CHAPTER 7
Swaps

Short Concept Questions

7.1 At the end of each accrual period the floating payment is calculated from a rate obtained by compounding the overnight rates observed in the period.

7.2 The OIS rates out to one year define zero rates directly. Those for maturities greater than a year define par yield bonds.

7.3 One.

7.4 Principal is usually exchanged in a currency swap but not in an interest rate swap.

7.5 The company can receive floating and pay 3.25%. It can therefore achieve a return of floating minus 0.25%.

7.6 The company can pay floating and receive 3.21%. It can therefore achieve a borrowing rate of floating plus 0.79%.

7.7 The spread over the reference rate in the floating-rate loan might be reset periodically.

7.8 Forward rates for each floating payment and risk-free rates.

7.9 The early exchanges have a negative value.

7.10 It can be valued as an exchange of bonds or by assuming that forward exchange rates will be realized.

Practice Questions

7.11

A has an apparent comparative advantage in fixed-rate markets but wants to borrow floating. B has an apparent comparative advantage in floating-rate markets but wants to borrow fixed. This provides the basis for the swap. There is a 1.4% per annum differential between the fixed rates offered to the two companies and a 0.5% per annum differential between the floating rates offered to the two companies. The total gain to all parties from the swap is therefore $1.4-0.5=0.9\%$ per annum. Because the bank gets 0.1% per annum of this gain, the swap should make each of A and B 0.4% per annum better off. This means that it should lead to A borrowing at SOFR – 0.3% and to B borrowing at 6.0%. The appropriate arrangement is therefore as shown in Figure S7.1.

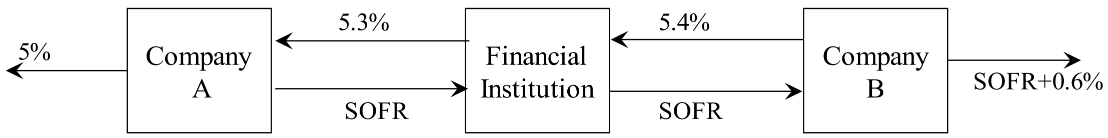

Figure S7.1: Swap for Problem 7.11

7.12

Consider the party paying floating. The first exchange involves paying \$1.2 million and receiving \$2.0 million in four months. It has a value of $0.8e^{-0.027\times0.3333}$ =\$0.7928 million. To value the second forward contract, we note that the forward interest rate is 3% per annum with semiannual compounding. The value of the forward contract is

$$100 \times (0.04 \times 0.5 - 0.03 \times 0.5)e^{-0.027\times0.8333} = \$0.4889 \text{ million}$$

The total value of the forward contracts is therefore \$0.7928+\$0.4889 = \$1.2817. This is the value of the swap to the party paying floating. For the party paying fixed, the value is −\$1.2817.

7.13

X has a comparative advantage in yen markets but wants to borrow dollars. Y has a comparative advantage in dollar markets but wants to borrow yen. This provides the basis for the swap. There is a 1.5% per annum differential between the yen rates and a 0.4% per annum differential between the dollar rates. The total gain to all parties from the swap is therefore $1.5-0.4=1.1$% per annum. The bank requires 0.5% per annum, leaving 0.3% per annum for each of X and Y. The swap should lead to X borrowing dollars at $9.6-0.3=9.3$ % per annum and to Y borrowing yen at $6.5-0.3=6.2$ % per annum. The appropriate arrangement is therefore as shown in Figure S7.2. All foreign exchange risk is borne by the bank.

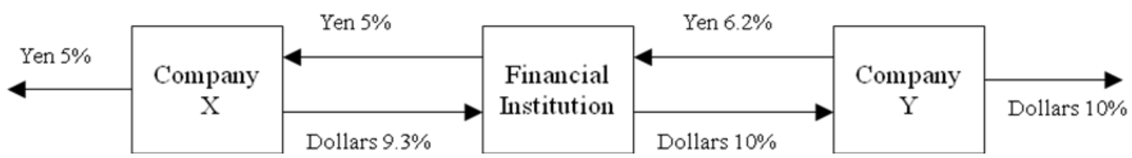

Figure S7.2: Swap for Problem 7.13

7.14

The swap involves exchanging the sterling interest of 20×0.10 or £2 million for the dollar interest of $30\times0.06=\$1.8$ million. The principal amounts are also exchanged at the end of the life of the swap. The value of the sterling bond underlying the swap is

