CHAPTER 6
Interest Rate Futures

Short Concept Questions

6.1 (a) actual/actual, (b) 30/360, (c) actual/360.

6.2 $P = 360(100 - Y)/n$ where P is the quoted price, Y is the cash price and n is the remaining life in days.

6.3 The dirty price is the cash price. The clean price is the quoted price. The cash price is the quoted price plus accrued interest.

6.4 The conversion factor for a bond is equal to the quoted price the bond would have per dollar of principal on the first day of the delivery month on the assumption that the interest rate for all maturities equals 6% per annum (with semiannual compounding). The bond maturity and the times to the coupon payment dates are rounded down to the nearest three months for the purposes of the calculation. The conversion factor defines how much an investor with a short bond futures contract receives when bonds are delivered. If the conversion factor is 1.2345, the amount investor receives is calculated by multiplying 1.2345 by the most recent futures price and adding accrued interest.

6.5 The futures contract price has increased by 54 basis points. A trader with a short position loses 54×\$25 or \$1,350.

6.6 It is settled to the three-month LIBOR two days before the third Wednesday of the delivery month.

6.7 It is settled at the end of the three month period to a rate obtained by compounding the SOFR overnight rates during the three-month period.

6.8 The convexity adjustment is the amount by which the futures rate has to be reduced to give an estimate of the forward rate for the period. The convexity adjustment is necessary because a) the futures contract is settled daily, and b) the futures contract expires at the beginning of the three months. Both of these lead to the futures rate being greater than the forward rate.

6.9 It only considers relatively small parallel shifts in the yield curve.

6.10 The futures rate. (The convexity adjustment is subtracted from this to obtain the forward rate.)

Practice Questions

6.11

There are 32 calendar days between July 7 and August 8. There are 184 calendar days between July 7 and January 7. The interest earned per \$100 of principal is therefore

3.5 × 32/184 = \$0.6087. For a corporate bond, we assume 31 days between July 7 and August 8 and 180 days between July 7 and January 7. The interest earned is 3.5 × 31/180 = \$0.6028.

6.12
There are 89 days between October 12 and January 9. There are 182 days between October 12 and April 12. The cash price of the bond is obtained by adding the accrued interest to the quoted price. The quoted price is $102\frac{7}{32}$ or 102.21875. The cash price is therefore

$$102.21875+\frac{89}{182}\times 3=\$103.69$$

6.13
The SOFR futures price has increased by 6 basis points. The investor makes a gain per contract of $25\times 6=\$150$ or \$300 in total.

6.14
From equation (6.4), the rate is

$$\frac{3.2\times 90+3\times 350}{440}=3.0409$$

or 3.0409%.

6.15
The value of a contract is $108\frac{15}{32}\times 1{,}000=\$108{,}468.75$. The number of contracts that should be shorted is

$$\frac{6{,}000{,}000}{108{,}468.75}\times\frac{8.2}{7.6}=59.7$$

Rounding to the nearest whole number, 60 contracts should be shorted. The position should be closed out at the end of July.

6.16
The cash price of the Treasury bill is

$$100-\frac{90}{360}\times 10=\$97.50$$

The annualized continuously compounded return is

$$\frac{365}{90}\ln\left(1+\frac{2.5}{97.5}\right)=10.27\%$$

6.17
The number of days between January 27 and May 5 is 98. The number of days between January 27 and July 27 is 181. The accrued interest is therefore

$$6\times\frac{98}{181}=3.2486$$

The quoted price is 110.5312. The cash price is therefore

$$110.5312+3.2486=113.7798$$

or \$113.78.

6.18

The cheapest-to-deliver bond is the one for which

$$\text{Quoted Price} - \text{Futures Price} \times \text{Conversion Factor}$$

is least. Calculating this factor for each of the 4 bonds, we get

$$\text{Bond 1}: 125.15625 - 101.375 \times 1.2131 = 2.178$$

$$\text{Bond 2}: 142.46875 - 101.375 \times 1.3792 = 2.652$$

$$\text{Bond 3}: 115.96875 - 101.375 \times 1.1149 = 2.946$$

$$\text{Bond 4}: 144.06250 - 101.375 \times 1.4026 = 1.874$$

Bond 4 is therefore the cheapest to deliver.

