CHAPTER 4
Interest Rates

Short Concept Questions

4.1 LIBOR is based on estimates of the rates at which banks can borrow, not actual transactions. As such it is open to manipulation.

4.2 In the U.S., SOFR which is a secured overnight rate, is replacing LIBOR. Rates replacing LIBOR in other currencies such as SONIA and ESTER are unsecured.

4.3 A repo rate is the rate implied by a transaction where it is agreed that assets will be sold and repurchased at a slightly higher price.

4.4 See Section 4.3.

4.5 5% quarterly compounded.

4.6 See equation (4.1).

4.7 The present value is Ae^{-RT} where T is the time until payment.

4.8 The par yield on a bond is the coupon rate that results in the bond's price being equal to its par value.

4.9 An FRA is an agreement to a future exchange. A predetermined fixed rate is exchanged for a reference rate with both being applied to the same principal for a period of time and the interest being paid in arrears.

4.10 Under expectations theory, forward interest rates reflect expected future short-term rates. Under liquidity preference theory, they are higher than expected future short-term interest rates.

Practice Questions

4.11
The rate with continuous compounding is

$$4\ln\left(1+\frac{0.07}{4}\right)=0.0694$$

or 6.94% per annum.

(a) The rate with annual compounding is

$$\left(1+\frac{0.07}{4}\right)^4-1=0.0719$$

or 7.19% per annum.

4.12

Suppose the bond has a face value of $100. Its price is obtained by discounting the cash flows at 5.2%. The price is

$$\frac{2}{1.026}+\frac{2}{1.026^2}+\frac{102}{1.026^3}=98.29$$

If the 18-month zero rate is R, we must have

$$\frac{2}{1.025}+\frac{2}{1.025^2}+\frac{102}{(1+R/2)^3}=98.29$$

which gives R=5.204%.

4.13

(a) With annual compounding, the return is

$$\frac{1100}{1000}-1=0.1$$

or 10% per annum.

(b) With semi-annual compounding, the return is R where

$$1000\left(1+\frac{R}{2}\right)^2=1100$$

i.e.,

$$1+\frac{R}{2}=\sqrt{1.1}=1.0488$$

so that $R=0.0976$. The percentage return is therefore 9.76% per annum.

(c) With monthly compounding, the return is R where

$$1000\left(1+\frac{R}{12}\right)^{12}=1100$$

i.e.

$$\left(1+\frac{R}{12}\right)=\sqrt[12]{1.1}=1.00797$$

so that $R=0.0957$. The percentage return is therefore 9.57% per annum.

(d) With continuous compounding, the return is R where:

$$1000e^R=1100$$

i.e.,

$$e^R=1.1$$

so that $R=\ln 1.1=0.0953$. The percentage return is therefore 9.53% per annum.

4.14

The forward rates with continuous compounding are as follows,

Qtr 2	3.4%
Qtr 3	3.8%
Qtr 4	3.8%
Qtr 5	4.0%
Qtr 6	4.2%

4.15

The value of the FRA is

$$1{,}000{,}000\times 0.25\times(0.045-0.040)\, e^{-0.036\times 1.25} = 1{,}195$$

or $1,195.

4.16

When the term structure is upward sloping, $c > a > b$. When it is downward sloping, $b > a > c$.

4.17

Duration provides information about the effect of a small parallel shift in the yield curve on the value of a bond portfolio. The percentage decrease in the value of the portfolio equals the duration of the portfolio multiplied by the amount by which interest rates are increased in the small parallel shift. The duration measure has the following limitation. It applies only to parallel shifts in the yield curve that are small.

4.18

The rate of interest is R where:

$$e^R = \left(1+\frac{0.08}{12}\right)^{12}$$

i.e.,

$$R = 12\ln\left(1+\frac{0.08}{12}\right)$$

$$= 0.0797$$

The rate of interest is therefore 7.97% per annum.

4.19

The equivalent rate of interest with quarterly compounding is R where

$$e^{0.04} = \left(1+\frac{R}{4}\right)^4$$

or

$$R = 4(e^{0.01}-1) = 0.0402$$

The amount of interest paid each quarter is therefore:

$$10{,}000\times\frac{0.0402}{4} = 100.50$$

or $100.50.

4.20

The bond pays \$2 in 6, 12, 18, and 24 months, and \$102 in 30 months. The cash price is

$$2e^{-0.04\times0.5}+2e^{-0.042\times1.0}+2e^{-0.044\times1.5}+2e^{-0.046\times2}+102e^{-0.048\times2.5}=98.04$$

4.21

The bond pays \$4 in 6, 12, 18, 24, and 30 months, and \$104 in 36 months. The bond yield is the value of y that solves

$$4e^{-0.5y}+4e^{-1.0y}+4e^{-1.5y}+4e^{-2.0y}+4e^{-2.5y}+104e^{-3.0y}=104$$

Using the *Solver* or *Goal Seek* tool in Excel, $y=0.06407$ or 6.407%.