$$\frac{2}{(1.07)^{1/4}}+\frac{22}{(1.07)^{5/4}}=22.182 \text{ million pounds}$$

The value of the dollar bond underlying the swap is

$$\frac{1.8}{(1.04)^{1/4}}+\frac{31.8}{(1.04)^{5/4}}=\$32.061 \text{ million}$$

The value of the swap to the party paying sterling is therefore
$32.061-(22.182\times1.55)=-\$2.321$million

The value of the swap to the party paying dollars is \$2.321 million. The results can also be obtained by viewing the swap as a portfolio of forward contracts. The continuously compounded interest rates in sterling and dollars are 6.766% per annum and 3.922% per annum. The 3-month and 15-month forward exchange rates are $1.55e^{(0.03922-0.06766)\times0.25}=1.5390$ and $1.55e^{(0.03922-0.06766)\times1.25}=1.4959$. The values of the two forward contracts corresponding to the exchange of interest for the party paying sterling are therefore
$(1.8-2\times1.5390)e^{-0.03922\times0.25}=-\$1.2656$ million
and
$(1.8-2\times1.4959)e^{-0.03922\times1.25}=-\$1.1347$ million
The value of the forward contract corresponding to the exchange of principals is
$(30-20\times1.4959)e^{-0.03922\times1.25}=+\$0.0787$ million
The total value of the swap is $-\$1.2656$ $-\$1.1347$ $+\$0.0787$ million or $-\$2.322$ million (which allowing for rounding errors is the same as that given by valuing bonds).

7.15
Credit risk arises from the possibility of a default by the counterparty. Market risk arises from movements in market variables such as interest rates and exchange rates. A complication is that the credit risk in a swap is contingent on the values of market variables. For example, suppose that a company has a single bilaterally cleared swap with a counterparty. The company's has credit risk only when the value of the swap to the company is positive.

7.16
The rate is not truly fixed because, if the company's credit rating declines, it will not be able to roll over its floating rate borrowings at floating plus 150 basis points. The effective fixed borrowing rate then increases. Suppose, for example, that the treasurer's spread over floating increases from 150 basis points to 200 basis points. The borrowing rate increases from 5.2% to 5.7%.

7.17
At the start of the swap, the contract has a value of approximately zero. As time passes, it is likely that the swap value will change. If at the time of a counterparty default the swap has a positive value to the bank and a negative value to the counterparty, the bank is likely to lose money. If the yield curve is upward sloping, the early exchanges are expected to be negative to the bank and the later exchanges are expected to be positive to the bank. This means that the swap is expected to have a positive value as time passes and, as a result, the bank's credit exposure is relatively high. When the yield curve is downward sloping the early exchanges are expected to be positive to the bank and the later exchanges are expected to be negative to the bank. This means that the swap is expected to have a negative value as time passes and, as a result, the bank's credit exposure is relatively low.

7.18
The spread between the interest rates offered to X and Y is 0.8% per annum on fixed rate investments and 0.0% per annum on floating rate investments. This means that the total apparent benefit to all parties from the swap is 0.8% per annum. Of this 0.2% per annum will go to the bank. This leaves 0.3% per annum for each of X and Y. In other words, company X

should be able to get a fixed-rate return of 8.3% per annum while company Y should be able to get a floating-rate return LIBOR + 0.3% per annum. The required swap is shown in Figure S7.3. The bank earns 0.2%, company X earns 8.3%, and company Y earns LIBOR + 0.3%.