6.19

There are 176 days between February 4 and July 30 and 181 days between February 4 and August 4. The cash price of the bond is, therefore:

$$110 + \frac{176}{181} \times 6.5 = 116.32$$

The rate of interest with continuous compounding is $2 \ln 1.06 = 0.1165$ or 11.65% per annum. A coupon of 6.5 will be received in 5 days or 0.01370 years time. The present value of the coupon is

$$6.5e^{-0.01370 \times 0.1165} = 6.490$$

The futures contract lasts for 62 days or 0.1699 years). The cash futures price if the contract were written on the 13% bond would be

$$(116.32 - 6.490)e^{0.1699 \times 0.1165} = 112.03$$

At delivery, there are 57 days of accrued interest. The quoted futures price if the contract were written on the 13% bond would therefore be

$$112.03 - 6.5 \times \frac{57}{184} = 110.01$$

Taking the conversion factor into account, the quoted futures price should be:

$$\frac{110.01}{1.5} = 73.34$$

6.20

If the bond to be delivered and the time of delivery were known, arbitrage would be straightforward. When the futures price is too high, the arbitrageur buys bonds and shorts an equivalent number of bond futures contracts. When the futures price is too low, the arbitrageur shorts bonds and goes long an equivalent number of bond futures contracts. Uncertainty as to which bond will be delivered introduces complications. The bond that appears cheapest-to-deliver now may not in fact be cheapest-to-deliver at maturity. In the case where the futures price is too high, this is not a major problem since the party with the short position (i.e., the arbitrageur) determines which bond is to be delivered. In the case where the futures price is too low, the arbitrageur's position is far more difficult since the bond to short is not known; it is unlikely that a profit can be locked in for all possible outcomes.

6.21
The forward interest rate for the time period between months 6 and 9 is 9% per annum with continuous compounding. This is because 9% per annum for three months when combined with $7\frac{1}{2}$% per annum for six months gives an average interest rate of 8% per annum for the nine-month period.
With quarterly compounding, the forward interest rate is

$$4(e^{0.09/4}-1)=0.09102$$

or 9.102%. This assumes that the day count is actual/365. With a day count of actual/360, the rate is $9.102\times 360/365=8.977$. The three-month SOFR quote for a contract maturing in six months is therefore

$$100-8.977=91.02$$

6.22
The forward rates calculated form the first two Eurodollar futures are 4.17% and 4.38%. These are expressed with an actual/360 day count and quarterly compounding. With continuous compounding and an actual/365 day count, they are
4ln(1 + (0.0417/4) × (365/360)) = 4.2057% and 4ln(1+(0.0438/4) × (365/360))=4.4164%
It follows from equation (6.4) that the 398 day rate is
(4 × 300 + 4.2057 × 98)/398 = 4.0506
or 4.0506%. The 489 day rate is
(4.0507 × 398 + 4.4167 × 91)/489 = 4.1187
or 4.1187%. We are assuming that the first futures rate applies to 98 days rather than the usual 91 days. The third futures quote is not needed.

6.23
Duration-based hedging procedures assume parallel shifts in the yield curve. Since the 12-year rate tends to move by less than the 4-year rate, the portfolio manager may find that he or she is over-hedged.

6.24
The company treasurer can hedge the company's exposure by shorting Eurodollar futures contracts. The Eurodollar futures position leads to a profit if rates rise and a loss if they fall. The duration of the commercial paper is twice that of the Eurodollar deposit underlying the Eurodollar futures contract. The contract price of a Eurodollar futures contract is 980,000. The number of contracts that should be shorted is, therefore,

$$\frac{4{,}820{,}000}{980{,}000}\times 2=9.84$$

Rounding to the nearest whole number 10 contracts should be shorted.

6.25
The treasurer should short Treasury bond futures contract. If bond prices go down, this futures position will provide offsetting gains. The number of contracts that should be shorted is

$$\frac{10{,}000{,}000\times 7.1}{91{,}375\times 8.8}=88.30$$

Rounding to the nearest whole number, 88 contracts should be shorted.

6.26
The answer in Problem 6.25 is designed to reduce the duration to zero. To reduce the duration from 7.1 to 3.0 instead of from 7.1 to 0, the treasurer should short

$$\frac{4.1}{7.1}\times 88.30 = 50.99$$

or 51 contracts.

6.27
You would prefer to own the Treasury bond. Under the 30/360 day count convention, there is one day between October 30 and November 1. Under the actual/actual (in period) day count convention, there are two days. Therefore, you would earn approximately twice as much interest by holding the Treasury bond.