4.22

Using the notation in the text, $m=2$, $d=e^{-0.07\times2}=0.8694$. Also

$$A=e^{-0.05\times0.5}+e^{-0.06\times1.0}+e^{-0.065\times1.5}+e^{-0.07\times2.0}=3.6935$$

The formula in the text gives the par yield as

$$\frac{(100-100\times0.8694)\times2}{3.6935}=7.0741$$

To verify that this is correct, we calculate the value of a bond that pays a coupon of 7.0741% per year (that is 3.5370 every six months). The value is

$$3.537e^{-0.05\times0.5}+3.537e^{-0.06\times1.0}+3.537e^{-0.065\times1.5}+103.537e^{-0.07\times2.0}=100$$

verifying that 7.0741% is the par yield.

4.23

The forward rates with continuous compounding are as follows:

Year 2: 4.0%

Year 3: 5.1%

Year 4: 5.7%

Year 5: 5.7%

4.24

Taking a long position in two of the 4% coupon bonds and a short position in one of the 8% coupon bonds leads to the following cash flows

$$\text{Year } 0: 90-2\times80=-70$$

$$\text{Year } 10: 200-100=100$$

because the coupons cancel out. \$100 in 10 years time is equivalent to \$70 today. The 10-year rate, R, (continuously compounded) is therefore given by

$$100=70e^{10R}$$

The rate is

$$\frac{1}{10}\ln\frac{100}{70}=0.0357$$

or 3.57% per annum.

4.25

If long-term rates were simply a reflection of expected future short-term rates, we would expect the term structure to be downward sloping as often as it is upward sloping. (This is based on the assumption that half of the time investors expect rates to increase and half of the time investors expect rates to decrease). Liquidity preference theory argues that long term rates are high relative to expected future short-term rates. This means that the term structure should be upward sloping more often than it is downward sloping.

4.26

The par yield is the yield on a coupon-bearing bond. The zero rate is the yield on a zero-coupon bond. When the yield curve is upward sloping, the yield on an *N*-year coupon-bearing bond is less than the yield on an *N*-year zero-coupon bond. This is because the coupons are discounted at a lower rate than the *N*-year rate and drag the yield down below this rate. Similarly, when the yield curve is downward sloping, the yield on an *N*-year coupon bearing bond is higher than the yield on an *N*-year zero-coupon bond.

4.27

A repo is a contract where an investment dealer who owns securities agrees to sell them to another company now and buy them back later at a slightly higher price. The other company is providing a loan to the investment dealer. This loan involves very little credit risk. If the borrower does not honor the agreement, the lending company simply keeps the securities. If the lending company does not keep to its side of the agreement, the original owner of the securities keeps the cash.

4.28

a) The bond's price is

$$8e^{-0.07}+8e^{-0.07\times 2}+8e^{-0.07\times 3}+8e^{-0.07\times 4}+108e^{-0.07\times 5}=103.05$$

b) The bond's duration is

$$\frac{1}{103.05}\left[8e^{-0.07}+2\times 8e^{-0.07\times 2}+3\times 8e^{-0.07\times 3}+4\times 8e^{-0.07\times 4}+5\times 108e^{-0.07\times 5}\right]$$

$$=4.3235 \text{ years}$$

c) Since, with the notation in the chapter

$$\Delta B=-BD\Delta y$$

the effect on the bond's price of a 0.2% decrease in its yield is

$$103.05\times 4.3235\times 0.002=0.89$$

The bond's price should increase from 103.05 to 103.94.

d) With a 6.8% yield the bond's price is

$$8e^{-0.068}+8e^{-0.068\times 2}+8e^{-0.068\times 3}+8e^{-0.068\times 4}+108e^{-0.068\times 5}=103.95$$

This is close to the answer in (c).

4.29

The 6-month Treasury bill provides a return of $6/94=6.383\%$ in six months. This is $2\times 6.383=12.766\%$ per annum with semiannual compounding or $2\ln(1.06383)=12.38\%$ per annum with continuous compounding. The 12-month rate is $11/89=12.360\%$ with annual compounding or $\ln(1.1236)=11.65\%$ with continuous compounding.

For the $1\frac{1}{2}$ year bond, we must have

$$4e^{-0.1238\times 0.5}+4e^{-0.1165\times 1}+104e^{-1.5R}=94.84$$

where R is the $1\frac{1}{2}$ year zero rate. It follows that

$$3.76+3.56+104e^{-1.5R}=94.84$$
$$e^{-1.5R}=0.8415$$
$$R=0.115$$

or 11.5%. For the 2-year bond, we must have

$$5e^{-0.1238\times0.5}+5e^{-0.1165\times1}+5e^{-0.115\times1.5}+105e^{-2R}=97.12$$

where R is the 2-year zero rate. It follows that

$$e^{-2R}=0.7977$$
$$R=0.113$$

or 11.3%.

