and the price of a 30-year bond with a 6.25% coupon is about 90.65. The spread is therefore,

$$\max[0, (98.5 \times 7 / 5.78 - 90.65) / 100]$$

or 28.64%. The rate paid by P&G is 35.39%.