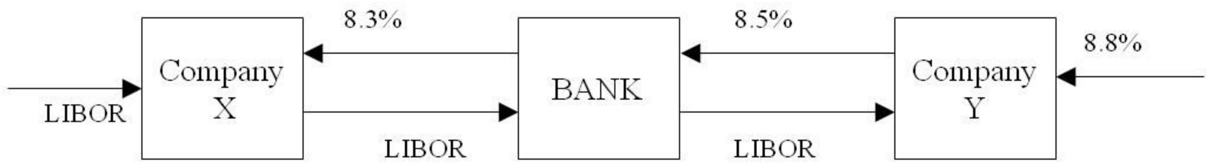

Figure S7.3 Swap for Problem 7.18

7.19
At the end of year 3 the financial institution was due to receive $200,000 (=0.5×4% of $10 million) and pay $150,000 (=0.5×3% of $10 million). The immediate loss is therefore $50,000. To value the remaining swap we assume that LIBOR forward rates are realized. All forward rates are 2% per annum. The remaining cash flows are therefore valued on the assumption that the floating payment is 0.5×0.02×10,000,000 = $100,000. The fixed payment is $200,000 and the net payment that would be received is 200,000−100,000=$100,000. The total cost of default is therefore the cost of foregoing the following cash flows:

3 year:	$50,000
3.5 year:	$100,000
4 year:	$100,000
4.5 year:	$100,000
5 year:	$100,000

Discounting these cash flows to year 3 at 1.8% per annum, we obtain the cost of the default as $441,120.

7.20
When interest rates are compounded annually

$$F_0 = S_0\left(\frac{1+r}{1+r_f}\right)^T$$

where F_0 is the T-year forward rate, S_0 is the spot rate, r is the domestic risk-free rate, and r_f is the foreign risk-free rate. As $r = 0.08$ and $r_f = 0.03$, the spot and forward exchange rates at the end of year 6 are

Spot:	0.8000
1 year forward:	0.8388
2 year forward:	0.8796
3 year forward:	0.9223
4 year forward:	0.9670

The value of the swap at the time of the default can be calculated on the assumption that forward rates are realized. The cash flows lost as a result of the default are therefore as follows:

Year	*Dollar Paid*	*CHF Received*	*Forward Rate*	*Dollar Equiv of CHF Received*	*Cash Flow Lost*
6	560,000	300,000	0.8000	240,000	-320,000
7	560,000	300,000	0.8388	251,600	-308,400
8	560,000	300,000	0.8796	263,900	-296,100
9	560,000	300,000	0.9223	276,700	-283,300
10	7,560,000	10,300,000	0.9670	9,960,100	2,400,100

Discounting the numbers in the final column to the end of year 6 at 8% per annum, the cost of the default is $679,800.

Note that, if company Y had no other business beside this swap, it would make no sense for the company to default just before the exchange of payments at the end of year 6 as the exchange has a positive value to company Y. In practice, company Y may be defaulting and declaring bankruptcy for reasons unrelated to this particular transaction.

7.21

Company A has a comparative advantage in the Canadian dollar fixed-rate market. Company B has a comparative advantage in the U.S. dollar floating-rate market. (This may be because of their tax positions.) However, company A wants to borrow in the U.S. dollar floating-rate market and company B wants to borrow in the Canadian dollar fixed-rate market. This gives rise to the swap opportunity.

The differential between the U.S. dollar floating rates is 0.5% per annum, and the differential between the Canadian dollar fixed rates is 1.5% per annum. The difference between the differentials is 1% per annum. The total potential gain to all parties from the swap is therefore 1% per annum, or 100 basis points. If the financial intermediary requires 50 basis points, each of A and B can be made 25 basis points better off. Thus a swap can be designed so that it provides A with U.S. dollars at Floating + 0.25% per annum, and B with Canadian dollars at 6.25% per annum. The swap is shown in Figure S7.4.

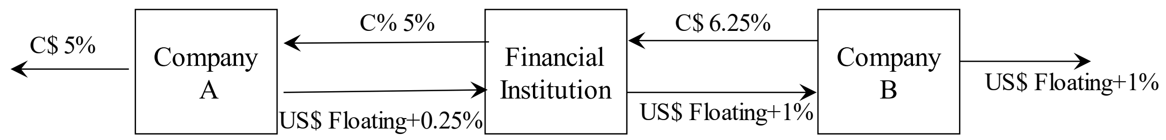

Figure S7.4 Swap for Problem 7.21

Principal payments flow in the opposite direction to the arrows at the start of the life of the swap and in the same direction as the arrows at the end of the life of the swap. The financial institution would be exposed to some foreign exchange risk which could be hedged using forward contracts.