4.30
The first exchange of payments is known. Each subsequent exchange of payments is an FRA where interest at 5% is exchanged for interest at LIBOR on a principal of $100 million. Interest rate swaps are discussed further in Chapter 7.

4.31
We must solve $1.11 = (1 + R/n)^n$ where R is the required rate and the number of times per year the rate is compounded. The answers are: a) 10.71%, b) 10.57%, c) 10.48%, d) 10.45%, e) 10.44%

4.32
The bond's theoretical price is
$20\times e^{-0.02\times0.5}+20\times e^{-0.023\times1}+20\times e^{-0.027\times1.5}+1020\times e^{-0.032\times2} = 1015.32$
The bond's yield assuming that it sells for its theoretical price is obtained by solving
$20\times e^{-y\times0.5}+20\times e^{-y\times1}+20\times e^{-y\times1.5}+1020\times e^{-y\times2} = 1015.32$
It is 3.18%.

4.33 (Excel file)
The answer (with continuous compounding) is 4.07%.

4.34
2.5% is paid every six months.

a) With annual compounding, the rate is $1.025^2-1=0.050625$ or 5.0625%
b) With monthly compounding, the rate is $12\times(1.025^{1/6}-1)=0.04949$ or 4.949%.
c) With continuous compounding, the rate is $2\times\ln 1.025=0.04939$ or 4.939%.

4.35
The duration of Portfolio A is

$$\frac{1\times2000e^{-0.1\times1}+10\times6000e^{-0.1\times10}}{2000e^{-0.1\times1}+6000e^{-0.1\times10}}=5.95$$

Since this is also the duration of Portfolio B, the two portfolios do have the same duration.

a) The value of Portfolio A is

$$2000e^{-0.1}+6000e^{-0.1\times10}=4016.95$$

When yields increase by 10 basis points, its value becomes

$$2000e^{-0.101}+6000e^{-0.101\times10}=3993.18$$

The percentage decrease in value is

$$\frac{23.77\times 100}{4016.95}=0.59\%$$

The value of Portfolio B is

$$5000e^{-0.1\times 5.95}=2757.81$$

When yields increase by 10 basis points, its value becomes

$$5000e^{-0.101\times 5.95}=2741.45$$

The percentage decrease in value is

$$\frac{16.36\times 100}{2757.81}=0.59\%$$

The percentage changes in the values of the two portfolios for a 10 basis point increase in yields are therefore the same.

b) When yields increase by 5%, the value of Portfolio A becomes

$$2000e^{-0.15}+6000e^{-0.15\times 10}=3060.20$$

and the value of Portfolio B becomes

$$5000e^{-0.15\times 5.95}=2048.15$$

The percentage reductions in the values of the two portfolios are:

$$\text{Portfolio A}: \frac{956.75}{4016.95}\times 100=23.82$$

$$\text{Portfolio B}: \frac{709.66}{2757.81}\times 100=25.73$$

Since the percentage decline in value of Portfolio A is less than that of Portfolio B, Portfolio A has a greater convexity.

4.36

In the Bond Price worksheet, we input a principal of 100, a life of 2 years, a coupon rate of 6% and semiannual settlement. The yield curve data from Table 4.2 is also input. The bond price is 98.38506. The DV01 is −0.018819. When the term structure rates are increased to 5.01, 5.81, 6.41, and 6.81, the bond price decreases to 98.36625. This is a reduction of 0.01881 which corresponds to the DV01. (The DV01 is actually calculated in DerivaGem by averaging the impact of a one-basis-point increase and a one-basis-point decrease.). The bond duration satisfies

$$\frac{\Delta B}{B}=-D\Delta y$$

In this case, $\Delta B = -0.01882$, $B = 98.38506$, and $\Delta y = 0.0001$ so that the duration is 10,000 × 0.01882/98.38506 = 1.91 years.

The impact of increasing all rates by 2% is to reduce the bond price by 3.691 to 94.694. The effect on price predicted by the DV01 is 200 × −0.01881 or −3.7638. The gamma is 0.036931 per % per %. In this case, the change is 2%. From equation (4.18), the convexity correction gamma is therefore

$$0.5 \times 0.036931 \times 2^2 = 0.0739$$

The price change estimated using DV01 and gamma is therefore −3.7638 + 0.0739 = 3.690 which is very close to the actual change.

The gamma is 0.036931 per % per %. Because 1% is 0.01, gamma is 10,000 × 0.036931. The convexity is gamma divided the bond price. This is 10,000 × 0.036931/98.38506 = 3.75.