6.28
We compound 3% (continuously compounded) for the first 60 days with 3.5% (quarterly compounded) for the next 90 days. 3.5% quarterly compounded is equivalent to 3.485% continuously compounded. The 150-day rate is (3 × 60 +3.485 × 90)/150 = 3.29% with continuous compounding.

6.29
Suppose that the contracts apply to the interest rate between times T_1 and T_2. There are two reasons for a difference between the forward rate and the futures rate. The first is that the futures contract is settled daily whereas the forward contract is settled once at time T_2. The second is that without daily settlement a futures contract would be settled at time T_1 not T_2. Both reasons tend to make the futures rate greater than the forward rate.

6.30
The actual number of days between the last coupon date (Jan 27) and today is 70. The number of days between the last coupon (Jan 27) and the next coupon (Jul 27) is 181. The accrued interest for the government bond is therefore (70/181)×3 = 1.16. The cash price of the bond is therefore 121.16. Using a 30/360 day count, the number of days between Jan 27 and today is 70 and the number of days between Jan 27 and Jul 27 is 180. The accrued interest for the corporate bond is therefore (70/180)×3 =1.17 so that the cash price is 121.17.

6.31
The cash bond price is currently

$$137+\frac{9}{184}\times 4 = 137.1957$$

A coupon of 4 will be received after 175 days or 0.4795 years. The present value of the coupon on the bond is $4e^{-0.05\times 0.4795}$=3.9053. The futures contract lasts 296 days or 0.8110 years. The cash futures price if it were written on the 8% bond would therefore be

$$(137.1957-3.9053)e^{0.05\times 0.8110}=138.8061$$

At delivery, there are 121 days of accrued interest. The quoted futures if the contract were written on the 8% bond would therefore be

$$138.8061-4\times\frac{121}{181}=136.1321$$

The quoted price should therefore be

$$\frac{136.1321}{1.2191} = 111.66$$

6.32

The Eurodollar futures contract price of 97.95 means that the Eurodollar futures rate is 2.05% per annum with quarterly compounding and an actual/360 day count. This becomes 2.05×365/360 = 2.0785% with an actual/actual day count. This is

$$4\ln(1+0.25\times0.020785)=0.020731$$

or 2.0731% with continuous compounding. The forward rate given by the 90-day rate and the 180-day rate is 2.4% with continuous compounding. This suggests an investor can profitably borrow money for 90 days and invest for 180. A short position in a Eurodollar futures ensures that the funds can be rolled over at about 2.07%. The overall borrowing rate for the 180 days is about 2.035% whereas the rate earned is 2.2%.

6.33

The U.S. Eurodollar futures contract maturing at time T enables an investor to lock in the forward rate for the period between T and T^* where T^* is three months later than T. If $\hat{r}$ is the forward rate, the U.S. dollar cash flows that can be locked in are

$$-Ae^{-\hat{r}(T^*-T)} \quad \text{at time} \quad T$$

$$+A \quad \text{at time} \quad T^*$$

where A is the principal amount. To convert these to Canadian dollar cash flows, the Canadian company must enter into a short forward foreign exchange contract to sell Canadian dollars at time T and a long forward foreign exchange contract to buy Canadian dollars at time T^*. Suppose F and F^* are the forward exchange rates for contracts maturing at times T and T^*. (These represent the number of Canadian dollars per U.S. dollar.) The Canadian dollars to be sold at time T are

$$Ae^{-\hat{r}(T^*-T)}F$$

and the Canadian dollars to be purchased at time T^* are

$$AF^*$$

The forward contracts convert the U.S. dollar cash flows to the following Canadian dollar cash flows:

$$-Ae^{-\hat{r}(T^*-T)}F \quad \text{at time} \quad T$$

$$+AF^* \quad \text{at time} \quad T^*$$

This is a Canadian dollar LIBOR futures contract where the principal amount is AF^*.

6.34

The number of short futures contracts required is

$$\frac{100,000,000\times 4.0}{122,000\times 9.0} = 364.3$$

Rounding to the nearest whole number, 364 contracts should be shorted.

a) This increases the number of contracts that should be shorted to

$$\frac{100,000,000\times 4.0}{122,000\times 7.0} = 468.4$$

or 468 when we round to the nearest whole number.

b) In this case, the gain on the short futures position is likely to be less than the loss on the bond portfolio. This is because the gain on the short futures position depends on the size of the movement in long-term rates and the loss on the bond portfolio depends on the size of the movement in medium-term rates. Duration-based hedging assumes that the movements in the two rates are the same.