7.22

The financial institution will have to buy 1.1% of the AUD principal in the forward market for each year of the life of the swap. Since AUD interest rates are higher than dollar interest rates, AUD is at a discount in forward markets. This means that the AUD purchased for year 2 is less expensive than that purchased for year 1; the AUD purchased for year 3 is less expensive than that purchased for year 2; and so on. This works in favor of the financial institution and means that its spread increases with time. The spread is always above 20 basis points.

7.23

Consider two offsetting plain vanilla interest rate swaps that a financial institution enters into with companies X and Y. We suppose that X is paying fixed and receiving floating while Y is paying floating and receiving fixed.

The quote suggests that company X will usually be less creditworthy than company Y. (Company X might be a BBB-rated company that has difficulty in accessing fixed-rate markets directly; company Y might be a AAA-rated company that has no difficulty accessing fixed or floating rate markets.) Presumably company X wants fixed-rate funds and company Y wants floating-rate funds.

The financial institution will realize a loss if company Y defaults when rates are high or if company X defaults when rates are low. These events are relatively unlikely since (a) Y is unlikely to default in any circumstances and (b) defaults are less likely to happen when rates are low. For the purposes of illustration, suppose that the probabilities of various events are as follows:

Default by Y:	0.001
Default by X:	0.010
Rates high when default occurs:	0.7
Rates low when default occurs:	0.3

The probability of a loss is

$$0.001 \times 0.7 + 0.010 \times 0.3 = 0.0037$$

If the roles of X and Y in the swap had been reversed the probability of a loss would be

$$0.001 \times 0.3 + 0.010 \times 0.7 = 0.0073$$

Assuming companies are more likely to default when interest rates are high, the above argument shows that the observation in quotes has the effect of decreasing the risk of a financial institution's swap portfolio. It is worth noting that the assumption that defaults are more likely when interest rates are high is open to question. The assumption is motivated by the thought that high interest rates often lead to financial difficulties for corporations. However, the empirical evidence on whether defaults are more likely when interest rates are high is mixed.

7.24

In an interest-rate swap a financial institution's exposure depends on the difference between a fixed-rate of interest and a floating-rate of interest. It has no exposure to the notional principal. In a loan the whole principal can be lost.

7.25

The bank is paying a floating-rate on the deposits and receiving a fixed-rate on the loans. It can offset its risk by entering into interest rate swaps (with other financial institutions or corporations) in which it contracts to pay fixed and receive floating.

7.26

Suppose that floating payments are made in currency A and fixed payments are made in currency B. The floating payments can be valued in currency A by (i) assuming that the forward rates are realized, and (ii) discounting the resulting cash flows at appropriate currency A discount rates. Suppose that the value is V_A. The fixed payments can be valued in currency B by discounting them at the appropriate currency B discount rates. Suppose that the value is V_B. If Q is the current exchange rate (number of units of currency A per unit of currency B), the value of the swap in currency A is $V_A - QV_B$. Alternatively, it is $V_A / Q - V_B$ in currency B.

7.27
The value in millions of dollars is
(0.03–0.034)×100/1.034+(0.03–0.034)/1.034^2 = –0.76

7.28
With continuous compounding the forward rate for the first exchange is 2.75% (the average of 2.7% and 2.8%. For the second exchange it is (3×4.5-2.8×1.5)/3 or 3.1%. For the final exchange it is (3.1×7.5-3×4.5)/3 or 3.25%. These rates become 2.7595%, 3.112%, and 3.2632% with quarterly compounding. The value of the swap in millions of dollars is therefore
$100[(0.027595-0.03)e^{-0.028\times0.125}+(0.0311-0.03)e^{-0.03\times0.375}+(0.032632-0.03)e^{-0.031\times0.6125}] =0.129$

7.29
(a) Company A can pay floating and receive 3.05% for three years. It can therefore exchange a loan at 3.45% into a loan at floating plus 0.40% or LIBOR plus 40 basis points
(b) Company B can receive floating and pay 3.30% for five years. It can therefore exchange a loan at floating plus 0.75% for a loan at 4.05%. But there is a danger that the spread is pays over floating on the loan increases during the five years.

7.30
The swap with a fixed rate of 3.2% is worth zero. The value of the first exchange to the party receiving fixed per dollar of principal is

$$\frac{0.032-0.030}{1.025} = 0.001951$$

The value of the second exchange is

$$\frac{0.032-0.032}{1.027^2} = 0.00$$

The value of the third exchange is

$$\frac{0.032-R}{1.029^3}$$

Hence

$$\frac{0.032-R}{1.029^3} = -0.001951$$

so that R = 0.034126 or 3.4126%.

A swap where 4% is received on a principal of \$100 million provides 0.8% of \$100 million or \$800,000 per year more than a swap worth zero. Its value is

$$\frac{800{,}000}{1.025}+\frac{800{,}000}{1.027^2}+\frac{800{,}000}{1.029^3} = 2{,}273{,}226$$

or about \$2.27 million.

7.31
We can value the swap as a series of forward rate agreements. Assuming the 2% is quarterly compounded and ignoring day count issues the payments received are \$0.5 million at times 1, 4, 7, and 10 months. The forward floating rate for the first payment is $(1.1\times2+1.4)/3 = 1.2\%$ or 1.202% with quarterly compounding. The forward floating rate for the second payment is $(4\times1.6-1.4)/3 = 1.667\%$ or 1.670% with quarterly compounding. The forward floating rate for the third payment is $(7\times1.7-4\times1.6)/3 = 1.833\%$ or 1.838% with quarterly compounding. The forward floating rate for the fourth period is $(10\times1.8-7\times1.7)/3 = 2.033\%$ or 2.038% with quarterly compounding.

The value of the swap in \$ millions is therefore

$$(0.5 - 1.202\times0.25)e^{-0.014\times1/12} + (0.5 - 1.670\times0.25)e^{-0.016\times4/12} + (0.5 - 1.838\times0.25)e^{-0.017\times7/12} + (0.5 - 2.038\times0.25)e^{-0.018\times10/12} = 0.328$$

7.32
The spread between the interest rates offered to A and B is 0.4% (or 40 basis points) on sterling loans and 0.8% (or 80 basis points) on U.S. dollar loans. The total benefit to all parties from the swap is therefore

$$80 - 40 = 40 \text{ basis points}$$

It is therefore possible to design a swap which will earn 10 basis points for the bank while making each of A and B 15 basis points better off than they would be by going directly to financial markets. One possible swap is shown in Figure S7.5. Company A borrows at an effective rate of 6.85% per annum in U.S. dollars.
Company B borrows at an effective rate of 10.45% per annum in sterling. The bank earns a 10-basis-point spread. The way in which currency swaps such as this operate is as follows. Principal amounts in dollars and sterling that are roughly equivalent are chosen. These principal amounts flow in the opposite direction to the arrows at the time the swap is initiated. Interest payments then flow in the same direction as the arrows during the life of the swap and the principal amounts flow in the same direction as the arrows at the end of the life of the swap.
Note that the bank is exposed to some exchange rate risk in the swap. It earns 65 basis points in U.S. dollars and pays 55 basis points in sterling. This exchange rate risk could be hedged using forward contracts.

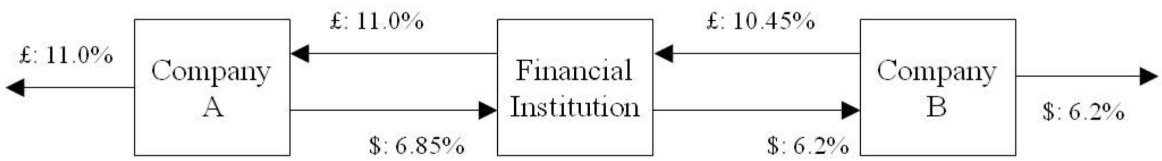

Figure S7.5 One Possible Swap for Problem 7.32

7.33
We know that exchanging 4% for floating is worth zero. Receiving 4.2% in exchange for LIBOR is therefore worth the present value of $0.5\times(0.042-0.04)\times\$10,000,000 = \$10,000$

received every six months for five years. This is

$$\sum_{i=1}^{10} 10{,}000(1+0.036/2)^{-i} = \$90{,}773$$